Spectral line analysis

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Spectral line analysis

GMRT P-band observations: 16 MHz split into 128 channels (115 km/s); maximize the continuum sensitivity

(Basu+12)
Spectral line analysis

VLA B, C and D array L-band observations: 1.56 MHz split into 128 channels (2.6 km/s); optimise frequency coverage and spectral resolution.

M81 Group: DSS/VLA image: Image courtesy of NRAO/AUI (Adler+96)
The objective of these lectures is to plan, observe and image spectral-line data.
Two-element interferometer

The component in which the signals are combined is called **Correlator**. The correlator is also responsible for providing *multi-frequency channel* output.

- $\vec{s}$ unit vector representing the direction in which two antennas are pointing
- $\vec{b}$ interferometer baseline
- $\tau_g$ geometric delay
  \[ = \frac{\vec{b} \cdot \vec{s}}{c} \]
- $\varphi = 2\pi \nu \tau_g$
Multi-frequency channel output:

- Essential for spectroscopy: resolve emission and absorption lines.
- Also most appropriate for RFI mitigation, and
- Making continuum images
Two-element interferometer
(Monochromatic signal)

\[ R = \langle V_1(t)V_2(t) \rangle \]
\[ V_1(t) = V_1 \cos(\omega t - \tau_g) \]
\[ V_2(t) = V_2 \cos(\omega t) \]

\[ R = \langle V_1V_2[\cos(\omega \tau_g) + \cos(2\omega t - \tau_g)] \rangle \]

\[ R_c = V_1V_2 \cos(\tau_g) = P \cos \left( \frac{2\pi \bar{b} \cdot \bar{s}}{\lambda} \right) \]

\( V_1 V_2 \) related to the power received.
Two-element interferometer
(Monochromatic signal, extended incoherent source)

Connection between — source brightness distribution — and interferometer output.

But only partial: interferometry is to measure both even and odd parts of the intensity i.e. both amplitude and phase.

\[ I(\bar{s}) : \text{Wm}^{-2}\text{Hz}^{-1}\text{Sr}^{-1} \]
\[ dr = A(\bar{s})I(\bar{s})\Delta\nu d\Omega \cos(2\pi\nu\tau_g) \]

For an incoherent source,
\[ r = \Delta\nu \int A(\bar{s})I(\bar{s})\cos 2\pi\nu\frac{\bar{b} \cdot \bar{s}}{c} d\Omega \]
Two-element interferometer
(Complex correlator)

The two outputs can be regarded as measuring the ‘real’ and ‘imaginary’ parts of the complex visibility. In addition, there is an improvement of $\sqrt{2}$ in the single-to-noise ratio.
Two-element interferometer
(Finite bandwidth)

Thus, the fringes are modulated by a sinc envelope.

\[
R_c = P \int_{\nu_0 - \Delta \nu/2}^{\nu_0 + \Delta \nu/2} \cos(\omega \tau_g) d\nu
\]

\[
= P \frac{\sin(\pi \Delta \nu \tau_g)}{\pi \Delta \nu \tau_g} \cos(2\pi \nu_0 \tau_g)
\]

Fringe pattern now is a combination of fringe patterns corresponding to a range of frequencies.
Correlator: filter banks

Conceptually, the simplest way to do spectroscopy is with analog filter banks.

Using many analog filters to make many narrow frequency channels.

Drawbacks: many filter banks are required; analog filters are costly and unstable.
Correlator: digital filter banks

Fortunately, signals are digitised —> use digital filter banks (FFT processor)

\[ V(u, v, \nu) = \int_{-\infty}^{\infty} V(u, v, \tau)e^{-2\pi i \nu \tau} d\tau \]

The same technique, which allows Fourier imaging, is used!
Basic idea

Need to channelize only a finite BW with finite resolution! Thus, can be determined using a finite number of lags

- Consider visibilities sampled at some fixed lag interval, $\Delta \tau$. One half of its inverse, $1/(2\Delta \tau)$, defines total frequency BW of the observation.
Any continuous band-limited signal can be reconstructed if sampled at the Nyquist rate.

**Sampling rate = \(1/2f\)**

Higher frequency components will be aliased to the lower frequencies in the sampled band.
If adequately sampled convolution with sinc provides exact interpolation of the original function from the samples.

(Thompson, Moran & Swenson)
Under-Sampling: Aliasing

If aliasing is avoided convolution with \textit{sinc} provides exact interpolation of the original function from the samples.

(Thompson, Moran & Swenson)
Basic idea

Need to channelize only a finite BW with finite resolution!

Thus, can be determined using a finite number of lags

- Consider visibilities sampled at some fixed lag interval, $\Delta \tau$. One half of its inverse, $1/(2\Delta \tau)$, defines total frequency BW of the observation.
- To determine $V(u,v,\nu)$ for $N$ channels need to measure $2N$ quantities, at least. For example, measure all lags between $-N\Delta \tau$ and $+(N-1)\Delta \tau$, a total $2N$ lags (a lag of zero included).

One half of the inverse of maximum lag, defines the spectral resolution, i.e. $1/(2N\Delta \tau) = BW/N$.
- XF or FX correlator.
XF and FX correlator

- **XF**
  - Take the window from the second data stream, slide it across, multiply and add at each instance or lag: result is visibility as a function of lag. FT this to get spectrum. Accumulate over integration time to improve signal-to-noise ratio.

- **FX**
  - Select a window in the two data streams, FFT each of them to get complex signal in frequency (-ve frequencies are discarded). Multiply the output from two data streams. Accumulate to improve signal-to-noise ratio.
Correlator response

- **XF**
  - The finite window in time domain has the effect of convolving frequency spectrum with the sinc function. Thus, any spectral feature within the spectrum is broadened by sinc function, and depending on its profile affects the neighbouring pixels.
  - Width of sinc function at FWHM = 1.2Δν/N (spectral resolution).
  - Hanning smoothing used to suppress the sidelobes (22% → 3%; but degrades the resolution).

- **FX**
  - Each of the station spectra already have sinc response. The cross power spectrum has sinc² response.
  - sinc² is narrower and has lower sidelobes!

Advantageous in suppressing effects of RFI.
Observing spectral lines

(1) Line frequency and reference frame
(2) Frequency set-up: know your correlation
(3) Calibration
Observing spectral lines (1)
Line frequency and reference frame

Determine observing frequency from:
(1) rest frequency of the line, and (2) velocity and rest frame of the object.
Observing spectral lines (1)
Line frequency and reference frame

<table>
<thead>
<tr>
<th>Rest Frame</th>
<th>Corrected for</th>
<th>Amplitude of Correction, km s(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geocentric</td>
<td>Earth rotation</td>
<td>0.5</td>
</tr>
<tr>
<td>Earth-Moon Barycentric</td>
<td>Effect of the Moon on the Earth</td>
<td>0.013</td>
</tr>
<tr>
<td>Heliocentric</td>
<td>Earth’s orbital motion</td>
<td>30</td>
</tr>
<tr>
<td>Solar System Barycentric</td>
<td>Effect of planets on the Sun</td>
<td>0.012</td>
</tr>
<tr>
<td>Local Standard of Rest</td>
<td>Solar motion</td>
<td>20</td>
</tr>
<tr>
<td>Galactocentric</td>
<td>Milky Way rotation</td>
<td>230</td>
</tr>
<tr>
<td>Local Group Barycentric</td>
<td>Milky Way motion</td>
<td>(\sim 100)</td>
</tr>
<tr>
<td>Virgo-centric</td>
<td>Local Group motion</td>
<td>(\sim 300)</td>
</tr>
<tr>
<td>Microwave Background</td>
<td>Local Supercluster motion</td>
<td>(\sim 600)</td>
</tr>
</tbody>
</table>

*Table 11–1. Velocity Rest Frames Derived from the Topocentric System*
Observing spectral lines (1)

Line frequency and reference frame

\( \nu = \nu_0 \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c} \cos \theta} \)

\[ \frac{v}{c} = \frac{\nu_0^2 - \nu^2}{\nu_0^2 + \nu^2} \quad \text{(For, } \theta = \pi) \]

\((v \ll c)\)

\[ \frac{\nu_{\text{radio}}}{c} = \frac{\nu_0 - \nu}{\nu_0} \quad \frac{\nu_{\text{optical}}}{c} = \frac{\nu_0 - \nu}{\nu} \]
Observing spectral lines (2)
(Know your correlator)

◆ Choose optimal observing set-up
◆ Frequency coverage vs spectral resolution (data rate)
◆ Continuum sensitivity
◆ Multiple spectral windows
◆ Beware of RFI environment
Observing spectral lines (3)
(Calibration)

- **Primary or flux calibrator**
- **Secondary or phase calibrator (close to the target)**
- **Also, bandpass calibrator (ideally point source with no spectral features)**
- **Calibrators and target to be observed with same spectral set-up.**
- **But consider the case of Galactic HI observations**
- **Observe the bandpass calibrator long enough (?)**
Calibrating bandpass

Gain of antenna (or baseline) as a function of frequency

- Bandpass is not flat, constant
  Need to be calibrated with regular observations of bandpass calibrator.
- Variations due to:
  - Front-end system: filters and amplifiers do not have a flat response.
  - Back-end filters: introduce frequency dependence
  - IF transmission system
  - RFI

Fortunately, most of these effects change slowly with time, such that observations of a bandpass calibrator every few hours are generally adequate.
Calibrating bandpass

Splitting time and frequency dependence of gain *(almost)*

\[ G_{ij}(\nu,t) = g_{ij}(t)b_{ij}(\nu,t) \]

Bandpass variations associated with individual antennas, therefore solve for antenna based gains
Calibrating bandpass: RFI

Gains and calibrated data, power tool to identify problems. Narrow-band RFI in a few baselines can appear as spikes in BPass.

Loop over calibration and flagging to determine best possible (stable) bandpasses.
Calibrating bandpass: RFI

Gains and calibrated data, power tool to identify problems. Narrow-band RFI appears as spikes in BPass.
Calibrating bandpass: errors

- Variation in BPass over time: interpolate over solutions in time.
- Source spectral index and frequency structure
  - Need a frequency dependent model of the calibrator
  - Noise on longer baselines may be higher
  - Determine which data to use for calibration!
- All of the above also important for continuum imaging: can lead to amplitude errors in map that cannot be fixed by self-calibration!
Continuum subtraction

Why do it?

◆ Easier to analyse line signal in the absence of continuum.
◆ Can skip deconvolution

Can be done in image and visibility domain: pros/cons
Continuum subtraction (1)

**IMAVG**

Subtract the dirty image made by combining all the line-free channels.

- Ignores the variation in \((u,v)\) coverage and beam across the band i.e. introduces errors due to chromatic aberration.
- Doesn’t take into account varying source properties across the band.
Continuum subtraction (2)

UVSUB

Fourier transform of the continuum model is subtracted in the visibility domain.

- Only method which can remove continuum over large areas of the sky.
- Also takes care of chromatic aberration.
- Accuracy comes from the quality of the continuum model. Any errors in the model propagate into data cube as systematic errors.
- Computationally expansive.
Continuum subtraction (3)

**UVLIN**

Low-order polynomial is fit to the line-free channels (baseline-based) to subtract the continuum.

- Computationally cheap and easy to use. No deconvolution involved.
- Deals with residual gain errors.
- Automatically corrects for spectral index.
- Can also be used for baseline-based spectral flagging.
- Produces a good continuum dataset.

But works well only over small field-of-view: the linear approximation to fit real and imaginary parts breaks down far away from the phase centre.

**Use a combination of UVSUB and UVLIN: always examine data.**
**Also see IMLIN.**
Doppler correction

Shifts due to heliocentric motion

RFI

Observed frequency (MHz)
Imaging spectral line cube

Similar to deconvolution of continuum maps, but need to deal with faint emission varying across the channels.

Importance of appropriate masking to recover all the emission and restore correctly without distorting the noise properties.
Imaging analysis - absorption towards a point source

4 MHz BW; 512 channels
~2 km/s resolution
Io = ~ 250 mJy
rms ~ 1.2 mJy/beam

<350K

<table>
<thead>
<tr>
<th>$z_{\text{abs}}$</th>
<th>FWHM</th>
<th>$\tau_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.312074</td>
<td>4±1</td>
<td>0.059±0.010</td>
</tr>
<tr>
<td>0.312058</td>
<td>13±2</td>
<td>0.035±0.008</td>
</tr>
<tr>
<td>0.311992</td>
<td>8±2</td>
<td>0.031±0.004</td>
</tr>
</tbody>
</table>

Column density, FWHM, Kinetic temperature, ......
Imaging analysis

Rotation curves, Gas morphology, kinematics and dynamics ..........

Walter et al. 2008
Summary

• Planning spectral line observations: correlation, frequency set-up
• Spectral line observations and analysis: observing frequency, calibration, continuum subtraction and doppler tracking.
• Imaging and analysing spectral line cubes.

References and further reading

• Bracewell: The Fourier Transform and its applications.
• Thompson, Moran & Swenson: Interferometry and Synthesis in Radio Astronomy.
• Synthesis Imaging in Radio Astronomy II: the NRAO lecture series.