ASTRONOMY AND ASTROPHYSICS-I: Assignment 4

Department of Physics
Savitribai Phule Pune University
July – December 2019

21 November 2019
To be returned on 03 December 2019 (leave with Hemant Lokhande or Reena Shrikumar)

- The deadline for the submission of the solutions of this assignment will be strictly enforced. No marks will be given if the assignment is not returned in time.
- You are free to discuss the solutions with friends, seniors and consult any books.
- Let me know if you find anything to be unclear or if you think that something is wrong in any of the questions.

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1. **Mass determination using binary stars:**

   (i) Consider two stars in a circular orbit about their centre of mass. Assuming that the orbital plane is perpendicular to the observer's line of sight (i.e., the orbital plane is parallel to the plane of the sky), show that the mass ratio is given by

   $$\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$$

   where α_1, α_2 are the angles subtended by the radii of the circular orbits of the two stars respectively. We can use this to measure the mass ratios for visual binaries.

   (ii) Show that the orbital period is given by

   $$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

   where $a = a_1 + a_2$ is the sum of the radii.

   (iii) Suppose we have measured α_1, α_2 and P, then what else do we need to measure so as to estimate the masses of each star? Write the explicit expressions for both m_1 and m_2 in terms of the measured quantities.

   [3+4+6]

2. **Importance of general relativistic effects in white dwarfs:**

   (i) For typical masses and sizes of white dwarfs, estimate whether general relativistic effects could be important.

   (ii) For stars having masses $\sim 1 M_\odot$, estimate the radius and density when general relativistic effects could be important, say, $GM/c^2R \sim 0.1$.

   [2+3]

3. **Superluminal motion:** Consider a source of radiation that moves with a speed v at an angle θ to the line of sight. We know that the apparent transverse speed of the source perpendicular to the line of sight is

   $$v_\perp = \frac{v \sin\theta}{1 - (v/c)\cos\theta}.$$

   (i) Show that for a given value of v, the apparent transverse speed $v_\perp$ is maximum when $\theta = \cos^{-1}(v/c)$, with the maximum value being γv where $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$.

   (ii) Show that in the limit $v \to c$, the maximum occurs for $\theta \approx \gamma^{-1}$.

   (iii) Show that the necessary condition for superluminal motion to occur is $v > c/\sqrt{2}$.

   [7+2+3]