Astronomical Techniques 1

(January - February 2015)

Assignment 2

Due on 7 March 2015; 1700 hrs. Total marks: 300, Weightage: 30%

General Instructions

1) You should try to obtain solutions to these questions by yourself. The less you talk to others, the more you will learn. (2) Most of the questions test a single concept introduced in class. For such questions, your answer should not be longer than 5-10 lines. (3) The deadline for submission of the answers is fixed. No credit for late submissions (4) I have tried to make the questions unambiguous and correct. But if you suspect that there is any error /ambiguity, feel free to say so in your answers. For a few problems, you may need to look up relevant numbers in the appropriate reference books. Please retain the correct number of significant figures in your answers. (5) Numbers within square brackets are the maximum marks for each question.

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(1) Four measures of the V-band photon rate of a certain source with a particular telescope/ instrument combination yield values of 1100, 1200, 1300, and 1400 photons per second. What is the estimate of the mean rate and the standard error in the mean rate? What is the best estimate of the variance of the parent distribution of the rate? Does it appear that the scatter in these observations is dominated by the photon statistics of the source? [10]

(2) For a CCD with a full well capacity of 80000 electrons per pixel and a 12 bit ADC, what gain value would you choose and why? [10]

(3) The efficiency of a large area survey can be estimated by its étendue $\epsilon = \Omega D^2 q$ where Ω is the solid angle of the field of view, D is the diameter of the telescope and q is the throughput quantum efficiency of the instrument. From the literature, determine the étendue of the Sloan Digital Sky Survey and the survey of the proposed Large Synoptic Survey Telescope (LSST). By what factor is the LSST more efficient? [5]

(4) The Sloan Digital Sky survey imaging camera reaches a 2σ limit for point source detection in the r band of 22.2 mag in 54.1 seconds of on-sky exposure. When imaging with IFOSC on the IUCAA telescope, one needs 30 minutes or more to reach this limiting magnitude on a moonless night. Enumerate the various factors that could contribute to the huge difference in performance between these telescopes of comparable size. Try to order the various factors in decreasing order of importance. [20]

(5) You are given a CCD image of a blank portion of the sky, devoid of stars and galaxies. In the idealized situation where QE=1 and all sources of noise *except* the shot noise from the background are negligible, describe a technique to estimate the gain of the CCD used, using the image itself. [10]

(6) You are given the following information about data obtained with a CCD. (1) for zero integration time, different frames give an rms variation of 1 ADU for any pixel (2) for equal length exposures giving an average count of 1000 ADU, different frames give a rms variation of 15 ADU for any pixel and (3) for (2) above the rms variation from pixel to pixel for any frame is 30 ADU. Find the gain, read noise and rms variation in response from pixel to pixel. [20]

(7) Using the calibration data for Vega and the UBV filter transmission curves (available for download on the course website), what would be the $(U - B)$ and $(B - V)$ colors for (a) a flat spectrum (in f_λ) source (b) a host white dwarf (a 10^5 K blackbody) and (c) a M type star (a 2500 K blackbody). Remember that in the $UBVRI$ system, colors and magnitudes of Vega are always zero. Note that this problem requires you to perform a numerical integration. [40]

(8) The night sky at a dark astronomical site has a typical brightness of 21 mag/arcsec^2 in the V band. Our eyes have a collecting area of about 1 cm^2, they detect light over a bandwidth of 2000 Å and see in a wide solid angle of about 1 steradian. At a dark site, what is the number of photons from the dark sky (excluding photons from stars and other astronomical objects) hitting our eyes every second? [20]

(9) If X and Y are *independent random variables* then show that Variance$(X + Y)$ = Variance(X) + Variance(Y) and Variance$(X - Y)$ = Variance(X) + Variance(Y). Would the above results hold if the variables X and Y were uncorrelated instead of being independent? [10]

(10) An extrasolar planet going around its host star, causes small variations in the radial velocity of the star as viewed from the earth. The amplitude of the observed variations depends upon the gravitational influence of the planet (its mass and distance from the star) and the angle of inclination of the planet's orbit relative to the line of sight. The measured radial velocity ($v \sin i$ versus time t) curve can be used to estimate the mass of the planet. If the inclination i is random, what is the distribution of $\sin i$? [10]

(11) You would like to study the kinematics of stars and gas in a nearby galaxy at moderate spectral resolution in order to constrain dynamical models. What type of spectrograph would you choose for such a project and why? Which specific spectrograph (and on which telescope) would you choose to observe with? [10]

(12) The nearby starburst galaxy M82 is estimated by several methods to have a star formation rate in its dusty central regions of 2-10 solar masses per year, which implies a supernova rate of 0.1 yr^{-1}. However, no new supernova has been detected by radio telescopes in the optically obscured starburst core for the last 20 years. What is the probability that this "supernova deficit" has occurred by chance? [10]

(13) The scandalous spherical aberration discovered to be present in the main optics of the Hubble Space Telescope when it was first launched seriously affected some science programs. Here, you are asked to estimate its impact on *faint point source* detection with the CCD detectors in the Wide Field Camera. Adopt the following parameters: primary diameter 2.4 m; focal ratio of the WFC f /12.9; total system throughput at 5500 Å (including filter and CCD) 15%; pixel size 15μm; readout noise per pixel 13 electrons rms; sky background 23 mag per arcsec^2. Ignore dark current, flat fielding errors, cosmic rays, and bias noise. Assume that the telescope without the aberration was *nearly* diffraction limited in the V band, with the Airy disk containing 70% of the light. (Ideally, it would contain 84%.) Estimate the corresponding limiting V magnitude for a 60 minute integration time. For our purposes, the limiting magnitude is defined as the magnitude of a star yielding a signal-to-noise ratio of 5 within the 70% encircled energy diameter. [25]

(14) In the presence of spherical aberration of the main mirror, the 70% encircled energy diameter became 2.8 arcsec at 5500 Å. Estimate the change in limiting V magnitude, for the same integration as in the previous problem, which resulted from this increase in image diameter. [10]

(15) You are observing a very faint source which produces only 1 detected photon per second against a bright background which produces 20 detected photons per second over the area of the source. How long do you have to integrate to obtain a signal-to-noise ratio for the source's flux (based on photon statistics alone) of 10? Assume that you are using an array detector which permits measurement of a very large number of simultaneous samples of the bright background. [10]

(16) A spectrograph has a reflection grating with 250 grooves mm^{-1}. Light is incident on the grating at an angle of incidence of 7 degrees. What wavelength of light will be diffracted at an angle of 10 degrees in the first order? What wavelength of photons will be diffracted at this angle in the second and third orders? What problems can this cause and how is this addressed? [10]

(17) In this question, we consider the case of SNR when the uncertainty on the background level is not negligible. Suppose we have a detector that records N_{tot} photons from a possible source in an observation of t seconds. We measure the background level in another observation in a nearby part of the sky where there are no sources and N_{bg} photons in a time t seconds. i) What is the uncertainty on N_{bg} ? ii) Write down an expression for N_{bg} and its uncertainty in terms of the background count rate b and the exposure time t. iii) Write down an expression for N_{tot} and its uncertainty in terms of the source count rate s and b and t. iv) Write down an expression for the number of source photons N_{src} and its uncertainty in terms of the source count rate s and b and t. v) Hence show that the signal to noise ratio of the source is $SNR = s/\sqrt{(s+2b)/t}$. [25]

(18) Pile-up occurs on X-ray CCDs when several photons hit the detector at the same place between two read-outs. In that case they are counted as one and their energies are summed. Pile-up thus affects both flux measurements and spectral characterisation of bright sources. An active galactic nucleus at a distance of 500 Mpc has a luminosity in X-rays of 1×10^{37} J, and is effectively a point source on the sky. You are planning to observe it with either Chandra (effective area 0.1 m^2, PSF=0.5 arcsec, CCD pixel size 0.5 arcsec, CCD readout cycle 3.2 s) or XMM (effective area 0.4 m^2, PSF=15 arcsec, CCD pixel size 4.4 arcsec, CCD readout cycle 0.2 s). Assuming that all of the photons received have energies of 1 keV, and treating the PSF as a **uniform circle** with the diameter given above, estimate whether pileup is likely to be a problem for either satellite. Why is pileup never an issue for optical observations? [25]

(19) One typically observes a star through a broadband filter, while the magnitude scale refers to a monochromatic flux. Consider a star whose spectrum is described by a power law $f_\lambda = A(\lambda/\lambda_0)^\gamma$, where A is the flux density at $\lambda = \lambda_0$, and λ_0 is the central wavelength of a filter which is a tophat: i.e., it lets 100% of the light through in the wavelength range $\lambda_0 - \Delta\lambda/2$ to $\lambda_0 + \Delta\lambda/2$, and no light through elsewhere. Calculate the flux of this star measured through the filter as a function of the power-law index γ. While the flux density at λ_0 is independent of γ, the flux through the filter varies appreciably. A typical range of γ is from -2 to +2; by what factor does the flux through the filter vary over this range? Do this calculation for $\lambda_0/\Delta\lambda = 0.02$ (an intermediate-band filter) and $\lambda_0/\Delta\lambda = 0.1$ (a broad-band filter). [20]