Extra-Galactic Astronomy I (Cosmology)
Project 3: Light element abundances from BBN
Total marks: 30

1 Background

In the Hot Big Bang model of cosmology, the Universe was so hot as very early times that it was NOT possible for protons and neutrons to combine and form different nuclei. The production of nuclei like the Deuterium, Tritium, Helium, and so on was possible only when the temperature was about a billion degrees K. The process is believed to have been completed a few minutes after the Big Bang. In this project, you will solve the relevant nuclear rate equations and compute the abundance of the light elements in the Universe as a function of time.

2 What to do

2.1 Setting up the equations

The solution of the abundance of different species would require knowledge of the radiation temperature T_γ as a function of time. This would be provided to you as a table in the file `temp-vs-t.dat`. You should plot the evolution of this temperature T_γ and understand the behaviour.

In this project, we will focus on the following six nucleons: hydrogen or proton (H or p), neutron (n), deuterium (D), tritium (T), He_3 (3) and He_4 (α). We shall ignore other light elements such as Lithium, Beryllium, etc.

If the (proper) number density of species i is denoted as n_i, then we define the *reduced* number density as

$$Y_i \equiv \frac{n_i}{n_B}, \tag{1}$$

where n_B is the baryon number density. Can you write n_B in terms of the quantity $\Omega_B h^2$ (where Ω_B is defined in terms of the density at the present epoch)?

We will concentrate only on the reactions that are most important. The following weak interactions represent conversion between the proton and neutron

$$\begin{aligned} p + \bar{\nu} &\rightleftharpoons n + e^+ \\ p + e^- &\rightleftharpoons n + \nu \end{aligned} \tag{2}$$

The corresponding reaction rates, denoted by $\lambda(n \to p)$ and $\lambda(p \to n)$ are given as a function of T_γ in the file `lambda-rates.dat`.

In addition, there are a number of electromagnetic reactions which are the following

$$\begin{aligned} n + p &\rightleftharpoons D + \gamma \\ D + D &\rightleftharpoons 3 + n \\ D + D &\rightleftharpoons T + p \\ 3 + n &\rightleftharpoons T + p \\ T + D &\rightleftharpoons \alpha + n \end{aligned} \tag{3}$$

We will denote the rate of the reaction $i + k \to k + l$ as $R(ij \to kl)$.

Given these reactions, the differential equations that determine the evolution of the nuclear species are given by

$$
\begin{aligned}
\dot{Y}_n &= -\lambda(n \to p)Y_n + \lambda(p \to n)Y_p \\
&\quad -Y_n Y_p \ R(np \to D\gamma) + Y_D Y_\gamma \ R(D\gamma \to np) \\
&\quad +Y_D Y_D \ R(DD \to 3n) - Y_3 Y_n \ R(3n \to DD) \\
&\quad -Y_3 Y_n \ R(3n \to Tp) + Y_T Y_p \ R(Tp \to 3n) \\
&\quad +Y_T Y_D \ R(TD \to \alpha n) - Y_\alpha Y_n \ R(\alpha n \to TD) \\
\dot{Y}_D &= Y_n Y_p \ R(np \to D\gamma) - Y_D Y_\gamma \ R(D\gamma \to np) \\
&\quad -2Y_D Y_D \ R(DD \to 3n) + 2Y_3 Y_n \ R(3n \to DD) \\
&\quad -2Y_D Y_D \ R(DD \to Tp) + 2Y_T Y_p \ R(Tp \to DD) \\
&\quad -Y_T Y_D \ R(TD \to \alpha n) + Y_\alpha Y_n \ R(\alpha n \to TD)
\end{aligned} \tag{4}
$$

We have written down the equations for n and D. Write down the rate equations for T, 3 and α. The proton abundance can be obtained from the conservation of mass

$$Y_n + Y_p + 2Y_D + 3Y_T + 3Y_3 + 4Y_\alpha = 1. \tag{5}$$

The fitting forms of the rates can be obtained from Esmailzadeh et al (1991), ApJ, **378**, 504 [see their equation (2.6) and (2.7) and also the Appendix] and Wagoner, Fowler & Hoyle (1967), ApJ, **148**, 3 [see their Table 2]. Make sure you get the units etc right.

2.2 Initial conditions

Evolve the system of differential equations starting from a epoch where $T_\gamma = 0.9 \times 10^{12}$ K. At these epochs, the rates of the reactions are so high that the abundances remain in thermal equilibrium. Hence you can assume $\dot{Y}_i = 0$ at this initial epoch. Then the differential equations become ordinary algebraic equations and you can find the roots. Use some multi-dimensional root finding algorithm to estimate the initial values of Y_i.

2.3 Solution of the equations

Given the initial conditions, you can solve the differential equations using some numerical routine. Note that the equations are quite "stiff", so standard routines may not work. Read about how to solve stiff equations and write an appropriate algorithm to solve them.

2.4 Results

Solve the rate equations for different values of $\Omega_B h^2$. In particular, take $\Omega_B h^2 = 0.01$ to 0.1 in logarithmic steps. Plot the values of $X_\alpha \equiv 4Y_\alpha$ and X_D/X_p as functions of $\Omega_B h^2$. Evolve the system till the two quantities attain saturation, i.e., their values do not change with time any more.

Which of the two quantities is better suited for constraining $\Omega_B h^2$?