Cosmology: Assignment 2

IUCAA-NCRA Graduate School
January - February 2018

18 January 2018
To be returned in the class on 30 January 2018

- The deadline for the submission of the solutions of this assignment will be strictly enforced. No marks will be given if the assignment is not returned in time.
- You are free to discuss the solutions with friends, seniors and consult any books. However, you should understand and be clear about every step in the answers. Marks may be reduced if you have not understood what you have written even though the answer is correct.
- Let me know if you find anything to be unclear or if you think that something is wrong in any of the questions.

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1. **Inflating universe:** Consider a spatially flat Friedmann universe with the metric

$$\mathrm{d}s^2 = c^2 \mathrm{d}t^2 - \mathrm{e}^{2t/b}(\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \mathrm{d}\phi^2),$$

where b is a constant.

(a) Calculate the Hubble parameter for this universe.

(b) Consider a galaxy at coordinate $r = r_1$ emitting a light signal at $t = t_1$. Let this signal be received by an observer at $r = 0$ at $t = t_0$. Show that

$$r_1 = bc \left[\mathrm{e}^{-t_1/b} - \mathrm{e}^{-t_0/b}\right].$$

(c) Show that a light signal emitted by the observer at $r = 0$, $t = 0$ asymptotically approaches the coordinate $r = bc$ but never reaches it.

[1+2+2]

2. **Unit conversions:** Convert the following quantities by inserting the appropriate factors of c, $\hbar$ and k_B:

(a) $T_0 = 2.725$ K to eV.

(b) $\rho_\gamma = \pi^2 T_0^4/15$ to eV^4.

(c) $\rho_\gamma = \pi^2 T_0^4/15$ to gm cm^{-3}.

(d) H_0^{-1} to h^{-1}cm.

(e) $m_{\mathrm{Planck}} = 1.2 \times 10^{19}$ GeV to K.

(f) $m_{\mathrm{Planck}} = 1.2 \times 10^{19}$ GeV to cm^{-1}.

(g) $m_{\mathrm{Planck}} = 1.2 \times 10^{19}$ GeV to s^{-1}.

[1+1+3+1+1+3+1]

3. **Non-relativistic matter:** Consider the equilibrium phase space distribution function for a non-degenerate gas

$$f(p,t) = \frac{g}{(2\pi)^3} \frac{1}{\mathrm{e}^{E(p)/T} \pm 1},$$

where symbols have their usual meaning. Show that in the non-relativistic limit ($T \ll m$), we get

(a)

$$n = g \left(\frac{mT}{2\pi} \right)^{3/2} \mathrm{e}^{-m/T}.$$

(b)

$$\rho = n \left(m + \frac{3T}{2} \right).$$

(c)

$$P = n\, T$$

[6+5+3]