Astrophysics: Assignment 5
HRI Graduate School
August - December 2011

26 September 2011
To be returned to the tutor by 21 October 2011

- The deadline for the submission of the solutions of this assignment will be strictly enforced. No marks will be given if the assignment is not returned in time.
- You are free to discuss the solutions with friends, seniors and consult any books. However, you should understand and be clear about every step in the answers. Marks may be reduced if you have not understood what you have written even though the answer is correct.
- Let me or your tutor know if you find anything to be unclear or if you think that something is wrong in any of the questions.

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1. **Dust extinction and optical depth:** Show that the change in apparent magnitude of a star due to extinction is given by 1.086 τ_λ, where τ_λ is the optical depth of the absorbing material along the line of sight.

   [2]

2. **Interstellar extinction:** If we assume for simplicity that dust particles are spherical and each has a radius a, then the geometrical cross section that a particle presents to a passing photon is just πa^2. We define the dimensionless **extinction coefficient** Q to be
$$Q = \frac{\sigma}{\pi a^2}$$
   where σ is the scattering cross section of the dust grain.

   Suppose that the interstellar medium in our Galaxy contains dust grains with uniform number density 10^{-6} m^{-3}, all with the same radius 10^{-7} m, and extinction efficiency $Q = 0.5$ at wavelength λ_0. Find the extinction in magnitudes at wavelength λ_0 for a star at a distance 1 kpc from the Earth.

   [2]

3. **Relation between collisional coefficients assuming thermodynamic equilibrium:** Consider a two-level atom where there are transitions between the states induced radiatively as well as collisionally. Assuming that the system is in thermodynamic equilibrium, derive the relations between the collisional coefficients.

   [4]

4. **Measuring cloud properties through brightness temperature measurements:** Consider a HI cloud in the ISM observed using the 21 cm transition line.

   (i) Show that, for 21 cm transition, the equation of radiative transfer can be written in terms of brightness temperature as
$$\frac{\mathrm{d}T_b}{\mathrm{d}\tau_\nu} = T_S - T_b$$

   (ii) Write down the general solution to the equation. Show that, for an isothermal cloud, the solution becomes
$$T_b = T_b(0)\ \mathrm{e}^{-\tau_\nu} + T_S\left(1 - \mathrm{e}^{-\tau_\nu}\right)$$

   (iii) Now imagine an observation when there is a background source behind the cloud. The quantity that is measured is the net absorption $T_{\mathrm{on}} = T_b(0) - T_b$. Write the expression for T_{on}.

   (iv) Next consider another observation of the same cloud with no background source. Then the emission from the cloud, T_{off}, can be measured. Write the expression for T_{off}.

   (v) Given T_{on} and T_{off}, show that one can estimate the optical depth and the spin temperature as
$$\begin{aligned} \tau_\nu &= -\ln\left[1 - \frac{T_{\mathrm{on}} + T_{\mathrm{off}}}{T_b(0)}\right] \\ T_S &= T_b(0)\frac{T_{\mathrm{off}}}{T_{\mathrm{on}} + T_{\mathrm{off}}} \end{aligned}$$

   [2 + 2 + 1 + 1 + 2]

5. **Information from Doppler broadening:** Two lines (Hα and OIII) were detected from a gaseous region with the wavelength $\lambda = (6563, 3729)$ Å and line widths $\Delta\lambda = (0.374, 0.154)$ Å. Assume that the line widths are due to a combination of turbulent velocities and thermal Doppler broadening (as can be deduced from their Gaussian shape). What is the temperature and turbulent velocity distribution of the line-emitting region?

[4]

6. **Limiting form of Voigt function:** Show that in the limit $\beta^2 \ll 1$, the Voigt function has the form

$$V(\alpha, \beta) = \mathrm{e}^{\alpha^2 - \beta^2} \; \mathrm{erfc}(\alpha)$$

What happens if we now take the limit $\alpha \to 0$?

[4]

7. **Neutral fraction in HII regions around stars:** Consider a OB star of radius 6×10^{11} cm and effective temperature $T_* = 4.2 \times 10^4$ K. Estimate the neutral fraction of hydrogen at a distance 5 pc from the star. Assume that the ISM consists only of hydrogen. Take the particle density and the temperature in the HII region to be 10 cm^{-3} and 2×10^4 K, respectively. You may ignore the frequency dependence of the ionization cross section in your calculation. Is this justified?

[5]

8. **Evolution of the radius of the HII region:** Show that the evolution of the radius $R(t)$ of a HII region around a star is of the form

$$R(t) = r_s \left(1 - \mathrm{e}^{-t/t_{\mathrm{rec}}}\right)^{1/3}$$

where r_s is the Stromgren radius and t_{rec} is the recombination time-scale. What is the form of t_{rec} and what would its value for ISM? Assume the luminosity of the star to be constant and the temperature of the HII region to be uniform.

[3]

9. **Electromagnetic waves in cold plasmas:** Consider an electron fluid with number density n_e. Let $\mathbf{E}$ be the self-consistent internal average electric field and $\mathbf{B}$ be the external magnetic field. Let an electromagnetic wave pass through this plasma.

(i) Write down the force equation in the approximation that the speed of the electron fluid is non-relativistic. What is the current $\mathbf{j}$ arising from the electron fluid? Also write down the Maxwell's equations which describe the time evolution of $\mathbf{E}$ and $\mathbf{B}$

(ii) Assume an electromagnetic field propagating along the z-direction. Then one can write

$$\mathbf{E} = \mathbf{E}(\omega)\; \mathrm{e}^{\mathrm{i}(kz - \omega t)}; \quad \mathbf{B} = \mathbf{B}(\omega)\; \mathrm{e}^{\mathrm{i}(kz - \omega t)}$$

What is $\mathbf{v}$ in terms of $\mathbf{E}(\omega)$?

(iii) Use the expansions of $\mathbf{E}$ and $\mathbf{B}$ in the Maxwell's equations and obtain the dispersion relation:

$$\omega^2 = \omega_p^2 + k^2 c^2$$

What is the form of ω_p?

(iv) What happens when $\omega < \omega_p$?

$[2 + 1 + 4 + 1]$