- A two element interferometer

# Astronomical Techniques II : Lecture 4

**Ruta Kale**

Low Frequency Radio Astronomy (Chp. 4)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy

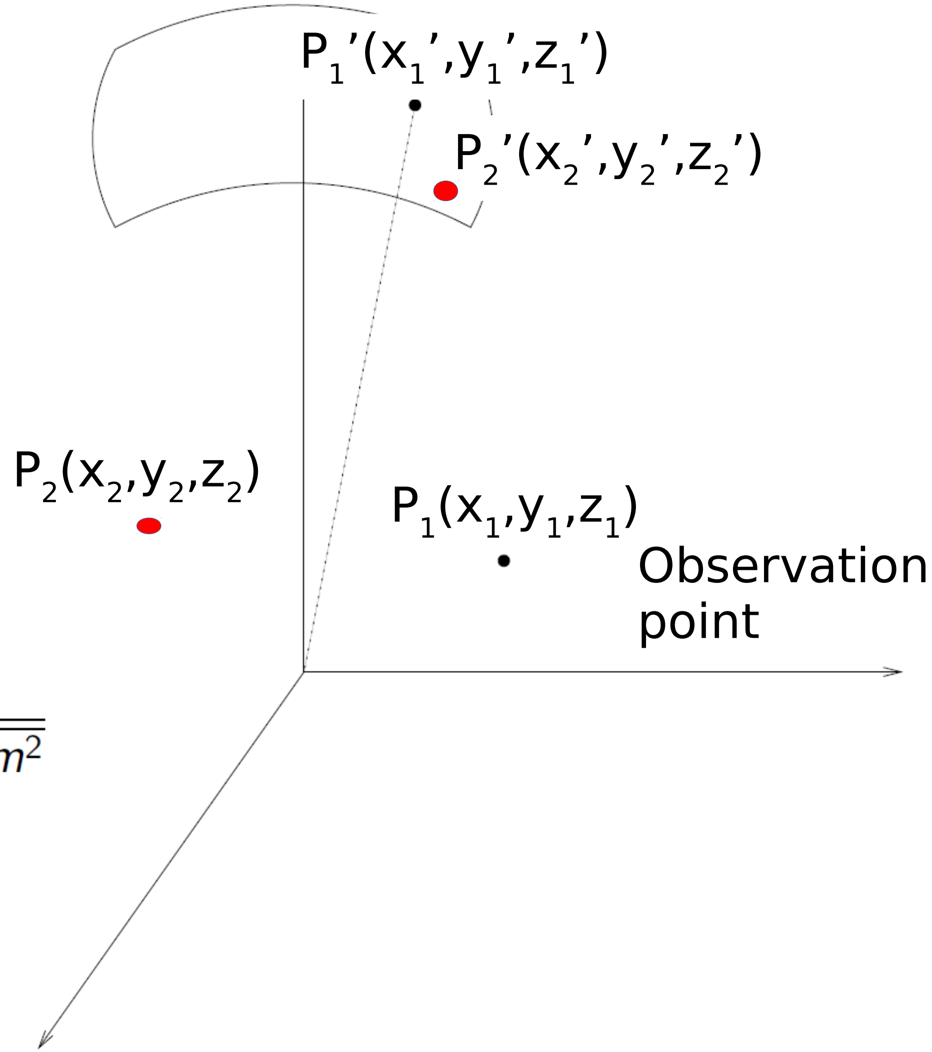
Synthesis imaging in radio astronomy II, Chp 2

Interferometry and synthesis in radio astronomy (Chp 2)

# Introduction

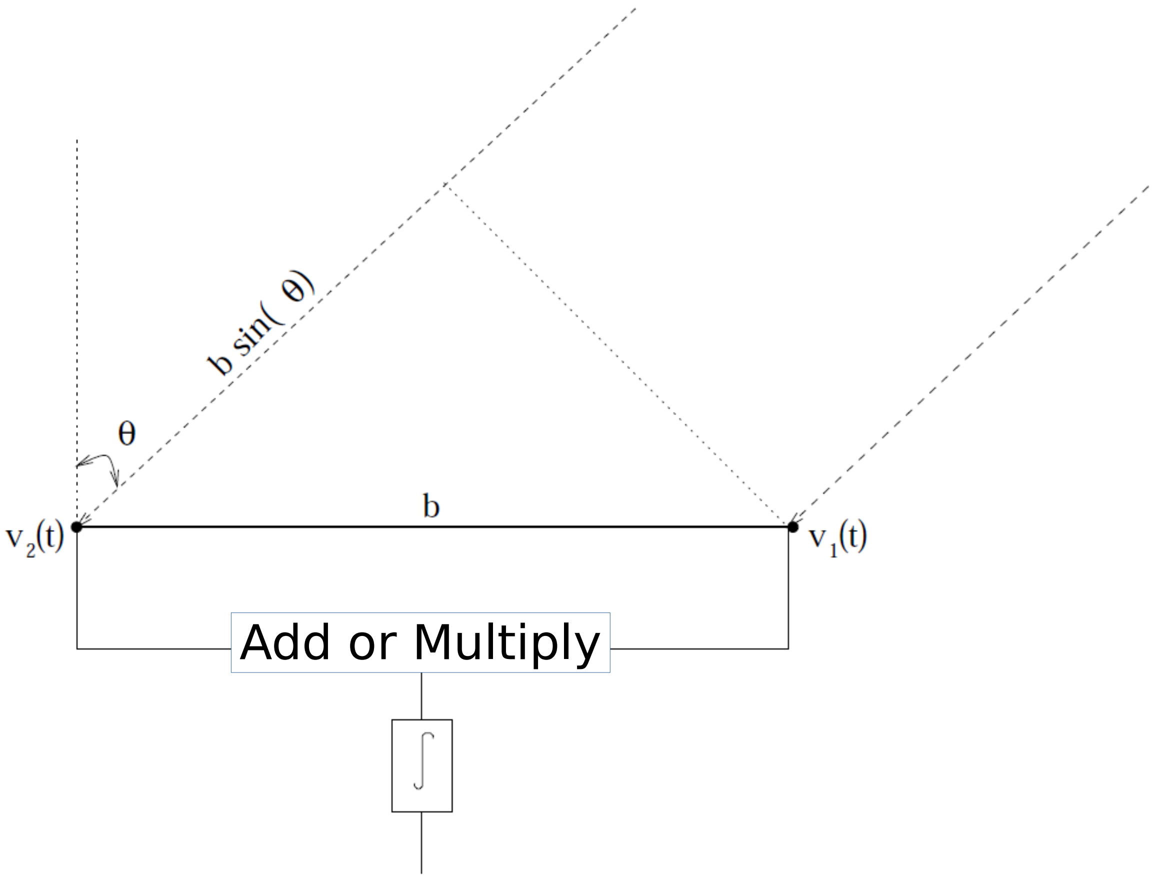
According to van Cittert-Zernicke theorem: the source brightness distribution can be derived if one can measure the mutual coherence function of the electric fields.

$$V(u, v, w) = \int I(l, m) e^{-i2\pi[lu+mv+nw]} \frac{dl\,dm}{\sqrt{1 - l^2 - m^2}}$$



# A two element interferometer

Assume the radiation emitted by the source is monochromatic having a frequency $\nu$

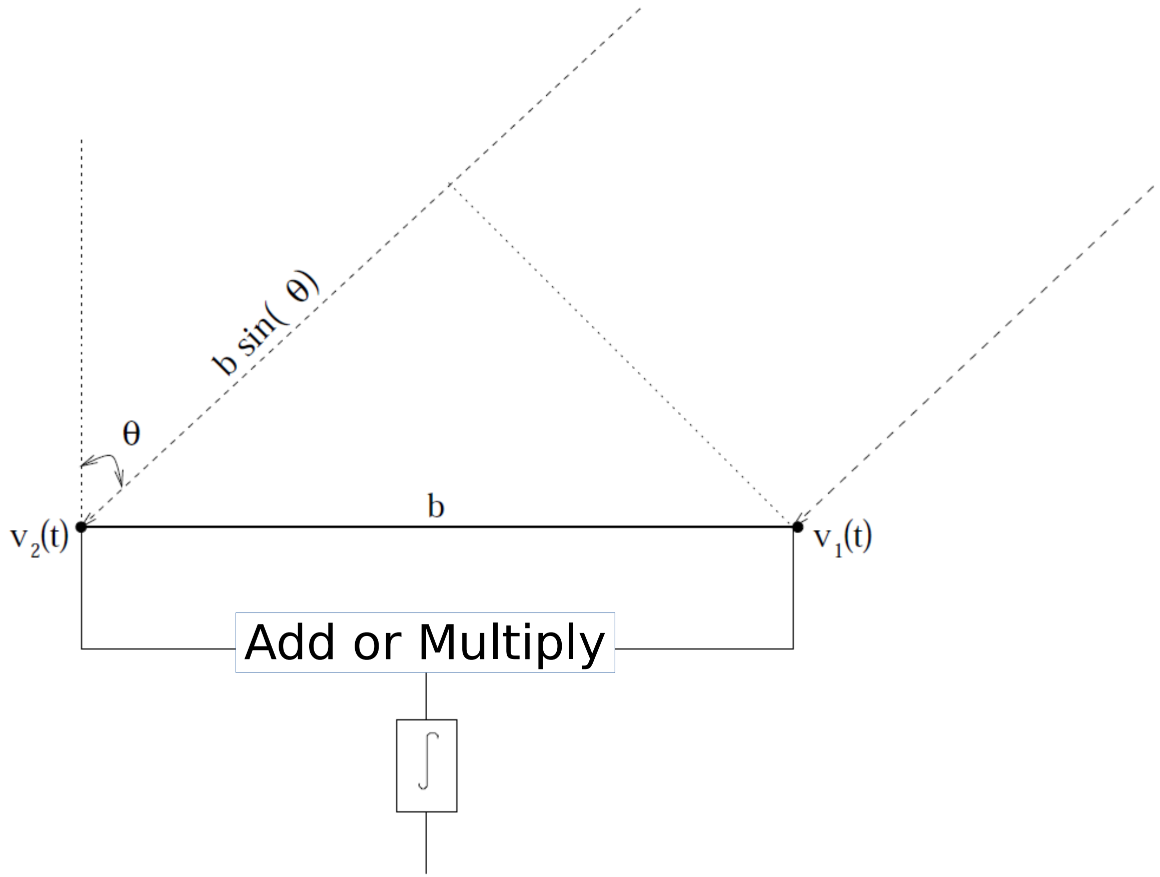


# A two element interferometer

Assume the radiation emitted by the source is monochromatic having a frequency $\nu$

The plane wave travels an extra distance to reach the second element – this is the *geometric delay*,

$$\tau_g = b \sin(\theta)/c$$



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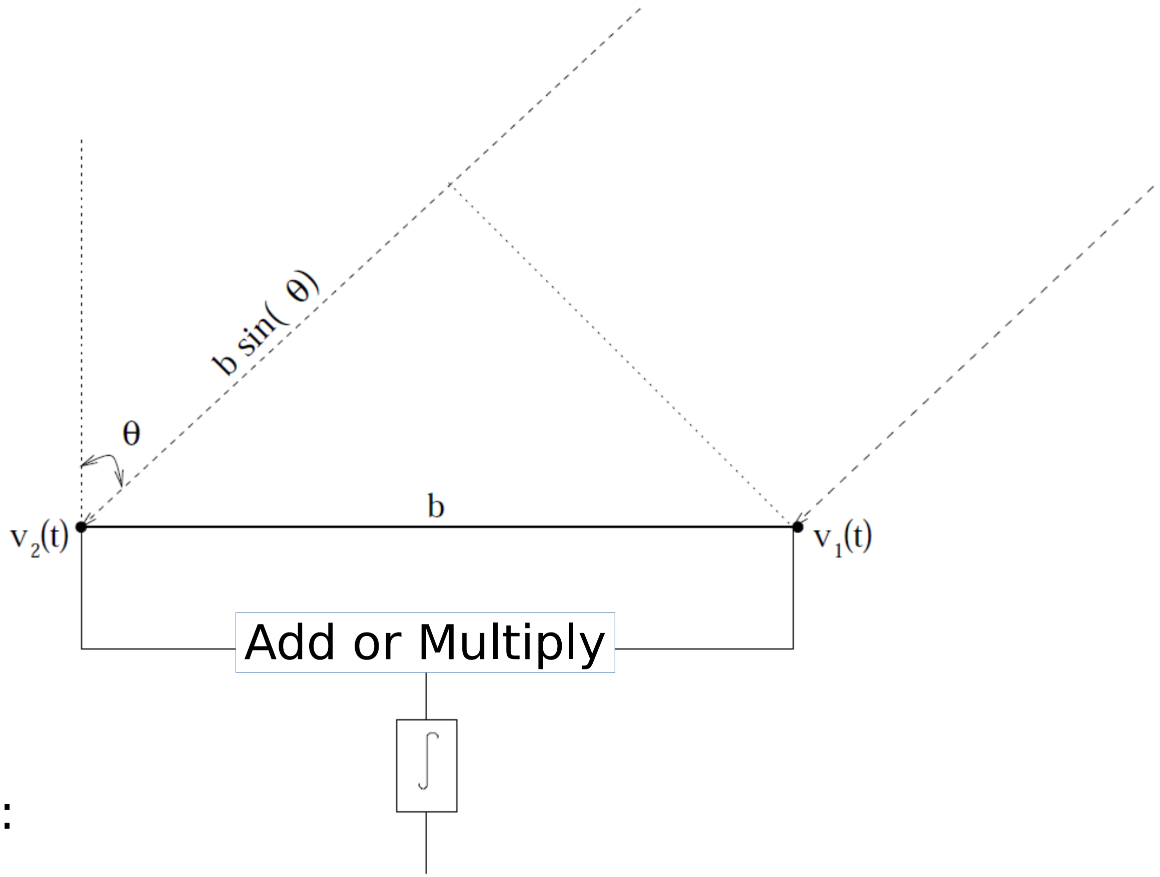


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The voltages at the two points:

$$v_1(t) = cos(2\pi\nu t) \quad \text{and} \quad v_2(t) = cos(2\pi\nu(t - \tau_g))$$

# A two element interferometer: adding

$$[v_1(t) + v_2(t)]^2 = [cos(2\pi\nu t) + cos(2\pi\nu(t - \tau_g))]^2$$

Squaring and then reducing the RHS using trigonometric identities and averaging:

$$\langle[v_1(t) + v_2(t)]^2\rangle = 1 + cos(2\pi\nu\tau)$$

$$= 1 + cos(2\pi\frac{b}{\lambda}sin(\theta))$$

The offset term : have to detect over and above the offset term that is dominated by noise that also varies and makes detection of sources difficult.
We will discuss multiplying interferometers henceforth.

# A two element interferometer: multiplying

Assuming averaging time is much longer than 1/Δν

$$r(\tau_g) = \frac{1}{\mathrm{T}}\int_{t-T/2}^{t+T/2} \cos(2\pi\nu t)\cos(2\pi\nu(t-\tau_g))dt$$

$$= \frac{1}{\mathrm{T}}\int_{t-T/2}^{t+T/2} (\cos(4\pi\nu t - 2\pi\tau_g) + \cos(2\pi\nu\tau_g))dt$$

$$\cos(4\pi\nu t)$$

$$= \cos(2\pi\nu\tau_g)$$

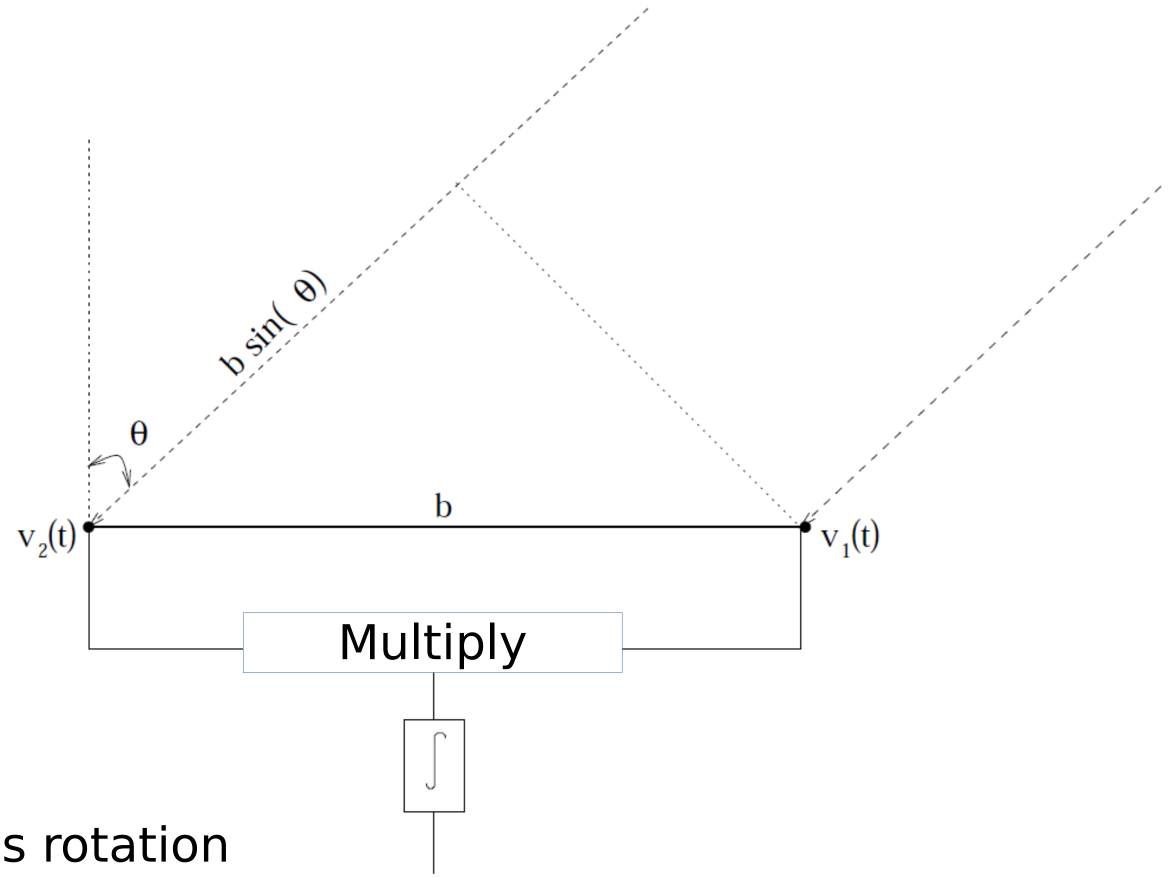
Averages out to zero

# A two element interferometer: multiplying

$\tau_g = b \sin(\theta)/c$

The theta changes with source rise and set. Assuming exactly east-west baseline vector, and source at declination 0 deg,

$$\theta = \Omega_E t$$




$\Omega_E$ angular frequency of Earth's rotation

= $7.29 \times 10^{-5}$ rad/s

# A two element interferometer: multiplying

$\tau_g$ = b sin($\theta$)/c

The theta changes with source rise and set. Assuming exactly east-west baseline vector, and source at declination 0 deg,

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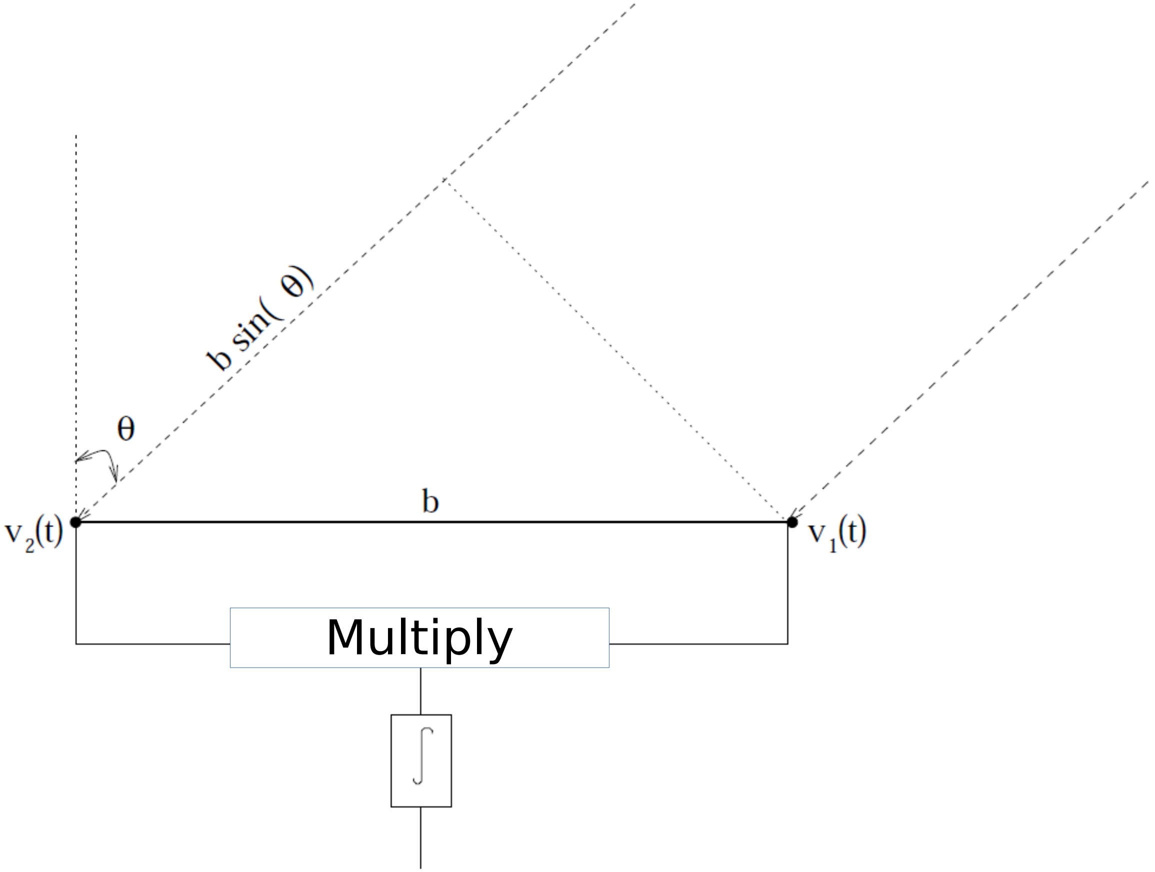
$$r(\tau_g) = \cos(2\pi\nu\tau_g)$$



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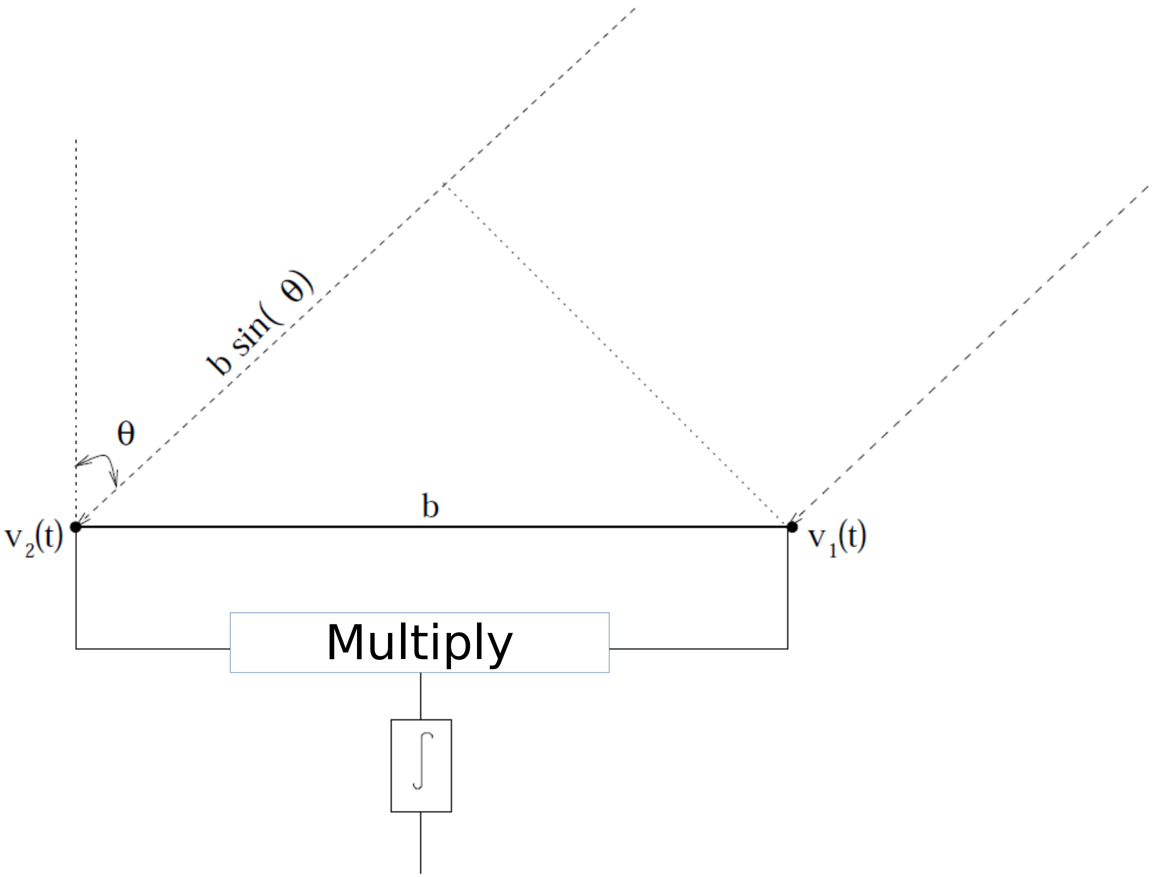


$$\theta = \Omega_E t$$

$$r(\tau_g) = \cos(2\pi\nu\tau_g)$$

$$r(\tau_g) = \cos(2\pi\nu \times b/c \times \sin(\Omega_E(t - t_z)))$$

$t_z$ is the time when the source is at the zenith.

# A two element interferometer: multiplying

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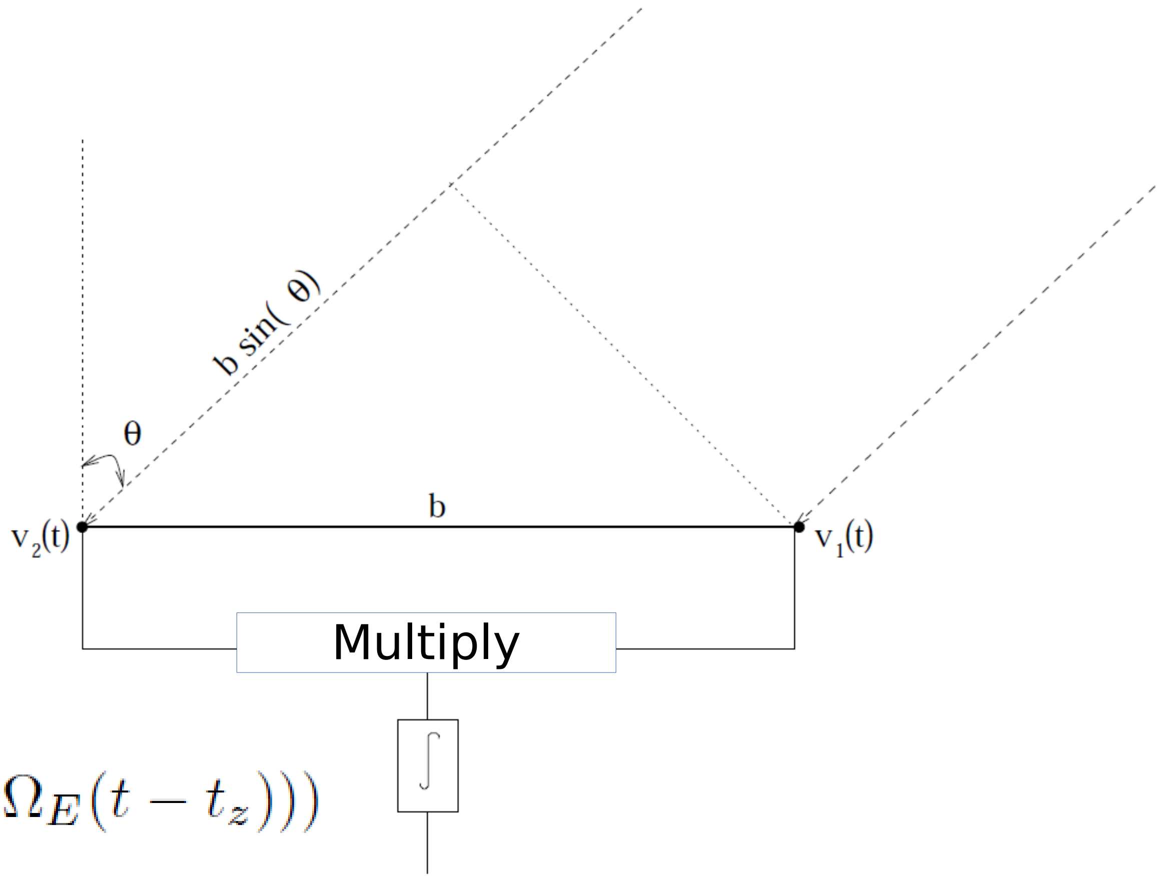
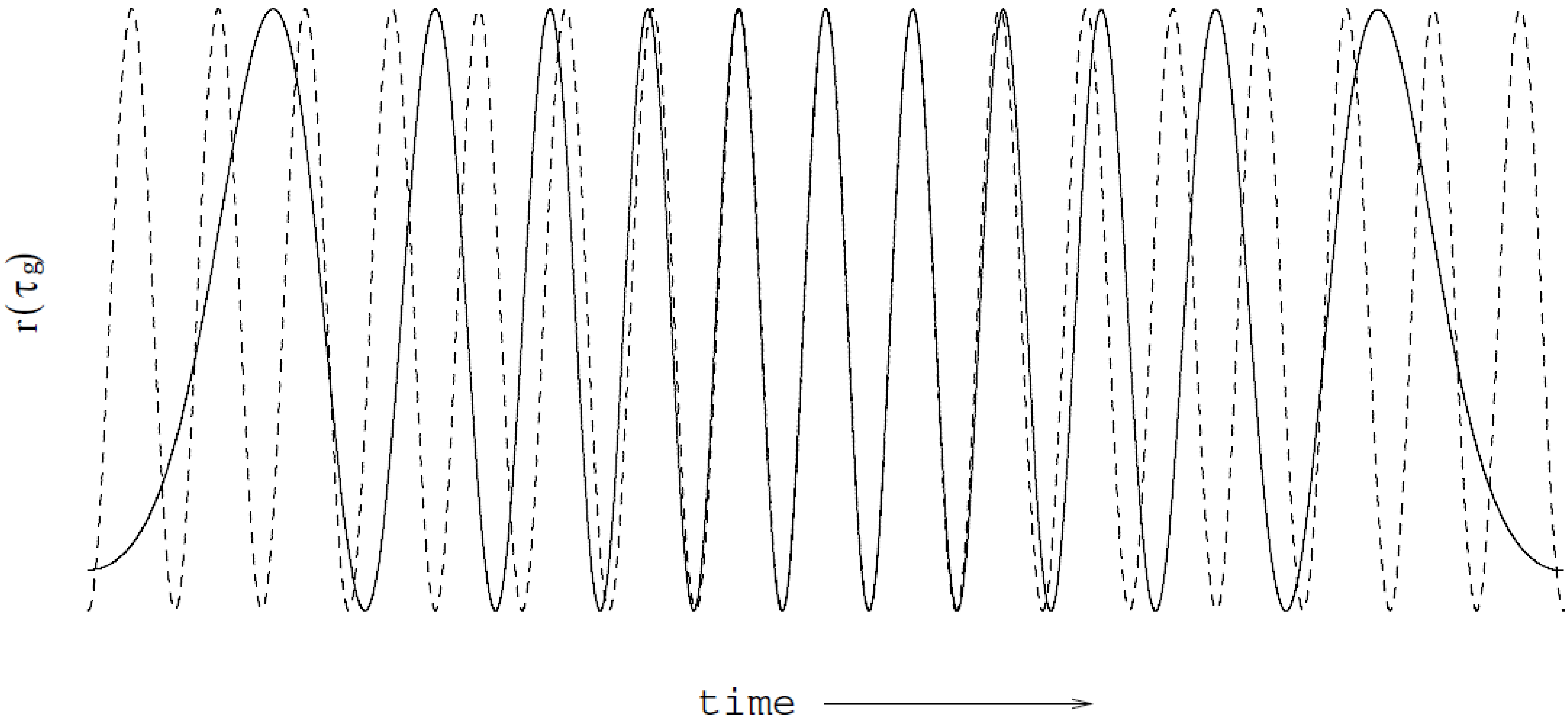



$$r(\tau_g) = \cos(2\pi\nu \ \times \ b/c \ \times \ \sin(\Omega_E(t - t_z)))$$

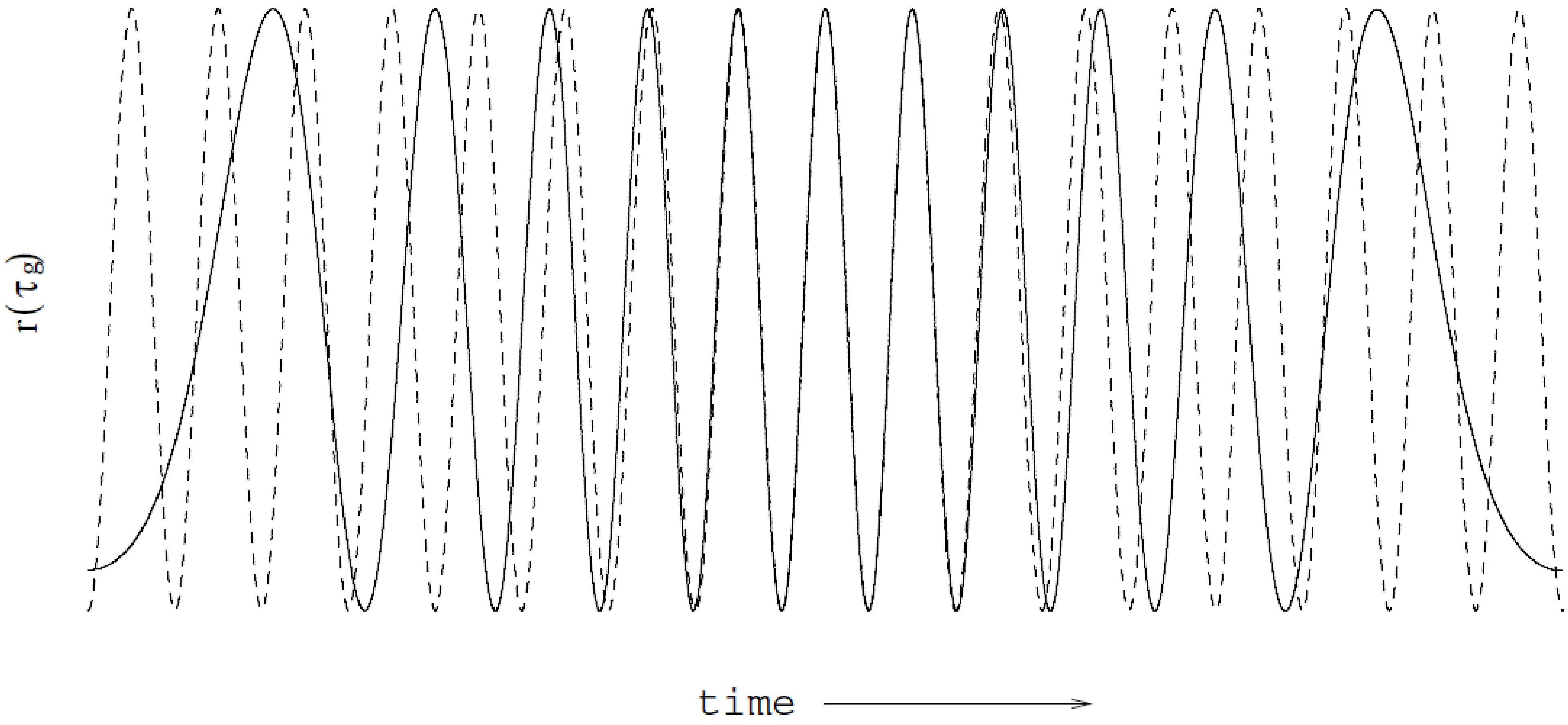
$r(\tau_g)$ *is called the fringe.*

# Fringe



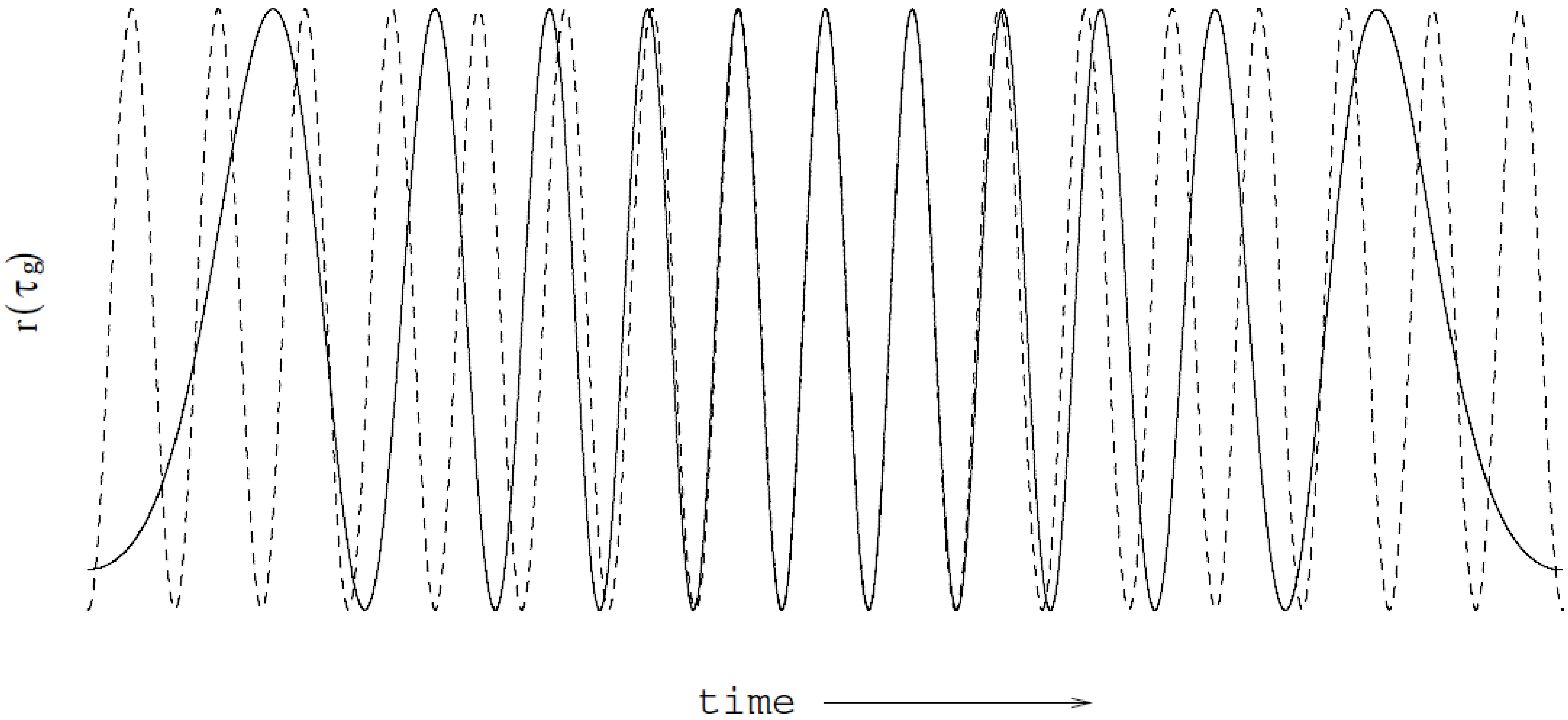

Solid line: observed output
Dashed line: pure sinusoid with frequency equal to the maximum instantaneous frequency of the fringe.

# Fringe




If the RA is known the time when the “fringe frequency” will peak can be predicted. Thus between measured fringe frequency peak and that expected one can accurately find the position of the source.

# Fringe




For a point source the fringe amplitude will remain the same. But if extended then the fringe amplitude will decrease due to waves arriving at a slightly different path differences from different parts of the source. For a very large source the fringe amplitude will be zero: source is *“resolved out”*.

# Resolution

Sources smaller than the fringe spacings will all appear as point sources. When the source size is such that the waves from different parts of the source give rise to the same phase lags, then the source will appear as a point source.

The minimum source size that can be resolved by the interferometer:

$$\pi \nu \Delta\theta b/c \lesssim \pi \quad \Longrightarrow \quad \Delta\theta \lesssim \lambda/b$$

The resolution of a two element interferometer with baseline length b is $\sim \lambda/b$

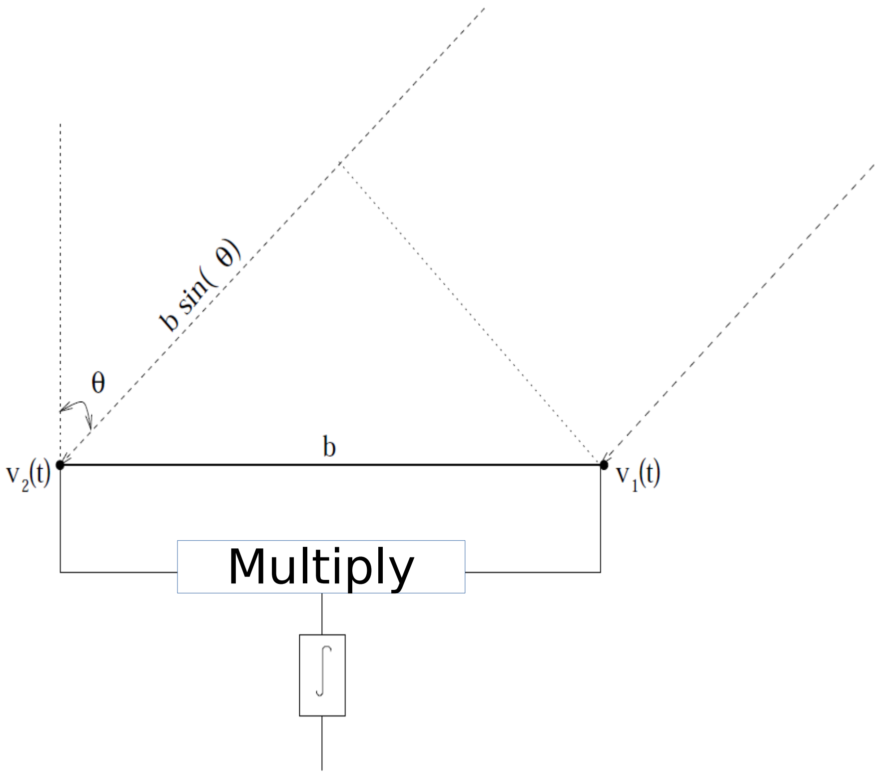
*Larger the b, higher will be the resolution.*

# A two element interferometer

One can infer the source position and size with a two element interferometer.

If we make measurements by varying the baseline length and orientations one will get different constraints on the source size and source brightness.

Using van Cittert Zernike theorem one can then infer the correct source brightness distribution on the sky.



# Quasi-mono-chromatic waves

In reality we have waves coming from a band $\Delta\nu$ around $\nu$.

Radiation if at one frequency arrives in phase, at an adjacent frequency it will be out of phase and for a large enough separation in frequencies, they may be 180 deg out of phase. Thus averaging all these together will decrease the amplitude of the fringe.

$$
\begin{aligned}
r(\tau_g) &= \frac{1}{\Delta\nu}\int_{\nu-\frac{\Delta\nu}{2}}^{\nu+\frac{\Delta\nu}{2}} \cos(2\pi\nu\tau_g)d\nu \\
&= \frac{1}{\Delta\nu} Re\left[\int_{\nu-\frac{\Delta\nu}{2}}^{\nu+\frac{\Delta\nu}{2}} e^{i2\pi\nu\tau_g} d\nu\right] \\
&= \cos(2\pi\nu\tau_g)\left[\frac{\sin(\pi\Delta\nu\tau_g)}{\pi\Delta\nu\tau_g}\right]
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\end{aligned}
$$

Sinc function here decreases rapidly as the bandwidth increases. Termed as “fringe washing”.

Need a way to average over large bandwidths without losing the fringe amplitude.

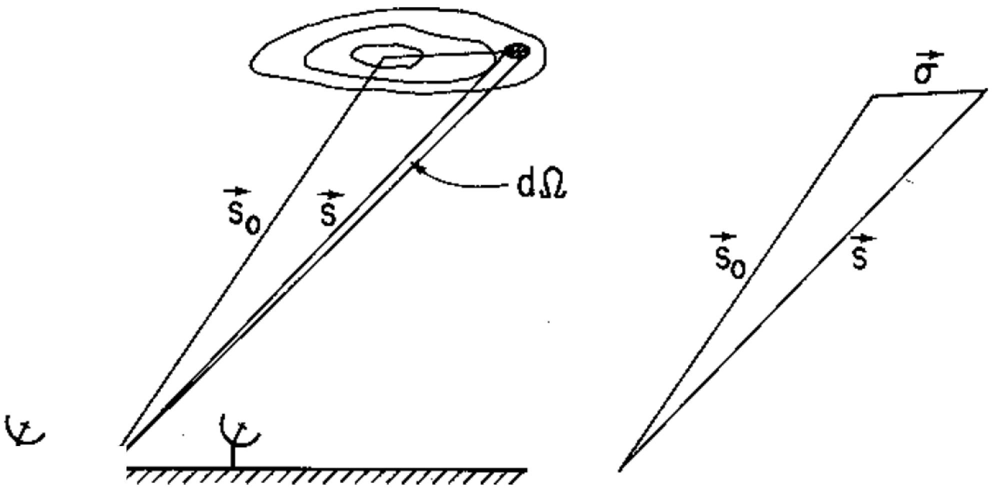
# Extended source

Consider a source at $s_0$ with some small extent. Any point on the source can be written as

$s = s_0 + \sigma$

$s_0 . \sigma = 0$

$\tau_g = s_0 . b$




$$r(\tau_g) = Re\left[\int I(\mathbf{s}) e^{\frac{-i2\pi \mathbf{s}.\mathbf{b}}{\lambda}} d\mathbf{s}\right]$$

$$= Re\left[e^{\frac{-i2\pi \mathbf{s_0}.\mathbf{b}}{\lambda}} \int I(\mathbf{s}) e^{\frac{-i2\pi \sigma.\mathbf{b}}{\lambda}} d\mathbf{s}\right]$$

$$= |\mathcal{V}| \cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}})$$

where $\mathcal{V} = |\mathcal{V}| e^{-i\Phi_{\mathcal{V}}}$

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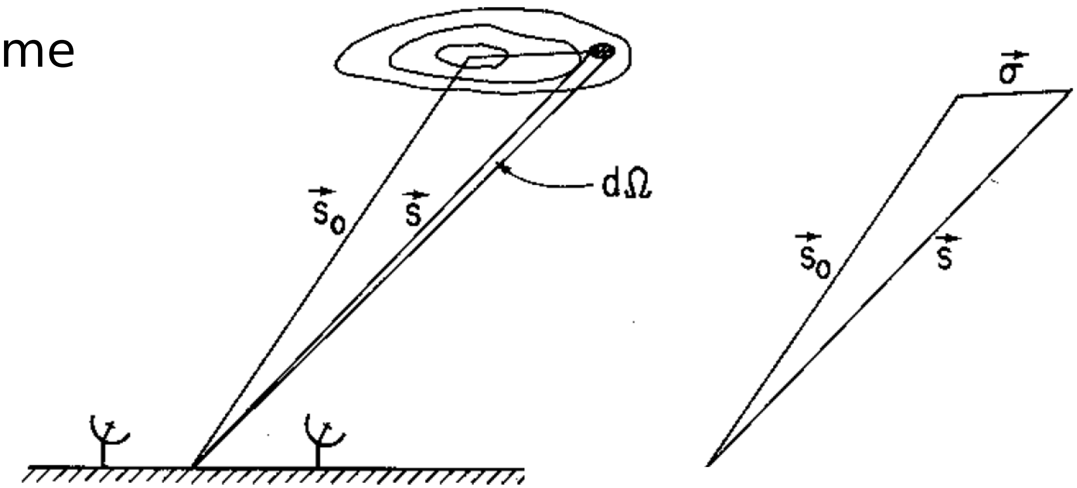


$$r(\tau_g) \;=\; |\mathcal{V}|\cos(2\pi\nu\tau_g + \Phi_\mathcal{V})$$ where $$\mathcal{V} = |\mathcal{V}|e^{-i\Phi_\mathcal{V}}$$

Only contains the variation of the fringe as a function of earth's rotation or source rise-set. If an equal delay is introduced in the signals' path we will have:

$$r(\tau_g) = |\mathcal{V}|\cos(\Phi_\mathcal{V})$$

This instrumental delay has to change continuously as $\tau_g$ changes: *delay tracking*

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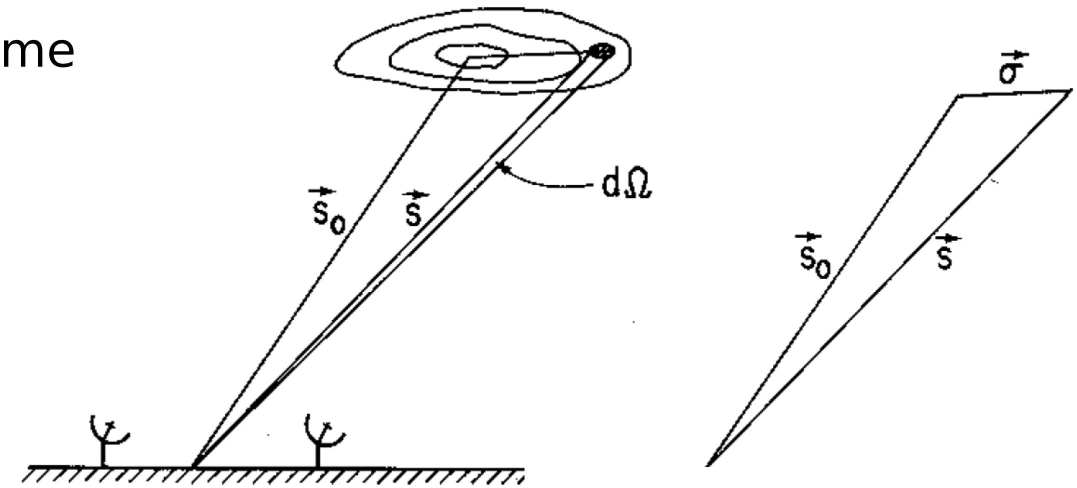


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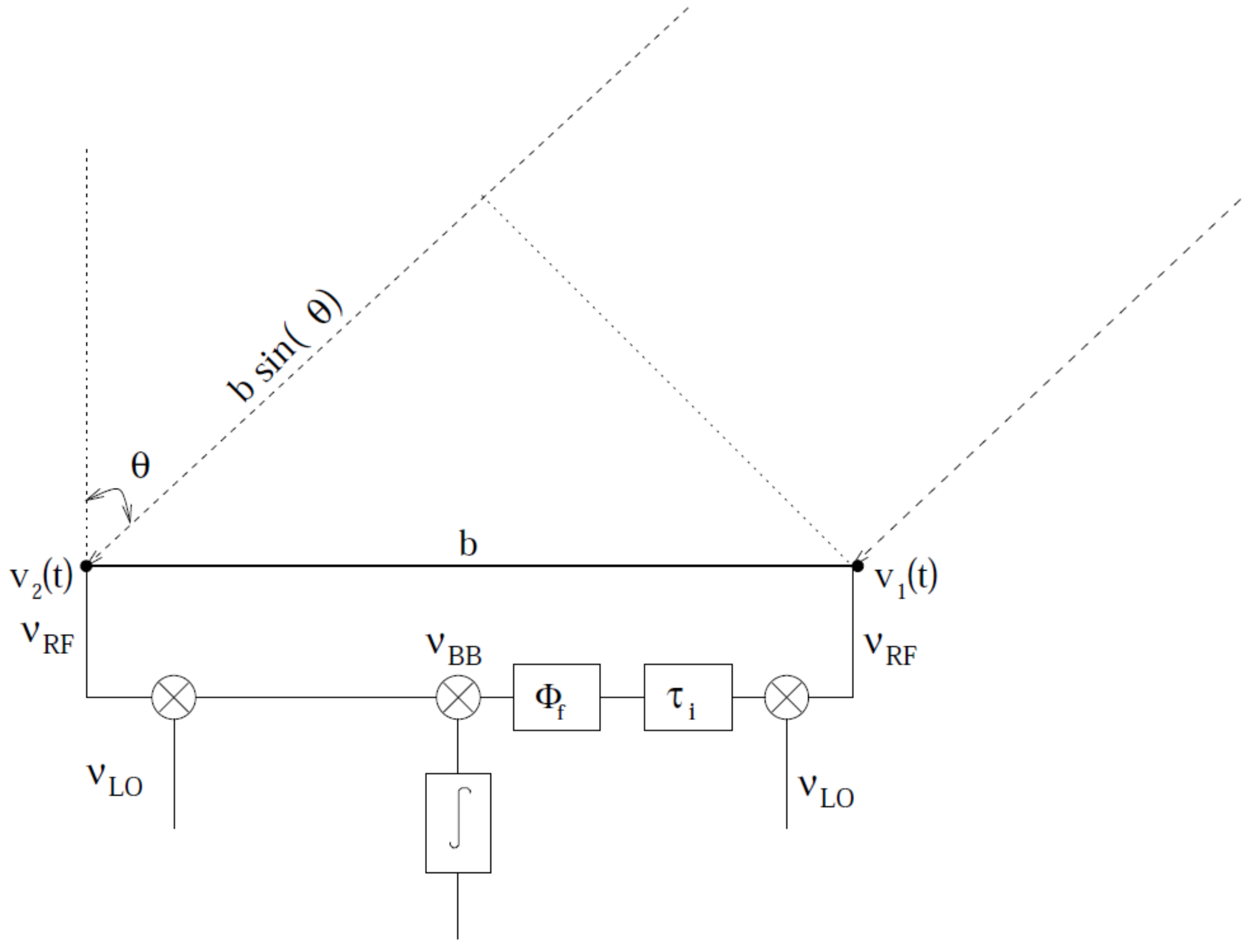
*While $\tau_g$ is in RF the delay tracking is in baseband and thus needs to be properly accounted.*

# Two element interferometer in practice



# Delay tracking and fringe stopping

$$\nu_{LO} = \nu_{RF} - \nu_{BB}$$

$$\tau_i = \tau_g + \Delta\tau$$

$$\begin{aligned} r(\tau_g) &= |\mathcal{V}| \left\langle \cos(\Phi_{\mathcal{V}} + 2\pi\nu_{BB}t - 2\pi\nu_{RF}\tau_g) \cos(2\pi\nu_{BB}(t - \tau_i) + \Phi_f) \right\rangle \\ &= |\mathcal{V}| cos(\Phi_{\mathcal{V}} + 2\pi(\nu_{RF} - \nu_{BB})\tau_g - \nu_{BB}\Delta\tau - \Phi_f) \\ &= |\mathcal{V}| \cos(\Phi_{\mathcal{V}} + 2\pi\nu_{LO}\tau_g - \nu_{BB}\Delta\tau - \Phi_f) \end{aligned}$$

$$\Phi_f = 2\pi\nu_{LO}\tau_g$$