ASTRONOMY AND ASTROPHYSICS: Assignment 1

FERGUSSON COLLEGE, PUNE
Savitribai Phule Pune University
January – April 2020

04 March 2020
To be returned in the class on 14 March 2020

- The deadline for the submission of the solutions of this assignment will be strictly enforced. No marks will be given if the assignment is not returned in time.
- You are free to discuss the solutions with friends, seniors and consult any books.
- Let me know if you find anything to be unclear or if you think that something is wrong in any of the questions.

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1. **Surface of a sphere:** The metric on a surface of a sphere of radius r is given by

$$\mathrm{d}s^2 = r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \; \mathrm{d}\phi^2\right)$$

   (a) Find all the Christoffel symbols Γ^i_{jk}.
   (b) Find all the components of the Ricci tensor R_{ij}.
   (c) Find the Ricci scalar R.

[6 + 4 + 2]

2. **Covariant derivatives do not commute in curved spacetimes:** Evaluate the commutator

$$\nabla_\mu \nabla_\nu A^\alpha - \nabla_\nu \nabla_\mu A^\alpha,$$

   where A^α is an arbitrary four-vector. Express your answer in terms of the Riemann curvature tensor.

[7]

3. **Godesics in Schwarzschild spacetime:** Consider the motion of a massive particle in a spacetime with metric

$$\mathrm{d}s^2 = -\left(1 - \frac{r_g}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - r_g/r} + r^2 \mathrm{d}\phi^2,$$

   where $r_g = 2GM$ (assuming $c = 1$), M being the mass of the spherically symmetric source giving rise to the gravitational field. Let us restrict to $r > r_g$.

   (a) Starting with the 'Lagrangian' $\mathcal{L} = g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$, work out the t and ϕ components of the geodesic equation. Here overdots denote derivatives with respect to the proper time τ, i.e., $\dot{} \equiv \mathrm{d}/\mathrm{d}\tau$.
   (b) Use the constraint equation $g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = -1$ to show

$$\frac{1}{2}\dot{r}^2 - \frac{GM}{r} + \frac{l^2}{2r^2} - \frac{GMl^2}{r^3} = \mathcal{E},$$

   where $\mathcal{E}$ and l are constants with usual meanings.

[3 + 3]