- A two element interferometer
- Aperture synthesis

Astronomical Techniques II : **Lecture 5**

Ruta Kale

Low Frequency Radio Astronomy (Chp. 4)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy

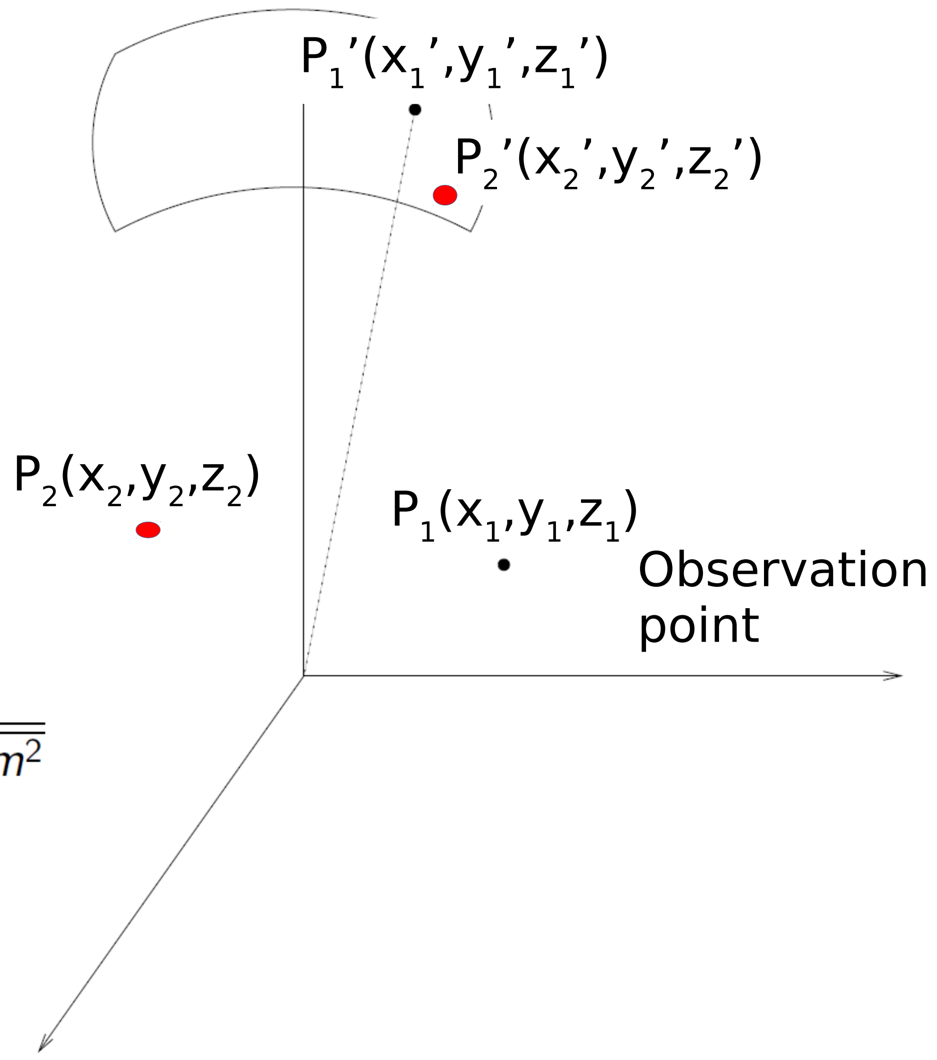
Synthesis imaging in radio astronomy II, Chp 2

Interferometry and synthesis in radio astronomy (Chp 2)

Introduction

According to van Cittert-Zernicke theorem: the source brightness distribution can be derived if one can measure the mutual coherence function of the electric fields.

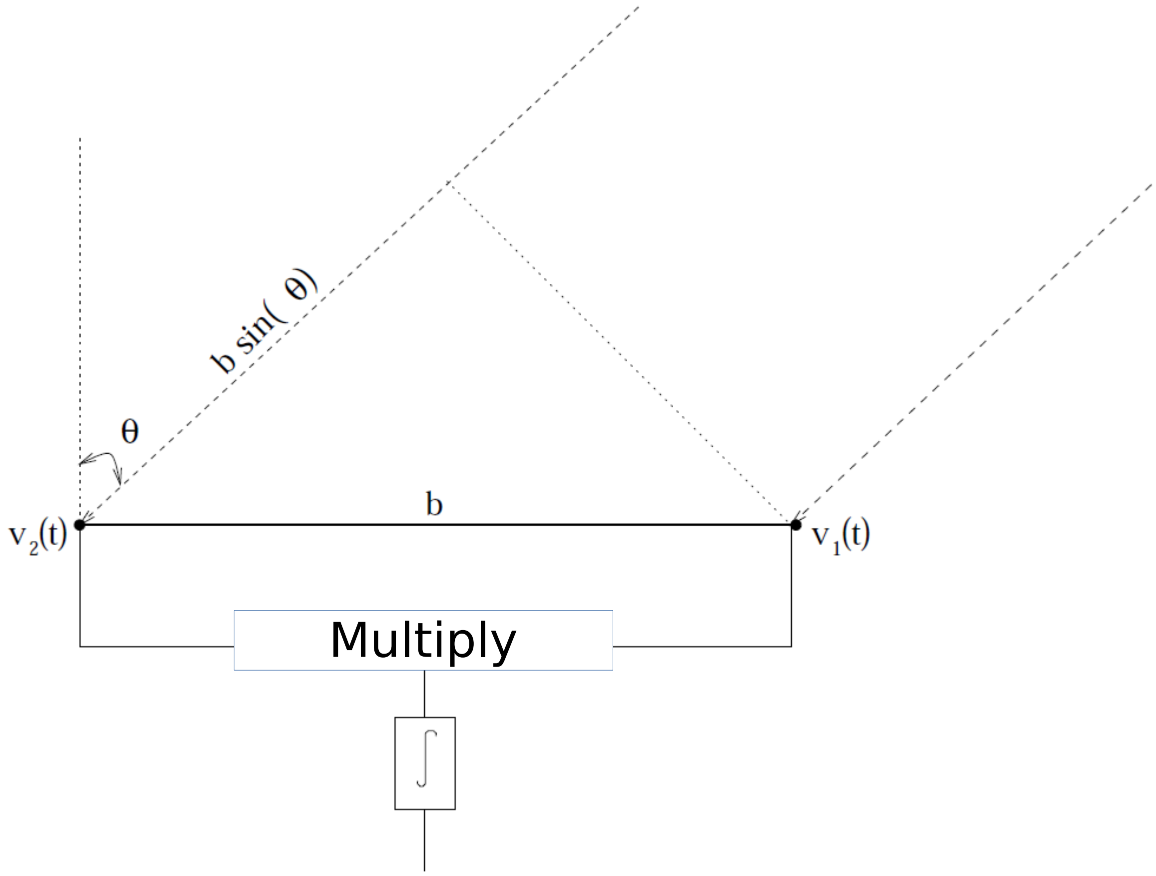
$$V(u,v,w) = \int I(l,m) e^{-i2\pi[lu+mv+nw]} \frac{dldm}{\sqrt{1-l^2-m^2}}$$



Two element interferometer: multiplying

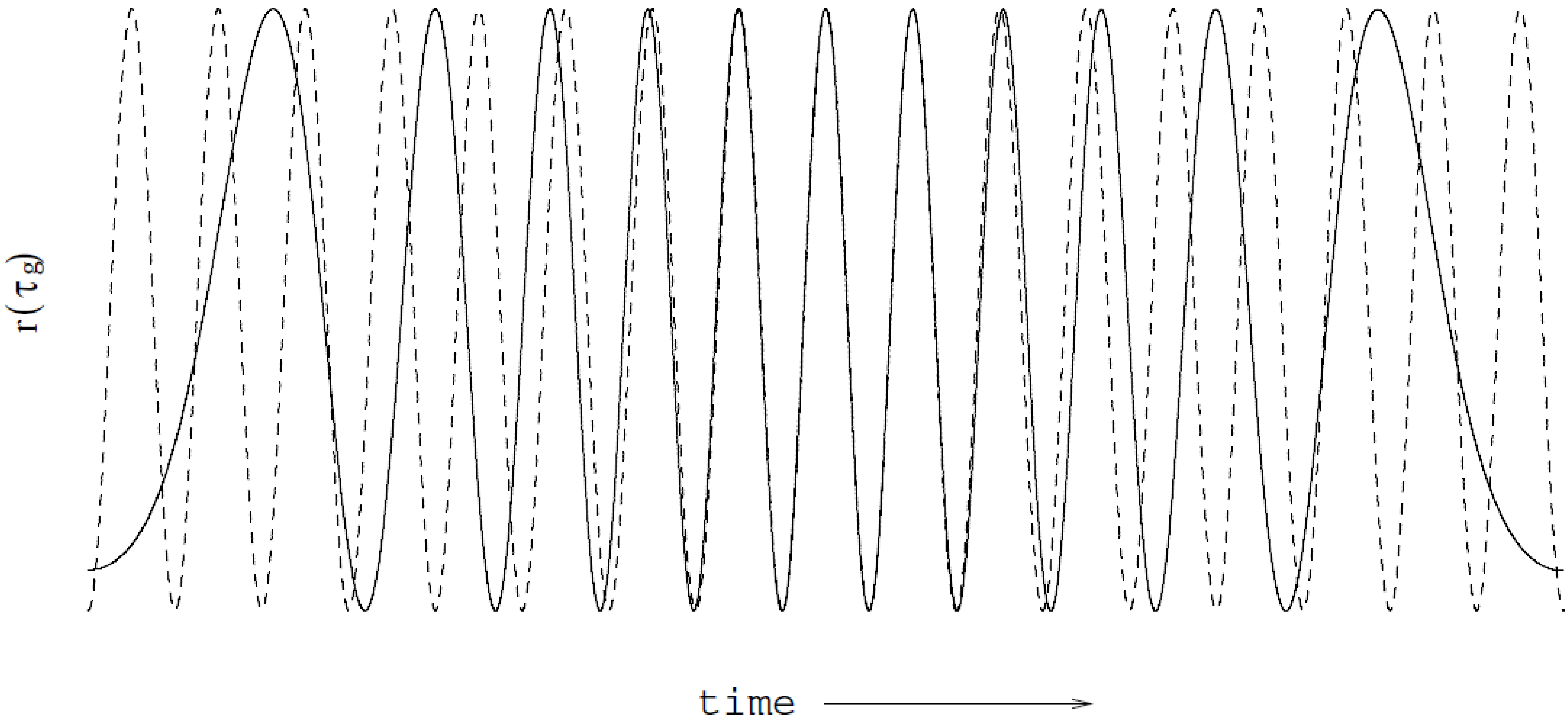
Geometric delay

$\tau_g = b\ sin(\theta)/c$



$$r(\tau_g) \quad = \quad |\mathcal{V}| \cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}})$$ where $$\mathcal{V} = |\mathcal{V}| e^{-i\Phi_{\mathcal{V}}}$$

Fringe

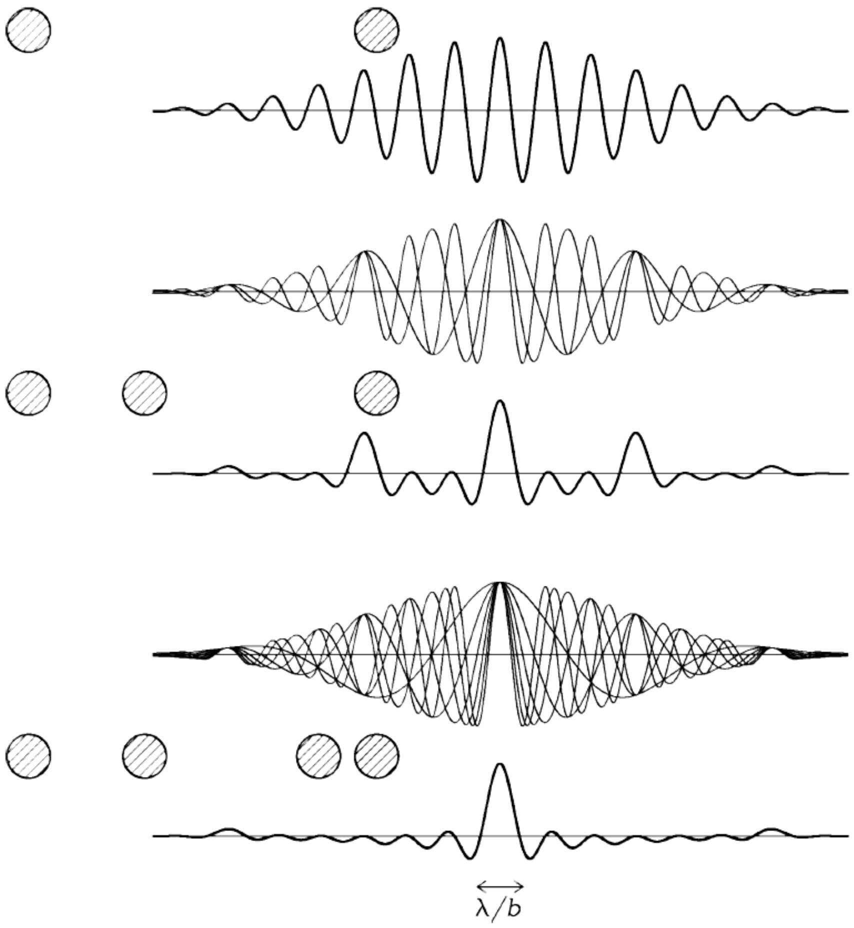


Solid line: observed output
Dashed line: pure sinusoid with frequency equal to the maximum instantaneous frequency of the fringe.

Response of baselines

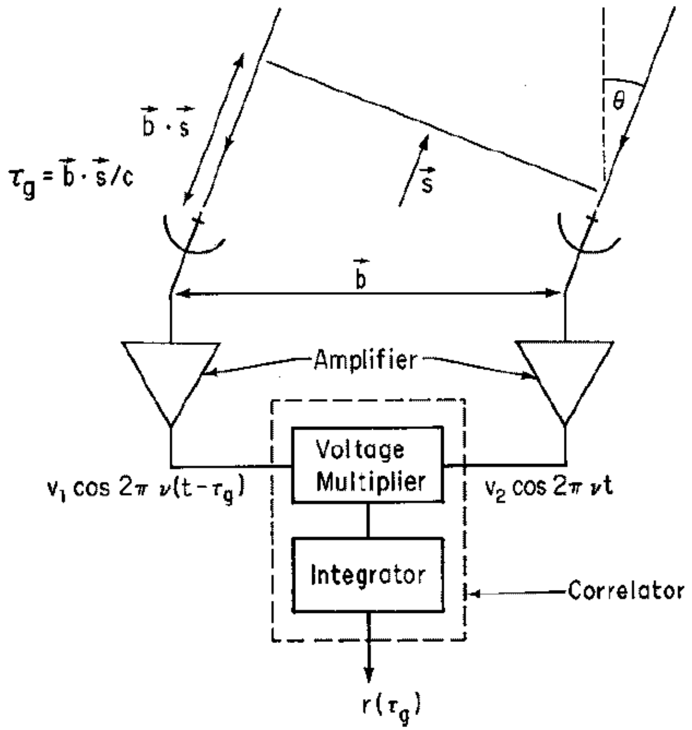
Instantaneous point source responses of interferometers with projected length *b* and two, three and four antennas distributed as shown.

Broad envelope:
Primary attenuation of the response of the individual antennas.

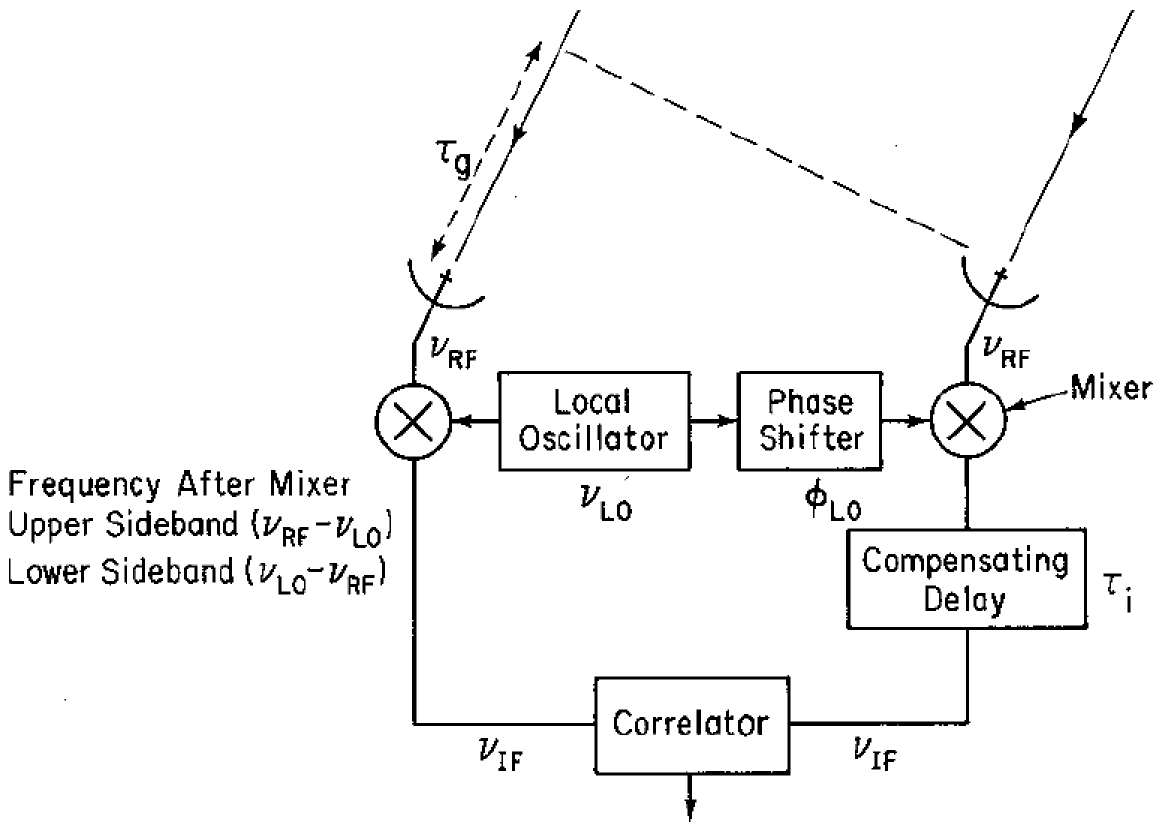


Two element interferometer

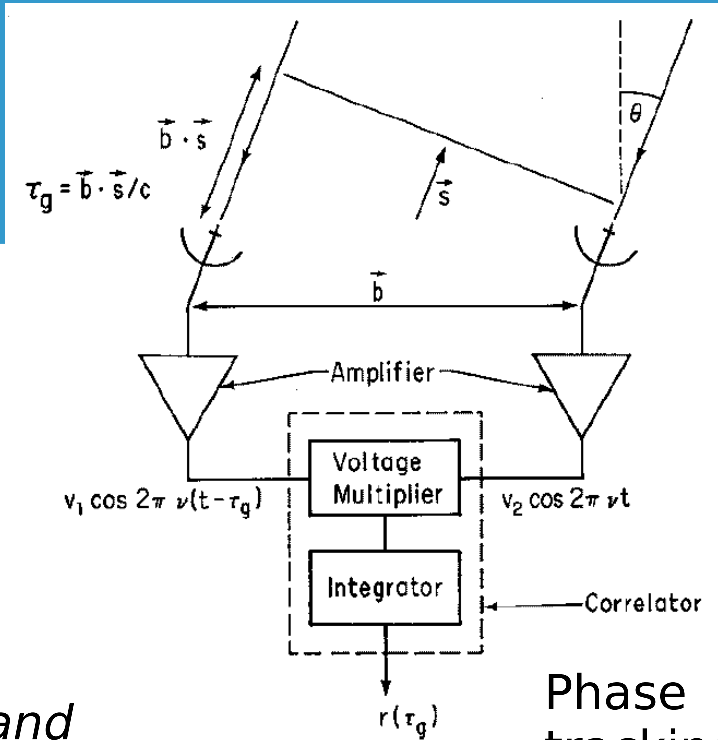
Geometric delay: varies as Earth rotates



Interferometer in practice including compensation for the geometric delay: delay correction



Two element interferometer



$$\langle V_1(t)V_2(t)\rangle$$

$$V_1(t) = v_1 \cos 2\pi\nu(t-\tau_g)$$

$$V_2(t) = v_2 \cos 2\pi\nu t$$

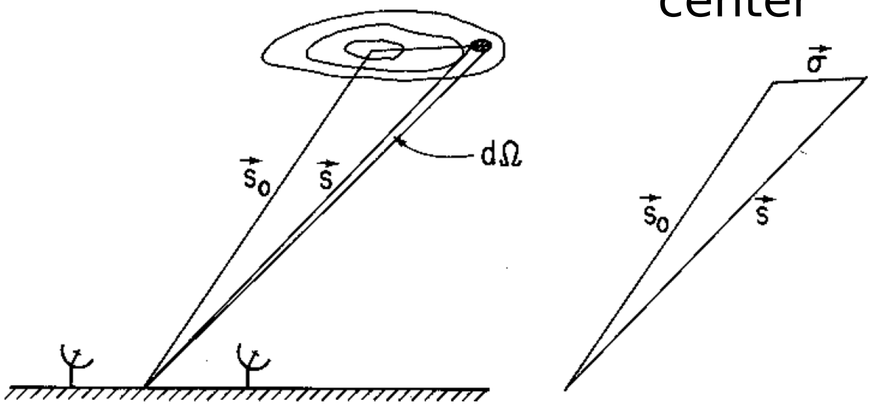
$$r(\tau_g) = v_1 v_2 \cos 2\pi\nu\tau_g\,.$$

Ignored the variation of A and I with bandwidth.

Phase tracking center

$$dr = A(\mathbf{s})I(\mathbf{s})\Delta\nu\, d\Omega \cos 2\pi\nu\tau_g$$

$$r = \Delta\nu \int_S A(\mathbf{s})I(\mathbf{s}) \cos \frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}}{c}\, d\Omega$$



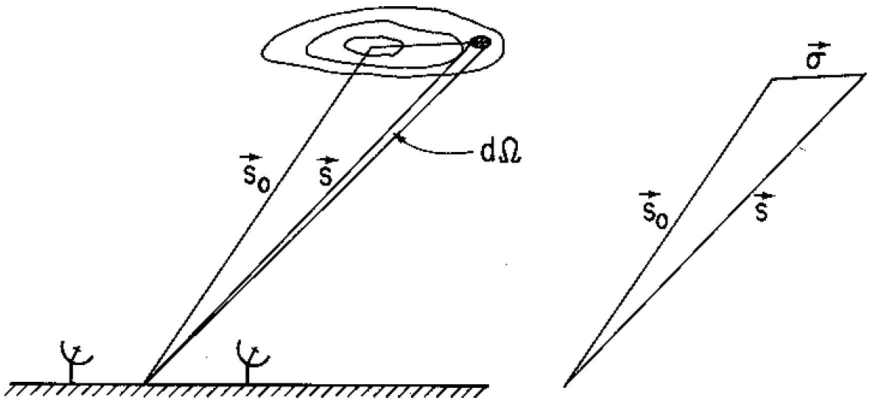
$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$

Assumed that both the antennas have the same effective collecting area.

Source is in the far field and is spatially incoherent.

$$r = \Delta\nu \int_S A(\mathbf{s}) I(\mathbf{s}) \cos \frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}}{c}\, d\Omega$$

$$\mathbf{s} = \mathbf{s}_0 + \boldsymbol{\sigma}$$



$$\begin{aligned} r \;=\; & \Delta\nu \cos\left(\frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}_0}{c}\right) \int_S A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \cos \frac{2\pi\nu\, \mathbf{b}\cdot\boldsymbol{\sigma}}{c}\, d\Omega \\ - \; & \Delta\nu \sin\left(\frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}_0}{c}\right) \int_S A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \sin \frac{2\pi\nu\, \mathbf{b}\cdot\boldsymbol{\sigma}}{c}\, d\Omega \end{aligned}$$

Visibility

$$
\begin{aligned}
r &= \Delta\nu \cos\left(\frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}_0}{c}\right) \int_S A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \cos\frac{2\pi\nu\, \mathbf{b}\cdot\boldsymbol{\sigma}}{c}\, d\Omega \\
&- \Delta\nu \sin\left(\frac{2\pi\nu\, \mathbf{b}\cdot\mathbf{s}_0}{c}\right) \int_S A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \sin\frac{2\pi\nu\, \mathbf{b}\cdot\boldsymbol{\sigma}}{c}\, d\Omega
\end{aligned}
$$

$\mathcal{A}(\boldsymbol{\sigma}) \equiv A(\boldsymbol{\sigma})/A_0$ Normalized antenna reception pattern.

$$
V \equiv |V| e^{i\phi_V} = \int_S \mathcal{A}(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) e^{-2\pi i\nu \mathbf{b}\cdot\boldsymbol{\sigma}/c}\, d\Omega
$$

Write the real and imaginary parts separately

Visibility

$$r = \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$
$$- \Delta\nu \sin\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$

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$$A_0|V|\cos\phi_V = \int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$ Real

$$A_0|V|\sin\phi_V = -\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\sin\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega$$ Imaginary

Visibility

$$
\begin{aligned}
r &= \Delta\nu \cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c}\right)\int_S A(\boldsymbol{\sigma})I(\boldsymbol{\sigma})\cos\frac{2\pi\nu\,\mathbf{b}\cdot\boldsymbol{\sigma}}{c}\,d\Omega \\
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Real

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Imaginary

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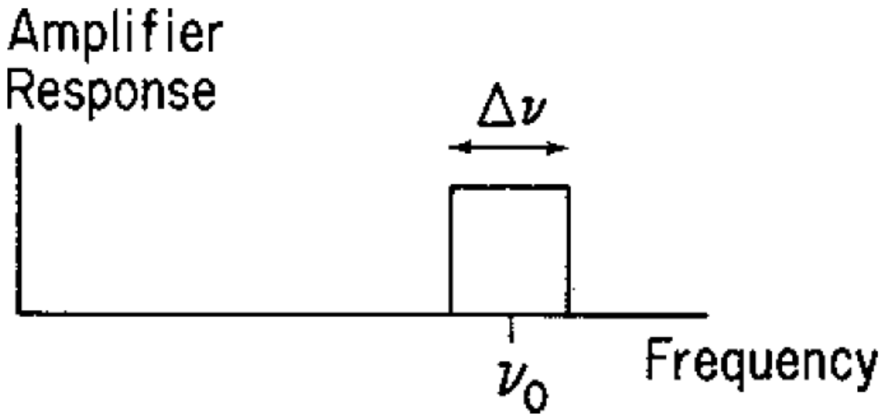
$$

$$
r = A_0\Delta\nu|V|\cos\left(\frac{2\pi\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} - \phi_V\right)
$$

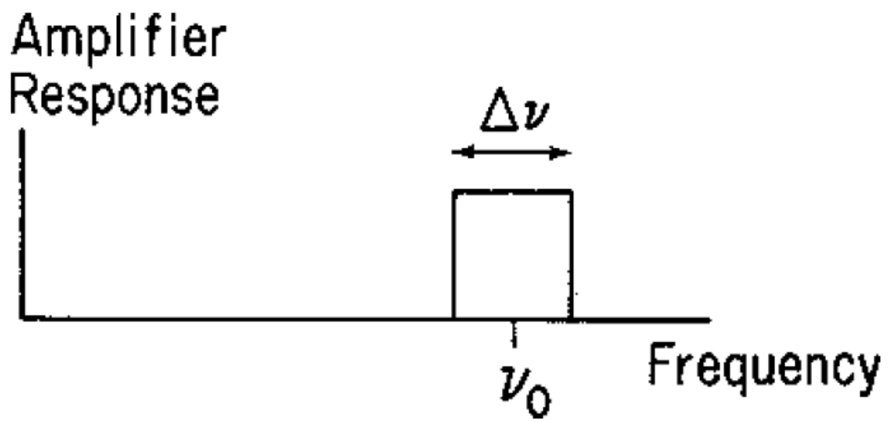
Effect of bandwidth



$$r = A_0|V| \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

$$= A_0|V|\Delta\nu \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

Effect of bandwidth



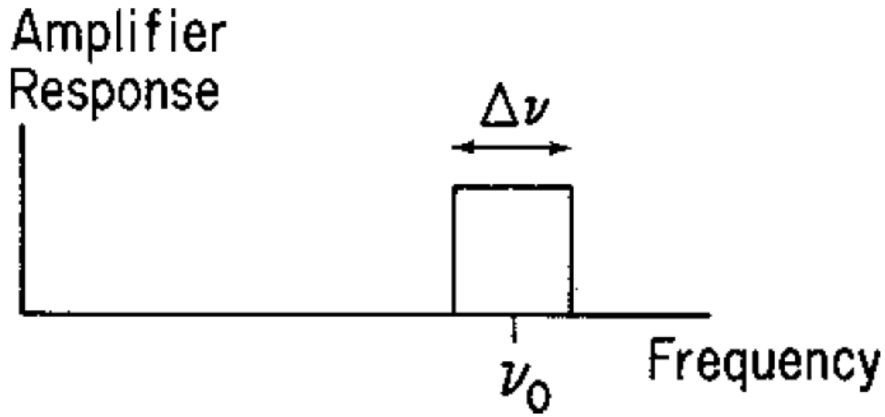
Modifies the fringe amplitude – maximum only when geometric delay is zero.

$$r = A_0 |V| \Delta\nu \frac{\sin \pi \Delta\nu \tau_g}{\pi \Delta\nu \tau_g} \cos\left(2\pi \nu_0 \tau_g - \phi_V\right)$$

$$|\pi \Delta\nu \tau_g| \ll 1 \qquad \frac{\sin \pi \Delta\nu \tau_g}{\pi \Delta\nu \tau_g} \approx 1 - \frac{(\pi \Delta\nu \tau_g)^2}{6}$$

Can calculate when the fringe amplitude is within 1% of the maximum.

Effect of bandwidth



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$$r = A_0 |V| \Delta\nu \frac{\sin \pi \Delta\nu \tau_g}{\pi \Delta\nu \tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

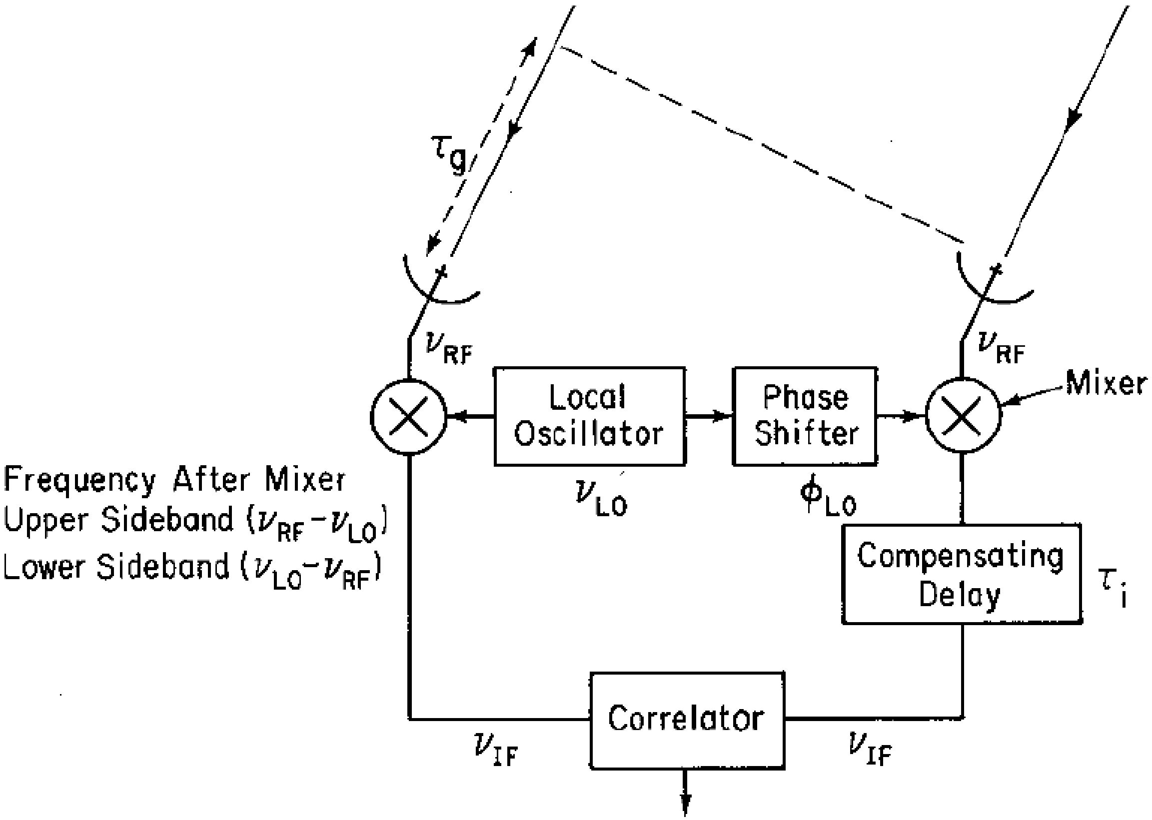
$$|\pi \Delta\nu \tau_g| \ll 1 \qquad \frac{\sin \pi \Delta\nu \tau_g}{\pi \Delta\nu \tau_g} \approx 1 - \frac{(\pi \Delta\nu \tau_g)^2}{6} > 0.99$$

Can calculate when the fringe amplitude is within 1% of the maximum for a given baseline length and bandwidth.

Delay tracking and frequency conversion

The geometric delay needs to be compensated in order to observe a source from rise to set.

RF converted to IF in a mixer and in one of the signals a delay to compensate the geometric delay is introduced.



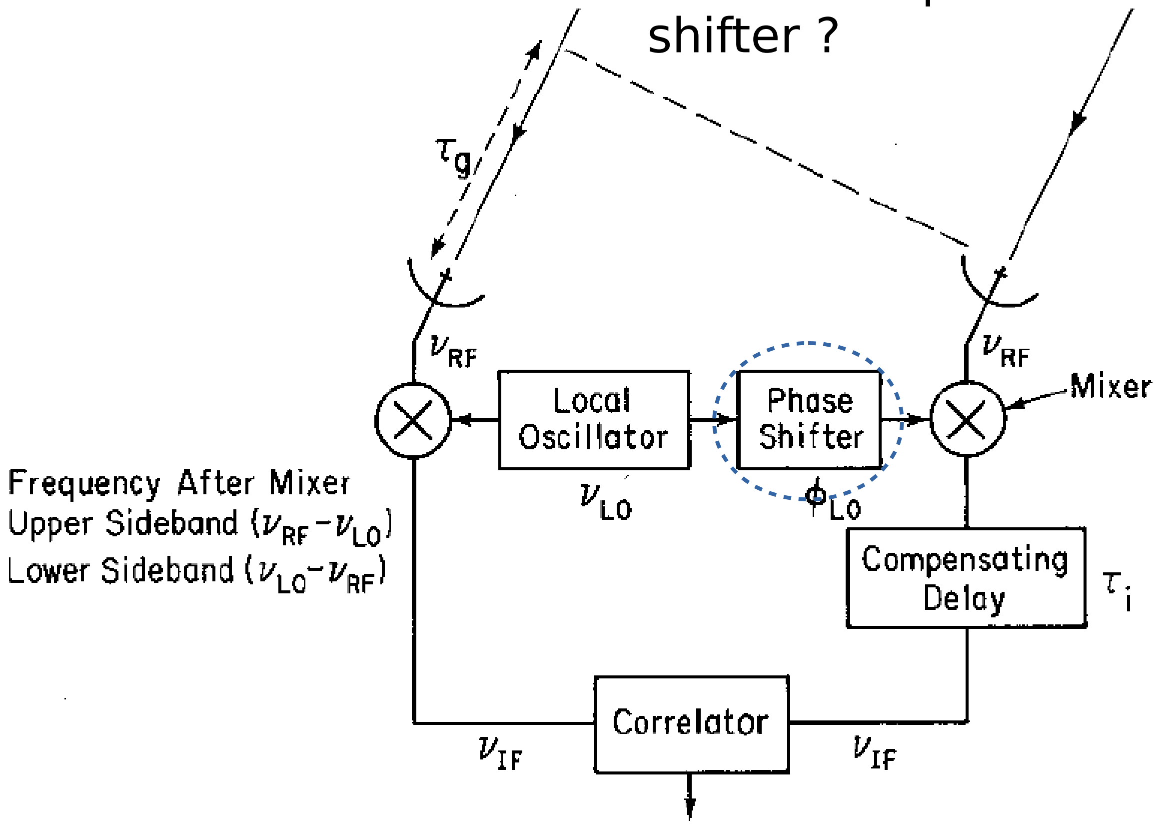
Low noise amplifier – not shown here but is present before the mixer in low frequency systems.

Delay tracking and frequency conversion

Role of the phase shifter ?

The geometric delay needs to be compensated in order to observe a source from rise to set.

RF converted to IF in a mixer and in one of the signals a delay to compensate the geometric delay is introduced.



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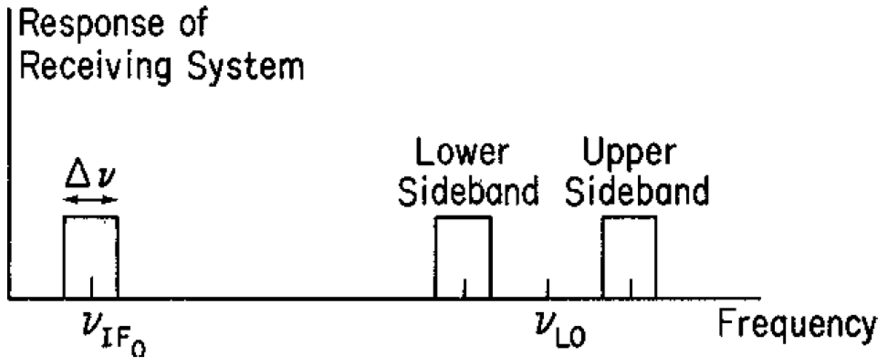
Delay tracking and frequency conversion

$$\nu_{\mathrm{RF}} = \nu_{\mathrm{LO}} \pm \nu_{\mathrm{IF}}$$

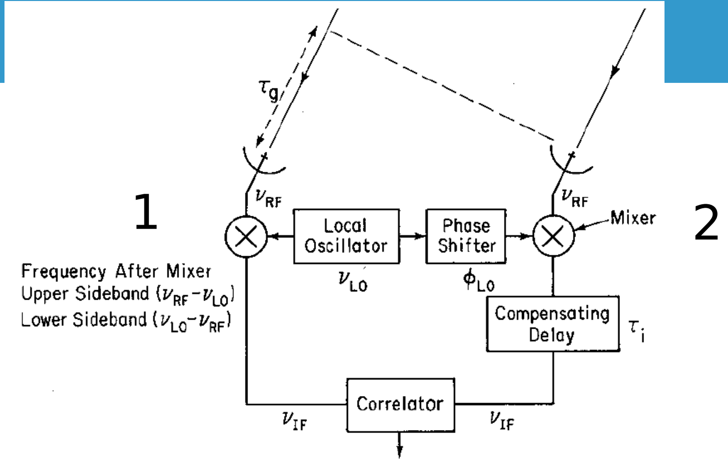


Upper side band and lower side band.

Both can be processed further (*double sideband system*) or using filters only one may be taken – called a *single sideband system*.

We need to see the phase changes before reaching the input of the correlator.

Delay tracking and frequency conversion



$$\nu_{\rm RF} = \nu_{\rm LO} \pm \nu_{\rm IF}$$

For USB:

$$\phi_1 \quad = \quad 2\pi\nu_{\rm RF}\tau_g = 2\pi(\nu_{\rm LO} + \nu_{\rm IF})\tau_g$$

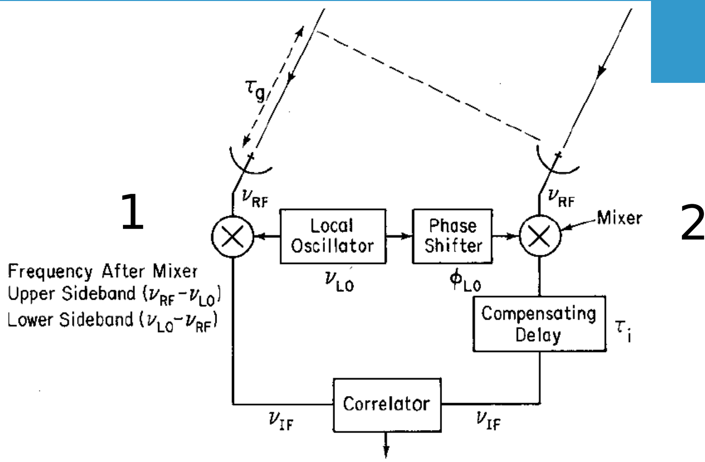
$$\phi_2 \quad = \quad 2\pi\nu_{\rm IF}\tau_i + \phi_{\rm LO}$$

$$r \quad = A_0|V|\Delta\nu\frac{\sin\pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g}\cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

Obtain the response by replacing the argument of cosine function wit

$\phi_1 - \phi_2 - \phi_V$ and integrating over IF from $\nu_{\rm IF_0} - \Delta\nu/2$ to $\nu_{\rm IF_0} + \Delta\nu/2$

Delay tracking and frequency conversion



$$\nu_{\rm RF} = \nu_{\rm LO} \pm \nu_{\rm IF}$$

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$$\phi_2 = 2\pi\nu_{\rm IF}\tau_i + \phi_{\rm LO}$$

$$r = A_0|V|\Delta\nu\frac{\sin\pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g}\cos(2\pi\nu_0\tau_g - \phi_V)$$

Obtain the response by replacing the argument of cosine function wit

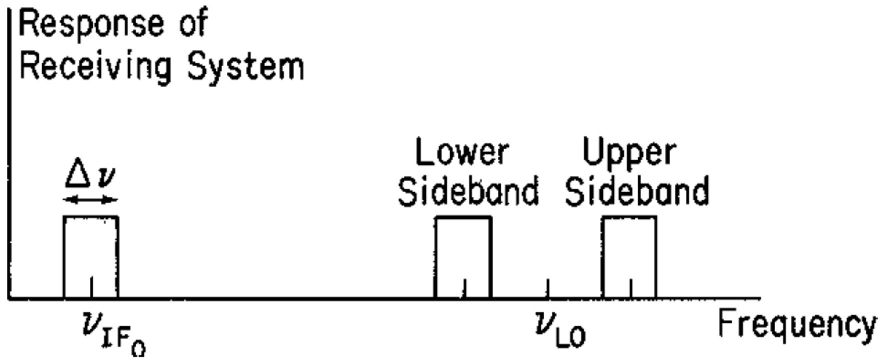
$\phi_1 - \phi_2 - \phi_V$ and integrating over IF from $\nu_{\rm IF_0} - \Delta\nu/2$ to $\nu_{\rm IF_0} + \Delta\nu/2$

$$r_u = A_0\Delta\nu|V|\frac{\sin\pi\Delta\nu\Delta\tau}{\pi\Delta\nu\Delta\tau}\cos[2\pi(\nu_{\rm LO}\tau_g + \nu_{\rm IF_0}\Delta\tau) - \phi_V - \phi_{\rm LO}]$$

$\Delta\tau = \tau_g - \tau_i$ Tracking error of the compensating delay

Delay tracking and frequency conversion

$$\nu_{\rm RF} = \nu_{\rm LO} \pm \nu_{\rm IF}$$



For LSB

$$\phi_1 = -2\pi(\nu_{\rm LO} - \nu_{\rm IF})\tau_g$$

$$\phi_2 = 2\pi\nu_{\rm IF}\tau_i - \phi_{\rm LO}$$

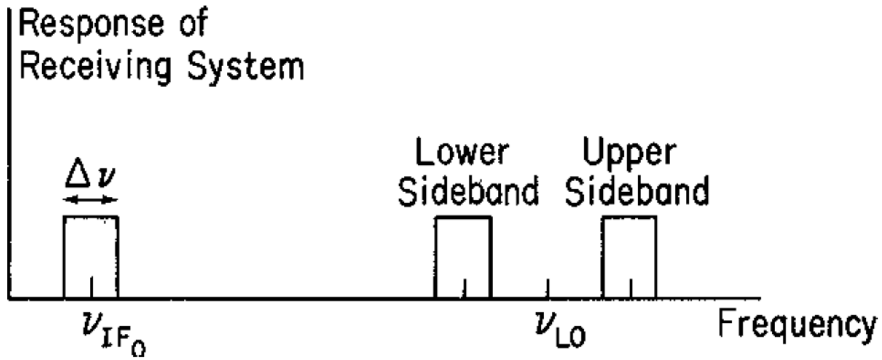
$$r_l = A_0 \Delta\nu |V| \frac{\sin \pi\Delta\nu\Delta\tau}{\pi\Delta\nu\Delta\tau} \cos[2\pi(\nu_{\rm LO}\tau_g - \nu_{{\rm IF}_0}\Delta\tau) - \phi_V - \phi_{\rm LO}]$$

$\Delta\tau = \tau_g - \tau_i$ Tracking error of the compensating delay

Delay tracking and fringe conversion

$$\nu_{\mathrm{RF}} = \nu_{\mathrm{LO}} \pm \nu_{\mathrm{IF}}$$

For a double sideband system:



$$r_u = A_0 \Delta\nu |V| \frac{\sin \pi\Delta\nu\Delta\tau}{\pi\Delta\nu\Delta\tau} \cos[2\pi(\nu_{\mathrm{LO}}\tau_g + \nu_{\mathrm{IF}_0}\Delta\tau) - \phi_V - \phi_{\mathrm{LO}}]$$

$$r_l = A_0 \Delta\nu |V| \frac{\sin \pi\Delta\nu\Delta\tau}{\pi\Delta\nu\Delta\tau} \cos[2\pi(\nu_{\mathrm{LO}}\tau_g - \nu_{\mathrm{IF}_0}\Delta\tau) - \phi_V - \phi_{\mathrm{LO}}]$$

$$\begin{aligned} r_d &= r_u + r_l \\ &= 2\Delta\nu A_0 |V| \frac{\sin(\pi\Delta\nu\Delta\tau)}{\pi\Delta\nu\Delta\tau} \cos(2\pi\nu_{\mathrm{LO}}\tau_g - \phi_V - \phi_{\mathrm{LO}}) \cos(2\pi\nu_{\mathrm{IF}_0}\Delta\tau) \end{aligned}$$

$$\Delta\tau = \tau_g - \tau_i$$ Tracking error of the compensating delay

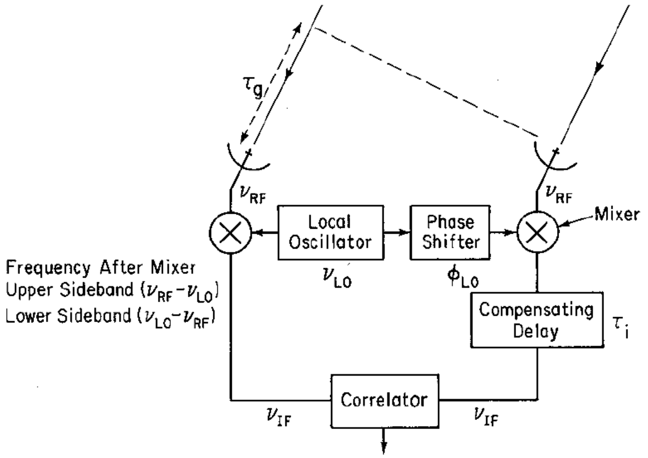
Fringe rotation/stopping

$$r_u = A_0 \Delta\nu |V| \frac{\sin \pi \Delta\nu \Delta\tau}{\pi \Delta\nu \Delta\tau} \cos[2\pi(\nu_{\rm LO}\tau_g + \nu_{{\rm IF}_0}\Delta\tau) - \phi_V - \phi_{\rm LO}]$$

If the term $(2\pi\nu_{\rm LO}\tau_g - \phi_{\rm LO})$ can be kept constant then the output will vary with changes in V and slow drifts in the instrument.

The control of LO phase shift is referred to as fringe stopping or fringe rotation.

The phase shifter allows to have this control and thus is introduced in the system.



Complex correlator: briefly

To measure the complex fringe both the real and imaginary components need to be measured. The imaginary part is just a $\pi/2$ phase shifted copy of the same.

Complex correlator: briefly

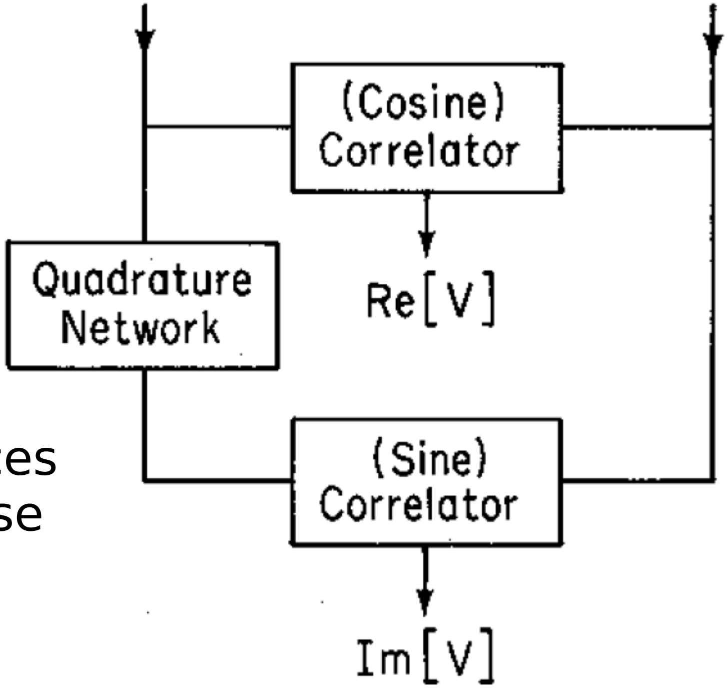
To measure the complex fringe both the real and imaginary components need to be measured. The imaginary part is just a $\pi/2$ phase shifted copy of the same.

For each antenna pair a second correlator with the shift is added in one of the inputs.

This is called complex correlator.

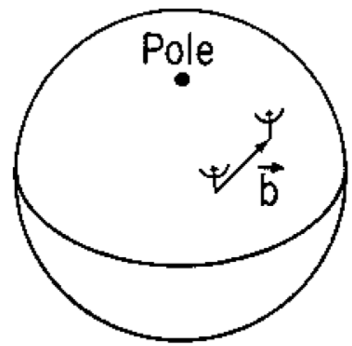
We will come to further details when we will discuss correlators.

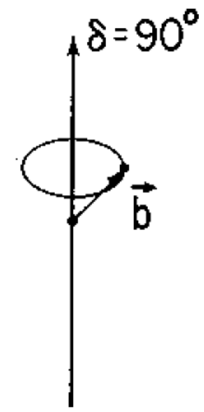
Introduces the phase shift

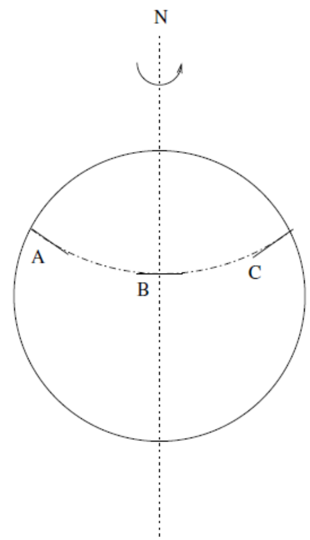


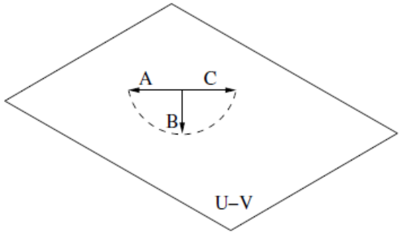
Coordinate systems

Baseline orientation; Track in the uv-plane.

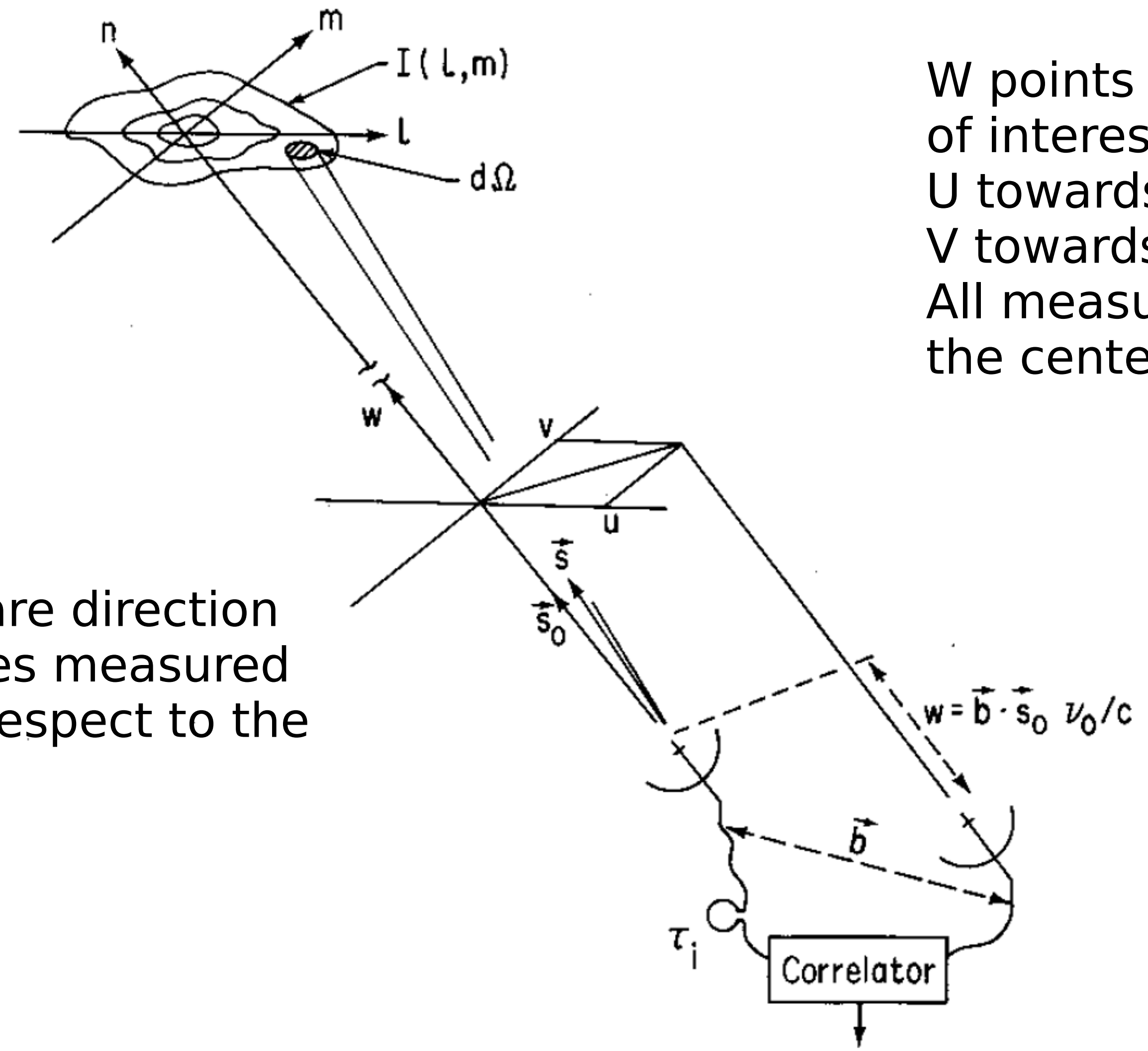








Coordinate systems



W points towards the direction of interest – phase centre.
U towards East
V towards North
All measured in wavelength of the center of the RF.

L, m are direction cosines measured with respect to the u,v.

$$\frac{\nu\,\mathbf{b}\cdot\mathbf{s}}{c} = ul + vm + wn$$

$$\frac{\nu\,\mathbf{b}\cdot\mathbf{s}_0}{c} = w,$$

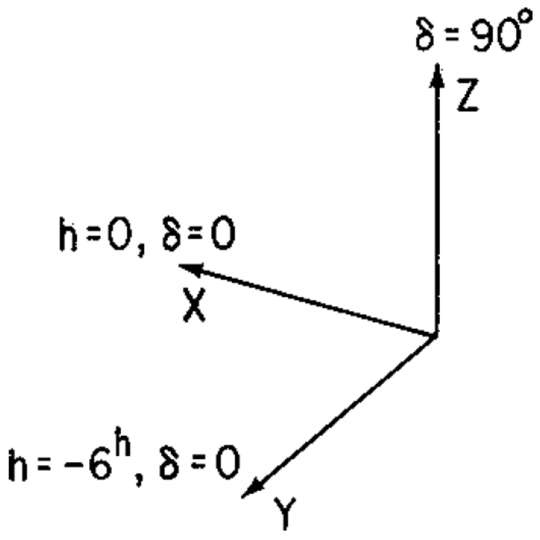
$$d\Omega = \frac{dl\,dm}{n} = \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

$$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{A}(l,m) I(l,m) e^{-2\pi i\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]} \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

Integrand taken as zero when $l^2 + m^2 \geq 1$

We have been through the conditions under which this is a 2-D Fourier transform.

Antenna spacings and u,v,w



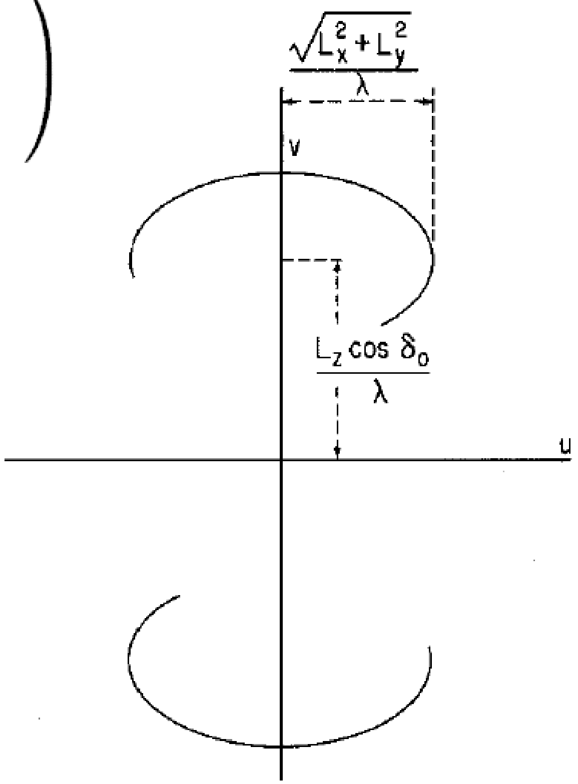
Coordinate system for baseline parameters:
X – direction of the meridian at the celestial equator
Y – towards East
Z- toward the North celestial pole

L_X, L_Y and L_Z are coordinate differences for the baseline, then

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$

H_0 and δ_0 are the hour angle and the declination of the phase reference position.

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$



What is the locus of a track in the uv-plane ?
Eliminating H_0 from the equations for u and v:

$$u^2 + \left(\frac{v - (L_Z/\lambda)\cos\delta_0}{\sin\delta_0} \right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$

$$V(-u, -v) = V^*(u, v)$$

Sampling in the uv-plane

Visibilities are sampled: the footprint in the uv-plane – *uv-coverage* is the sampling function.

For a point source at the phase centre the visibility is a constant as a function of u and v.

The FT of the sampling function is then the response to a point source – *the synthesized beam.*