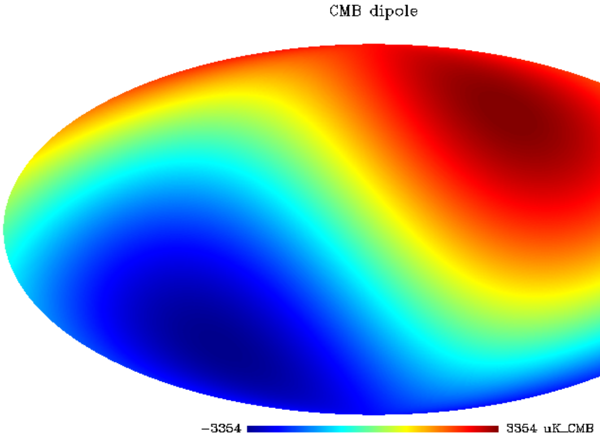
Cosmology
Lecture 18
Velocity field

Tirthankar Roy Choudhury
National Centre for Radio Astrophysics
Tata Institute of Fundamental Research
Pune

NCRA • TIFR

Effect of peculiar velocities

- Until now, we have ignored the effect of peculiar velocities on the cosmological observables.
- A very prominent effect of the peculiar motion of the Earth is the dipole anisotropy of the CMB.



Planck data

- While doing cosmology with CMB, one subtracts out the dipole first.

Observed redshifts

- The presence of peculiar velocities affect the redshift of an object and hence the determination of distances using the standard Hubble-Lemaitre law.
- The observed redshift $z_{\rm obs}$ of an object (as determined, e.g., from wavelengths of spectral lines) consists of a cosmological component $z_{\rm cos}$ and a component z_p arising from the radial component of the peculiar velocity v_r.
- The relation between comoving distance and the cosmological redshift $z_{\rm cos}$ is

$$\chi(z_{\rm cos}) = \int_0^{z_{\rm cos}} \frac{c\,\mathrm{d}z}{H(z)},$$

  and the redshift arising from the peculiar velocity v_r is

$$z_p = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1.$$

  The total redshift is

$$1 + z_{\rm obs} = (1 + z_{\rm cos})(1 + z_p)$$

- In case of non-relativistic velocities and small redshifts, the relation simplifies to

$$z_{\rm obs} \approx \frac{H_0 r}{c} + \frac{v_r}{c}.$$

  Note that this is valid only in the nearby universe and has to be modified for high redshifts.

Density field from peculiar velocities

- To estimate the peculiar velocity v_r, one needs the redshift $z_{\rm obs}$ and an independent estimate of the distance r.
- For nearby galaxies, we can use the Tully-Fisher relation for spiral galaxies $L \propto V_{\rm max}^3$, where $V_{\rm max}$ is the maximum rotation velocity. One can also use the diameter - velocity dispersion $D_n \propto \sigma^{1.2}$ for ellipticals, where D_n is the diameter at which the mean surface brightness drops to some fiducial value.
- One thus has the redshift $z_{\rm obs}$ and r from observations, allowing to estimate v_r.
- In the linear regime, the peculiar velocity field is related to the density field through the continuity equation

$$\dot{\delta}(\vec{x}, t) = \dot{D}\, \delta(\vec{x}, t_0) = -\frac{1}{a} \vec{\nabla} \cdot \vec{v}(\vec{x}, t) \quad \Longrightarrow \quad \vec{\nabla} \cdot \vec{v}(\vec{x}, t) = -a \frac{\dot{D}}{D}\, \delta(\vec{x}, t) = -a\, H(a)\, f(a)\, \delta(\vec{x}, t).$$

Thus the knowledge of the velocity field can be used to estimate the density field.

Cosmology using large-scale velocity field

- To calculate the non-radial components of $\vec{v}$, note that in the linear (and quasi-linear) regime we can write $\vec{v} = \vec{\nabla} V$ (the rotational component decaying rapidly). Then $v_r = \partial V / \partial r$, thus giving

$$V(\vec{r}) = \int_0^r \mathrm{d}r' \; v_r(r', \theta, \phi).$$

One can thus obtain V and correspondingly the velocity field $\vec{v}$.

- The density contrast is then

$$\delta(\vec{x}) = -\frac{\vec{\nabla} \cdot \vec{v}(\vec{x})}{a\, H(a)\, f(a)}.$$

- Once can thus estimate the density field, and then compare with, e.g., the density field δ_{gal} of the observed galaxies.
- Usually at large scales $\delta_{\mathrm{gal}} = b_{\mathrm{gal}}\, \delta$, where b_{gal} is the galaxy bias. Hence

$$\delta_{\mathrm{gal}} = -\frac{\vec{\nabla} \cdot \vec{v}\, b_{\mathrm{gal}}}{H\, a\, f(a)},$$

which can be used to constrain

$$\beta = \frac{f(a)}{b_{\mathrm{gal}}} \approx \frac{\Omega_m^{0.6}(a)}{b_{\mathrm{gal}}}.$$

This can be used to put constraints on cosmology as well as galaxy formation.

Redshift distance

- The measurement of distances using Tully-Fisher or similar relations become impossible at high redshifts. For these high redshift galaxies, we only have information about the total redshift z_{obs}. This can affect our inference on the distances and hence the measurement of correlation function.
- We can assign a distance to the galaxy using the Hubble-Lemaitre law (assuming it to be nearby)

$$s = \frac{c}{H_0}\, z_{\text{obs}}.$$

- Since $cz_{\text{obs}} \approx H_0 r + v_r$ we get

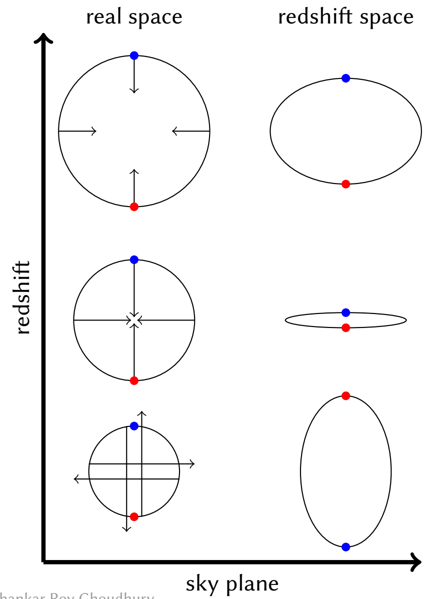
$$s = r + \frac{v_r}{H_0}.$$

- In vector notation, we write

$$\vec{s} = \vec{r} + \frac{\vec{v} \cdot \hat{r}}{H_0} \hat{r}.$$

- The quantity s is called the **redshift distance**, while r is the “real distance”.
- Clearly $s < r$ when $v_r < 0$. The object will seem nearer in the redshift space when it is moving towards us compared to the Hubble expansion.

Redshift space distortion



- Let us now understand physically the effect of redshift space using evolution of a spherical shell.
- For a large radius within which the overdensity is small, the expansion of the mass shell is decelerated but its peculiar velocity is still too small to compensate for the Hubble expansion. In redshift space the mass shell will then appear squashed along the line-of-sight when observed from a distance much larger than its size.
- A mass shell with linear overdensity $\delta \sim 1$ is just turning around at the time it is observed, so its peculiar infall velocity is exactly equal to the Hubble expansion velocity across its radius. In redshift space this shell appears completely "collapsed" to an observer at large distance.
- A mass shell which has already turned around has a peculiar infall velocity which exceeds the Hubble expansion across its radius. If this infall velocity is less than twice the Hubble expansion velocity, the shell appears flattened along the line-of-sight, but with the nearer side having larger redshift distance than the farther side.
- At smaller radii the peculiar infall velocities of collapsing shells are much larger than the relevant Hubble velocites and are randomised by scattering effects. The structure then appears to be elongated along the line-of-sight in redshift space (a **"finger-of-God"** pointing back to the observer).

Density contrast in redshift space

- It is usually the case that the scale of perturbations is much smaller than the distance from us.
- In that case one can use the plane parallel approximation and construct a local Cartesian coordinate system. We choose the z-direction along the line of sight, hence

$$\vec{s} = \vec{r} + \frac{v_z}{H_0} \hat{z}.$$

- Using the conservation of mass (or equivalently the conservation of galaxy counts), we can write

$$\rho_{\rm gal}^{(s)}(\vec{s})\, \mathrm{d}^3 s = \rho_{\rm gal}(\vec{r})\, \mathrm{d}^3 r \quad \Longrightarrow \quad \left[1 + \delta_{\rm gal}^{(s)}(\vec{s})\right] \mathrm{d}^3 s = \left[1 + \delta_{\rm gal}(\vec{r})\right] \mathrm{d}^3 r.$$

- Since the coordinate transformation involves only the z-component, the Jacobian is simply

$$\left| \frac{\partial s^a}{\partial r^b} \right| = 1 + \frac{1}{H_0} \frac{\partial v_z}{\partial r_z}.$$

- Hence

$$1 + \delta_{\rm gal}^{(s)}(\vec{s}) = \left(1 + \frac{1}{H_0} \frac{\partial v_z}{\partial r_z}\right)^{-1} \left[1 + \delta_{\rm gal}(\vec{r})\right].$$

- In the linear approximation $\delta_{\rm gal} \ll 1,\ v_z \ll c$, then

$$1 + \delta_{\rm gal}^{(s)}(\vec{s}) \approx 1 - \frac{1}{H_0} \frac{\partial v_z}{\partial r_z} + \delta_{\rm gal}(\vec{r}) \quad \Longrightarrow \quad \delta_{\rm gal}^{(s)}(\vec{s}) \approx \delta_{\rm gal}(\vec{r}) - \frac{1}{H_0} \frac{\partial v_z}{\partial r_z}.$$

Kaiser effect

- In the Fourier space

$$\delta^{(s)}_{\rm gal}(\vec{k}) \approx \delta_{\rm gal}(\vec{k}) - \frac{\mathrm{i}k_z}{H_0} v_z(\vec{k})$$
$$= \delta_{\rm gal}(\vec{k}) - \frac{\mathrm{i}k_z}{H_0}\; H\,a\,f(a)\; \frac{\mathrm{i}k_z}{k^2}\delta(\vec{k}) \approx \delta_{\rm gal}(\vec{k}) - \frac{\mathrm{i}k_z}{H_0}\; H_0\, f(a)\; \frac{\mathrm{i}k_z}{k^2}\frac{\delta_{\rm gal}(\vec{k})}{b_{\rm gal}}$$
$$= \delta_{\rm gal}(\vec{k}) \left[1 + \frac{k_z^2}{k^2}\,\frac{f(a)}{b_{\rm gal}}\right] = \delta_{\rm gal}(\vec{k}) \left(1 + \beta \frac{k_z^2}{k^2}\right).$$

- One can define $\mu_k \equiv \dfrac{k_z}{k}$, such that $\theta = \cos^{-1}\mu_k$ is the angle between $\vec{k}$ and the line of sight. In that case

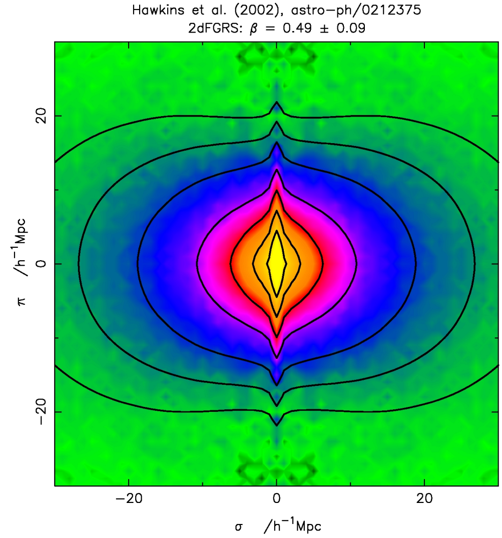
$$\delta^{(s)}_{\rm gal}(\vec{k}) = \delta_{\rm gal}(\vec{k}) \left(1 + \beta\mu_k^2\right).$$

  This relates the density contrasts in the real and redshift spaces.
- The power spectra are related by

$$P^{(s)}_{\rm gal}(\vec{k}) = P_{\rm gal}(k) \left(1 + \beta\mu_k^2\right)^2,$$

  showing that the effect of redshift space is to make the power spectrum anisotropic. This effect at large scales is known as the **Kaiser effect**.
- In general the function $P^{(s)}_{\rm gal}(\vec{k}) \equiv P^{(s)}_{\rm gal}(k, \mu_k)$ can be expanded in terms of Legendre polynomials. The corresponding coefficients can be measured from observations, which in turn can be used to estimate β.

Observing the Kaiser effect



$$P(k) \longrightarrow P(k_{\parallel}, k_{\perp}), \quad k = \sqrt{k_{\parallel}^2 + k_{\perp}^2}.$$

Spherically averaged power spectrum

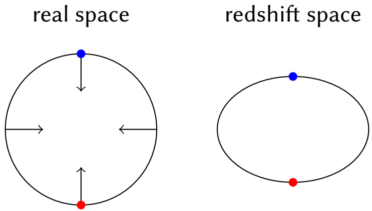
- A quantity of interest is the “spherically averaged power spectrum” defined as

$$P^{(s)}_{\mathrm{gal},0}(k) \equiv \frac{1}{2}\int_{-1}^{1} \mathrm{d}\mu_k \, P^{(s)}_{\mathrm{gal}}(\vec{k}),$$

  which is nothing but the power spectrum in the redshift space integrated over all possible angles.
- This is shown to be

$$P^{(s)}_{\mathrm{gal},0}(k) = P_{\mathrm{gal}}(k) \times \frac{1}{2}\int_{-1}^{1} \mathrm{d}\mu_k \, \left(1+\beta\mu_k^2\right)^2 = P_{\mathrm{gal}}(k)\left(1+\frac{2}{3}\beta+\frac{1}{5}\beta^2\right).$$

- This shows that the amplitude of fluctuations increase in the redshift space. This is expected as the peculiar velocity tends to move the points towards high density regions in the redshift space.



- It is possible to write down equivalent expressions for correlation functions ξ in the real and redshift spaces as well.

Finger-of god effect

- At small scales, the Kaiser effect is not sufficient to capture the effects of redshift space distortions. The main effect in these scales arise from the pairwise velocity dispersion along the line of sight in virialized objects.
- It turns out that the effect can be modelled as

$$\delta_{\text{gal}}^{(s)}(\vec{k}) = \delta_{\text{gal}}(\vec{k})\, \mathrm{e}^{-k^2 \mu_k^2 \sigma^2 / 2H^2},$$

  where σ is the velocity dispersion.
- We can find the value of σ as compared to the Hubble velocity as follows:

$$\begin{aligned}
\sigma^2 &\sim \frac{GM}{R_{\text{vir}}} \\
&= \frac{G}{H^2} \frac{M}{R_{\text{vir}}^3} \left(H R_{\text{vir}}\right)^2 = \frac{1}{2} \frac{8\pi G}{3H^2} \frac{3M}{4\pi R_{\text{vir}}^3} v_H^2 \\
&= \frac{1}{2} \frac{1}{\rho_c} \rho_{\text{vir}}\, v_H^2 = \frac{1}{2} \frac{\bar{\rho}}{\rho_c} \frac{\rho_{\text{vir}}}{\bar{\rho}}\, v_H^2 \\
&\sim \frac{1}{2} \Omega_{m,0}\, 200\, v_H^2.
\end{aligned}$$

- This implies

$$\sigma \sim \sqrt{\Omega_{m,0}}\, 10\, v_H.$$

  So, haloes appear $\sim 5 - 10$ times longer along the line of sight compared to the sky plane. This is the finger-of-God effect discussed earlier.

Observing the finger-of-God effect