

# Introduction to Astronomy and Astrophysics I

## Lecture 5

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# Some concerns

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What kind of questions will be asked in the exam?

Your notes are sparse. Where can we read more elaborately about the topics covered in class?

# If you don't know coding, now is the time to learn

I strongly recommend learning Python. It is the most commonly used language in a lot of different areas.

See: <https://web.iucaa.in/~ace/programs/>

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- logarithmic
- base  $100^{1/5}$
- inverted - brighter objects have lower magnitude
- measurements relative
- For small changes, a  $x$  fractional change in flux produces a  $x$  change in magnitude

# Vega magnitude

The most widely used magnitude system through the year 2000 was based on a set of normalizing constants derived from the spectrum of the bright star Vega. We have now moved to *absolute* systems based on calibrations in terms of physical flux units.

# The STMAG system - flux per unit wavelength

$$m_{\lambda}(\lambda) = -2.5 \log_{10} F_{\lambda}(\lambda) - 21.1$$

where  $F_{\lambda}(\lambda)$  is the spectral flux density per unit wavelength of a source at the top of the Earth's atmosphere in units of  $\text{erg s}^{-1} \text{cm}^{-2} \text{\AA}^{-1}$ .

This is also known as the *STMAG* system because it is standard for the Hubble Space Telescope. For more details, see the Synphot User's Guide at STScI.

# AB magnitudes - flux per unit frequency

The corresponding system based on flux per unit frequency is

$$m_{\nu}(\lambda) = -2.5 \log_{10} F_{\nu}(\lambda) - 48.6$$

where  $F_{\nu}(\lambda)$  is in units of  $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$ .

This is also known as the *AB* system. This system has been adopted by the Sloan Digital Sky Survey, GALEX, JWST.

# Jansky- used in FIR and radio astronomy

$$1 \text{ Jy} = 10^{-23} \text{ erg/sec/Hz/cm}^2$$

The conversion between AB magnitude  $m$  and flux density  $f$  of a star in Janskies is:  $f = 3631 \text{ Jy} \times 10^{-0.4m}$

# Caveats

- narrow band fluxes measured using broad bands (*UBVRIJHK* - calibration problems)
- calibration done not with laboratory standards, but with *standard stars*
- Many filter systems - Johnson *UBVRI*, Gunn *ugriz*

# Surface brightness

is the magnitude or flux per unit solid angle - mag/arcsec<sup>2</sup> (optical), Jy/beam (radio), MJy/str (IR)

# Absolute magnitude - luminosity measure

$$M = m - 5((\log_{10} D_L) - 1)$$

for cosmological distances,  $D_L$  is the luminosity distance.



# Distance Modulus

The distance modulus equation can be written as:

$$\mu = m - M = 5 \log_{10}(D_L) - 5 \quad (1)$$

- $\mu$  is the distance modulus.
- $D_L$  is the distance in parsecs.

# Interpreting the Distance Modulus

- If  $\mu > 0$ , the star is farther than 10 parsecs.
- If  $\mu < 0$ , the star is closer than 10 parsecs.
- Distance modulus can be rearranged to solve for distance:

$$D_L = 10^{(\mu+5)/5} \quad (2)$$

# Color in astronomy

$$B - V = m_B - m_V$$

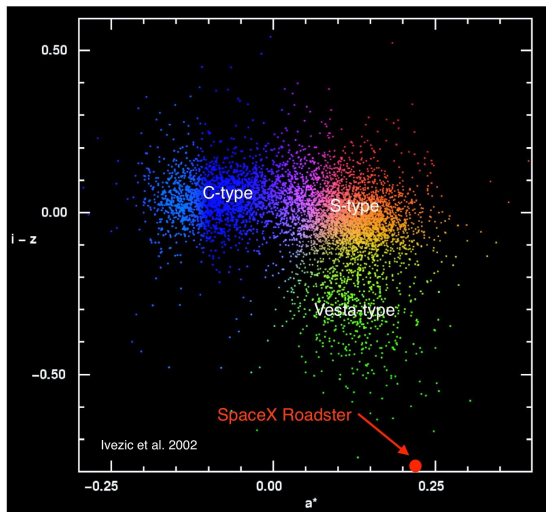
$$R - K = m_R - m_K$$

Shorter wavelength filter is always placed first, by convention. **What does a low value of color indicate? Can color be negative? Why is color important?**

# How to find this car in space?



# How to find Musk's roadster?

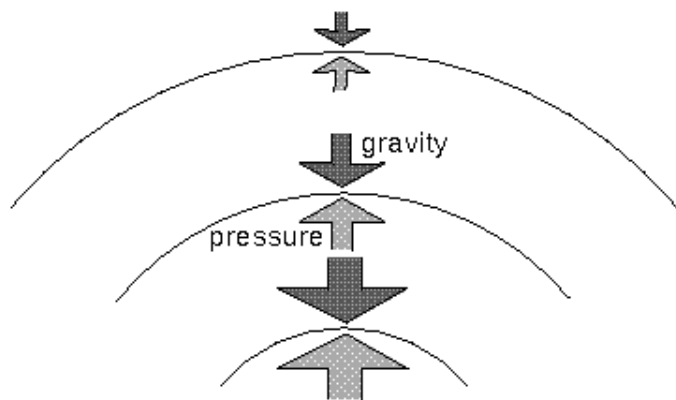


# What is a star?

- ① it is bound by self-gravity
- ② it is powered by an internal energy source

If any of these conditions is violated, the star ceases to be a star.

# Why are stars in equilibrium?



Deeper layers have more gravity compression, so they have greater outward pressure to compensate.

# Hydrostatic equilibrium

The diagram illustrates the hydrostatic equilibrium equation with handwritten annotations in various colors:

- DENSITY** (pink oval) points to  $\rho$ .
- MASS WITHIN A SPHERE OF RADIUS  $r$**  (green box) points to  $M_r$ .
- ACCELERATION** (orange box) points to  $\frac{d^2 r}{dt^2}$ .
- GRAVITATIONAL CONSTANT** (black box) points to  $G$ .
- PRESSURE GRADIENT** (blue box) points to  $\frac{dP}{dr}$ .

$$\rho \frac{d^2 r}{dt^2} = -G \frac{M_r \rho}{r^2} - \frac{dP}{dr}$$



# Hydrostatic equilibrium

The diagram shows the hydrostatic equilibrium equation with several handwritten annotations in colored boxes and circles:

- DENSITY** (pink oval) points to the  $\rho$  in the acceleration term.
- MASS WITHIN A SPHERE OF RADIUS  $r$**  (green box) points to the  $M_r$  in the gravitational term.
- ACCELERATION** (orange box) points to the  $\frac{d^2 r}{dt^2}$  term.
- GRAVITATIONAL CONSTANT** (black box) points to the  $G$  in the gravitational term.
- PRESSURE GRADIENT** (blue box) points to the  $\frac{dP}{dr}$  term.

$$\rho \frac{d^2 r}{dt^2} = -G \frac{M_r \rho}{r^2} - \frac{dP}{dr}$$

assumptions - no magnetic field, star is not rotating, GR not important!

# Eddington Luminosity

- Named after Sir Arthur Eddington (1882-1944)
- Represents the maximum luminosity a star can achieve while maintaining hydrostatic equilibrium
- Balance between radiation pressure outward and gravitational force inward
- Crucial concept in understanding stellar evolution and black hole accretion

# Key Physical Principles

- Radiation pressure: Outward force from photons
- Gravitational force: Inward pull of gravity
- Electron scattering: Primary interaction between photons and matter
- Hydrostatic equilibrium: Balance of forces in a star

# Deriving the Classical Eddington Luminosity

- 1 Start with force balance:

$$F_{\text{rad}} = F_{\text{grav}}$$

- 2 Radiation force (assuming Thomson scattering):

$$F_{\text{rad}} = \frac{L\sigma_T}{4\pi r^2 c}$$

- 3 Gravitational force:

$$F_{\text{grav}} = \frac{GMm_p}{r^2}$$

# Deriving the Classical Eddington Luminosity (cont.)

- 4 Equate forces and solve for L:

$$\frac{L\sigma_T}{4\pi r^2 c} = \frac{GMm_p}{r^2}$$

$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T}$$

- 5 Where:

- $L_{\text{Edd}}$ : Eddington luminosity
- $G$ : Gravitational constant
- $M$ : Mass of the object
- $m_p$ : Proton mass
- $c$ : Speed of light
- $\sigma_T$ : Thomson scattering cross-section

# Beyond Thomson Scattering

- Classical Eddington limit assumes only Thomson scattering
- In reality, other radiation processes contribute:
  - Bound-free absorption (photoionization)
  - Free-free absorption (bremsstrahlung)
- These processes can significantly modify the Eddington limit
- Particularly important in high-temperature, high-density environments

# Modified Eddington Luminosity

- Total opacity:  $\kappa = \kappa_T + \kappa_{bf} + \kappa_{ff}$
- Modified Eddington luminosity:

$$L_{\text{Edd,mod}} = \frac{4\pi GMc}{\kappa}$$

- Where:
  - $\kappa_T$ : Thomson scattering opacity
  - $\kappa_{bf}$ : Bound-free opacity
  - $\kappa_{ff}$ : Free-free opacity
- Generally,  $L_{\text{Edd,mod}} < L_{\text{Edd,classical}}$

# Importance in Stellar Physics

- Sets an upper limit on stellar masses
- Explains instabilities in very massive stars
- Influences late stages of stellar evolution
- Crucial in understanding supernovae mechanisms



# Applications in (Supermassive) Black Hole Physics

- Limits accretion rates onto black holes
- Explains self-regulation in active galactic nuclei (AGN)
- Used to estimate black hole masses from observed luminosities
- Important in understanding quasar feedback and galaxy evolution

# Further Reading

Further reading: "Accretion Power in Astrophysics" by Frank, King, & Raine

# Most stellar equations can be derived from simple conservation laws

- Conservation of mass
- Conservation of momentum
- Conservation of energy

# Energy Transport in stars

- Conduction
- Radiation
- Convection - “convection in stars proceeds under rather malicious conditions: turbulent motion transports enormous fluxes of energy in a very compressible gas, which is stratified in density, pressure, temperature, and gravity over many powers of ten.” – KWW

# What is an equation of state?

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Ideal gas equation of state:  $P = nkT/V$

Non-relativistic degenerate electron gas:  $P = K(n/V)^{5/3}$

Relativistic degenerate electron gas:  $P = K'(n/V)^{4/3}$

Why do we speak of electrons as “electron gas”?

# Polytropic equations of state

If  $\rho$  is independent of temperature, then  $\rho = \rho(P)$  and we can write

$$P = K\rho^\gamma = K\rho^{1+1/n}$$

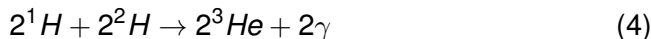
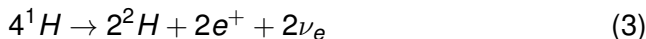
The value of the polytropic index  $n$  depends on whether the degenerate gas is relativistic or non-relativistic.  $K$  can be a constant or can vary from star to star, depending on the equation of state. Polytropic approach makes many simplifying assumptions and is now out of fashion.

# Textbook - Stellar Structure and Evolution

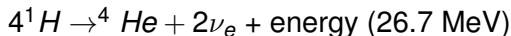
by Kippenhahn, Weigert and Weiss (Springer: 2012)  
Comprehensive coverage on all aspects of stellar structure and Evolution.



# Fusion: Hydrogen to Helium- via proton chain

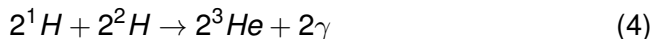
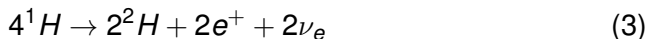


Net reaction is:

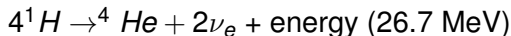


Sun produces  $4 \times 10^{26}$  joules per second. How many of the above fusion reactions are happening per second, if the Sun's energy comes entirely from the above process?

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9.36e+37

Read about the solar neutrino problem and how it was solved.

# How long will the Sun last?

The Sun converts loses about 4 million tonnes of mass every second. The efficiency of the fusion process is only about 0.7%. How long before the Sun runs out of Hydrogen if it started out with 80% hydrogen?

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$$2e30 * 0.8 / (4e9 * (86400 * 365 * 1e9)) * 0.007$$

89 GYr

# Chemical composition of the Sun

$X$  mass fraction of hydrogen 0.70

$Y$  mass fraction of helium 0.28

$Z$  mass fraction of metals 0.02

By definition,  $X + Y + Z = 1$ .  $Z$  is called metallicity.

# Temperature and luminosity relation for stars

For black bodies,

$$\sigma T_{\text{eff}}^4 = L/4\pi R^2$$

where  $\sigma$  is the Stefan-Boltzmann constant.

$$\text{So, } L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

Stellar spectra are well approximated by a black body with

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Thus, it natural to expect some correlation between luminosity and temperature. When the sun becomes a red giant it's radius will be about 2 AU. It's surface temperature will be about 3000 K. What will be it's luminosity relative to its present luminosity?

# Translating observations to physical parameters using physical laws

