Cosmology
Lecture 9

Neutrino decoupling and dark matter freeze-out

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Major milestones of the hot Big Bang

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Event	t (approx)	z (approx)	T (approx)	Description
Inflation	10^{-34} s ?	$\to \infty$	10^{16} GeV ?	perturbations generated
Baryogenesis	?	?	?	explain the observed baryon density through some mechanism without assuming any initial asymmetry in matter and antimatter.
EW phase transition	10^{-11} s	10^{15}	100 GeV	electroweak force "breaks" into weak and electromagnetic
QCD phase transition	10^{-5} s	10^{12}	100 MeV	quarks & gluons bind into protons & neutrons
Dark matter freeze-out	?	?	?	dark matter particles decouple
Neutrino decoupling	1 s	10^{10}	1 MeV	neutrinos decouple
Electron-positron annihilation	10 s	10^{9}	0.5 MeV	$e^{\pm}$ annihilate into photons
Big Bang nucleosynthesis	3 m	10^{8}	100 keV	nuclei of light elements form
Matter-radiation equality	6×10^{4} y	3500	1 eV	
Recombination	4×10^{5} y	1100	0.2 eV	neutral hydrogen atoms form
Photon decoupling	4×10^{5} y	1100	0.2 eV	photons decouple from matter, CMB originates
Formation of first stars	10^{8} y	15	5 meV	first galaxies form
Dark energy-matter equality	10^{10} y	0.4	0.3 meV	
Present	1.4×10^{10} y	0	0.2 meV	

Universe at $k_B T \lesssim 1$ GeV

- Let us apply the concepts discussed so far to study the evolution of the universe for $T \lesssim 10^{13}$ K ~ 1 GeV.
- Particles with $m > 1$ GeV (like protons and neutrons) would have become nonrelativistic and hence would not contribute significantly towards the energy density or entropy.
- The only particles which would be relativistic are photons ($m_\gamma = 0$), electrons, positrons ($m_e \sim 0.5$ MeV), three species of neutrino and anti-neutrino ($m_\nu \lesssim 0.1$ eV).
- Neutrino equilibrium maintained by weak interactions, e.g., $e^+ + e^- \longleftrightarrow \nu_e + \bar{\nu}_e$ and $e^- + \bar{\nu}_e \longleftrightarrow e^- + \bar{\nu}_e$.
- The values of g_*, g_{*S} are given by (note that neutrinos are spin-1/2 particles but they have $g_\nu = 1$ because all of them have left-handed helicity)

$$g_* = g_{*S} = g_{*,\mathrm{th}} = \sum_B g_B + \frac{7}{8} \sum_F g_F = g_\gamma + \frac{7}{8} \left[g_e + g_{\bar{e}} + 3(g_\nu + g_{\bar{\nu}})\right] = \frac{43}{4} = 10.75.$$

Decoupling of neutrinos

- The weak interaction rate for neutrinos is $\Gamma_W \sim G_{\rm Fermi}^2 T^5$, where $G_{\rm Fermi} = \alpha / m_\xi^2 \approx 1.17 \times 10^{-5} \text{GeV}^{-2}$ is the Fermi constant ($m_\xi \approx 50$ GeV is the mass of the gauge-vector boson).
- Using the form of the Hubble parameter derived earlier, one can show that

$$\frac{\Gamma_W}{H} \sim \left(\frac{k_B T}{1.4 \text{MeV}} \right)^3 \sim \left(\frac{T}{1.6 \times 10^{10} \text{K}} \right)^3$$

- So, the interaction rate of neutrinos becomes lower than expansion rate when $T < T_D \approx 1$ MeV. The corresponding time is

$$t = 2.42 \times 10^{-6} \text{s} \; g_*^{-1/2} \left(\frac{k_B T}{\text{GeV}} \right)^{-2} \text{s} \sim \text{s}.$$

- At these lower temperatures, the neutrinos are decoupled from the rest. Since they are almost massless, they are relativistic at the time of decoupling (they will remain relativistic even now if they are massless).
- After neutrino freeze-out, electrons and positrons remain in equilibrium with radiation via annihilation and pair-production. The entropy of the neutrinos will be conserved separately. Since the neutrinos remain relativistic during decoupling, the total g_{*S} remains conserved.

Degrees of freedom

- The value of g_{*S} contributed by electron, positrons and photons (after neutrino decoupling) would be

$$g_{*S} = \sum_B g_B + \frac{7}{8} \sum_F g_F = g_\gamma + \frac{7}{8} \left(g_e + g_{\bar{e}}\right) = \frac{11}{2} = 5.5.$$

- The same for the neutrinos would be

$$g_{*S} = \frac{7}{8} 3 \left(g_\nu + g_{\bar{\nu}}\right) = \frac{21}{4} = 5.25,$$

  the total being 10.75, as before. The two components will be conserved separately.
- The temperatures of the two components would keep on falling as a^{-1}, thus neutrinos and photon-electron-positron plasma would continue to have the same temperature even though they do not interact.

Annihilation of $e^{\pm}$ pairs

- However, the photon temperature T can be different if the value of g_{*S} changes somehow, e.g., if species in equilibrium become non-relativistic.
- When $T = T_\gamma \lesssim 0.5$ MeV, the photons do not have enough energy to produce $e^+ e^-$ pairs and hence the electrons and positrons would annihilate and thus disappear. Once this happens, the value of g_{*S} would be

$$g_{*S} = g_\gamma + \frac{7}{8} \times 3 \left(g_\nu + g_{\bar{\nu}}\right) \left(\frac{T_\nu}{T}\right)^3 = 2 + \frac{21}{4} \left(\frac{T_\nu}{T}\right)^3 .$$

- Because g_{*S} decreases (the contribution of e^-, e^+, γ plasma goes from 5.5 to 2) during $e^+ e^-$ annihilation at $a = a_{\rm ann}$, we expect T to rise (which is because of pairs dumping energies into photons). Note that since the neutrinos have already decoupled, they cannot receive any of the entropy released by the electron-positron pairs.

Neutrino and photon temperatures

Since entropy is conserved, we expect them to be equal before ($a = a_-$) and after ($a = a_+$) the annihilation

$$g_{*S}(a_-)\ T^3(a_-)\ a_-^3 = g_{*S}(a_+)\ T^3(a_+)\ a_+^3 \qquad \text{conservation of entropy}$$

$$\frac{43}{4}(T_-\ a_-)^3 = 2(T_+\ a_+)^3 + \frac{21}{4}\left(\frac{T_\nu}{T}\right)^3\Bigg|_{a=a_+}(T_+\ a_+)^3 \qquad g_{*S} \text{ for } \gamma, e^\pm, \nu_e, \bar{\nu}_e$$

$$\frac{43}{4}(T_{\nu-}\ a_-)^3 = 2(T_+\ a_+)^3 + \frac{21}{4}(T_{\nu+}\ a_+)^3 \qquad T_\nu = T = T_\gamma \text{ before annihilation}$$

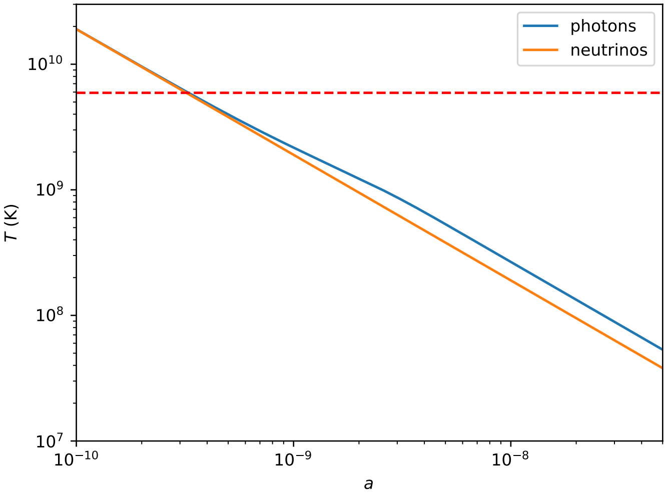
$$\frac{43}{4}(T_{\nu+}\ a_+)^3 = 2(T_+\ a_+)^3 + \frac{21}{4}(T_{\nu+}\ a_+)^3 \qquad T_\nu \propto a^{-1} \text{ throughout}$$

$$\frac{11}{2}T_{\nu+}^3 = 2T_+^3$$

$$T_\nu(a_+) = \left(\frac{4}{11}\right)^{1/3} T(a_+)$$

Clearly the photon temperature is higher than the neutrinos after annihilation.

Numerical solution to the temperature evolution



The neutrino background

- Thus $T_\nu < T$ when the pair-production is halted at $T_\gamma \lesssim 0.5$ MeV. Since both T_ν and T fall as a^{-1} afterwards, the above relation is maintained till today.
- The neutrino temperature today is $T_\nu = (4/11)^{1/3} \times 2.73$ K ≈ 1.95 K.
- The neutrinos form a homogeneous background which can be detected today. However, since the neutrinos have only weak interaction, detecting the small background is a big challenge.
- At present epoch, photons and (massless) neutrinos (plus antineutrinos) are relativistic, however they are at different temperatures, hence

$$g_*(t_0) = \sum_{B \in \text{bosons}} g_B \left(T_B/T\right)^4 + (7/8) \sum_{F \in \text{fermions}} g_F \left(T_F/T\right)^4 = 2 \times 1 + (7/8)(3 \times 2 \times 1) \left(4/11\right)^{4/3} \approx 3.36,$$

$$g_{*S}(t_0) = \sum_{B \in \text{bosons}} g_B \left(T_B/T\right)^3 + (7/8) \sum_{F \in \text{fermions}} g_F \left(T_F/T\right)^3 = 2 \times 1 + (7/8)(3 \times 2 \times 1)(4/11) \approx 3.91.$$

- The energy density at the present epoch is

$$\rho_R(t_0) = \frac{\pi^2}{30} \left(\frac{k_B^4}{\hbar^3}\right) g_* T_0^4 \approx 8.1 \times 10^{-34} \text{ gm cm}^{-3}.$$

- Then $\Omega_{r,0} \equiv \Omega_{R,0} \approx 4.3 \times 10^{-5} h^{-2} \approx 1.68\, \Omega_{\gamma,0}$ because $\rho_{\gamma,0} = (\pi^2/15)\left(k_B^4/\hbar^3\right) T_0^4$.
- The contribution to density by neutrinos will be given by the above only if the neutrinos are still relativistic. If the neutrinos have mass, then when $T \lesssim m_\nu$ it is possible for neutrinos to contribute much more to the density because there would be a lower bound in the contribution to density made by a neutrino. A neutrino with mass m_ν has to contribute at least m_ν to density.

Non-equilibrium evolution

- It is possible to work out the effect of decoupling and departure from equilibrium more rigorously. A nice application of this would be to study the freezing out of dark matter and calculate the amount of dark matter present.
- For this, we need to assume something about the dark matter particles, which will be speculative. Let us start with the hypothesis that the dark matter particles have only weak interaction.
- We also assume that at sufficiently early times the dark matter particle X and its anti-particle $\bar{X}$ can annihilate and produce two light particles l and $\bar{l}$, i.e., $X + \bar{X} \longleftrightarrow l + \bar{l}$.
- These light particles are assumed to be tightly coupled to the rest of the cosmic plasma. Hence they will assume their equilibrium densities $n_l = n_l^{(0)}$ and $n_{\bar{l}} = n_{\bar{l}}^{(0)}$.
- In case X are neutrinos, l would be electrons.
- We also assume that $n_X \approx n_{\bar{X}}$, i.e., the asymmetry in the dark matter and anti-matter is very small (which follows if the chemical potential is small).
- We then have (see the previous lecture)

$$\frac{\dot{n}_{0,X}}{n_{0,X}} = -\,\Gamma_X \left[1 - \frac{n_{0,X}^{(0)} n_{0,\bar{X}}^{(0)}}{n_{0,l}^{(0)}\, n_{0,\bar{l}}^{(0)}} \, \frac{n_{0,l} n_{0,\bar{l}}}{n_{0,X} n_{0,\bar{X}}}\right] = -\langle \sigma v \rangle n_{\bar{X}} \left[1 - \frac{n_{0,X}^{(0)} n_{0,\bar{X}}^{(0)}}{n_{0,X} n_{0,\bar{X}}}\right],$$

$$\frac{\mathrm{d} n_{0,X}}{\mathrm{d} t} = -a^{-3} \, \langle \sigma v \rangle n_{0,X}^2 \left[1 - \frac{\left(n_{0,X}^{(0)}\right)^2}{n_{0,X}^2}\right] = -a^{-3} \, \langle \sigma v \rangle \left[n_{0,X}^2 - \left(n_{0,X}^{(0)}\right)^2\right].$$

The temperature evolution

- We will assume for simplicity that during most of the relevant epochs related to the dark matter decoupling and freeze-out the temperature $T \propto a^{-1}$. We then have

$$\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mathrm{d}\ln T}{\mathrm{d}t} = -\frac{\mathrm{d}\ln a}{\mathrm{d}t} = -\frac{\dot{a}}{a} = -H$$

- Let us introduce a new dimensionless variable

$$x \equiv \frac{m_X}{k_B T}$$

such that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{m_X}{k_B T^2}\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{x}{T}\frac{\mathrm{d}T}{\mathrm{d}t} = H\,x.$$

- Since we are in the radiation dominated era, the evolution of H is given by

$$H \propto a^{-2} \propto T^2 \Longrightarrow H(T) = \frac{H(m_X/k_B)}{x^2} \equiv \frac{H_m}{x^2},$$

H_m is the Hubble parameter when $x = 1$, i.e., $k_B T = m_X$.

The Riccati equation

- Hence
$$\frac{\mathrm{d}n_{0,X}}{\mathrm{d}x} = \frac{\mathrm{d}n_{0,X}}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{1}{H\,x}\,a^{-3}\,\langle\sigma v\rangle\left[n_{0,X}^2 - \left(n_{0,X}^{(0)}\right)^2\right] = -\frac{\langle\sigma v\rangle}{H_m}\,\frac{x}{a^3}\,\left[n_{0,X}^2 - \left(n_{0,X}^{(0)}\right)^2\right].$$
- Let us define another dimensionless variable
$$Y_X \equiv \left(\frac{\hbar^3}{k_B^3}\right)\frac{n_{0,X}}{(aT)^3} = \left(\frac{\hbar^3}{k_B^3}\right)\frac{n_X}{T^3}.$$
- Since the combination aT does not evolve, we get
$$\frac{\mathrm{d}Y_X}{\mathrm{d}x} = -\frac{\hbar^3/k_B^3}{a^3\,T^3}\,\frac{\langle\sigma v\rangle}{H(m_X)}\,\frac{x}{a^3}\,\frac{a^6\,T^6}{\hbar^6/k_B^6}\left[Y_X^2 - \left(Y_X^{(0)}\right)^2\right]$$
$$= -\left(\frac{k_B^3}{\hbar^3}\right)\frac{\langle\sigma v\rangle}{H_m}\,xT^3\,\left[Y_X^2 - \left(Y_X^{(0)}\right)^2\right] = -\frac{\langle\sigma v\rangle m_X^3}{\hbar^3\,H_m}\,\frac{1}{x^2}\,\left[Y_X^2 - \left(Y_X^{(0)}\right)^2\right].$$
- We are thus left with solving the differential equation
$$\frac{\mathrm{d}Y_X}{\mathrm{d}x} = -\frac{\lambda}{x^2}\,\left[Y_X^2 - \left(Y_X^{(0)}\right)^2\right],$$

  where
$$\lambda \equiv \frac{\langle\sigma v\rangle m_X^3}{\hbar^3\,H_m} \longrightarrow \frac{\langle\sigma v\rangle m_X^3 c^3}{\hbar^3\,H_m}.$$

  Equations of the above form are known as **Riccati equations**.

Equilibrium densities

- To solve the equation, we need to know how λ is related to x or T. For some models, the cross section $\langle\sigma v\rangle$ depends on T, while for some it is independent. We will work with the simple case where λ can be taken as a constant.
- The equilibrium number density is given by

$$n_X^{(0)} = \frac{g_X}{2\pi^2\hbar^3}\int_{m_X}^{\infty}\frac{\mathrm{d}E\, E\sqrt{E^2 - m_X^2}}{\mathrm{e}^{E/k_B T} \pm 1}.$$

- This gives

$$Y_X^{(0)} = \left(\frac{\hbar}{k_B}\right)^3 \frac{n_X^{(0)}}{T^3} = \frac{g_X}{2\pi^2}\int_x^{\infty}\frac{\mathrm{d}y\, y\sqrt{y^2 - x^2}}{\mathrm{e}^y \pm 1}.$$

- The number density can be written in closed form for the ultra- and non-relativistic case. For the former ($x \ll 1$), we have

$$n_X^{(0)} = \frac{\zeta(3)}{\pi^2}\left(\frac{k_B}{\hbar}\right)^3 g_{B,F} T^3 \Longrightarrow Y_X^{(0)} = g_{B,F}\frac{\zeta(3)}{\pi^2},$$

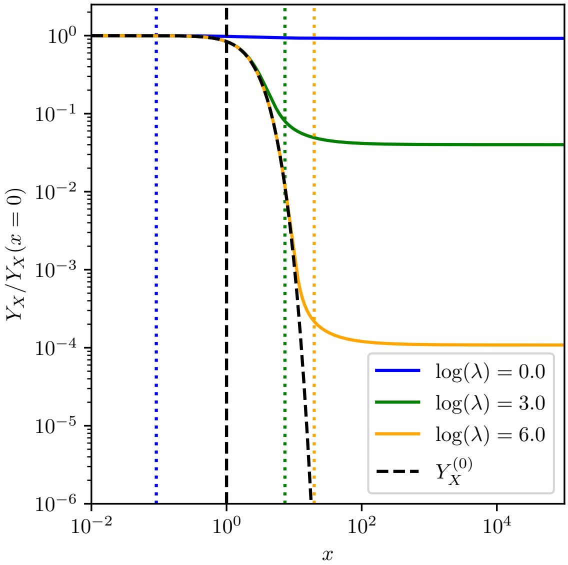
  where $g_{B,F} = g_X,\ 3g_X/4$ for bosons, fermions.
- For the non-relativistic case ($x \gg 1$),

$$n_X^{(0)} = g_X\left(\frac{m_X k_B T}{2\pi\hbar^2}\right)^{3/2} \mathrm{e}^{-m_X/k_B T} \Longrightarrow Y_X^{(0)} = \frac{g_X}{(2\pi)^{3/2}}\, x^{3/2}\, \mathrm{e}^{-x}.$$

Relics

- Returning back to the Riccati equation, we can see that for $x \to 0$, the interaction term $\lambda/x^2 \to \infty$, and hence the system will try to attain its equilibrium value $Y_X(x \to 0) \simeq Y_X^{(0)}$.
- As x increases, the annihilations become less efficient $\lambda/x^2 \to 0$ and hence $Y_X \longrightarrow$ constant, the freeze-out value. Let the freeze-out occur at $x = x_D$, then $Y_{X,\infty} \equiv Y_X(x \to \infty) = Y_X(x_D)$.
- The frozen out species are usually known as "relics".
- There are three possible types of relics:
  1. **Hot relics:** the particle remains relativistic during decoupling and is relativistic even today (e.g., massless neutrinos).
  2. **Warm relics:** the particle remains relativistic during decoupling and is non-relativistic even today (e.g., massive neutrinos).
  3. **Cold relics:** the particle has become non-relativistic before decoupling (e.g., cold dark matter).
- In the case of the hot/warm relics, $x_D \ll 1$. For cold relics, $x_D \gg 1$.
- Note that we are studying what are known as "thermal relics". It is possible that there were non-thermal relics too (i.e., particles that were never in equilibrium with the cosmic plasma), e.g., axion-like particles inspired by string theory.

Numerical solution



Hot and warm relics

- For the hot/warm relics, $x_D \ll 1$, so the equilibrium density $Y_X^{(0)}$ does not evolve till $x = x_D$.
- In that case, the Riccati equation implies a solution $Y_X = Y_X^{(0)} = \text{constant}$.
- Hence the relic density is given by $Y_{X,\infty} = Y_X^{(0)}(x_D) = g_{B,F}\dfrac{\zeta(3)}{\pi^2}$.
- The number density at decoupling is $n_X(x_D) = \left(\dfrac{k_B^3}{\hbar^3}\right) Y_X^{(0)}(x_D)\, T_D^3 = \dfrac{\zeta(3)}{\pi^2}\left(\dfrac{k_B^3}{\hbar^3}\right) g_{B,F} T_D^3$.
- If the species has decoupled, the number density will evolve as a^{-3}, hence the relic density today ($a = 1$) would be $n_{X,0} = n_X(x_D)\, a_D^3 = \dfrac{\zeta(3)}{\pi^2}\left(\dfrac{k_B^3}{\hbar^3}\right) g_{B,F} T_D^3 a_D^3$.
- It is useful to express this quantity in terms of the CMB temperature today T_0. We have already seen that $T \propto a^{-1} g_{*S}^{-1/3}$ (conservation of entropy). Hence

$$T_D^3 a_D^3 = T_0^3\, \frac{g_{*S}(T_0)}{g_{*S}(T_D)}$$

- Since $g_{*S}(T_0) \approx 3.91$, the number density of the hot/warm relic at present is given by

$$n_{X,0} = 3.91\frac{\zeta(3)}{\pi^2}\left(\frac{k_B^3}{\hbar^3}\right)\frac{g_{B,F}}{g_{*S}(T_D)}\, T_0^3,$$

  which is inversely proportional to $g_{*S}(T_D)$, the entropy state of the universe during decoupling of the species.
- Note that in this case, the relic density is "relatively" insensitive to the details of decoupling.

Present density of hot/warm relics

- For the hot relics, the number density is given by its temperature

$$n_{X,0} = \frac{\zeta(3)}{\pi^2} \left(\frac{k_B^3}{\hbar^3}\right) g_{B.F} T_{X,0}^3 \implies T_{X,0}^3 = \frac{3.91}{g_{*S}(T_D)} T_0^3.$$

- As an example, for massless neutrinos we have $g_{*S}(T_D) = 10.75$, hence $T_{\nu,0}^3 = 0.36 T_0^3$, identical to the result we obtained earlier. The relic energy density is $\rho_{X,0} \propto T_{X,0}^4$.
- Warm relics have a small mass, hence we have

$$\rho_{X,0} = n_{X,0}\, m_X = 3.91 \frac{\zeta(3)}{\pi^2} \left(\frac{k_B^3}{\hbar^3}\right) \frac{g_{B,F}}{g_{*S}(T_D)} T_0^3\, m_X.$$

- If we put in the appropriate values, we get the corresponding density parameter as

$$\Omega_{X,0} = \frac{\rho_{X,0}}{\rho_{c,0}} = 0.0765\, h^{-2} \frac{g_{B,F}}{g_{*S}(T_D)} \left(\frac{m_X}{1\text{eV}}\right).$$

- Since we know from observations that total matter $\Omega_{m,0} h^2 \lesssim 1$, we get a conservative bound

$$m_X \lesssim 13.1 \text{ eV} \frac{g_{*S}(T_D)}{g_{B,F}}.$$

For neutrinos $g_{B,F} = 2 \times 3/4$ (accounting for anti-particles) and so $m_\nu \lesssim 94$ eV. However, structure formation studies provide a much tighter bound on the mass.

Cold relics

- The calculation is not straightforward for cold relics because the equilibrium density $Y_X^{(0)}$ begins to evolve with x as soon as the species becomes non-relativistic.
- One can obtain approximate solutions by assuming that the species retains the equilibrium density till $x = x_D$ and freezes out at $x > x_D$. Then $Y_{X,\infty} = Y_X(x_D) = Y_X^{(0)}(x_D)$.
- The value of x_D can be estimated by demanding $\Gamma_X(x_D) = H(x_D)$.
- These calculations, when compared with the present bounds on dark matter density $\Omega_{\mathrm{DM},0}$, leads to a heavy $m_X \sim \mathrm{GeV} - \mathrm{TeV}$ having weak interaction-like cross sections as possible dark matter candidate. These are known as **Weakly Interacting Massive Particles (WIMPs)**.
- Unfortunately, no such particles have been detected in the particle physics collider experiments or otherwise.