- Correlators

Astronomical Techniques II : Lecture 7

Ruta Kale

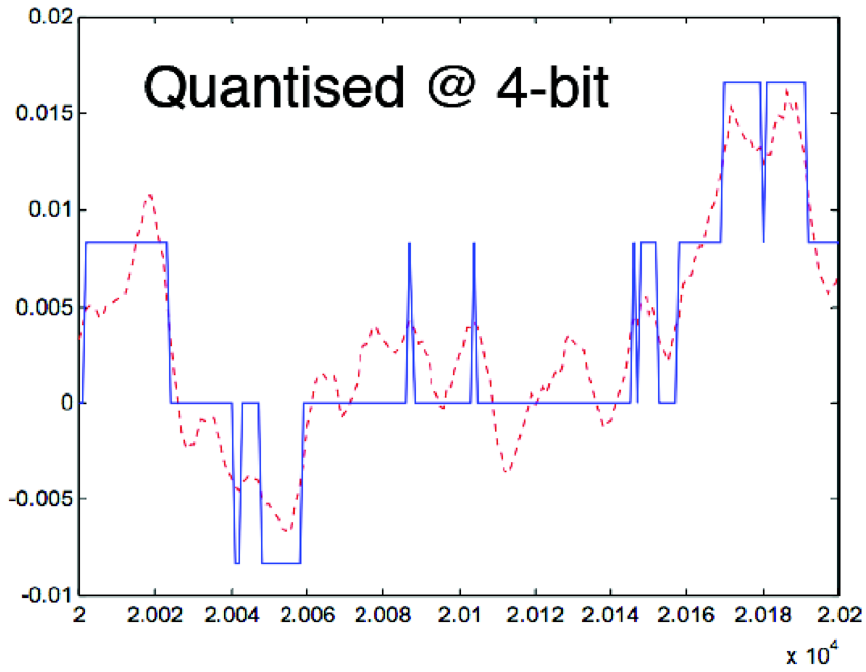
- Low Frequency Radio Astronomy (Chp. 8, 9)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy
- Synthesis imaging in radio astronomy II, Chp. 4
- Interferometry and synthesis in radio astronomy (Chp. 8)
- Talk by Adam Deller (https://nmt.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=42804ec9-3b6c-4b40-8978-a920010eb3fa)

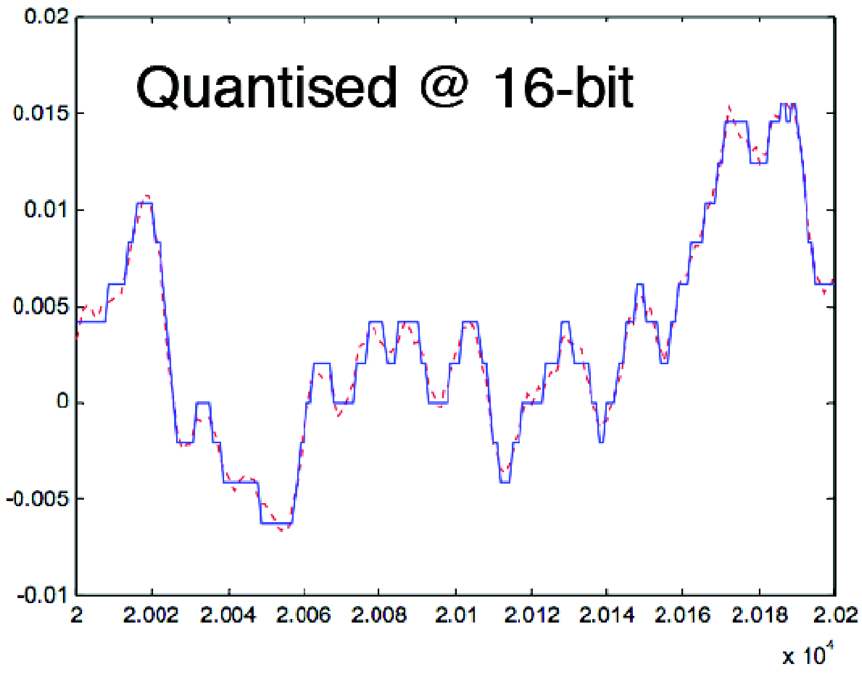
Before getting into correlators

- Sampling quantization
- Digital delays
- Discrete cross correlation and power spectral density (Van Vleck correction)

Sampling quantization

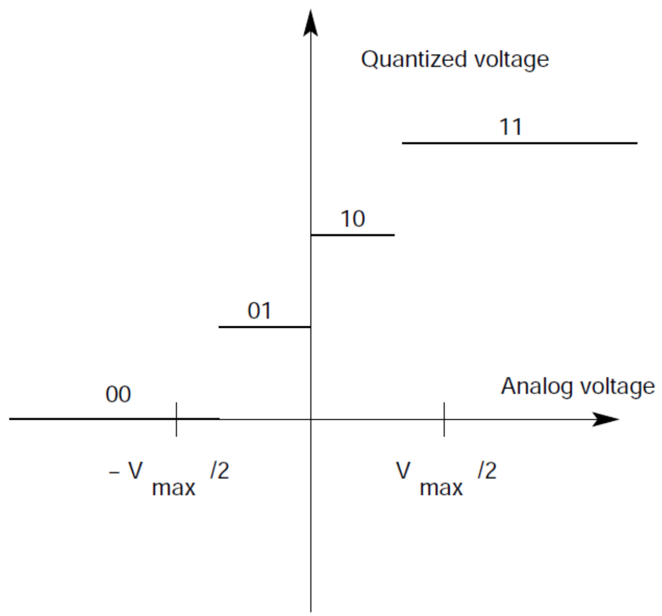
- Digital systems represent values over a limited number of bits.
- Quantized values are an integer multiple of the quantization step, q.
- Leads to distortion of signal: *quantization noise*.

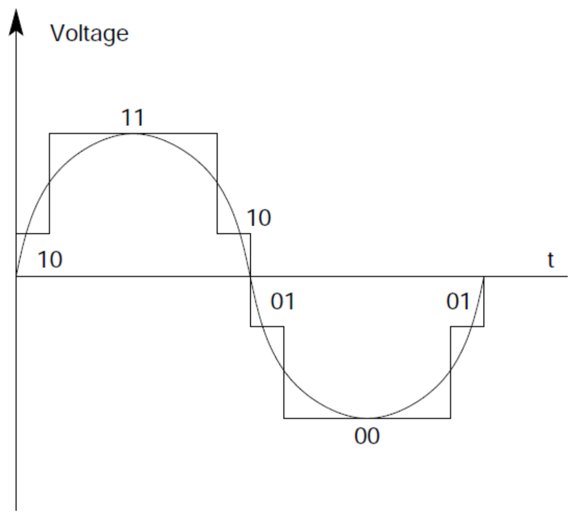




Figures: J. Roy

Sampling quantization





$$e(t) = s(t) - s_q(t)$$

Difference of the original and the quantized signal.

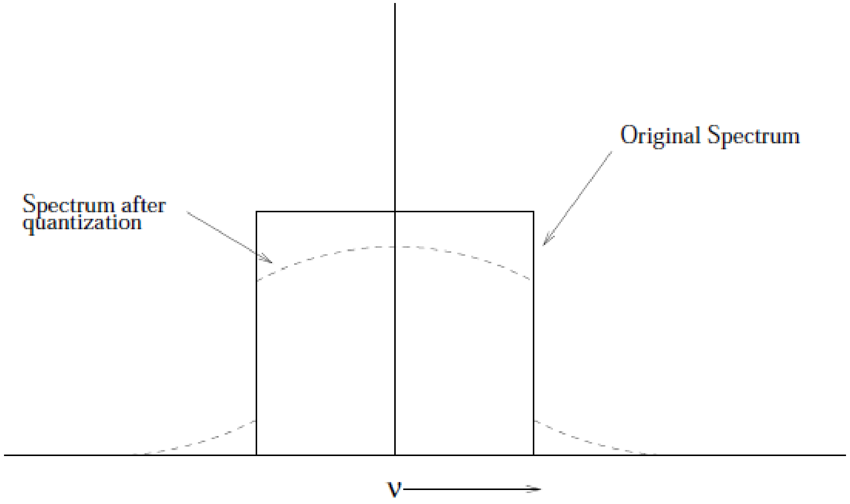
Quantization noise

$$e(t) = s(t) - s_q(t)$$

The variance of quantization noise under conditions of uniform quantization can be approximated to $q^2/12$.

The spectrum of a Nyquist sampled signal after quantization, extends beyond the bandwidth.

Oversampling can reduce the aliased power.



Dynamic range

Quantizer operates over a limited range of input voltage amplitude: referred to a the dynamic range.

E. g. For quantization with M bits the largest value represented is 2^M -1 for binary representation.

Amplitudes exceeding the range get clipped.

The minimum change in the signal that can be expressed is limited to the quantization step q.

Digital delay

Let s(t) be a signal sampled with sampling frequency f_s. Then the digitized signal with a delay of m samples,

$$s(n-m)$$

Corresponds to a delay of m x $1/f_s$

Delay only at the level of integer multiple of the inverse of the sampling frequency can be corrected in this setup.

Any delay smaller than this will remain uncompensated. Such delay produces a phase gradient in the FT.

The fractional delay is removed by introducing phase gradients.

Discrete correlation and power spectral density

Cross-correlation of two signals s_1 and s_2

$$R_c(\tau) = \ < s_1(t) s_2(t+\tau) >$$

In practice the estimator is:

$$m \times 1/f_s$$

$$R(m) = \frac{1}{N} \sum_{n=0}^{N-1} s_1(n) s_2(n+m) \ \ 0 \leq m \leq M$$

m denotes the samples by which the signal s_2 is delayed and M denotes the maximum delay.

R(m) is a random variable whose expectation value converges to

$$R_c(\tau = \tfrac{m}{f_s}) \qquad\qquad N \rightarrow \infty$$

The correlation estimated using digitized samples deviates from that with infinite amplitude precision.

The relation between the true correlation and that measured can be written as:

$$\hat{R}_c(m/f_s) = \mathbf{F}(\hat{R}(m))$$

Normalized correlation functions – normlization to square root of zero lag autocorrelations of s_1 and s_2. *F is the correction function.*

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For one-bit quantization this correction has the form:

“Van Vleck correction”

$$\hat{R}_c(m/f_s) = \sin(\frac{\pi}{2}\hat{R}(m))$$

Non-linear correction – needs to be applied before any other operation on the cross correlation.

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For derivation see : Chp. 8 of Interferometry and Synth. Imaging in RA

Cross correlators

Lag correlators (XF):
Cross-correlation followed by Fourier transformation.

FX correlators:
Fourier transformation followed by cross-correlation.

Digital implementation

$$\begin{aligned} r_R(\tau_g) &= |\mathcal{V}| \cos(2\pi\nu\tau_g - 2\pi\nu_{BB}\tau_i + \Phi_{\mathcal{V}}) \\ &= |\mathcal{V}| \cos(2\pi\nu_{LO}\tau_g - 2\pi\nu_{BB}\Delta\tau_i + \Phi_{\mathcal{V}}) \end{aligned}$$

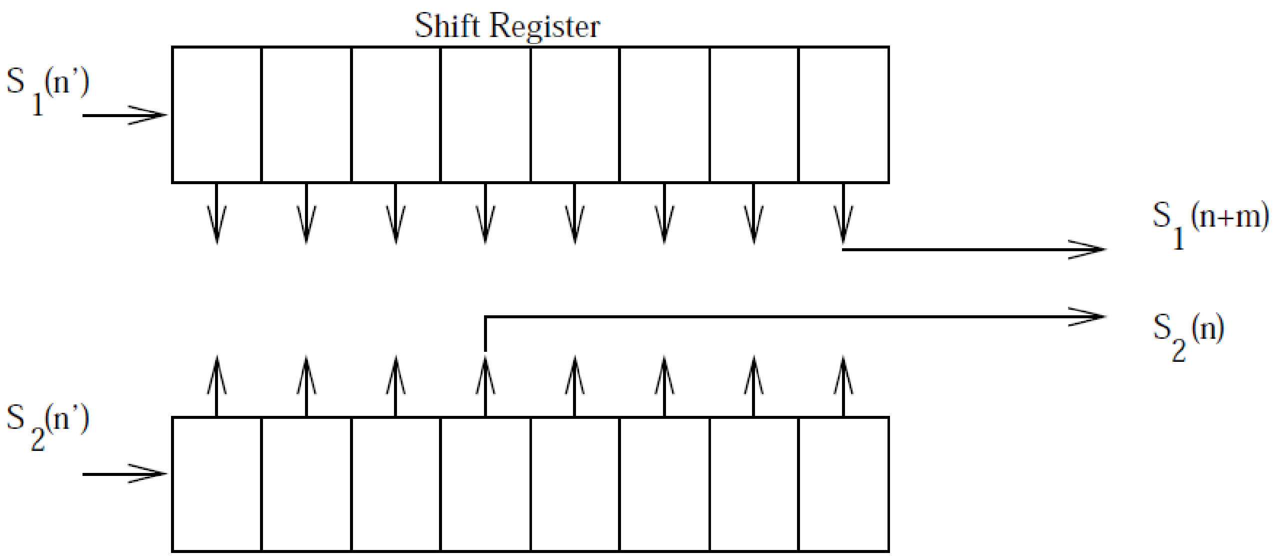
Delays:

Delay compensation in baseband as opposed to in the RF

Finite precision of delay compensation

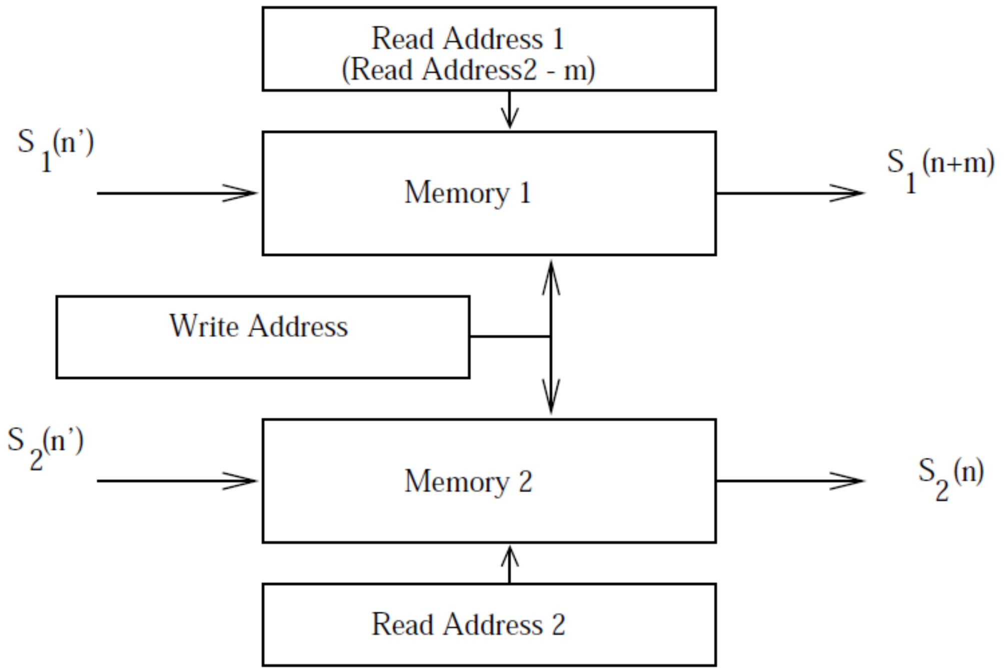
Both need dynamic compensation.

Delay compensation using shift registers



Delay implementation using shift registers

Delay compensation using memory



Delay implementation using Memory

Offset between read pointer and write pointer adjusted to achieve delay compensation.

Fractional delays

Introducing delays in the sampling clocks.

Or introducing a phase gradient after FT.

Fringe stopping is done by changing the phase of the LO such that,

$$2\pi\nu_{LO}\tau_g - \phi_{LO} = 0$$

Achieved digitally by multiplying the sampled time series with the factor:

$$e^{-j\phi_{LO}}$$

Fractional delays

$$\phi_{LO}(t) = 2\pi\nu_{LO}\tau_g = 2\pi\nu_{LO}\frac{b\sin(\Omega t)}{c}$$

The fringe. Omega is the angular rotation speed of earth.

$$\phi_{LO}(t) = \phi_{LO}(t_0) + 2\pi\nu_{LO}\frac{b\Omega\cos(\Omega t_0)}{c}\Delta t$$

For a short time interval this is the approximation.

$\phi_{LO}(t)$ is the phase of an oscillator with frequency $\nu_{LO}\frac{b\Omega\cos(\Omega t_0)}{c}$

After a short time intervals this has to be updated: digital implementation of this is called as the number controlled oscillator (NCO).

Complex correlator

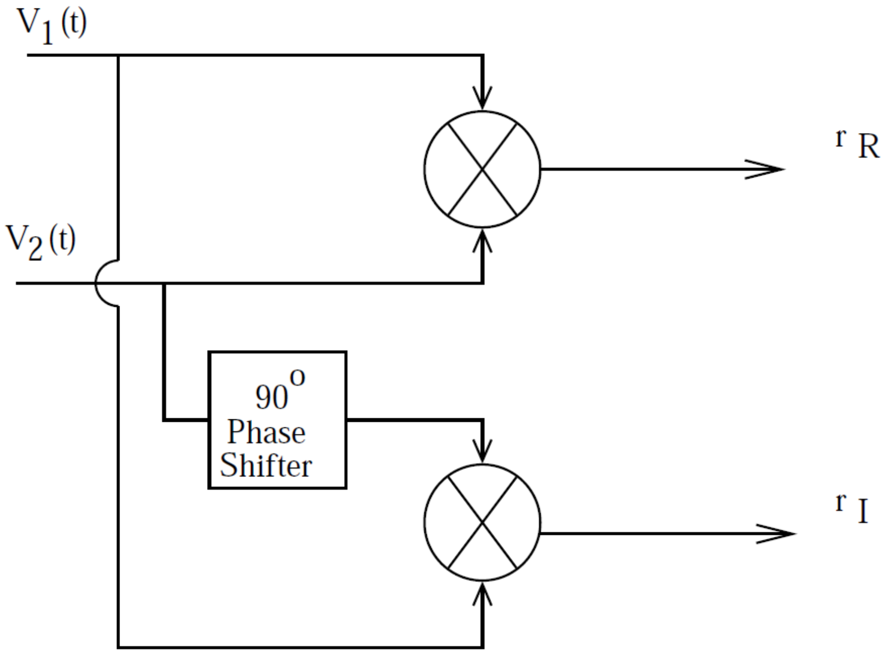
$$r_R = |\mathcal{V}|\cos(\Phi_{\mathcal{V}})$$

$$r_I(\tau_g) = |\mathcal{V}|\cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}} + \pi/2)$$

After compensating the delay,

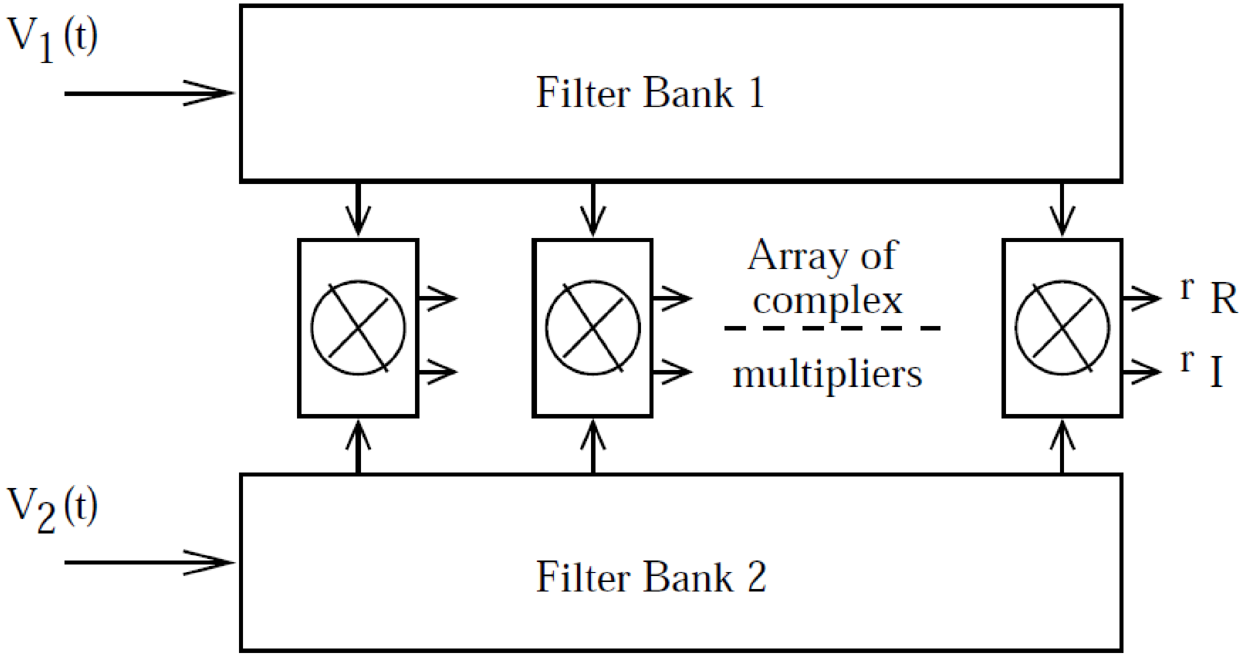
$$\begin{aligned} r_I &= |\mathcal{V}|\cos(\Phi_{\mathcal{V}} + \pi/2) \\ &= |\mathcal{V}|\sin(\Phi_{\mathcal{V}}) \end{aligned}$$

$$\begin{aligned} |\mathcal{V}| &= \sqrt{r_R^2 + r_I^2} \\ \Phi_{\mathcal{V}} &= \tan^{-1}\left(\frac{r_I}{r_R}\right) \end{aligned}$$



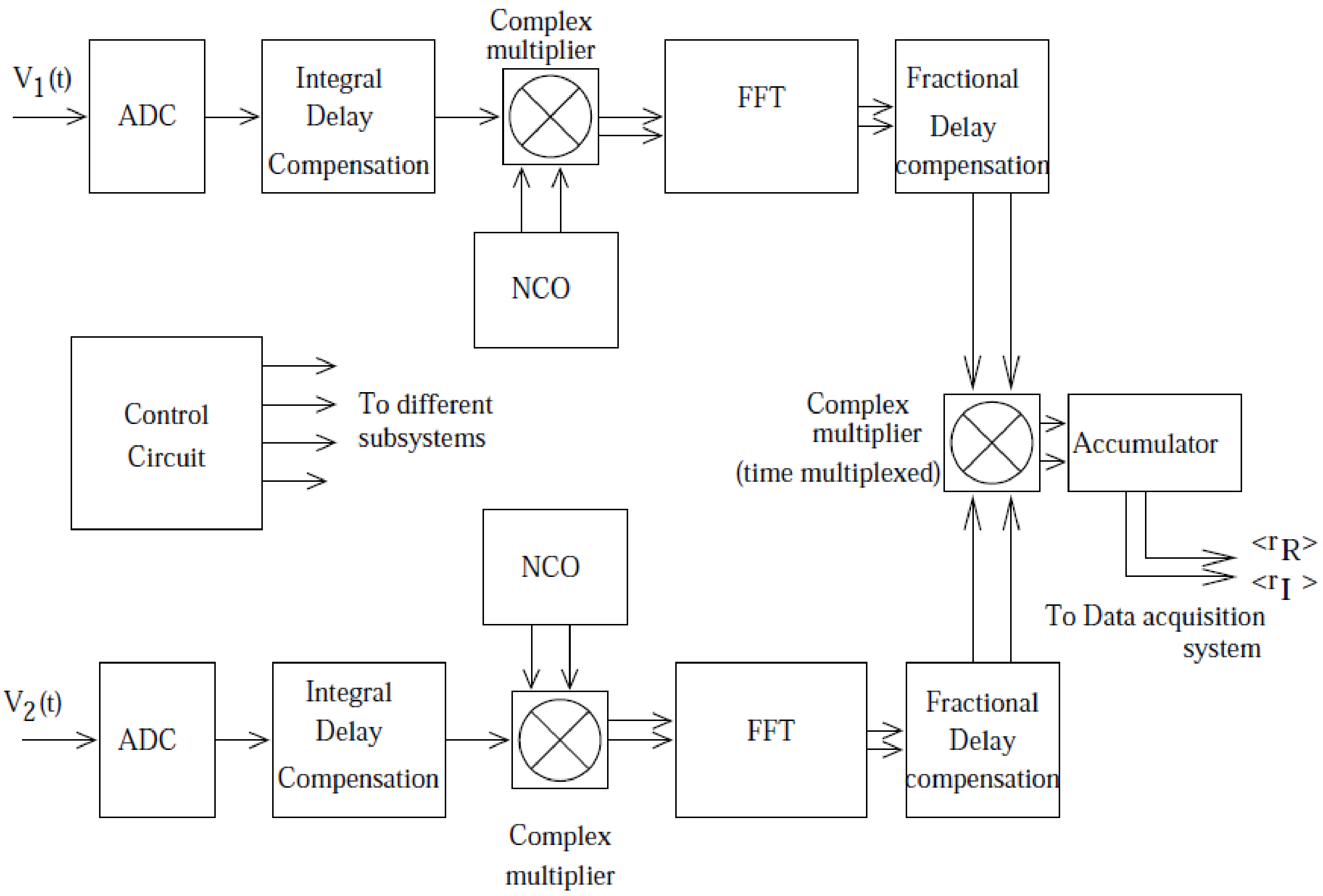
$$\mathcal{V} = r_R + j r_I$$

FX correlator (schematic)

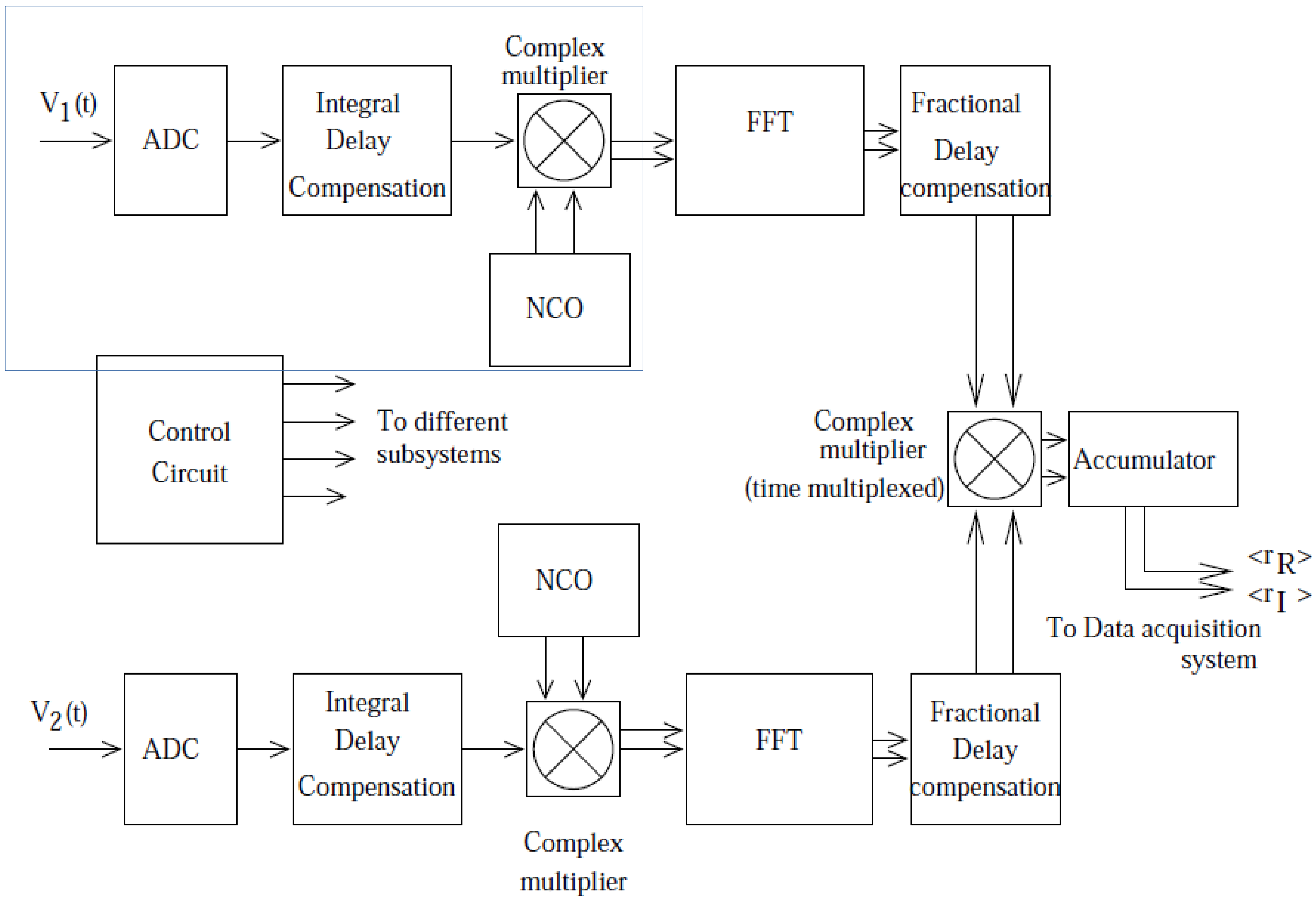


The band limited signal is decomposed into spectral components using a filter bank and each filter output is cross correlated with the corresponding one from the other using a complex multiplier.

FX correlator: implementation



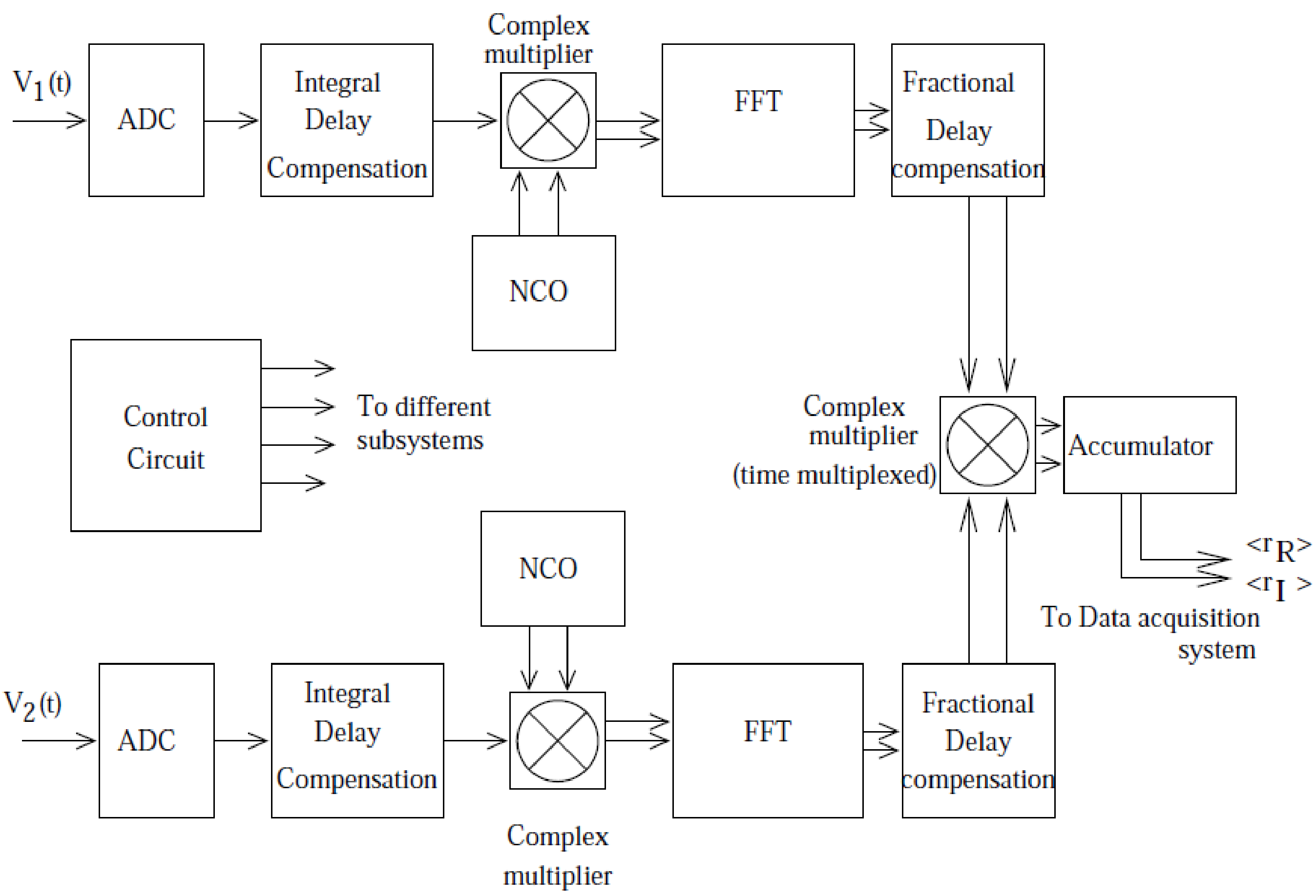
FX correlator: implementation



- Signal is digitized.
- Integral delays are compensated for.
- These then multiplied with NCO output for fringe stopping.

NCO: Number controlled oscillator

FX correlator: implementation



- Signal is digitized.
- Integral delays are compensated for.
- These then multiplied with NCO output for fringe stopping.
- Then passed through FFT block to realise a filter bank.
- Phase gradients applied for fractional delay compensation.
- Then multiplied with the similar output of second antenna.

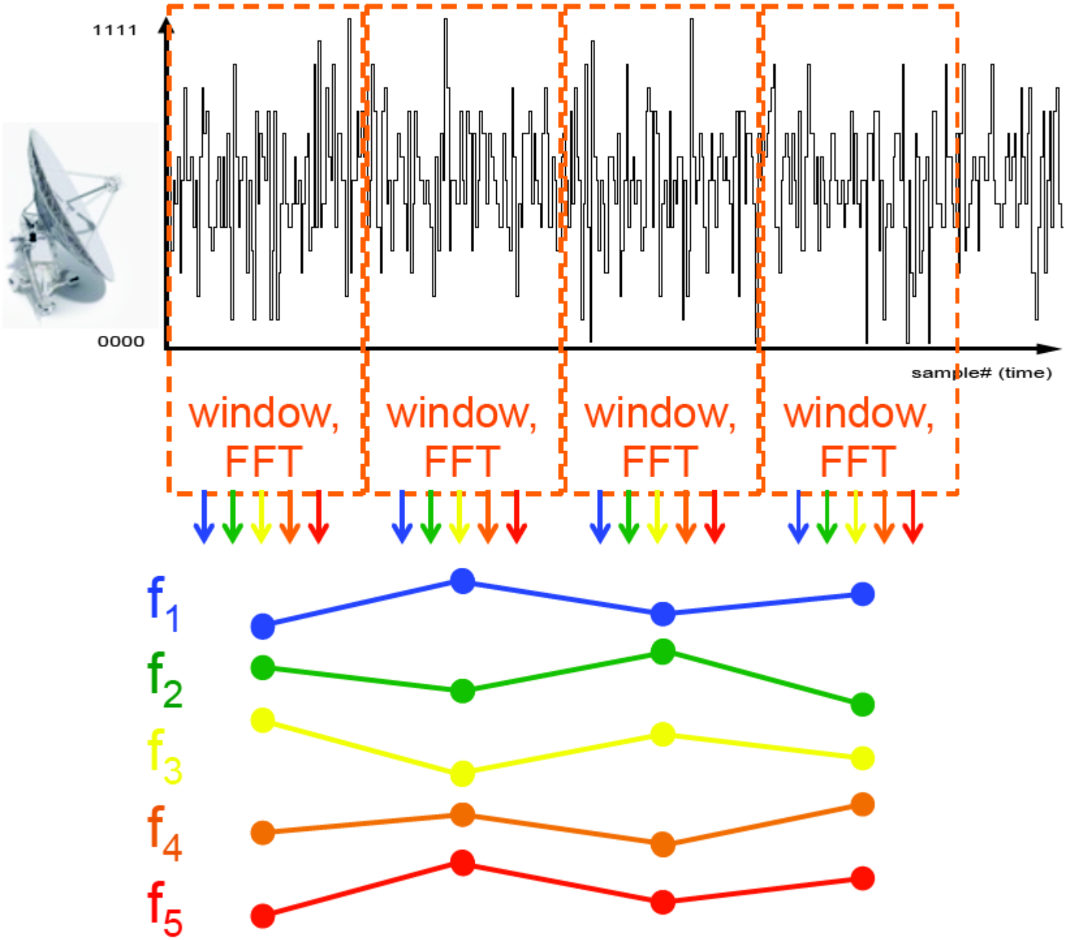


Figure credit: Adam Deller

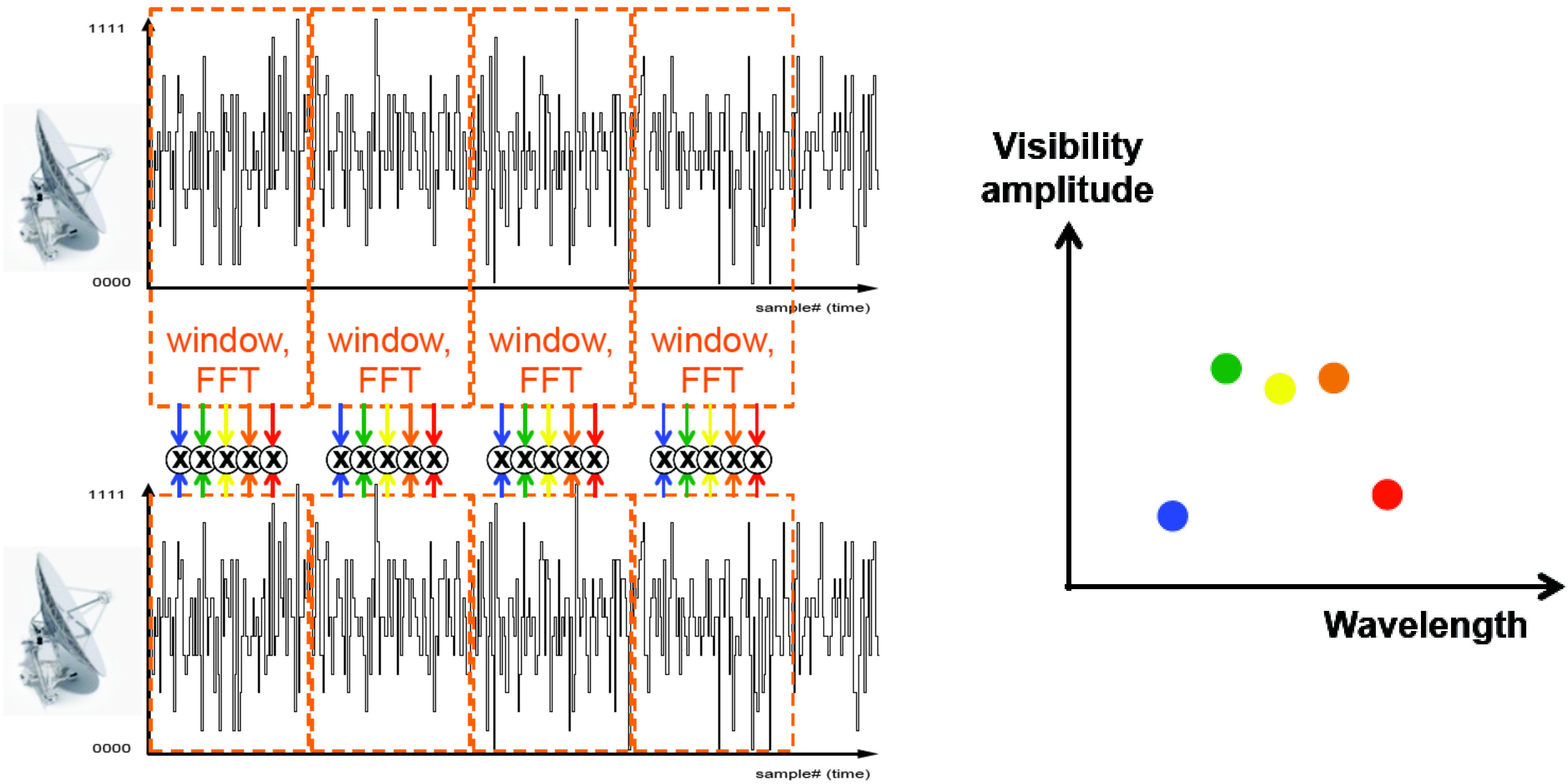
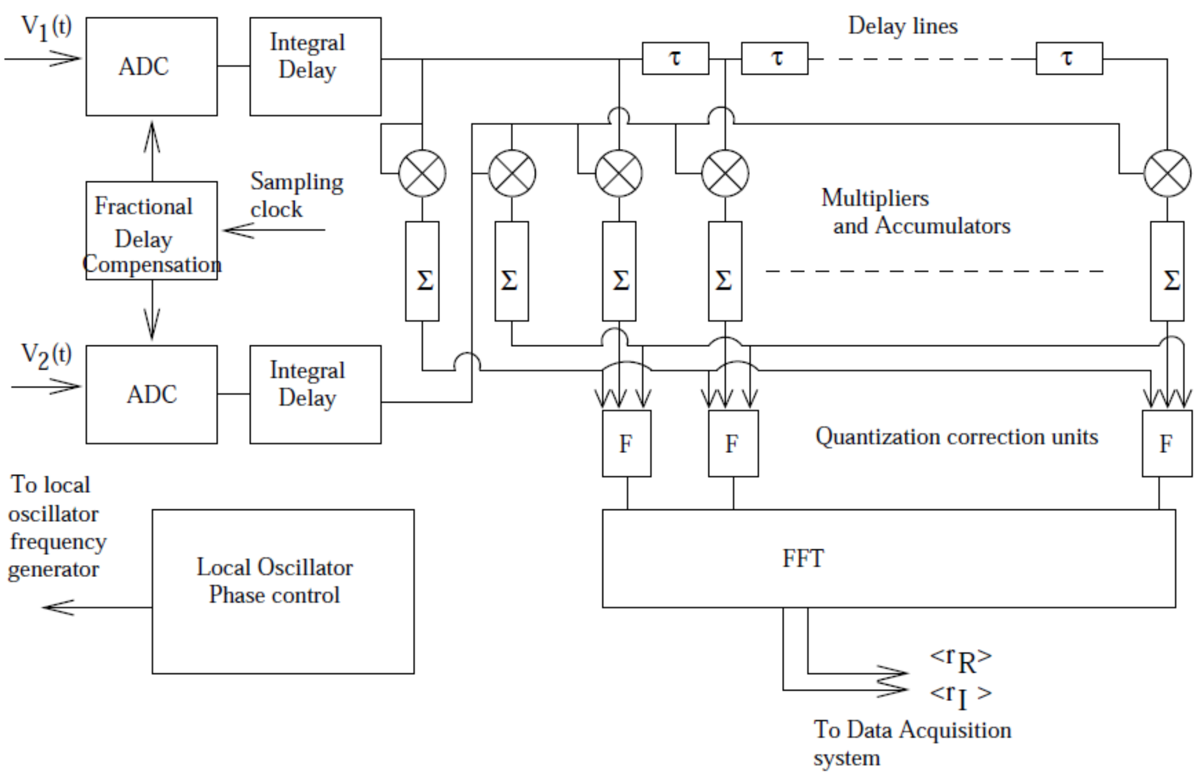


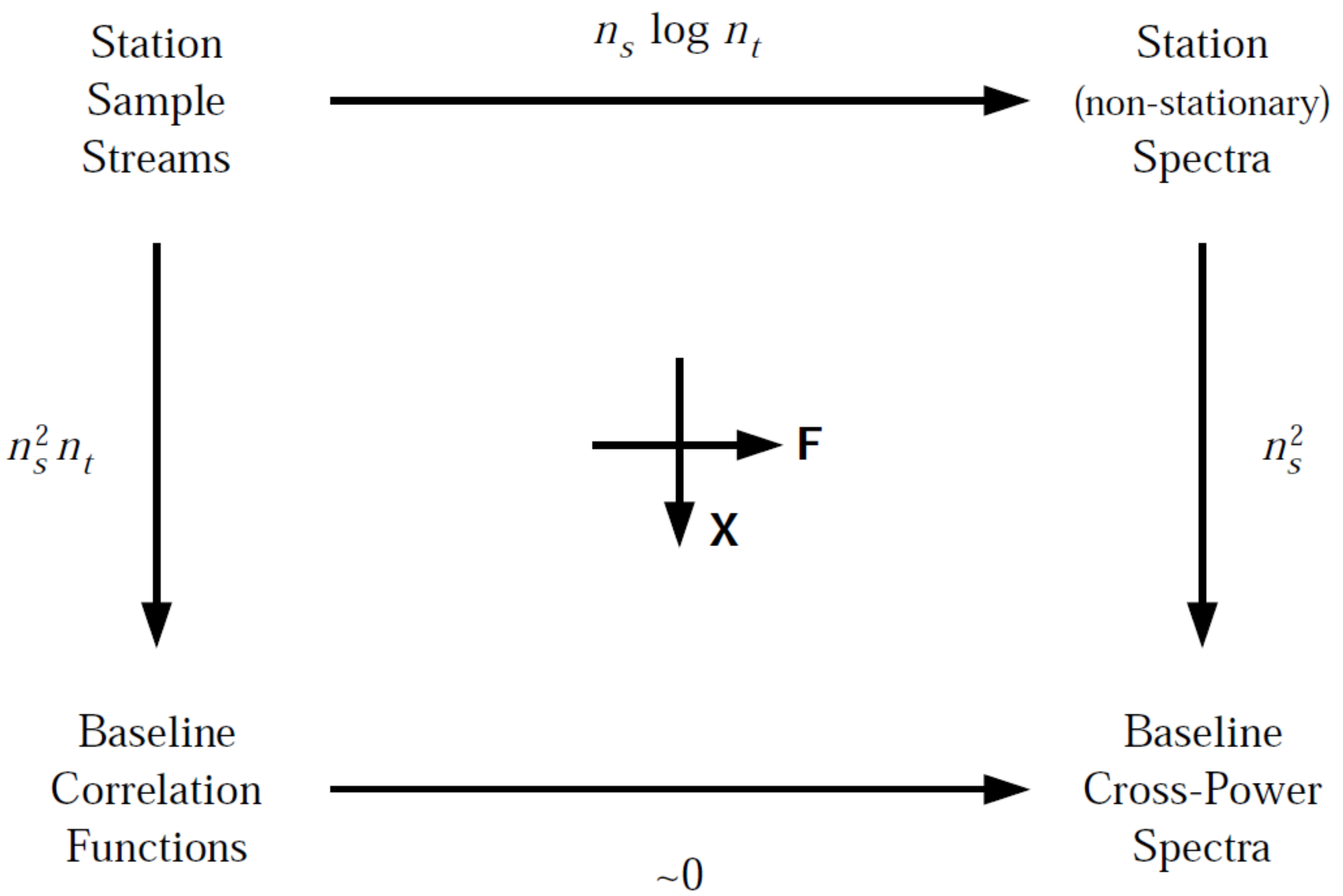
Figure credit: Adam Deller

XF correlator



- Signal is digitized.
- Integral delays are compensated for.
- Fractional delays corrected using sampling clock.
- Multipliers and delay lines.
- Quantization correction applied to normalied correlations.
- Cross correlation spectrum is obtained using DFT.

FX and XF data processing paths



n_s antennas/stations and n_t samples

Examples

GMRT, ALMA have an FX correlator.

VLA, IRAM have an XF correlator

IRAM: Institut de Radio Astronomie
Millimetrique

Coherent and incoherently phased arrays

- Phasing an array : Combine signals from the elements with proper delay and phase adjustment so that the beam can be steered in the chosen direction.

- Coherent addition: Add the voltages and then the sum is put through a square-law detector to produce proportional to the power in the summed signal. The sensitivity is n-times the single element. But the beam is narrow – corresponding to that of the array. Good for point source sensitivity but covers small area in the sky.

- Incoherent addition: One can first square the voltages and then combine – called incoherent addition as the phase information is lost. The beam corresponds to that of the single element. Useful to observe large portions of the sky – e.g. search for pulsars.