Cosmology
Lecture 20
Galaxies

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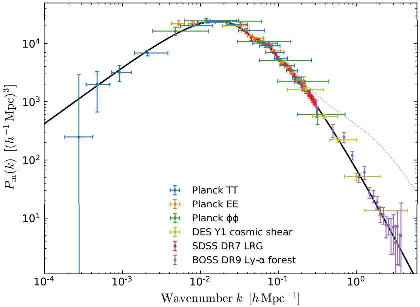
Cosmology using galaxy surveys

- Measuring the location, redshift and other properties (e.g., luminosity, colour, spectral composition) of galaxies is an important task in cosmology. These are done using **galaxy redshift surveys**.
- The large-scale distribution allows one to measure the shape of the power spectrum (e.g., power-law slope and turnover, BAO features) and hence constrain cosmological parameters.



Courtesy ESA and Planck Collaboration

- A set of galaxy surveys rely on **gravitational lensing** of light. These are useful to study the distribution of dark matter between the lensed galaxy and us.

Galaxy formation: cooling

- The galaxies would form when baryons are attracted within the collapsed dark matter halo. Hence the locations of galaxies would correspond to the high density regions of the dark matter field.
- The gas, when attracted into the dark matter potential well, acquires kinetic energy and thus gets heated up. The typical temperature of the gas (assuming it to be hydrogen) would be

$$T_{\rm vir} \sim \frac{G M m_p}{k_B R_{\rm vir}}.$$

  This temperature is $\sim 10^4$ K for $10^8 M_\odot$ dark matter haloes, and is smaller for lighter haloes.
- The overdensity ~ 200 is not sufficient to trigger nuclear reactions, hence one needs to condense the gas further for stars to form.
- The condensation is possible only if the pressure of the gas can be reduced, i.e., one needs to cool the gas.
- In case the gas is made of atomic hydrogen, the only way to dissipate energy is via atomic transitions which is possible only for $T_{\rm vir} > 10^4$ K. If the as is cooler, the atoms simply remain in the ground state.
- The process of cooling and condensation will be determined by the Jeans criterion.

Jeans criterion for collapse

- A spherically symmetric gas cloud evolves as

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{Gm(r)}{r^2} - \frac{1}{\rho}\frac{\mathrm{d}P(r)}{\mathrm{d}r}.$$

- The cloud will expand (collapse) if the second (first) term on the right dominates.
- The pressure term dominates in a characteristic time-scale $t_s \sim R/c_s$, where c_s is the sound speed. The scaling, in terms of the temperature, is given by

$$t_s \sim \frac{R\sqrt{\rho}}{\sqrt{P}} \sim \frac{R(\mu m_p)^{1/2}}{(k_B\ T)^{1/2}} \sim M^{1/3}\rho^{-1/3}(\mu m_p)^{1/2}(k_B T)^{-1/2}$$

- The gravity term dominates at a time-scale (known as the free-fall time) $t_{\rm ff} \sim (G\rho)^{-1/2}$.
- Clouds with $t_{\rm ff} < t_s$ will collapse under self-gravity. This implies that the condition for collapse is

$$(G\rho)^{-1/2} < M^{1/3}\rho^{-1/3}(\mu m_p)^{1/2}(k_B T)^{-1/2} \quad \Longrightarrow \quad M > \rho^{-1/2}\left(\frac{k_B T}{G\mu m_p}\right)^{3/2}.$$

- Clouds with masses above some threshold value $M > M_J$ will collapse under self-gravity. This phenomenon is known as **Jeans instability** and the threshold mass M_J is called **Jeans mass**. Its actual expression is

$$M_J = \left(\frac{3}{4\pi\rho}\right)^{1/2}\left(\frac{5 k_B T}{G\mu m_p}\right)^{3/2}.$$

  For interstellar medium, it turns out to be $M_J \sim 10^5 M_\odot$.

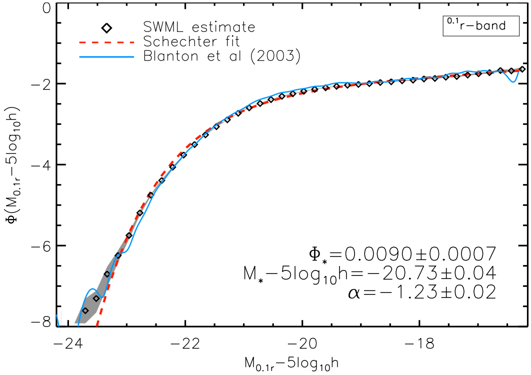
Fragmentation

- Now, as the cloud collapses, its density increases. Whether the Jeans mass ($M_J \propto \rho^{-1/2} T^{3/2}$) decreases will depend on how the temperature changes.
- According to the virial theorem, kinetic (thermal) and potential (gravitational) energies are related $E_T = |E_G|/2$.
- Now, as the cloud collapses, its gravitational energy $|E_G| \propto M^2/R$ increases. Hence, if virial theorem holds (which is true if the star is in quasi-hydrostatic equilibrium which, in turn, holds if the mass of the cloud is not too large than the Jeans mass), then E_T also increases.
- An increase in E_T will result in enhanced excitations of atoms/molecules, which will de-excite and hence produce radiation. This radiation can be trapped within the system (optically thick) or can escape away (optically thin).
- In early parts of collapse, the density of the cloud is small enough so that it is optically thin. The system will hence re-adjust itself by radiating away the excess energy and the temperature may either remain constant (isothermal) or decrease (cooling). For proto-stellar collapse, it is usually isothermal.
- In that case, the Jeans mass decreases. This will lead to formation of smaller clouds, which is called **fragmentation**.
- This process cannot continue forever. Since ρ increases, the cloud will become optically thick at some point. At this stage, the energy cannot escape the cloud and hence it evolves adiabatically $P \propto V^{-\gamma} \propto \rho^{\gamma} \implies T \propto P/\rho \propto \rho^{\gamma-1}$.
- For an ideal, mono-atomic gas, we have $\gamma = 5/3$, so $T \propto \rho^{2/3}$. Then $M_J \propto \rho^{-1/2} T^{3/2} \propto \rho^{1/2}$. This means that the Jeans mass starts increasing. At this point, the collapse halts.
- It is possible to estimate the Jeans mass at the moment when the gas cloud makes a transition to adiabatic evolution. This will be the smallest mass scale that can form because of fragmentation. It can be shown to be $\sim M_{\odot}$.

Galaxy luminosity function

- One can study the models of galaxy formation using the **galaxy luminosity function** $\Phi(L)\,\mathrm{d}L$, defined as the number of galaxies per unit volume in the luminosity range $(L, L + \mathrm{d}L)$.
- It can also be expressed in terms of the absolute magnitude $M = -2.5 \log_{10}(L/L_1)$.

Montero-Dorta & Prada (2009)
SDSS DR6



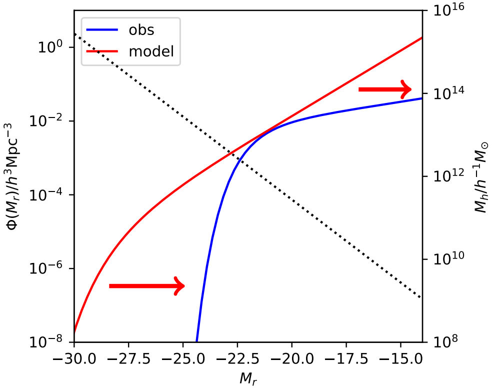
$M_{0.1r\odot} \approx 4.76$

- Schechter fit to the data: $\Phi(L) = \frac{\Phi_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha} \mathrm{e}^{-L/L_*}$, or

$$\Phi(M) = 0.4 \ln(10)\, \Phi_*\, 10^{-0.4(M-M_*)(\alpha+1)} \exp\left[-10^{-0.4(M-M_*)}\right].$$

Relating the halo mass function to the galaxy luminosity function

- As we have discussed, galaxies form inside dark matter haloes. Hence the galaxy numbers must be related to the halo numbers.
- Consider a simple scenario where
  - each halo forms one galaxy,
  - the luminosity of the galaxy is proportional to the halo mass
- In that case, we write $L_r = \zeta\, M_h$, and then compute $\Phi(L)$ from the theoretical $n(M_h) \equiv \mathrm{d}n/\mathrm{d}M_h$. We treat ζ as a free parameter which is tuned to find a good match to the data.
- We need the bright galaxies to be fainter, and also the faint galaxies to be fainter.
- More detailed models of galaxy formation too suffer from this difficulty.

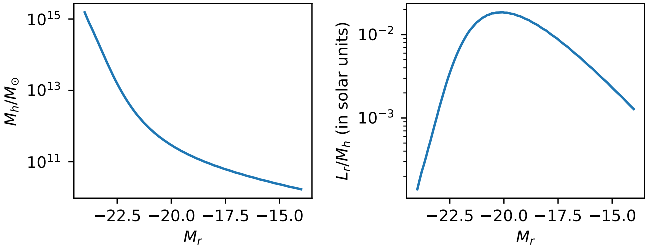


Abundance matching

- Suppose instead we leave the $L_r - M_h$ relation free (except that it must be monotonic). We find the relation that will allow perfect match with the data. This is done through the implicit relation

$$\int_{L_r}^{\infty} \mathrm{d}L' \, \Phi(L') = \int_{M_h}^{\infty} \mathrm{d}M' \, n(M').$$

- This technique is called the **abundance matching**.



- Clearly, the low-mass haloes and the high-mass haloes form stars inefficiently. These are because of **radiative/reionization feedback** and **supernova feedback** in low-mass haloes, and **AGN feedback** in high-mass haloes.

Galaxies as high density regions

- Let us now study the clustering properties of the high-density regions which are potential sites of galaxies. In terms of linear density contrast, such regions would correspond to $\delta \gtrsim \delta_c$.
- Suppose one measures the clustering properties of galaxies, then one can ask how is it related to the dark matter power spectrum.
- Let there be two points with *linearly extrapolated* density contrasts $\delta_1 \equiv \delta(\vec{x}_1),\ \delta_2 \equiv \delta(\vec{x}_2)$.
- The joint probability of the first point having a δ between $(\delta_1, \delta_1 + \mathrm{d}\delta_1)$ and the second between $(\delta_2, \delta_2 + \mathrm{d}\delta_2)$ is

$$\mathcal{P}(\delta_1, \delta_2) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2\sigma^2(1-\rho^2)}\left(\delta_1^2 + \delta_2^2 - 2\rho\delta_1\delta_2\right)\right],$$

where

$$\rho = \frac{\xi(r)}{\sigma^2}.$$

Probability of high density regions

- Since we are interested in high density regions, let us compute the probability that both δ_1 and δ_2 are above some threshold value $\nu\sigma$. This is given by

$$\mathcal{P}_2(\delta_1 > \nu\sigma, \delta_2 > \nu\sigma) = \int_{\nu\sigma}^{\infty} \mathrm{d}\delta_1 \int_{\nu\sigma}^{\infty} \mathrm{d}\delta_2 \mathcal{P}(\delta_1, \delta_2)$$

$$= \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \int_{\nu\sigma}^{\infty} \mathrm{d}\delta_1 \int_{\nu\sigma}^{\infty} \mathrm{d}\delta_2 \exp\left[-\frac{1}{2\sigma^2(1-\rho^2)}\left(\delta_1^2 + \delta_2^2 - 2\rho\delta_1\delta_2\right)\right].$$

- In order to simplify the problem, let us assume $\rho \ll 1$, i.e., thr clustering is weak. Then we can write the exponent as

$$-\frac{1}{2\sigma^2(1-\rho^2)}\left(\delta_1^2 + \delta_2^2 - 2\rho\delta_1\delta_2\right) \approx -\frac{1}{2\sigma^2}[\delta_1^2 + (\delta_2 - \rho\delta_1)^2 - \rho^2\delta_1^2] \approx -\frac{1}{2\sigma^2}[\delta_1^2 + (\delta_2 - \rho\delta_1)^2].$$

- Then the probability becomes

$$\mathcal{P}_2(\delta_1 > \nu\sigma, \delta_2 > \nu\sigma) \approx \frac{1}{2\pi\sigma^2} \int_{\nu\sigma}^{\infty} \mathrm{d}\delta_1 \, \mathrm{e}^{-\delta_1^2/2\sigma^2} \int_{\nu\sigma}^{\infty} \mathrm{d}\delta_2 \exp\left[-\frac{(\delta_2 - \rho\delta_1)^2}{2\sigma^2}\right]$$

$$= \frac{1}{2\pi\sigma}\sqrt{\frac{\pi}{2}} \int_{\nu\sigma}^{\infty} \mathrm{d}\delta_1 \, \mathrm{e}^{-\delta_1^2/2\sigma^2} \left[1 + \mathrm{erf}\left(\frac{\delta_1\rho - \nu\sigma}{\sigma\sqrt{2}}\right)\right].$$

Correlation of high density regions

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- If we further assume $\nu \gg 1$ (i.e., the regions have extremely high densities compared to the variance) and use the asymptotic approximation $\mathrm{erf}(x) \to -1 - \exp(-x^2)/\sqrt{\pi}x$, then

$$\mathcal{P}_2(\delta_1 > \nu\sigma, \delta_2 > \nu\sigma) \approx \frac{1}{2\pi\sigma}\sqrt{\frac{\pi}{2}}\int_{\nu\sigma}^{\infty} \mathrm{d}\delta_1\ \mathrm{e}^{-\delta_1^2/2\sigma^2}\left[\exp\left(-\frac{(\delta_1\rho - \nu\sigma)^2}{2\sigma^2}\right)\frac{\sqrt{2}}{\sqrt{\pi}\nu}\right] \approx \frac{1}{2\pi\nu^2}\mathrm{e}^{-\nu^2}\mathrm{e}^{\nu^2\rho}.$$

- The probability of one point to be above the threshold is

$$\mathcal{P}_1(\delta > \nu\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\int_{\nu\sigma}^{\infty} \mathrm{d}\delta\ \mathrm{e}^{-\delta^2/2\sigma^2} \approx \frac{1}{\sqrt{2\pi}\ \nu}\mathrm{e}^{-\nu^2/2}.$$

- So, the correlation of high density regions is given by

$$\frac{\mathcal{P}_2}{\mathcal{P}_1^2} - 1 \equiv \xi_\nu(r) = \mathrm{e}^{\rho\nu^2} - 1 = \exp\left[\frac{\nu^2}{\sigma^2}\xi(r)\right] - 1$$

Bias of high density regions

- If $\rho\nu^2 \ll 1$, then

$$\xi_\nu(r) \approx \rho\nu^2 = \frac{\nu^2}{\sigma^2}\xi(r).$$

  For $\nu > \sigma$ (which corresponds to the density contrasts being larger than σ^2), we get $\xi_\nu > \xi$, showing that sufficiently high density regions are more clustered.

- One can thus write the correlation of high density regions as

$$\xi_\nu(r) = b_\nu^2\, \xi(r),$$

  where b_ν is called the **bias**.

- Thus galaxies (and other tracers of the matter distribution) are *biased* with respect to the underlying dark matter distribution.
- A similar relation holds for the power spectrum $P_\nu(k)$ as well.
- In general, the bias depends on the nature of the tracers under consideration. It is usually scale-independent at sufficiently large scales, but can be quite complicated at smaller scales.

Detailed calculation of the bias

- The bias can be calculated more rigorously using the mass functions.
- Analogous to the correlation function of matter

$$\xi_{mm}(\vec{x}_1 - \vec{x}_2) = \langle \delta_m(\vec{x}_1)\, \delta_m(\vec{x}_2) \rangle, \quad \delta_m(\vec{x}) = \frac{\rho_m(\vec{x}_1)}{\bar{\rho}_m} - 1,$$

  we can write for haloes

$$\xi_{hh}(\vec{x}_1 - \vec{x}_2 | M_1, M_2) = \langle \delta_h(\vec{x}_1, M_1)\, \delta_h(\vec{x}_2, M_2) \rangle, \quad \delta_h(\vec{x}, M) = \frac{n_M(\vec{x})}{\bar{n}_M} - 1.$$

  Note that $\bar{n}_M$ is the globally averaged mass function given by the standard form (e.g., Press-Schechter), while $n_M(\vec{x})$ is the mass function at a point $\vec{x}$.
- It can be shown that to the linear order

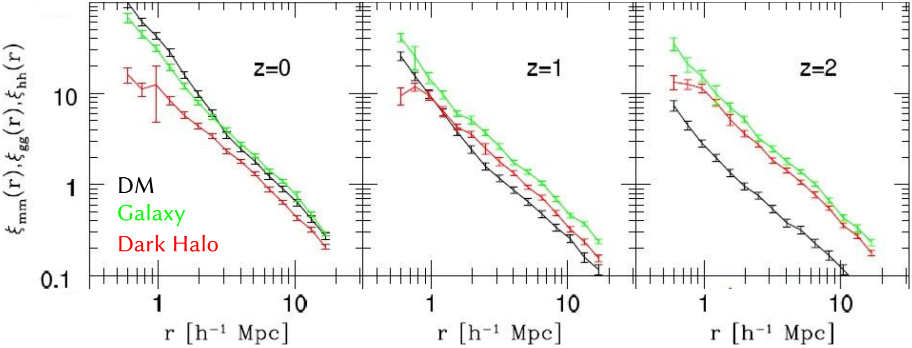
$$\delta_h(\vec{x}, M) = \frac{\nu^2 - 1}{\delta_c}\, \delta_m(\vec{x}), \quad \nu \equiv \frac{\delta_c(z)}{\sigma(M)}.$$

- It is customary to define the **linear halo bias** through the relation $\delta_h(\vec{x}, M) = b(M)\, \delta_m(\vec{x})$.
- Note that the value of ν is higher for larger M. Hence the bias will be higher for larger mass (i.e., rarer) haloes.
- Hence we can write

$$\xi_{hh}(\vec{x}_1 - \vec{x}_2 | M_1, M_2) = b(M_1)\, b(M_2)\, \xi_{mm}(\vec{x}_1 - \vec{x}_2).$$

  The full correlation function can be calculated by integrating over the mass function.
- A similar relation exists for the power spectrum as well.

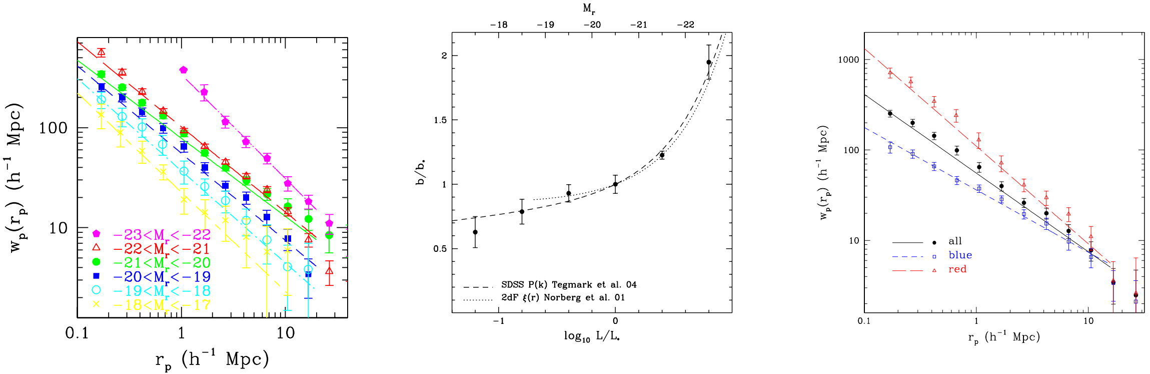
Results from simulations



Yoshikawa et al (2001)

Results from observations (SDSS)

- Projected correlation function $w_p(r_p) = 2 \int_0^\infty \mathrm{d}r_{\parallel}\, \xi(\vec{x}) = 2 \int_0^\infty \mathrm{d}r_{\parallel}\, \xi(r_{\parallel}, r_p)$



Zehavi et al (2005)