January – February 2014

- All the necessary steps must be shown. Include explanations where required.
- The answers should be brief and to the point. Irrelevant and unnecessary statements will be penalized.
- The paper contains a bonus question of 3 marks. In principle, you can score more than 100% in this quiz if you answer enough questions correctly.

The ${}^{0}_{0}$ and ${}^{\alpha}_{\alpha}$ components of the Einstein equations in a perturbed FRW universe (under linear approximation) are given by

$$3\frac{a'}{a}\phi' + \left(k^2 + 3\frac{a'^2}{a^2}\right)\phi = -4\pi G a^2 \bar{\rho}\delta, \qquad \phi'' + 3\frac{a'}{a}\phi' + 2\frac{a''}{a}\phi - \frac{a'^2}{a^2}\phi = 4\pi G a^2 p.$$

Consider the universe to be spatially flat containing only one component of matter having a equation of state $P = w\rho$ with $w \neq -1$.

1. If the equation state holds for the unperturbed as well as perturbed density and pressure, show that $p = w\bar{\rho}\delta$.
2. Use the fact that $\bar{\rho} \propto a^{-3(1+w)}$ to obtain the η-dependence of a. Also obtain the quantities a'/a and a''/a.
3. Eliminate δ and p from the Einstein equations and obtain a second-order differential equation for $\phi(\eta)$.
4. **Bonus:** Solve the differential equation for large scales.

[1+5+4+3 (bonus)]