Extra-Galactic Astronomy - I
Cosmology

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Part I - Lecture 10

Big Bang Nucleosynthesis

- Production mechanism of light nuclei
- Predictions
- Observations
- Historical notes

Equilibrium expectations

Consider a non-relativistic nuclear species $A(Z)$ with mass A, charge Z in kinetic equilibrium:

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_A - m_A}{T}\right)$$

If reactions producing nucleus A out of Z protons and $A–Z$ neutrons are fast enough, then chemical equilibrium holds:

$$\mu_A = Z\mu_p + (A - z)\,\mu_n$$

Eliminate chemical potentials and define binding energy B_A :

$$B_A \equiv Z m_p + (A - Z) m_n - m_A$$

$$n_A = g_A A^{3/2} 2^{-A} \left(\frac{2\pi}{m_N T}\right)^{3(A-1)/2} n_p^Z n_n^{A-Z} \exp\left(B_A/T\right)$$

where $m_N \approx m_p \approx m_n \approx m_A/A$.

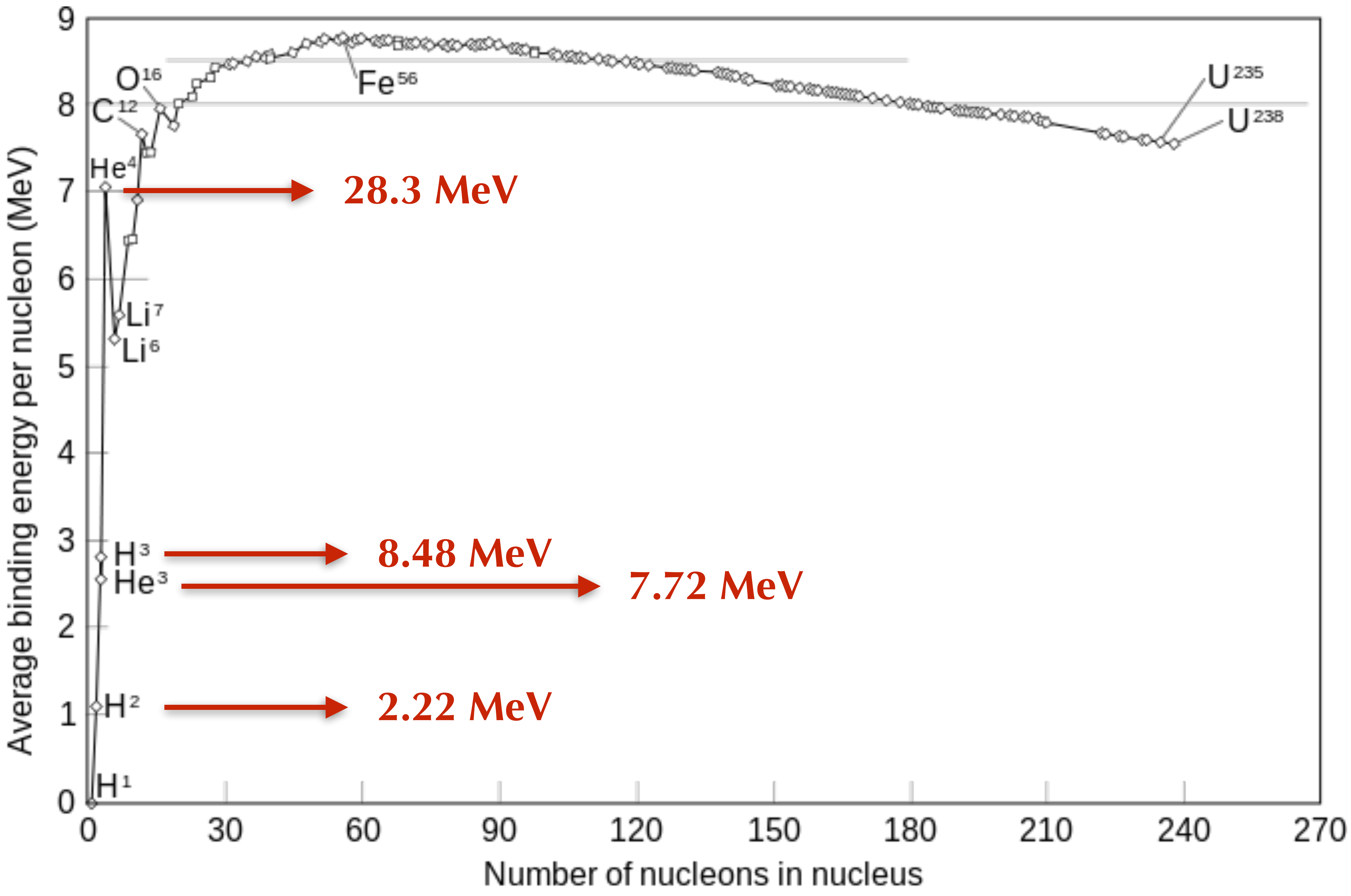
Defining $n_N = n_n + n_p + \sum A n_A$
and $X_A \equiv A n_A / n_N$

$$\begin{aligned} X_A = g_A\, \zeta(3)^{A-1}\, \pi^{(1-A)/2}\, 2^{(3A-5)/2} A^{5/2} \\ \times\, \eta^{A-1} (T/m_N)^{3(A-1)/2} \\ \times\, X_p^Z X_n^{A-Z} \exp\left(B_A/T\right) \end{aligned}$$

where

$$\eta \equiv \frac{n_N}{n_\gamma} = 2.68 \times 10^{-8}\ \left(\Omega_B h^2\right)$$

Binding energies



28.3 MeV

8.48 MeV

7.72 MeV

2.22 MeV

Neutron-to-proton ratio

After nucleosynthesis, essentially all neutrons incorporated into Helium. Hence initial ratio of neutrons to protons will help determine final abundances.

At $T >> 1$Mev, weak interactions maintain balance between neutrons & protons:

$$n \longleftrightarrow p + e^- + \bar{\nu}$$

$$\nu + n \longleftrightarrow p + e^-$$

$$e^+ + n \longleftrightarrow p + \bar{\nu}$$

with rates $\Gamma / H \simeq 0.8 \, (T / 1\text{MeV})^3$

Ignoring chemical potentials of electrons and electron neutrinos, in chemical equilibrium we have:

$$n/p \equiv n_n/n_p = X_n/X_p = \exp\left(-Q/T\right)$$

where $Q \equiv m_n - m_p = 1.293$MeV. So for T >> 1Mev, $\boldsymbol{X_n \simeq X_p \simeq 0.5}$

Neutron-to-proton ratio

Just before *T* ~ 1Mev, bulk of neutrinos decouple from plasma.

Soon after, e^+e^- pairs annihilate and disappear (transferring entropy to photons). Reactions converting neutrons to protons and vice-versa freeze-out, leaving a neutron-to-proton ratio

$$(n/p)_{\text{freeze-out}} = \exp(-Q/T_F) \simeq 1/6$$

Thereafter, occasional weak interactions occur (eventually dominated by free neutron decay), reducing the ratio from ~1/6 to ~1/7.

At this time, the other nuclear species are still in equilibrium, with tiny abundances.

Nucleosynthesis

In equilibrium we have

$$X_A = g_A\,\zeta(3)^{A-1}\,\pi^{(1-A)/2}\,2^{(3A-5)/2}A^{5/2} \\ \times \eta^{A-1}(T/m_N)^{3(A-1)/2} \\ \times X_p^Z X_n^{A-Z}\exp\left(B_A/T\right)$$

so initially (around $T \gg 1$MeV) we have **$X_A \sim \eta^{A-1} \ll 1$**, and we expect $X_A \sim 1$ around temperature $T_{\rm Nuc}$

$$T_{\rm Nuc} \simeq \frac{B_A/(A-1)}{\ln(\eta^{-1}) + 1.5\ln(m_N/T)}$$

Note that these are order 0.1-0.3Mev for d, …, ^4He, … ^{12}C, despite binding energies per nucleon ~ 1-8 Mev. This is mainly due to smallness of **η**.

Deuterium bottleneck

If equilibrium were maintained throughout, then ^4He would appear first. However, number densities are too low for anything but 2-body processes to be rapid enough compared to expansion. So the following chain leads to build-up of elements

$$\begin{aligned} \text{first :} \quad & p + n \longleftrightarrow d + \gamma \\ \text{next :} \quad & d + d \longleftrightarrow {}^3H + p \\ & d + d \longleftrightarrow {}^3He + n \\ \text{next :} \quad & d + {}^3H \longleftrightarrow {}^4He + n \\ & d + {}^3He \longleftrightarrow {}^4He + p \end{aligned}$$

First step is fast enough, even well below 0.1Mev, that deuterium abundances are well-approximated by equilibrium expression $X_d \sim \boldsymbol{\eta}\, X_p\, X_n\, \exp(B_d / T)$. However, smallness of B_d means that deuterons are rare well after equilibrium ^4He would be abundant, i.e., nucleosynthesis must wait until $T \sim T_d \sim 0.1$ MeV $< T_{4He}$ for ^4He to be produced.

End of nucleosynthesis

Beyond $T \sim T_d$, ^4He (the most tightly bound of light elements) is rapidly produced and eats up essentially all free neutrons.

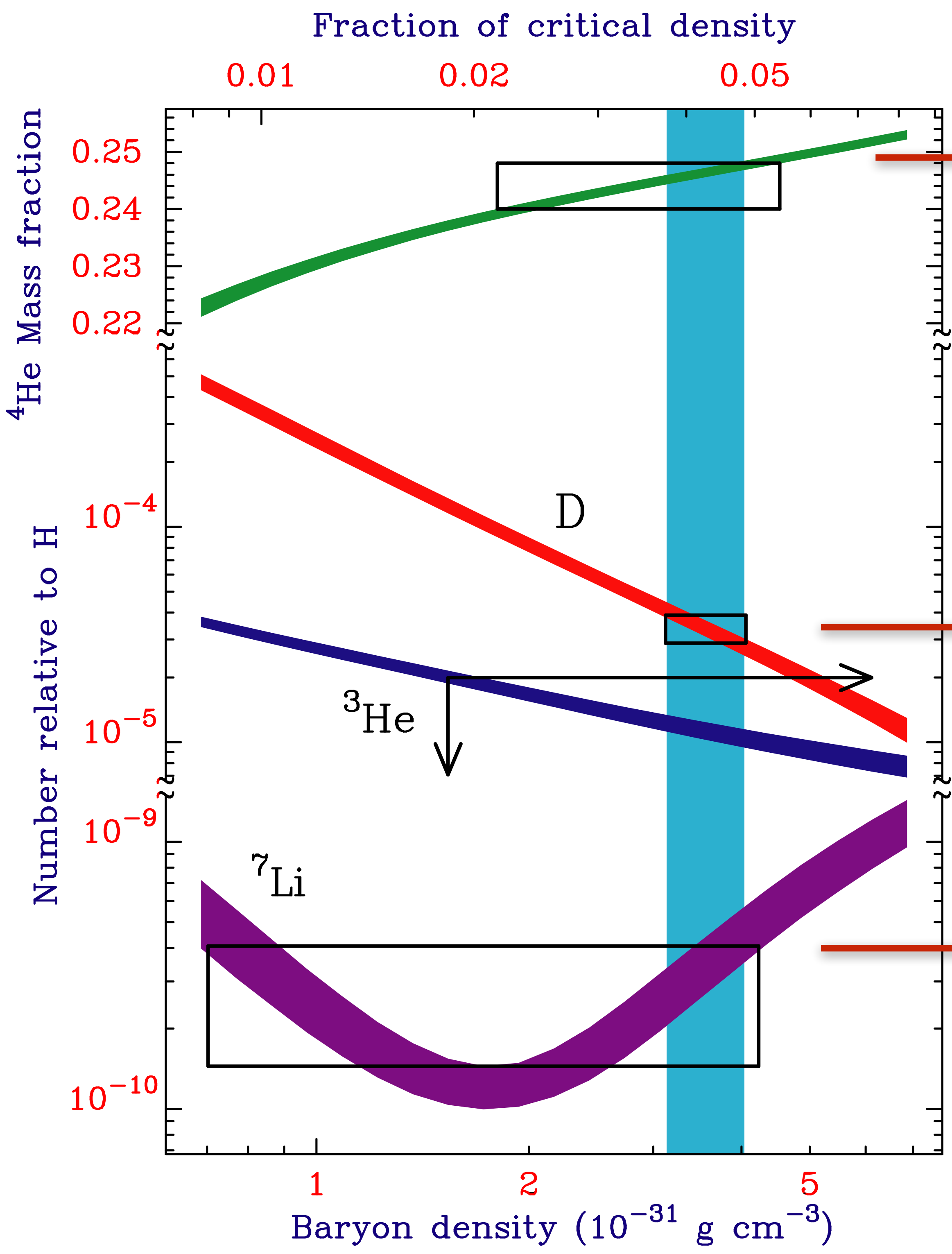
Small amount of ^7Li is produced by two processes:

$$\begin{aligned}
\text{direct:} \quad & {}^3H + {}^4He \longleftrightarrow {}^7Li + \gamma \\
\text{indirect:} \quad & {}^3He + {}^4He \longleftrightarrow {}^7Be + \gamma \\
& e^- + {}^7Be \longleftrightarrow {}^7Li + \nu \qquad \textit{(much later)}
\end{aligned}$$

Although ^{12}C, ^{16}O, etc. have binding energies larger than ^4He, production of these is hampered for two reasons: (a) no stable isotopes of mass numbers 5 and 8, (b) significant Coulomb barrier suppression at low temperatures.
(In stars, this gap is bridged by the triple-alpha reaction, but number densities are too low for this to occur in the early universe.)

Finally, substantial amounts of d and ^3He are left `unburnt', once their rates (proportional to their relative abundances and to $\boldsymbol{\eta}$) become too slow compared to Hubble expansion. Dependence on $\boldsymbol{\eta}$ means that primordial abundances probe $\Omega_B h^2$.

Predictions



Burles, Nollett, Turner (1999)

Mass fraction

$Y_P \simeq 4n_{He} / n_N = 4(n_n/2)/(n_n+n_p)$

$= 2\ (n/p)_{Nuc} / (1 + (n/p)_{Nuc}) \simeq 0.25$

where $(n/p)_{Nuc} \simeq 1/7$

— weak dependence on $\Omega_B h^2$

— theoretical error dominated by uncertainty in neutron half-life: 10.5 ± 0.2 min

Rates proportional to η, hence freeze-out abundances inversely proportional to η.

Initial decrease because directly produced ^7Li is destroyed by $p + {^7Li} \longleftrightarrow {^4He} + {^4He}$.

For larger η, indirect process becomes progressively more efficient.

Observations: ^4He

Observation of recombination lines in spectra of HII regions allows determination of ^4He mass fraction Y, using low-metallicity Blue Compact Dwarf (BCD) galaxies.

Primordial value Y_P inferred from extrapolating measurements of Y and metallicity (e.g., O/H) to zero metallicity.

Many issues involved: effects of collisional excitation, fluorescence reddening; accounting for $^4He^{++}$, unseen neutral ^4He, etc.
*[See, e.g. Thuan & Izotov (1998) Space Science Reviews, **84**, 83-94]*

Current estimates:
$Y_P = 0.2477 \pm 0.0029 \;\Rightarrow \boldsymbol{\eta} = (5.8 \pm 1.8) \times 10^{-10}$

*[Peimbert+ (2007) ApJ, **666**, 636-646]*

Observations: ^3He

Several local techniques for measuring ^3He:

— observations of $^3He^+$ hyperfine line in galactic HII regions:
$^3He / H < 1.1 \times 10^{-5}$ *[Bania, Rood & Balser (2002) Nature, **415**, 54]*

— measurements of pre-solar abundances in oldest meteorites, abundance in solar wind (from gas rich meteorites, lunar soil, foil placed on Moon by Apollo astronauts):
$[(d + {}^3He)/H] \simeq (3.6 \pm 0.6) \times 10^{-5}$

Problem is, ^3He can be both produced and destroyed in stars. Detailed calculations suggest that amount of ^3He ejected into ISM by stars should be small. Allowing for conservative estimates on amount of ^3He burned by stars then gives

$[(d + {}^3He)/H]_P \lesssim 8 \times 10^{-5} \Rightarrow \boldsymbol{\eta} > 4 \times 10^{-10}$

Observations: Deuterium

Early techniques: UV absorption studies of local ISM; deuterated molecules (DCO, DHO) in ISM; deuterated molecules in atmosphere of Jupiter; pre-solar abundances from meteorites/solar wind (see previous): broad range $d/H \sim 1\text{-}4 \times 10^{-5}$ with varying precision.

Due to low value of B_d, deuterium easily destroyed in any astrophysical environment. So all astrophysical observations strictly provide only *lower limit* on primordial abundance. Existence of astrophysical deuterium is amongst strongest evidence supporting hot Big Bang model.

More recently (since late 1990's), from measurements of d and H absorption lines in QSO spectra (absorption due to high-redshift intergalactic clouds), e.g.:
$d/H = (2.78 \pm 0.41) \times 10^{-5}$ *[Kirkman+ (2003) ApJS, **149**, 1]*
$d/H = (2.8 \pm 0.7) \times 10^{-5}$ *[Noterdaeme+ (2012) A&A, **542**, L33]*

Since absorption occurs in low metallicity systems, expect these to be tracing nearly primordial distribution.

Observations: ^7Li

Early observations of atmospheres of unevolved old stars in Galactic halo gave: $^7Li / H = (2\text{-}2.4 \pm {\sim}0.1) \times 10^{-10}$, lower than the expected 3×10^{-10} for $\Omega_B h^2 = 0.022$.

Since ^7Li is produced in stars and in interaction of cosmic rays with matter, this lower value might pose a problem to BBN. However, it is also plausible that ^7Li abundance has been *depleted* by convection in stellar atmospheres (supported by observations of ^7Li in atmospheres of stars with varying temperatures in same globular cluster).

Understanding this discrepancy is an active field of interest.

Historical notes

Dramatis personae:

— **George Gamow**, Ralph Alpher, Robert Herman (late 1940's, early 50's):
initially wanted to produce *all* elements in BBN
— **Fred Hoyle**, with Margaret & Geoffrey Burbidge, **William Fowler** (1950's, 1960's):
stellar nucleosynthesis
[recall Hoyle rejected Lemaître's ideas, preferred continuous creation of hydrogen]
— **P. J. E. Peebles** (1966)
modern theory of primordial nucleosynthesis
— Ya. B. Zel'dovich (1965)
related calculations, not known outside Iron Curtain until much later
— **R. V. Wagoner**, W. A. Fowler & F. Hoyle (1967)
detailed calculations, extended to more nuclides and reactions

For a historical account, see *The First Three Minutes* by Steven Weinberg.