Introduction to Astronomy and Astrophysics I

Lecture 8

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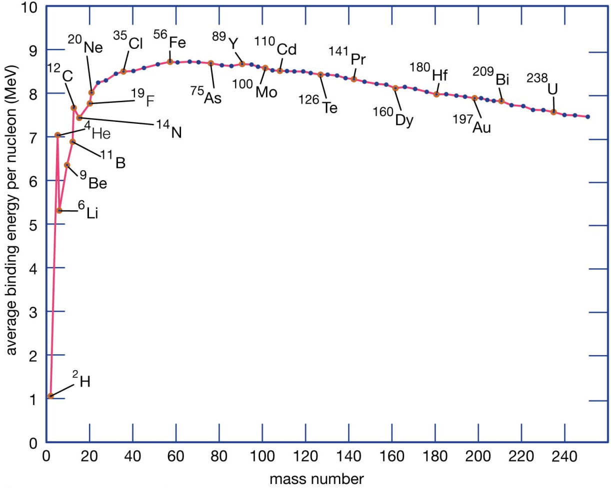
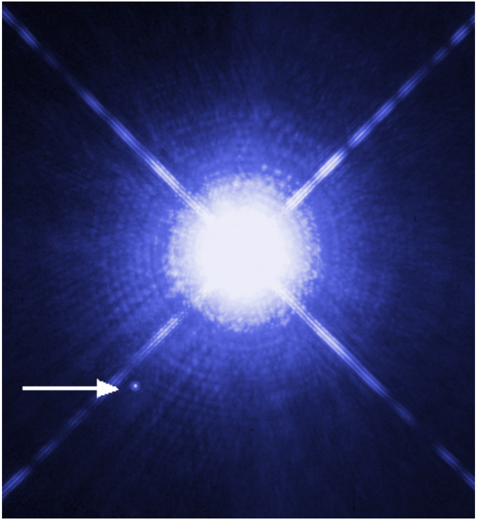
Binding energy per nucleon



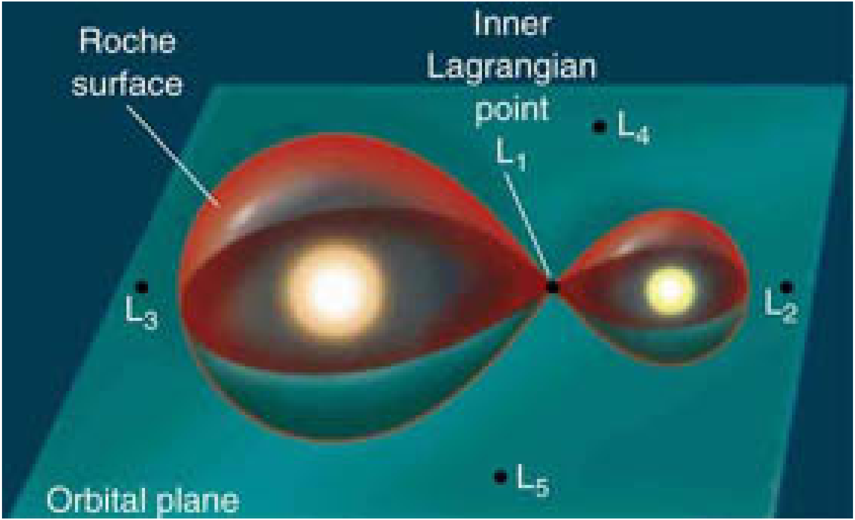
Sirius B - White Dwarf



Roche Lobes of a binary star system

- Roche potential (Φ): Defines the gravitational potential in a rotating frame.
- Equipotential surfaces: Roche lobes correspond to critical equipotential surfaces.

Roche Lobe


Roche
surface
Inner
Lagrangian
point
L_1
L_4
L_3
L_2
L_5
Orbital plane

Roche Lobe Radius

- Approximation by Eggleton (1983) (1% accuracy over all mass ranges):
- Roche lobe radius (R_L) formula:

$$\frac{R_L}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$$

- Where $q = \frac{M_2}{M_1}$ and a is the binary separation.

Type Ia supernova

caused by accretion of matter from a normal star onto a white dwarf companion. Luminosity in a relatively narrow range, calibratable from light curve.

Type Ia explosion

Type 1a explosion

White Dwarfs: Importance in Astrophysics

- Cosmochronology:
  - Age dating of stellar populations
  - Cooling sequence as a "cosmic clock"
- Type Ia Supernovae:
  - Standard candles for cosmology
  - Led to discovery of accelerating universe expansion
- Binary evolution:
  - Cataclysmic variables like Type 1a
  - manifest as low mass X-ray binaries
- Planetary systems:
  - Evidence of past and present planetary systems
  - Insight into late stages of planetary evolution Why late?

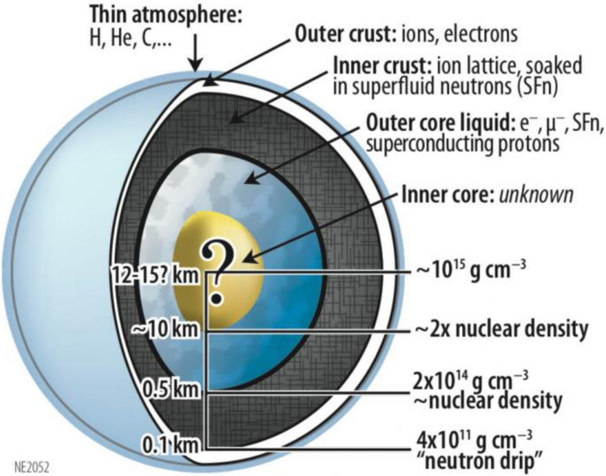
Neutron Stars: Formation

- Result of core-collapse supernovae
- Progenitor mass range: $8M_\odot < M < 20M_\odot$ (approximately)
- Formation process:
  1. Iron core reaches Chandrasekhar limit
  2. Core collapse and neutronization
  3. Bounce and shockwave
  4. Neutrino emission
  5. Ejection of outer layers
- Remnant: hot, rapidly rotating neutron star

Neutron Star Structure

- Radius: $\sim 10 - 14$ km
- Mass: $\sim 1.4 - 3M_{\odot}$
- Layers:
  - Outer crust: nuclei + electrons
  - Inner crust: neutron-rich nuclei + free neutrons
  - Outer core: neutron superfluid + protons + electrons
  - Inner core: exotic matter? (uncertain)
- Supported by neutron degeneracy pressure

Neutron star structure



Neutron Star Properties

- Extreme density: $\rho \sim 10^{14} - 10^{15}$ g/cm^3
- Rapid rotation: periods from ms to seconds
- Strong magnetic fields: $10^8 - 10^{15}$ Gauss
- Equation of State (EoS):
  - as always, relates pressure to density
  - Determines mass-radius relationship
  - Active area of research
- Maximum mass: $\sim 2.5 - 3M_\odot$ (EoS dependent)

Neutron Star Equation of State

- Challenges in determining EoS:
  - Extreme conditions not reproducible on Earth
  - Quantum many-body problem
  - Uncertainty in composition of inner core
- Constraints from observations:
  - Mass measurements (binary pulsars)
  - Radius estimates (X-ray bursts, cooling curves)
  - Gravitational wave observations

The TOV Equation

$$\frac{dP(r)}{dr} = -\frac{G\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]}{r\left[r - 2GM(r)/c^2\right]} \tag{1}$$

- $P(r)$: Pressure at radius r.
- $\rho(r)$: density at radius r.
- $M(r)$: Mass enclosed within radius r.

Newtonian Limit of the TOV Equation

The Tolman-Oppenheimer-Volkoff (TOV) equation can be rewritten as:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left[1 + \frac{P(r)}{\rho(r)c^2}\right]\left[1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right]\left[1 - \frac{2GM(r)}{rc^2}\right]^{-1} \quad (2)$$

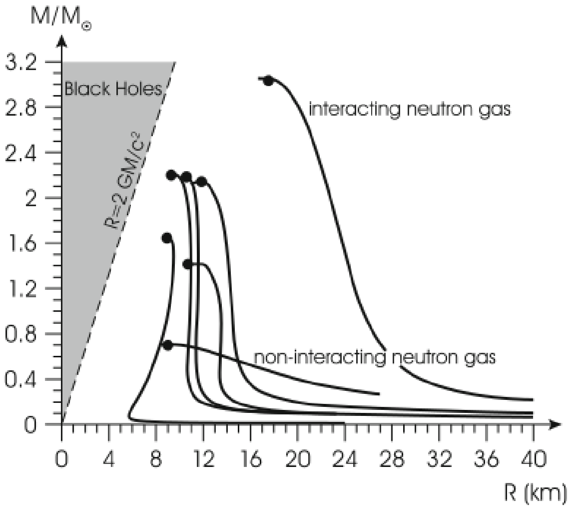
Newtonian Limit:
If the speed of light $c \to \infty$:

$$\frac{dp(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} \quad (3)$$

Interpretation:

- The first relativistic correction: $\frac{P(r)}{\rho(r)c^2}$, representing pressure effects.
- The second relativistic correction: $\frac{4\pi r^3 P(r)}{M(r)c^2}$, affecting the enclosed mass.
- The third relativistic correction: $\frac{2GM(r)}{rc^2}$, gravitational redshift factor.

Neutron star mass radius relationship



Credit: Kippenhahn et al. (2012)

Pulsars: Rotating Neutron Stars

- Highly magnetized, rotating neutron stars
- Lighthouse model:
  - Misaligned magnetic and rotation axes
  - Beamed emission along magnetic axis
  - Observed as periodic pulses
- Types:
  - (Normal) Radio pulsars
  - X-ray pulsars
  - Millisecond pulsars

Pulsar Properties and Evolution

- Spin periods: $P \sim 1$ ms – 10 s
- Spin-down rate: $\dot{P} \sim 10^{-21} - 10^{-12}$ s/s
- Magnetic field strength:

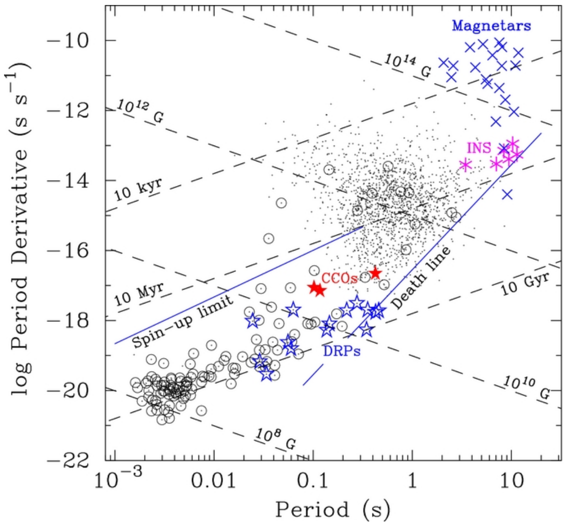
$$B \sim 3.2 \times 10^{19} \sqrt{P\dot{P}} \text{ Gauss}$$

- Characteristic age:

$$\tau_c = \frac{P}{2\dot{P}}$$

- Evolution on P-$\dot{P}$ diagram

P - $\dot{P}$ diagram



Neutron Star Observations

- Radio observations:
  - Primary method for detecting pulsars
  - Dispersion measure gives distance estimate
- X-ray observations:
  - Thermal emission from surface
  - X-ray pulsars in binary systems
- Gamma-ray observations:
  - Magnetospheric emission
  - Fermi Gamma-ray Space Telescope discoveries
- Gravitational waves:
  - Neutron star mergers (GW170817)
  - Constraints on neutron star equation of state

Exotic Neutron Stars

- Magnetars:
  - Extremely strong magnetic fields ($10^{14} - 10^{15}$ G)
  - Soft Gamma Repeaters (SGRs)
  - Anomalous X-ray Pulsars (AXPs)
- Quark stars (hypothetical):
  - Core composed of deconfined quark matter
  - Possible "strange" quark matter
  - Observational evidence still lacking

Neutron Stars: Astrophysical Importance

- Testing grounds for extreme physics:
  - Quantum chromodynamics
  - General relativity in strong-field regime
- Precise timekeepers:
  - Tests of general relativity (Hulse-Taylor binary)
  - Potential for gravitational wave detection
- Binary systems:
  - X-ray binaries
  - Source of gravitational waves
- Nuclear physics:
  - Constraints on nuclear equation of state
  - Insight into behavior of matter at extreme densities