Astrophysics: Mid-term Examination

HRI Graduate School
August - December 2011

29 September 2011
Duration: 3 hours

- The paper is of 60 marks. Attempt all the questions.
- You are free to consult your class notes during the examination.
- Let me know if you find anything to be unclear or if you think that something is wrong in any of the questions.

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1. A star has a parallax angle of 0.25 arc second and an apparent magnitude 2. What is its absolute magnitude? Given the fact that the Sun has a luminosity 4×10^{33} ergs s^{-1} and has an absolute magnitude of about 5, find the luminosity of the star.

[4]

2. A spherical source of radiation (with radius R) has a uniform intensity I_ν.

(i) Show that the total flux of radiation from the source at a distance $r > R$ from the centre of the source will be

$$F_\nu(r) = \pi I_\nu \left(\frac{R}{r}\right)^2$$

What is the flux $F_\nu(R)$ emergent from the source?

(ii) What is the energy density of radiation $u_\nu(r)$ at a distance r expressed in terms of I_ν?

(iii) What is the pressure $P_\nu(r)$ in terms of I_ν?

(iv) How is P_ν related to u_ν at $r = R$? What happens to the relation when $r \gg R$?

$[5 + 3 + 3 + 2 = 13]$

3. (i) Show that the radiative transfer equation for thermal emission (i.e., material with $S_\nu = B_\nu$) in the Rayleigh-Jeans regime has the form

$$\frac{\mathrm{d}T_b}{\mathrm{d}\tau_\nu} = -T_b + T$$

where T is the temperature of the material (not necessarily uniform).

(ii) Write down the general solution to the equation.

(iii) What happens to the solution when T is uniform throughout the medium?

(iv) What happens when the medium in optically thin? What happens in the optically thick case?

$[3 + 3 + 1 + 2 = 9]$

4. Solve the Lane-Emden equation

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2 \frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right) = -\theta^n; \qquad \theta(0) = 1, \qquad \left.\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right|_{\xi=0} = 0$$

for $n = 0$. Find the form of $\theta(\xi)$ and the value of ξ_1 (defined so that $\theta(\xi_1) = 0$).

[4]

5. Assume that the density in a star varies linearly with radius as

$$\rho(r) = \rho_c \left(1 - \frac{r}{R}\right)$$

where ρ_c is the central density and R is the radius of the star.

(i) How is the mass M of the star related to its radius R?

(ii) Show that the pressure is given by

$$P(r) = \frac{5\pi G \rho_c^2 R^2}{36}\left(1 - \frac{24}{5}\frac{r^2}{R^2} + \frac{28}{5}\frac{r^3}{R^3} - \frac{9}{5}\frac{r^4}{R^4}\right)$$

(iii) Show that the central pressure P_c can be written as

$$P_c = \frac{5}{4\pi}\frac{GM^2}{R^4}$$

(iv) Assuming that the star consists of ideal gas, show that the temperature is given by

$$T(r) = \frac{5\pi G\mu m_p}{36k_B}\rho_c R^2 \left(1 + \frac{r}{R} - \frac{19}{5}\frac{r^2}{R^2} + \frac{9}{5}\frac{r^3}{R^3}\right)$$

[3 + 5 + 2 + 4 = 14]

6. Consider the collision of two neutron stars of equal mass obeying the mass radius relationship $RM^{1/3}$ = constant.

(i) Compute the binding energy (in ergs) which is released in the process, assuming the same form of the mass radius relationship holds for the final object and no mass is lost during the interaction.

(ii) What will be the value of this energy (in ergs) if the neutron stars had mass $1M_\odot$ and radius 10 km?

(iii) What do you expect the final object to be? Give reasons.

[3 + 1 + 2 = 6]

7. Consider two stars of mass m_1 and m_2 in which m_1 transfers mass to m_2 through filling the Roche lobe. This process conserves mass and angular momentum but not energy since the accretion dissipates energy into heat. Assume that the stars are on circular orbit whose radii are also changing with time due to accretion. From conservation of angular momentum and mass, show that

$$\frac{\dot{a}}{a} = 2\dot{m}_1\left(\frac{1}{m_1} - \frac{1}{m_2}\right),$$

where $a = a_1 + a_2$ is the sum of the orbit radii. Hence show that if the lighter star is losing mass the stars gradually separate away while if the heavier star is losing mass the stars come closer catastrophically.

[5]

8. A pulsar is rotating in a way that it completes 716 rotations in a second.

(i) What is the minimum density (in cgs units) of that star?

(ii) Obtain the constraint on the pulsar mass as a function of its radius.

(iii) What is the minimum stellar mass (in units of $M_\odot$) if radius is 12 km?

[2 + 2 + 1 = 5]