- Two element interferometer
- Correlators

Astronomical Techniques II : **Lecture 8**

Ruta Kale

- Low Frequency Radio Astronomy (Chp. 8, 9)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy
- Synthesis imaging in radio astronomy II, Chp. 4
- Interferometry and synthesis in radio astronomy (Chp. 8)
- Talk by Adam Deller (https://nmt.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=42804ec9-3b6c-4b40-8978-a920010eb3fa)

Sampling in the uv-plane

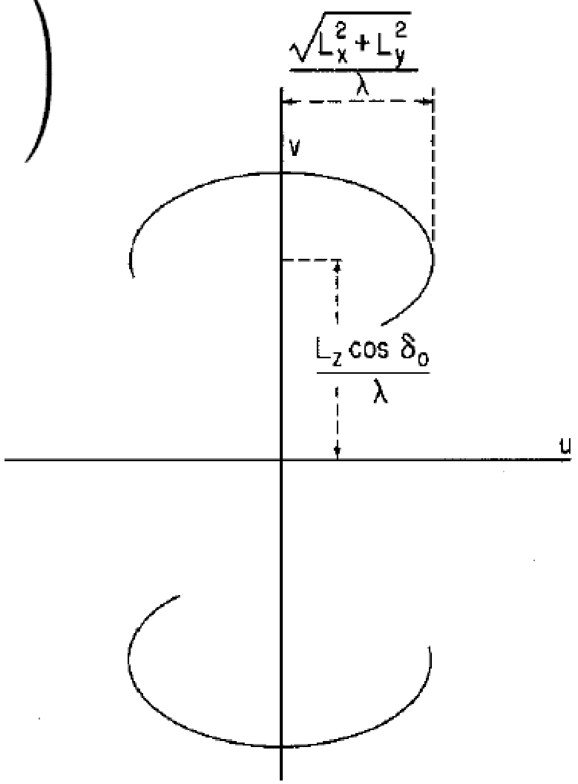
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$

Visibilities are sampled: the footprint in the uv-plane – *uv-coverage* is the sampling function.

For a point source at the phase centre the visibility is a constant as a function of u and v.

The FT of the sampling function is then the response to a point source – *the synthesized beam.*

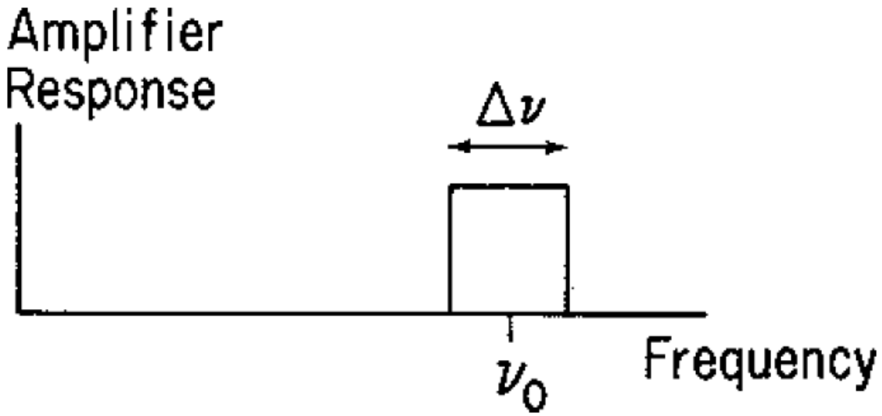
The sampling in the uv-plane decides the shape of the synthesized beam.



$$u^2 + \left(\frac{v - (L_Z/\lambda)\cos\delta_0}{\sin\delta_0}\right)^2 = \frac{L_X^2 + L_Y^2}{\lambda^2}$$

$$V(-u,-v) = V^*(u,v)$$

Effect of bandwidth



$$\tau_g = b \sin(\theta)/c$$

$$r = A_0|V| \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \cos\left(2\pi\nu\tau_g - \phi_V\right)\, d\nu$$

$$= A_0|V|\Delta\nu \frac{\sin \pi\Delta\nu\tau_g}{\pi\Delta\nu\tau_g} \cos\left(2\pi\nu_0\tau_g - \phi_V\right)$$

- Bandwidth leads to a modulation of the fringe with a sinc function.
- Introduction of delay tracking to remove this effect: however it is only valid for the delay tracking centre.

Bandwidth smearing

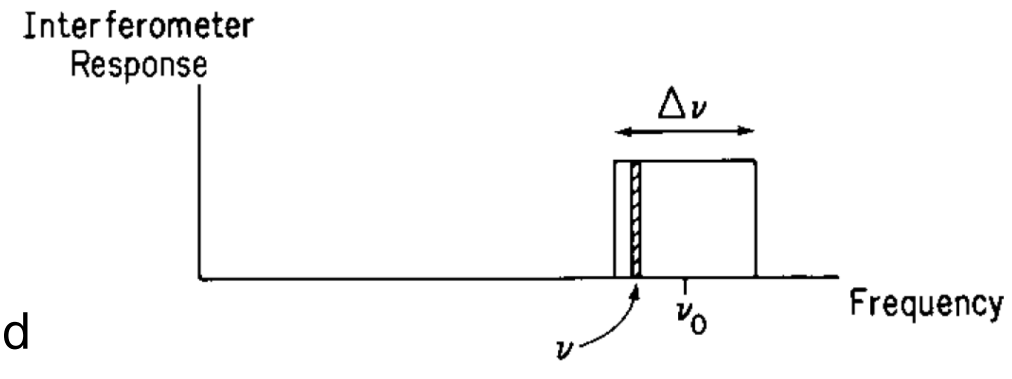
The bandwidth over which the signal that is delay tracked only at the central frequency is averaged and this lead to blurring in the image.

u_0,v_0 for the central frequency and u and v for another frequency.

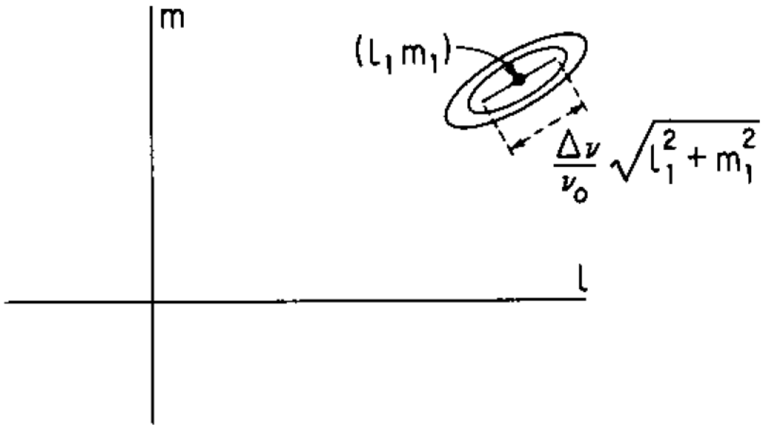
$$V\left(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v\right) \rightleftharpoons \left(\frac{\nu}{\nu_0}\right)^2 I\left(\frac{\nu}{\nu_0}l, \frac{\nu}{\nu_0}m\right)$$

Range of variation in the coordinates decided by ν/ν_0

Introduces a *radial smearing* proportional to their distance from the tracking centre.



(a)

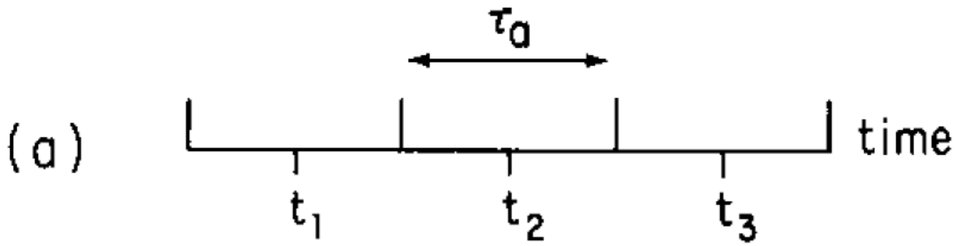


(b)

Will become significant when it becomes of the order of the synthesized beam.

Time average smearing

Data are separated into time intervals τ_a



Data within a time interval $\pm\tau_a/2$ are all clubbed into one sample.

Consider a telescope at the north pole: uv-tracks are concentric circles. Rotating at the angular velocity of the Earth ω_e

The time offset τ in assigning the uv-coordinates will result in visibility rotation by an amount $\omega_e\tau$

Time average smearing

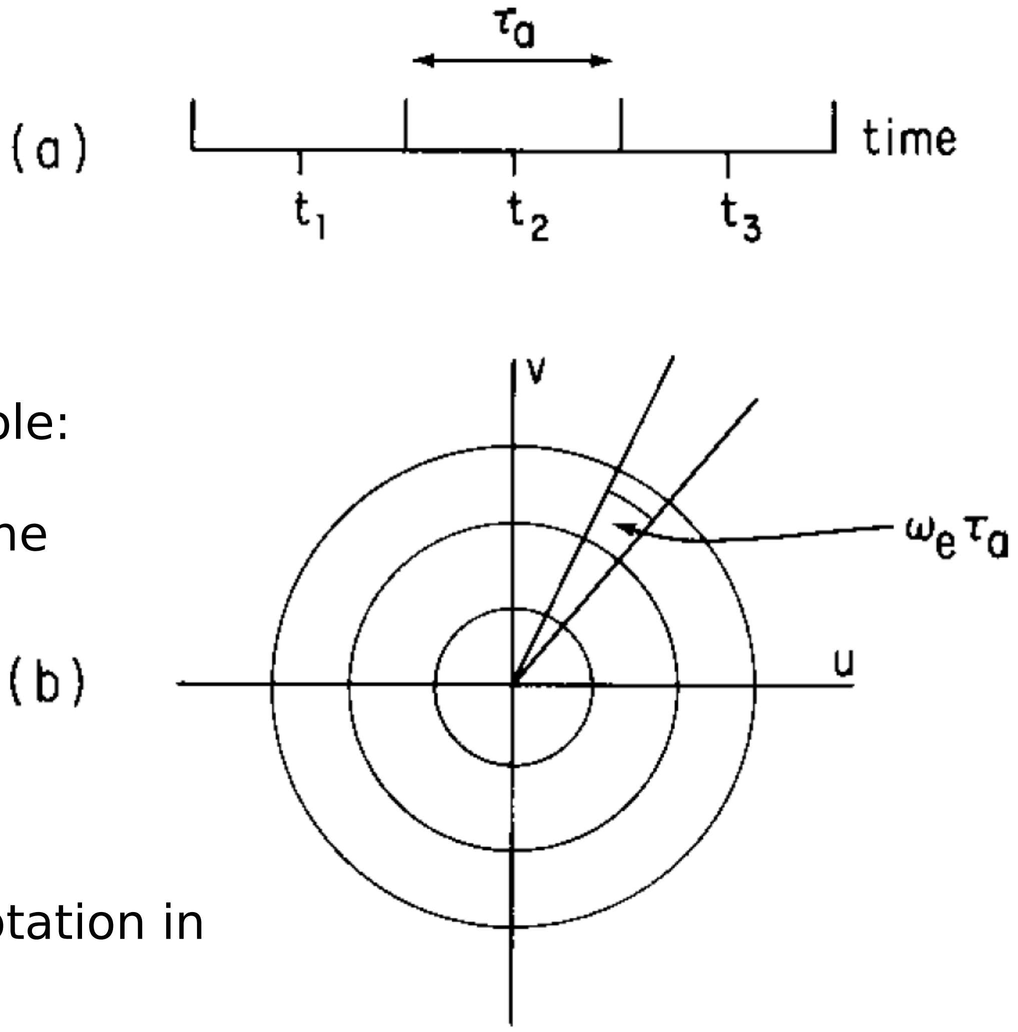
Data are separated into time intervals τ_a

Data within a time interval $\pm\tau_a/2$ are all clubbed into one.

Consider a telescope at the north pole: uv-tracks are concentric circles. Rotating at the angular velocity of the Earth ω_e

The time offset of assigning the coordinates will be $\omega_e \tau_a$

Rotation in one domain results in rotation in another for the FT.



Time average smearing

Data are separated into time intervals τ_a

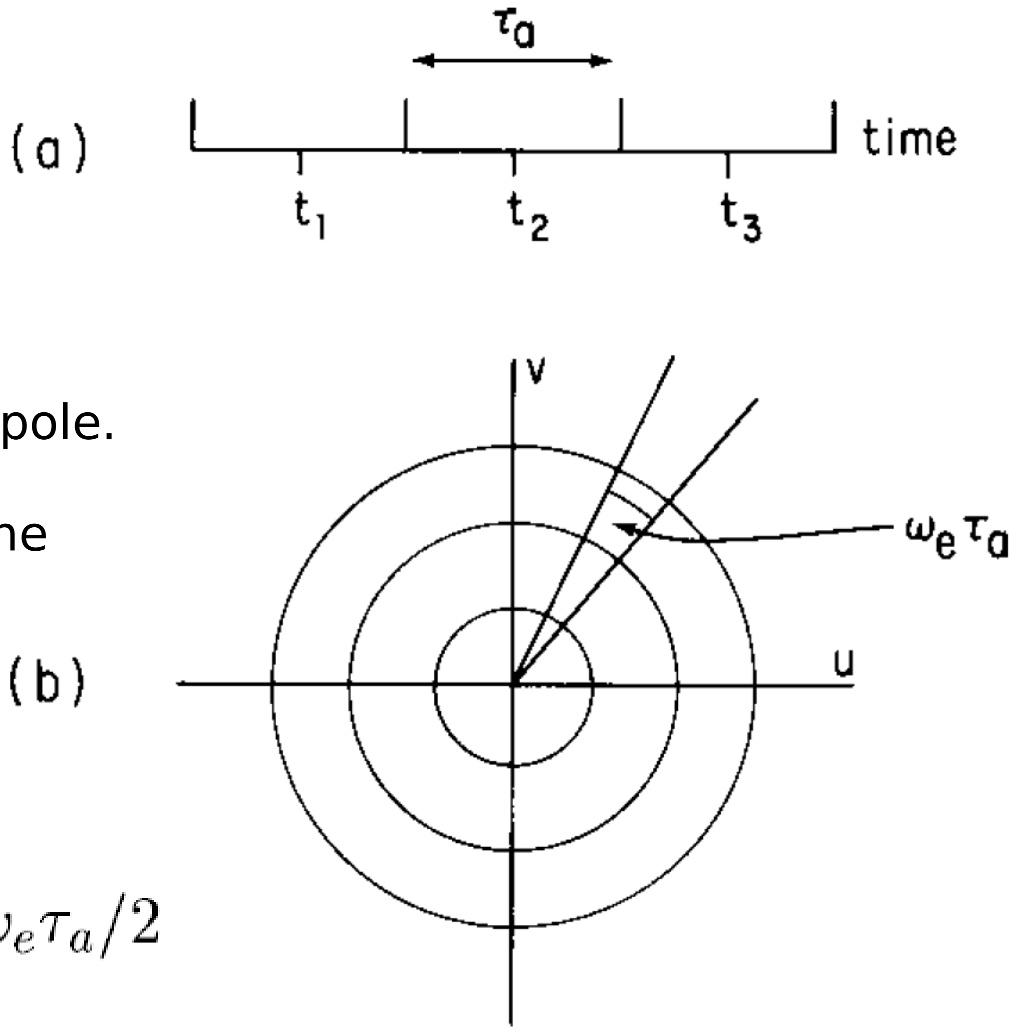
Data within a time interval $\pm\tau_a/2$ are all clubbed into one.

Easily visualised for a source at the pole.
uv-tracks are concentric circles.
Rotating at the angular velocity of the Earth ω_e

The time offset of assigning the coordinates will be $\omega_e\tau_a$

Image offsets distributed over $\pm\omega_e\tau_a/2$

$$\omega_e\tau_a\sqrt{l^2+m^2}$$



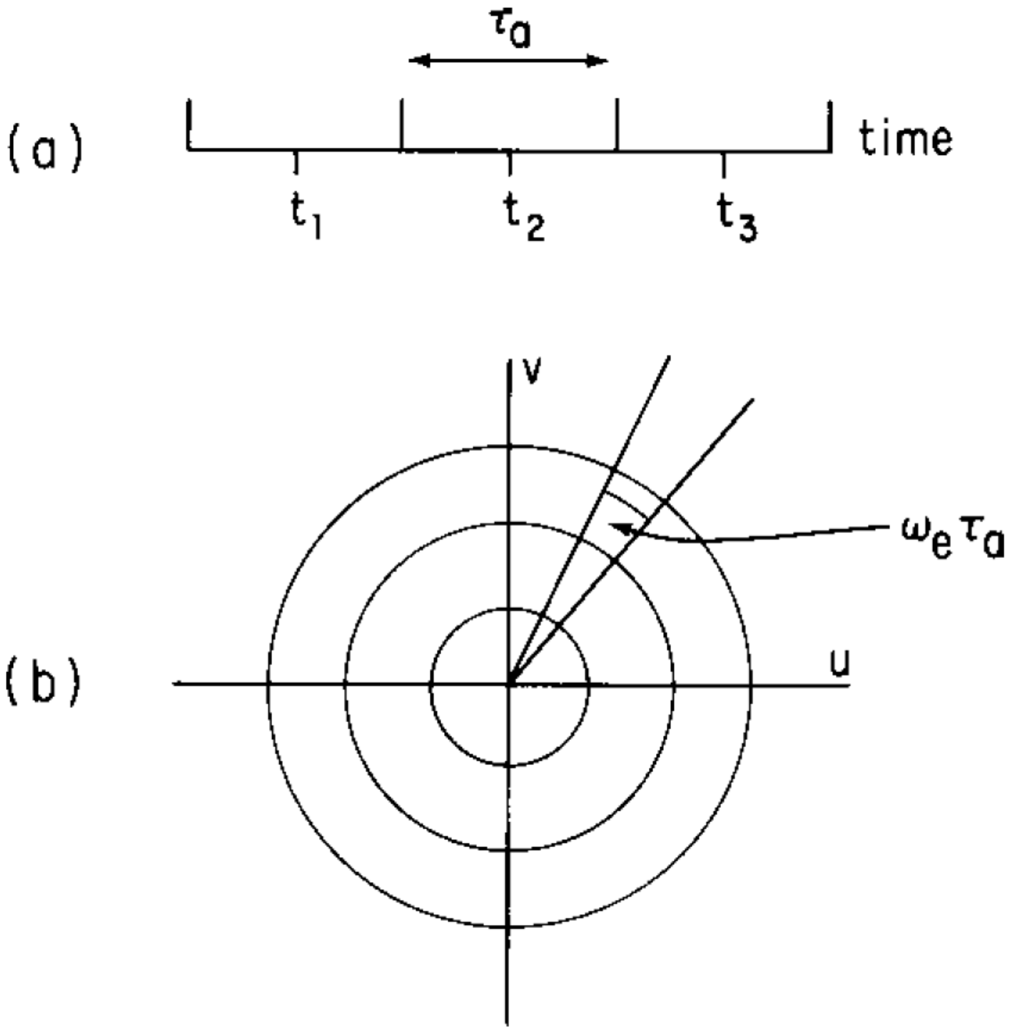
Time average smearing

Data are separated into time intervals τ_a

Data within a time interval $\pm\tau_a/2$ are all clubbed into one.

In general for a non-polar source and a non E-W baseline the effect is not a simple rotation.

Full treatment of BW and time average smearing in:
Chapter 18, Synthesis Imaging in Radio Astronomy
Also see Sec. 6.4 in Interferometry and synthesis in radio astronomy



Short summary

- FT of the aperture gives the antenna pattern.
- What we measure is the convolution of antenna pattern with the sky.

- For an interferometer we have the “synthesized” aperture: uv-coverage.
- FT of the uv-coverage gives the synthesized beam.
- And we measure visibilities of the sky convolved with the synthesized beam and attenuated by the response of the individual antenna called the primary beam.
- To obtain a good image, one needs a well sampled aperture. And the design of antenna configurations is aimed towards obtaining the best uv-coverage with minimum number of elements.

Sampled visibility

Consider observation of a source. Measurements of visibilities with various baseline lengths are made. Sampling can be represented by a series of delta functions as:

Visibility domain

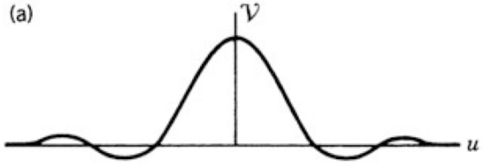
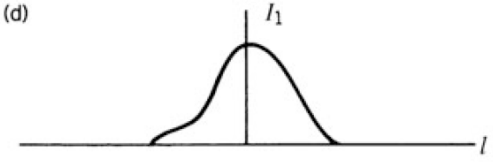
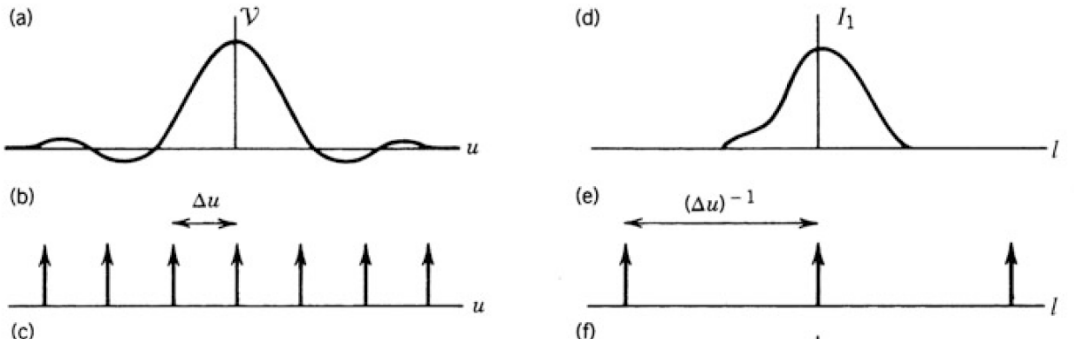


Image domain



Sampled visibility

Consider observation of a source. Measurements of visibilities with various baseline lengths are made. Sampling can be represented by a series of delta functions as:



$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function

Bracewell and Roberts 1954

Sampled visibility

Consider observation of a source. Measurements of visibilities with various baseline lengths are made. Sampling can be represented by a series of delta functions as:

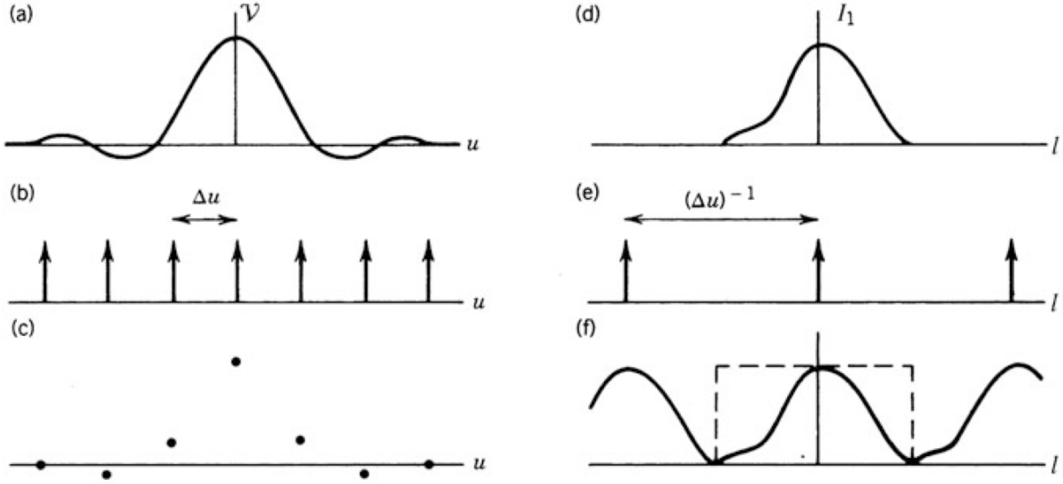
$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function

Bracewell and Roberts 1954

FT of sampling:

$$\mathrm{III}(l\Delta u) = \frac{1}{\Delta u} \sum_{p=-\infty}^{\infty} \delta\left(l - \frac{p}{\Delta u}\right)$$



In the *l* domain the FT of sampled visibility is the convolution of the FT of V(u) with $\mathrm{III}(l\Delta u)$

(FT of V(u) is I(*l*)).

Sampled visibility

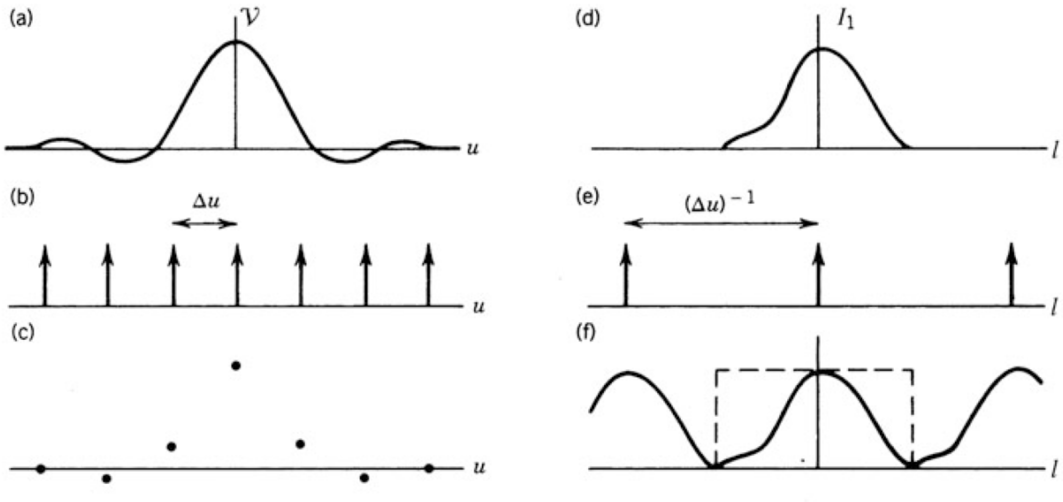
Consider observation of a source. Measurements of visibilities with various baseline lengths are made. Sampling can be represented by a series of delta functions as:

$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function

FT of sampling:

$$\mathrm{III}(l\Delta u) = \frac{1}{\Delta u} \sum_{p=-\infty}^{\infty} \delta\left(l - \frac{p}{\Delta u}\right)$$



In the *l* domain the FT of sampled visibility is the convolution of the FT of V(u) with $\mathrm{III}(l\Delta u)$

So long as I(*l*) is a source of finite extent, I(*l*) will not overlap so long as I(l) is non-zero only in the range of l that is not greater than $(\Delta u)^{-1}$

Sampled visibility

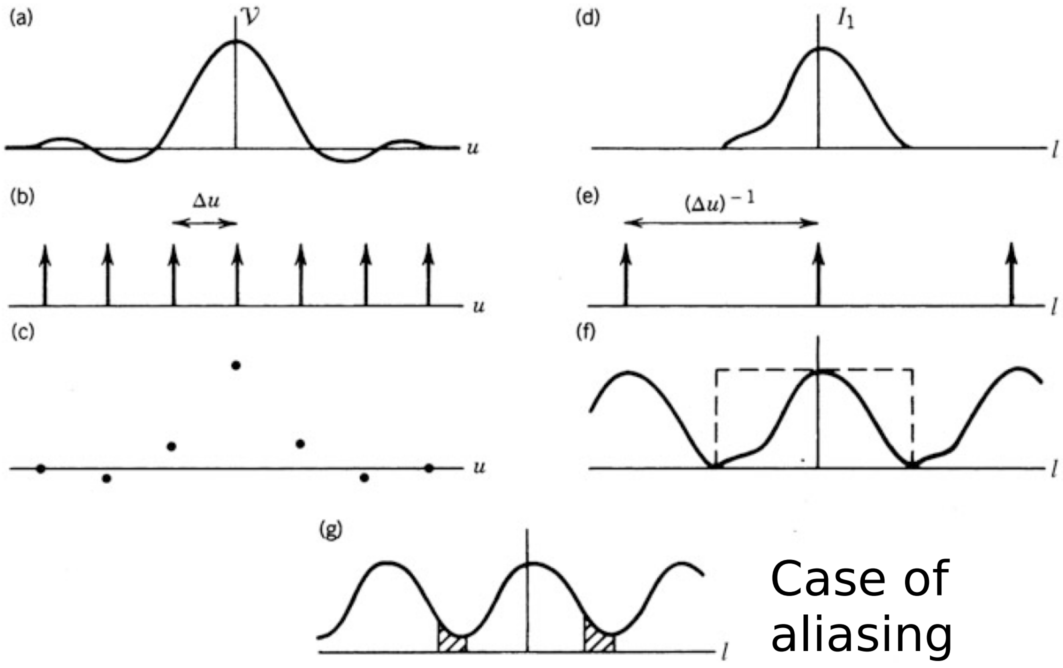
Consider observation of a source. Measurements of visibilities with various baseline lengths are made. Sampling can be represented by a series of delta functions as:

$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function

FT of sampling:

$$\mathrm{III}(l\Delta u) = \frac{1}{\Delta u} \sum_{p=-\infty}^{\infty} \delta\left(l - \frac{p}{\Delta u}\right)$$

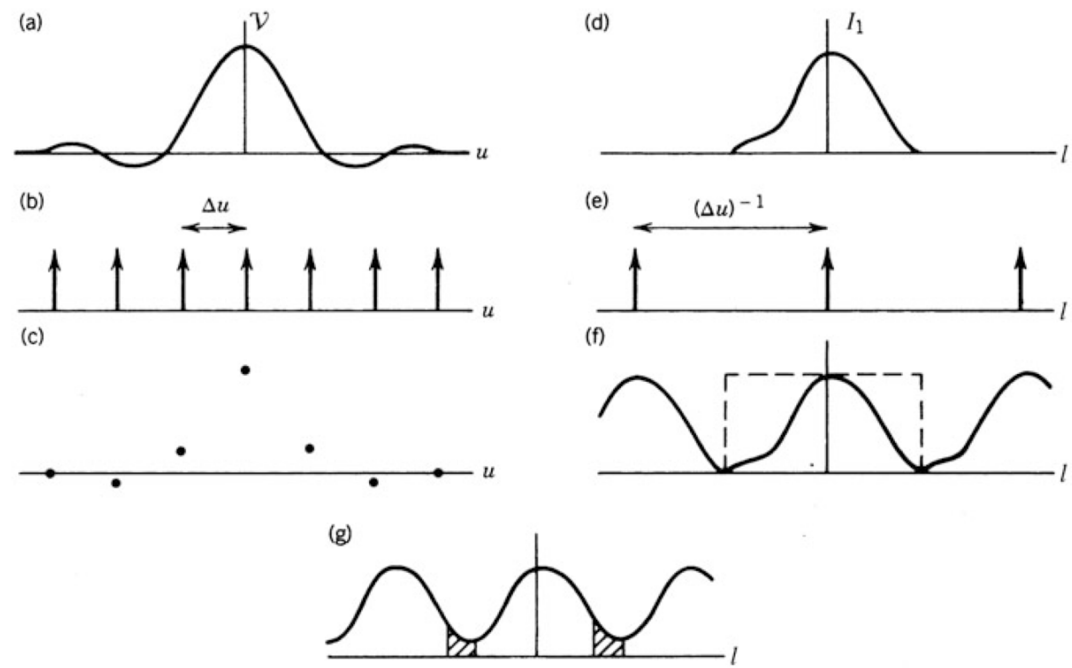


Case of aliasing

To avoid aliasing, the sampling interval should not be greater than the reciprocal of the interval in l in which I(l) is non-zero.

Sampled visibility

Consider observation of a source. Measurements of visibilities with various baseline lengths are made. Sampling can be represented by a series of delta functions as:



$$\left[\frac{1}{\Delta u}\right] \text{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

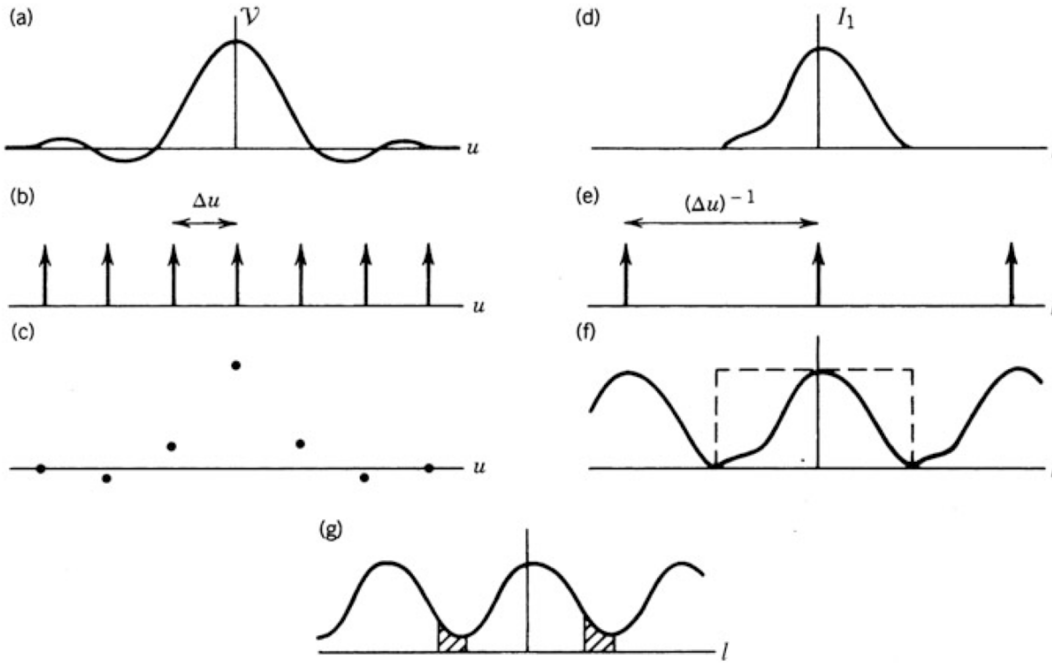
Shah function

Interpolation in u-domain will correspond to removing the replications in the *l* domain.
This corresponds to a rectangular function in l domain – which is the sinc function in the u-domain.

$$\frac{\sin \pi u / \Delta u}{\pi u}$$

Sampled visibility

Consider observation of a source. Measurements of visibilities with various baseline lengths are made. Sampling can be represented by a series of delta functions as:



$$\left[\frac{1}{\Delta u}\right] \mathrm{III}\left(\frac{u}{\Delta u}\right) = \sum_{i=-\infty}^{\infty} \delta(u - i\Delta u)$$

Shah function

Sampling theorem for visibilities:

If the intensity distribution is nonzero only within an interval of width l_w, $I_1(l)$ *is fully specified by sampling the visibility function at points spaced* $\Delta u = l_w^{-1}$ *in u – the critical sampling interval.*

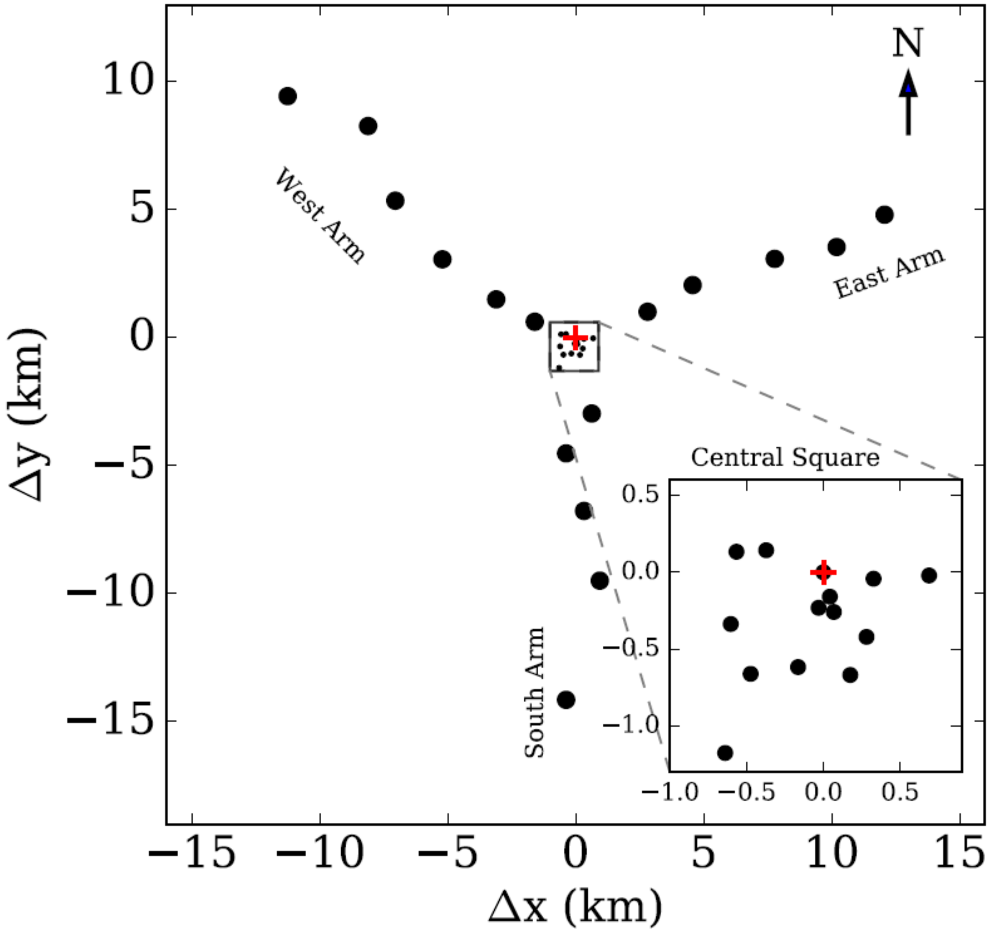
Aperture synthesis

Choice of antenna configuration made to optimize sampling of the visibility function in the u,v space.

Aperture synthesis

Choice of antenna configuration made to optimize sampling of the visibility function in the u,v space.

GMRT array configuration

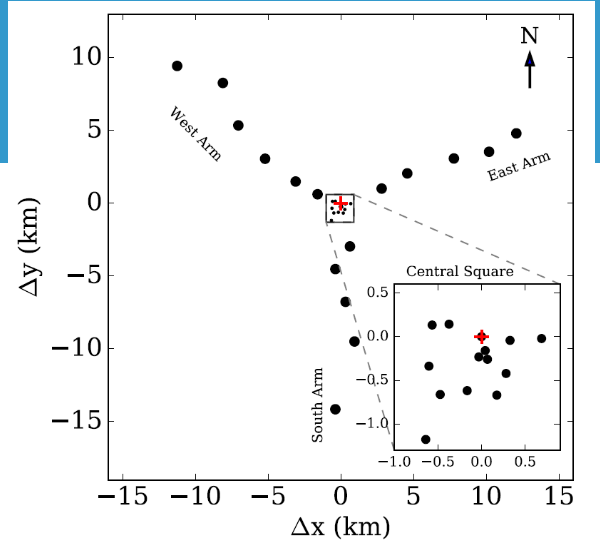


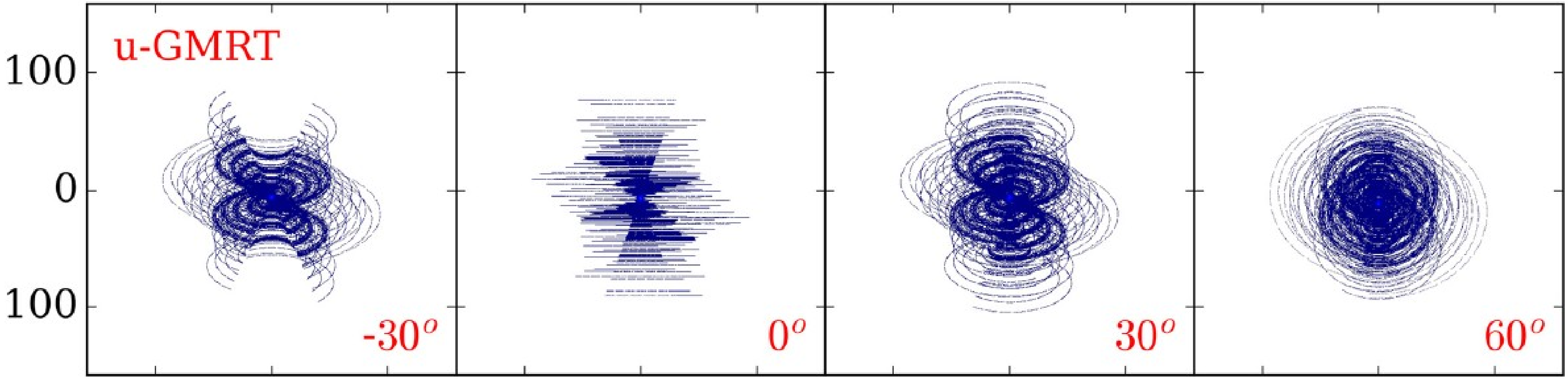
Latitude ~+19 degrees

Patra et al 2019

Aperture synthesis

GMRT array configuration and uv-coverage

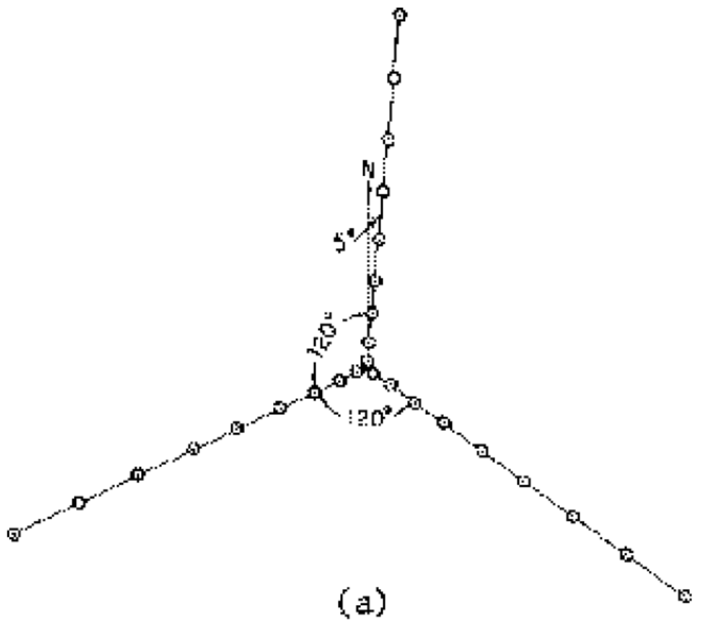




u (kilolambda)

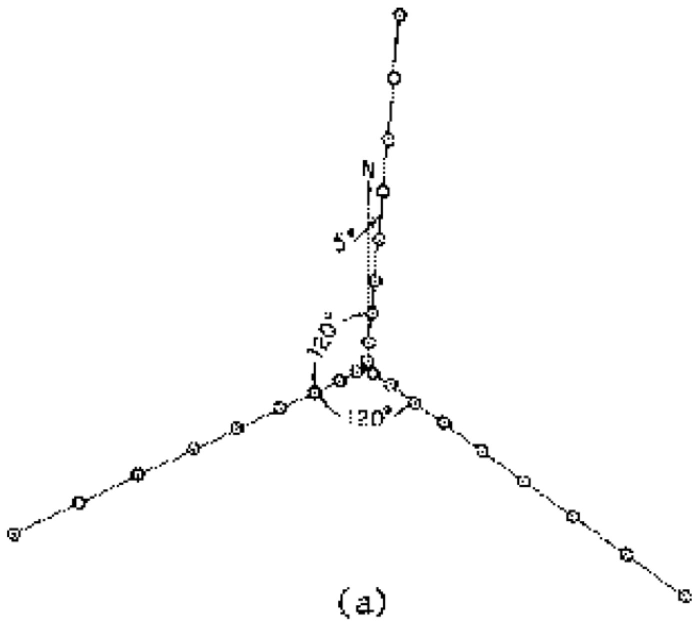
Patra et al 2019

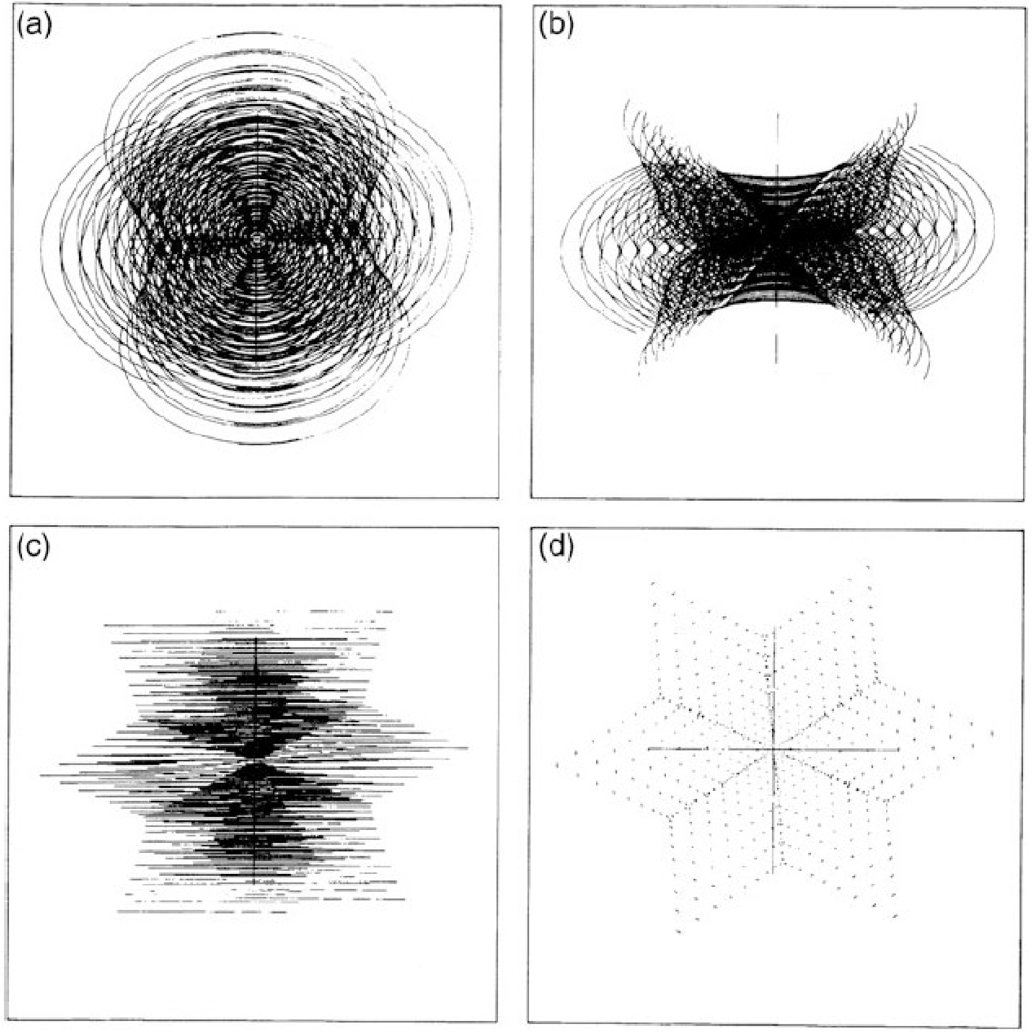
Aperture synthesis



Jansky Very Large Array configuration
Latitude ~ 34 degrees

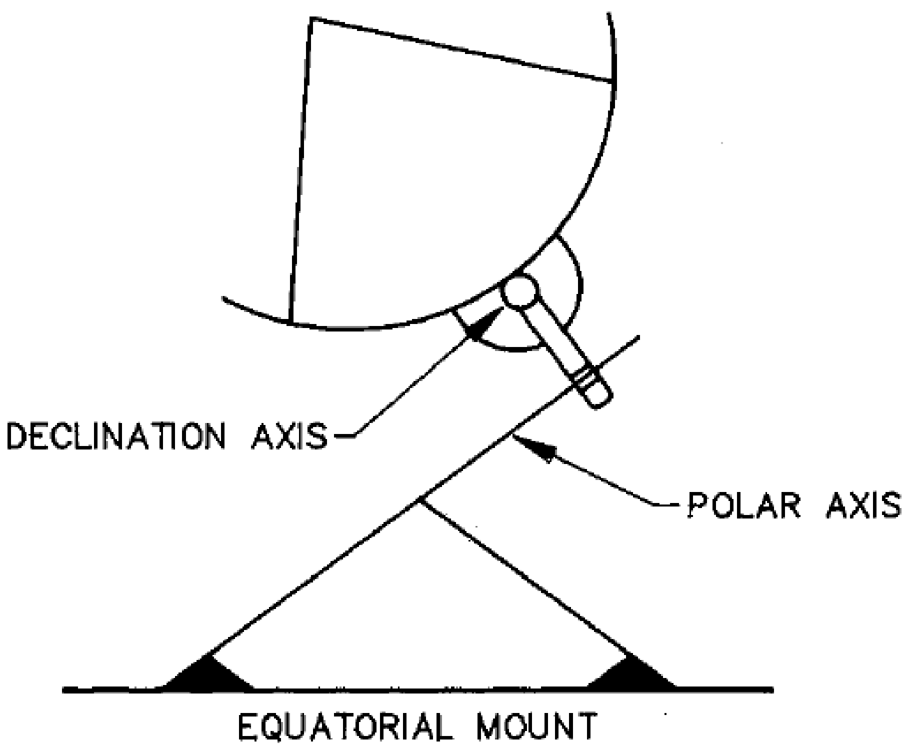
Aperture synthesis

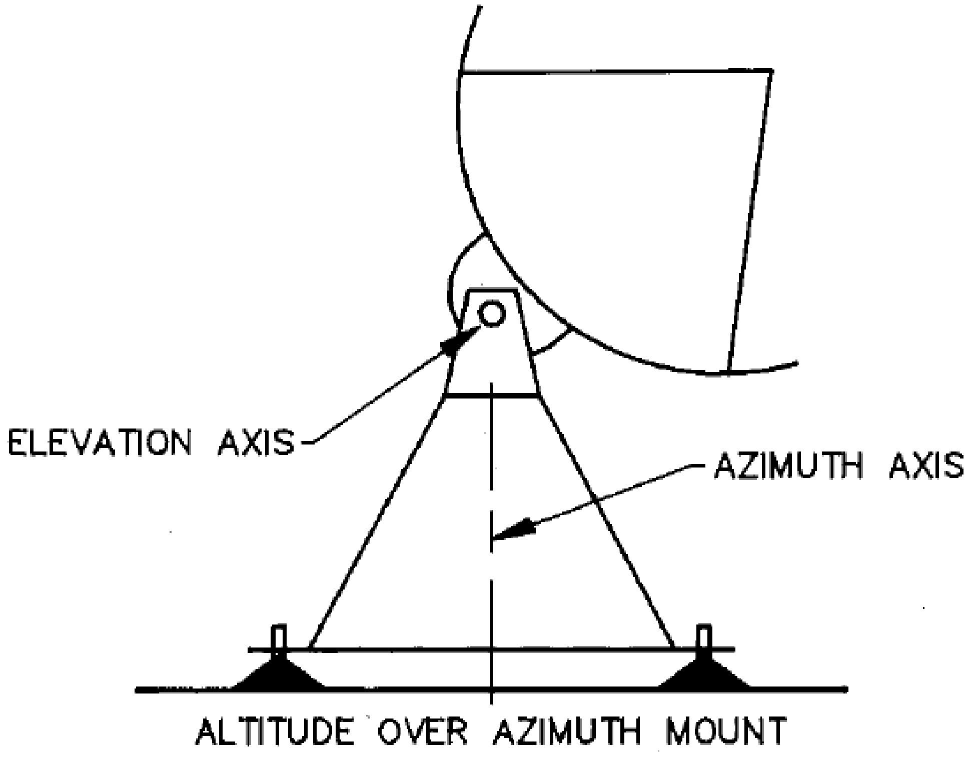




Declinations +45, +30 and 0 degrees shown in a, b and c. d is the snapshot coverage looking at the zenith.

Telescope mounts





Equatorially mounted telescopes

Westerbrok Synthesis Radio Telescope (WSRT), The Netherlands
An East-West Array

Ooty Radio Telescope

Images: Wikipedia

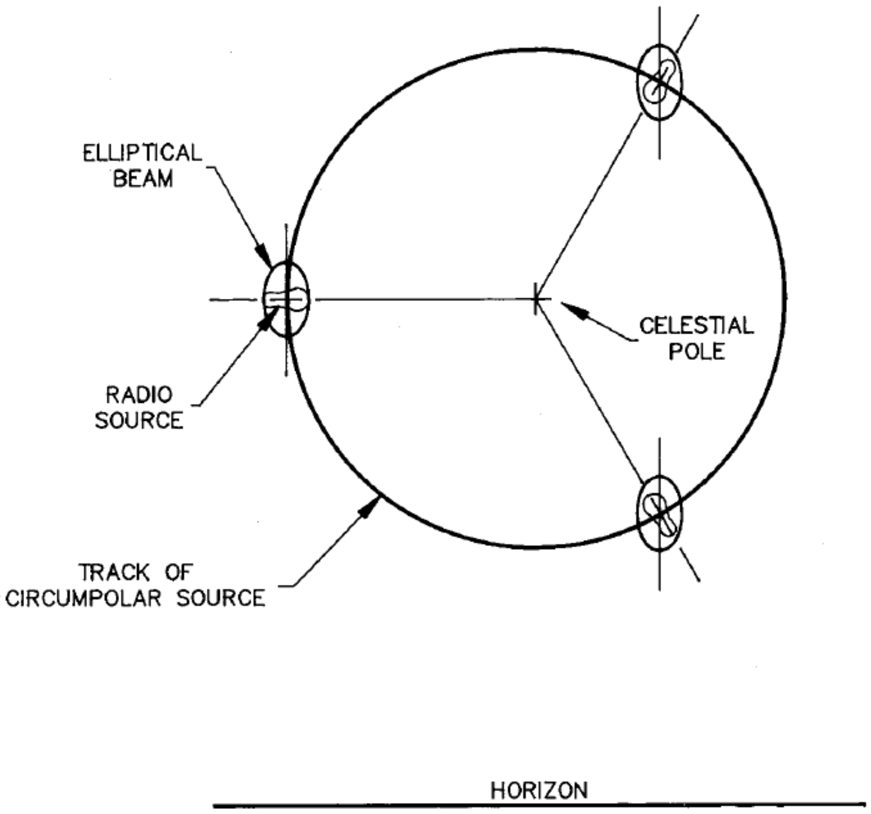
Alt-Az mounted telescopes

GMRT

JVLA

Alt-Az mount and rotation of beam

Circum polar source observed with a non-circular beam of a alt-az mounted antenna.



Alt-Az mount and rotation of beam

Circum polar source observed with a non-circular beam of a alt-az mounted antenna.

ASKAP : the feeds are rotated to compensate for this rotation.

Image: CSIRO

Before getting into correlators

- Sampling quantization
- Digital delays
- Discrete cross correlation and power spectral density (Van Vleck correction)

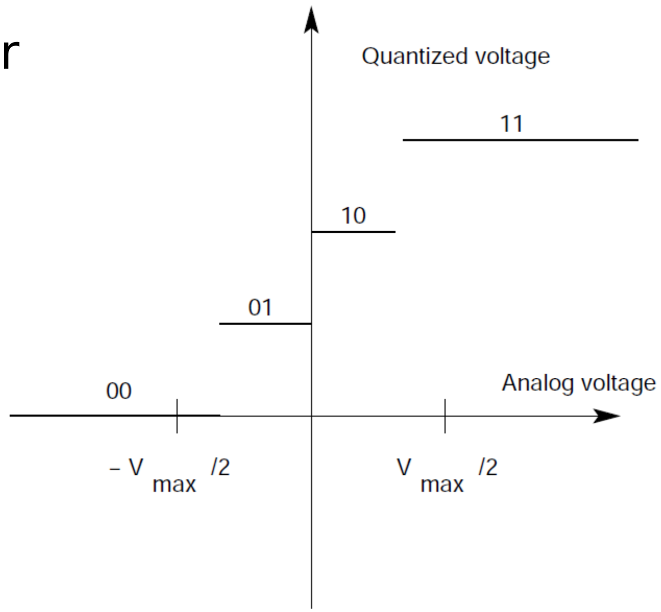
Sampling quantization

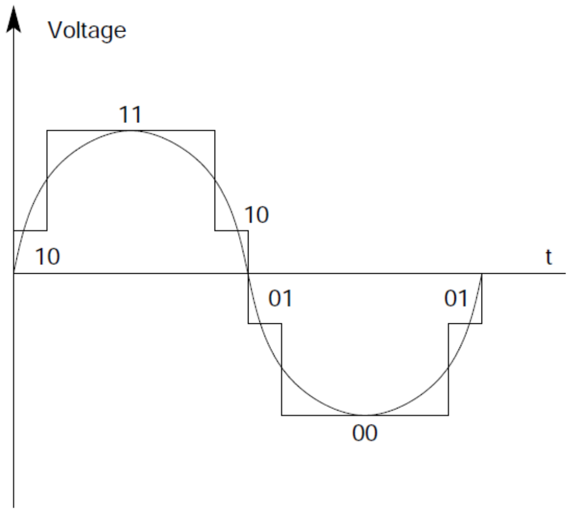
- Digital systems represent values over a limited number of bits.
- Quantized values are an integer multiple of the quantization step, q.
- Leads to distortion of signal: *quantization noise*.

Sampling quantization

- Digital systems represent values over a limited number of bits.
- Quantized values are an integer multiple of the quantization step, *q*.
- Leads to distortion of signal: *quantization noise*.

A 2-bit four level quantizer



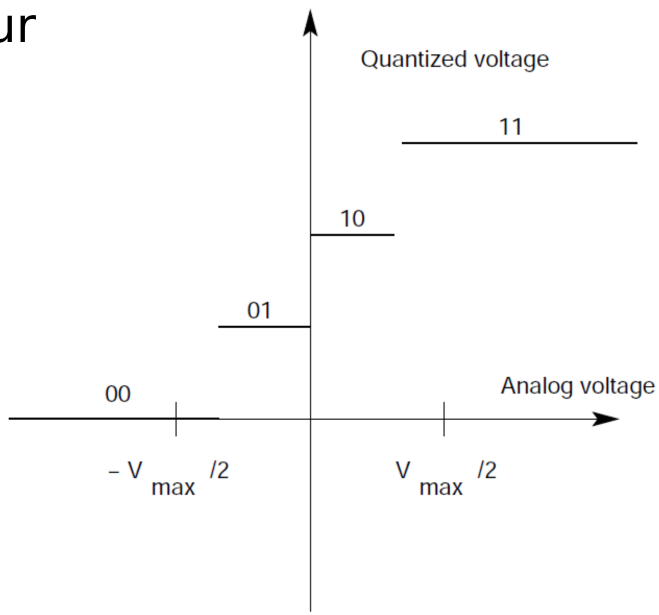


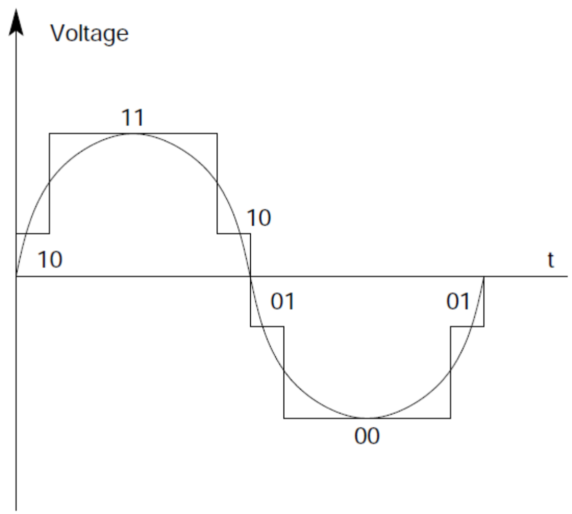
$q = V_{max}/2^2$

V_{max} can be expressed to within an error of $\pm q/2$

Sampling quantization

A 2-bit four level quantizer





Quantization distorts the sampled signal affecting both the amplitude and spectrum of the signal.

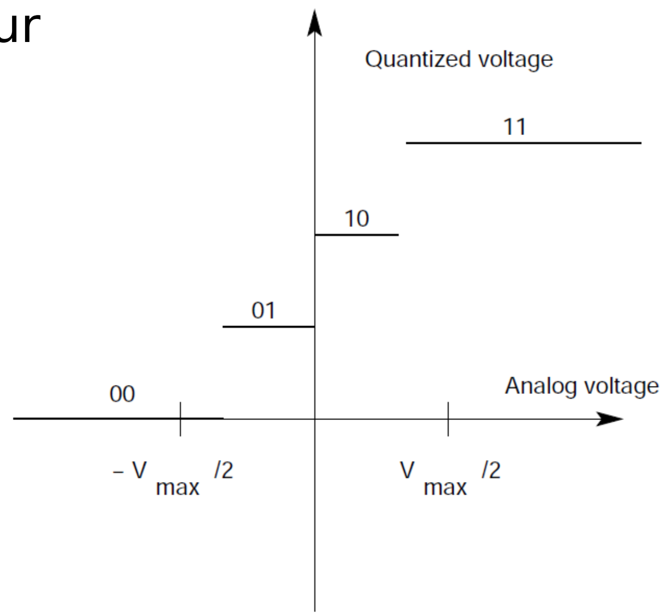
The amplitude distortion:

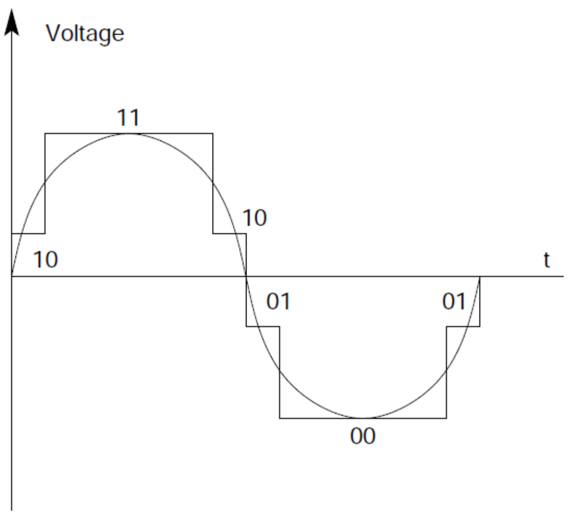
$$e(t) = s(t) - s_q(t)$$

Difference of the original and the quantized signal.

Sampling quantization

A 2-bit four level quantizer





Quantization distorts the sampled signal affecting both the amplitude and spectrum of the signal.

The amplitude distortion:

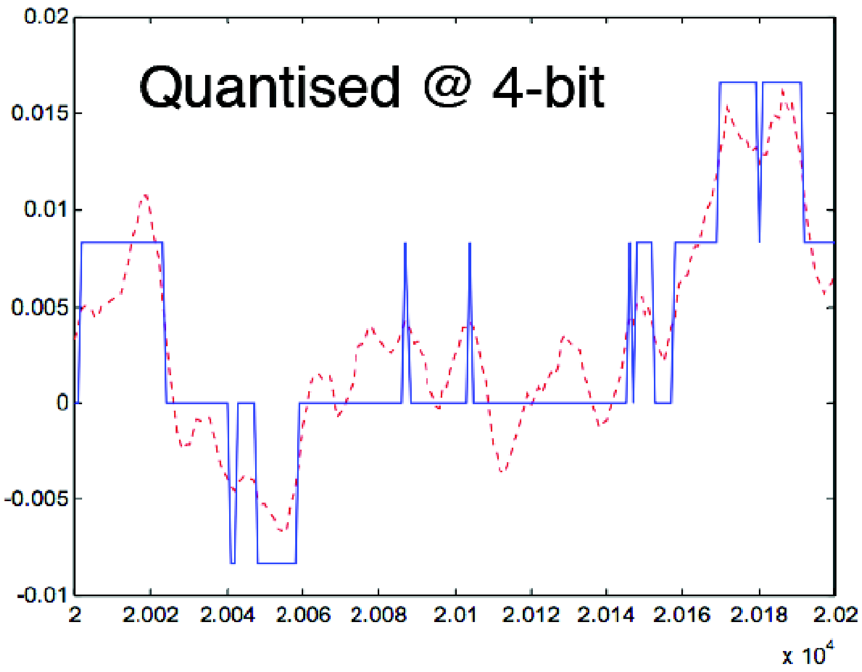
$$e(t) = s(t) - s_q(t)$$

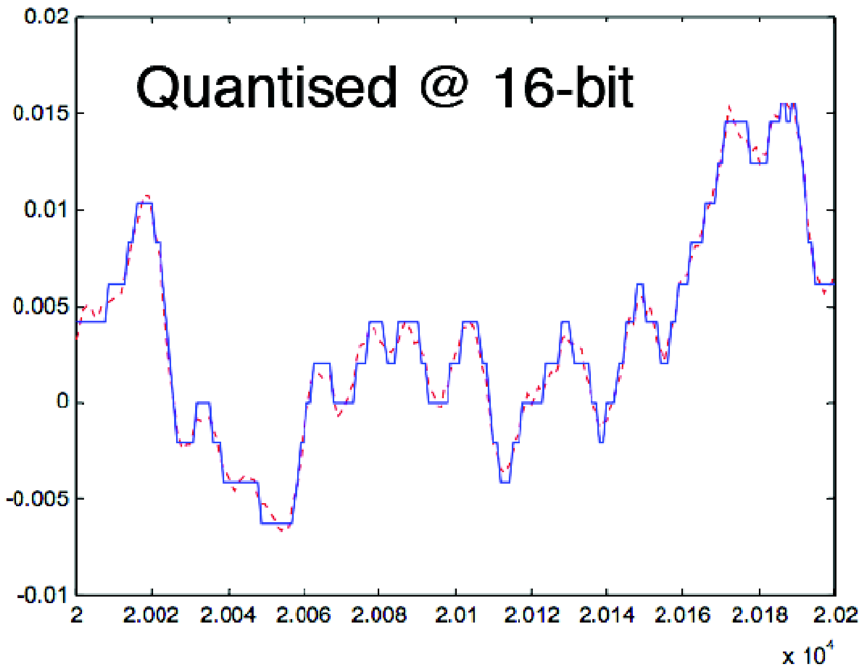
Difference of the original and the quantized signal.

Quantization noise

Sampling quantization

- Digital systems represent values over a limited number of bits.
- Quantized values are an integer multiple of the quantization step, *q*.
- Leads to distortion of signal: *quantization noise*.

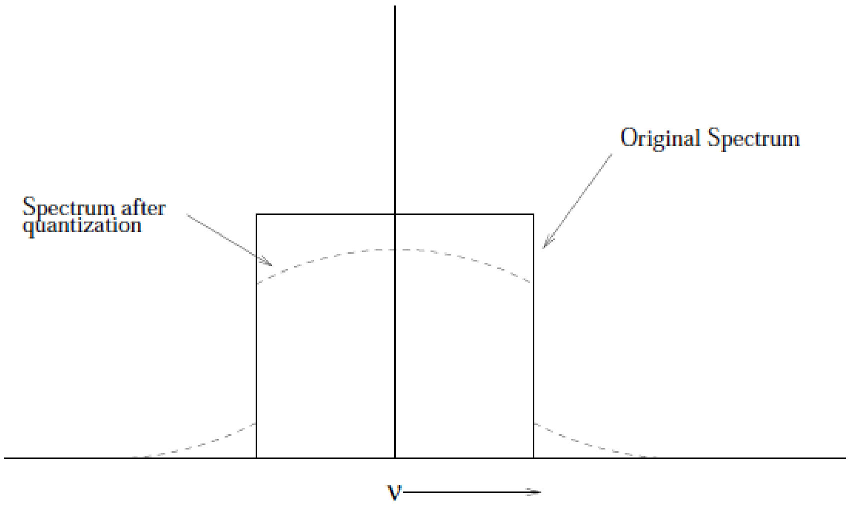




Figures: J. Roy

Quantization noise

$$e(t) = s(t) - s_q(t)$$



The variance of quantization noise under conditions of uniform quantization can be approximated to $q^2/12$.

The spectrum of a Nyquist sampled signal after quantization, extends beyond the bandwidth.

The spectral density of quantization noise within the bandwidth ($\Delta\nu$) can be considered uniform and is $q^2/12\Delta\nu$

Oversampling can reduce the aliased power.

Dynamic range

Quantizer operates over a limited range of input voltage amplitude: referred to a the dynamic range.

E. g. For quantization with M bits the largest value represented is 2^M -1 for binary representation.

Amplitudes exceeding the range get clipped.

The minimum change in the signal that can be expressed is limited to the quantization step q.

For an ideal quantizer with uniform quantization the dynamic range is $3/2 * 2^{2M}$.

Digital delay

Let s(t) be a signal sampled with sampling frequency f_s. Then the digitized signal with a delay of m samples,

$$s(n - m)$$

Corresponds to a delay of m x $1/f_s$

Delay only at the level of integer multiple of the inverse of the sampling frequency can be corrected in this setup.

Any delay smaller than this will remain uncompensated. Such delay produces a phase gradient in the FT.

The fractional delay is removed by introducing phase gradients.

Discrete correlation and power spectral density

Cross-correlation of two signals s_1 and s_2

$$R_c(\tau) = \ < s_1(t) s_2(t+\tau) >$$

In practice the estimator is:

$$m \times 1/f_s$$

$$R(m) = \frac{1}{N} \sum_{n=0}^{N-1} s_1(n) s_2(n+m) \quad 0 \leq m \leq M$$

m denotes the samples by which the signal s_2 is delayed and M denotes the maximum delay.

R(m) is a random variable whose expectation value converges to

$$R_c(\tau = \frac{m}{f_s}) \qquad\qquad N \rightarrow \infty$$

The correlation estimated using digitized samples deviates from that with infinite amplitude precision.

The relation between the true correlation and that measured can be written as:

$$\hat{R}_c(m/f_s) = \mathbf{F}(\hat{R}(m))$$

Normalized correlation functions – normlization to square root of zero lag autocorrelations of s_1 and s_2. *F is the correction function.*

The correlation estimated using digitized samples deviates from that with infinite amplitude precision.

The relation between the true correlation and that measured can be written as:

$$\hat{R}_c(m/f_s) = \mathbf{F}(\hat{R}(m))$$

Normalized correlation functions – normlization to square root of zero lag autocorrelations of s_1 and s_2. *F is the correction function.*

For one-bit quantization this correction has the form:

“Van Vleck correction”

$$\hat{R}_c(m/f_s) = \sin(\frac{\pi}{2}\hat{R}(m))$$

The correlation estimated using digitized samples deviates from that with infinite amplitude precision.

The relation between the true correlation and that measured can be written as:

$$\hat{R}_c(m/f_s) = \mathbf{F}(\hat{R}(m))$$

Normalized correlation functions – normlization to square root of zero lag autocorrelations of s_1 and s_2. *F is the correction function.*

For one-bit quantization this correction has the form:

“Van Vleck correction”

$$\hat{R}_c(m/f_s) = \sin(\frac{\pi}{2}\hat{R}(m))$$

Non-linear correction – needs to be applied before any other operation on the cross correlation.

The correlation estimated using digitized samples deviates from that with infinite amplitude precision.

The relation between the true correlation and that measured can be written as:

$$\hat{R}_c(m/f_s) = \mathbf{F}(\hat{R}(m))$$

Normalized correlation functions – normlization to square root of zero lag autocorrelations of s_1 and s_2. *F is the correction function.*

For one-bit quantization this correction has the form:

"Van Vleck correction"

$$\hat{R}_c(m/f_s) = \sin(\frac{\pi}{2}\hat{R}(m))$$

Non-linear correction – needs to be applied before any other operation on the cross correlation.

For derivation see : Chp. 8 of Interferometry and Synth. Imaging in RA

Cross correlators

Lag correlators (XF):
Cross-correlation followed by Fourier transformation.

FX correlators:
Fourier transformation followed by cross-correlation.

Digital implementation

$$\begin{aligned} r_R(\tau_g) &= |\mathcal{V}|\cos(2\pi\nu\tau_g - 2\pi\nu_{BB}\tau_i + \Phi_{\mathcal{V}}) \\ &= |\mathcal{V}|\cos(2\pi\nu_{LO}\tau_g - 2\pi\nu_{BB}\Delta\tau_i + \Phi_{\mathcal{V}}) \end{aligned}$$

Delays:

Delay compensation in baseband as opposed to in the RF

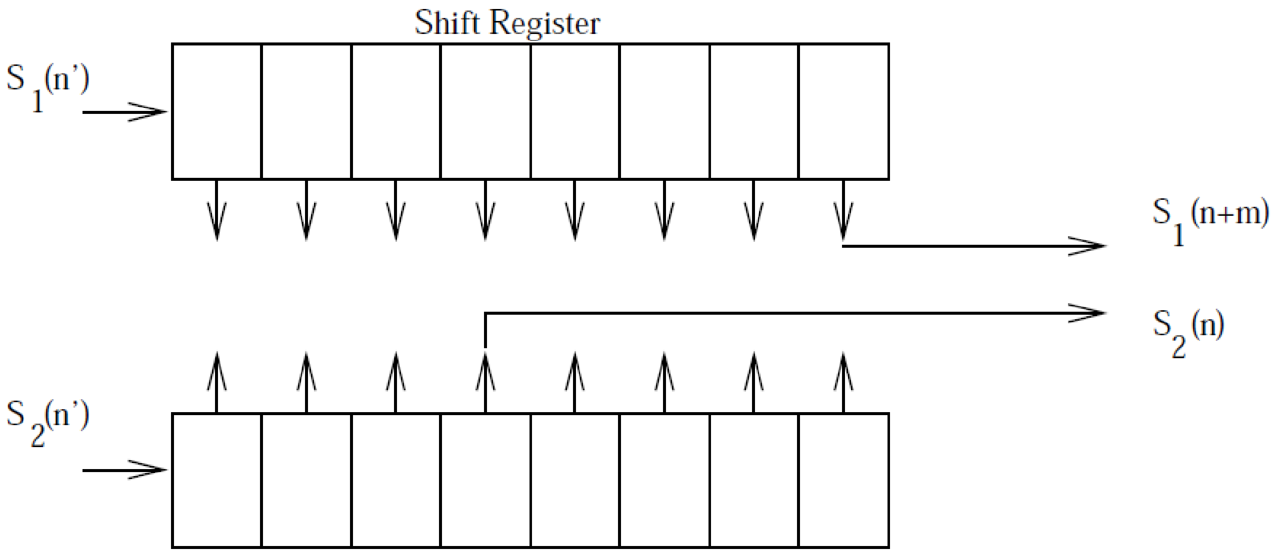
Constant across the band, but the geometric delay is changing with time

Finite precision of delay compensation

Depends on frequency

Both need dynamic compensation.

Delay compensation using shift registers

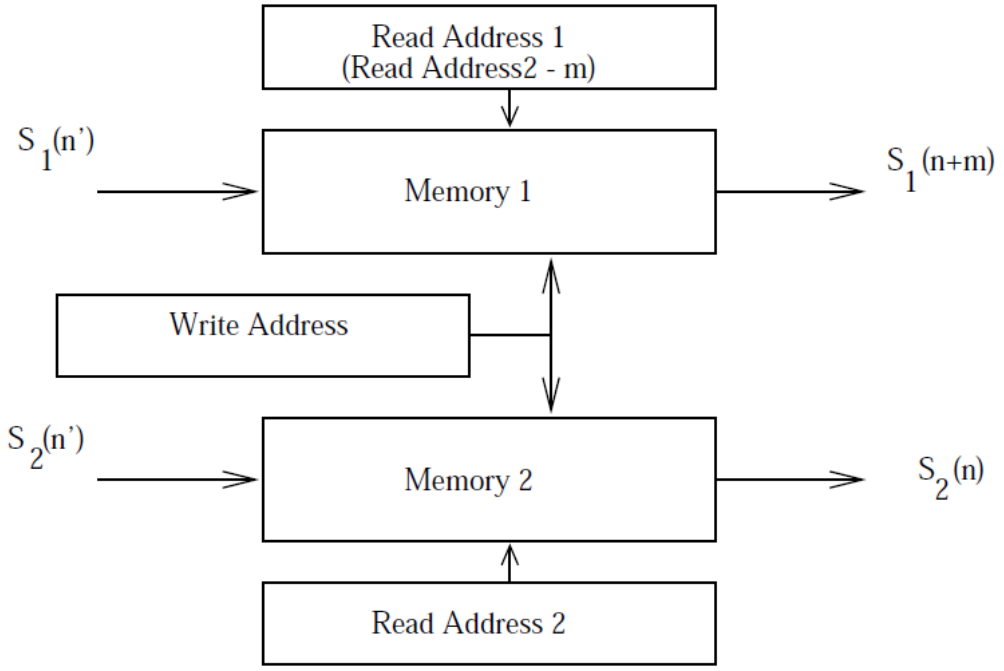


Delay implementation using shift registers

The length of the shift registers is adjusted to compensate the delay.

Delay compensation using memory

Signals from both the antennas are written to RAM and read out.



Delay implementation using Memory

Offset between read pointer and write pointer adjusted to achieve delay compensation.

Fractional delays

Introducing delays in the sampling clocks.

Or introducing a phase gradient after FT.

Fringe stopping is done by changing the phase of the LO such that,

$$2\pi\nu_{LO}\tau_g - \phi_{LO} = 0$$

Achieved digitally by multiplying the sampled time series with the factor:

$$e^{-j\phi_{LO}}$$

Fractional delays

$$\phi_{LO}(t) = 2\pi\nu_{LO}\tau_g = 2\pi\nu_{LO}\frac{b\sin(\Omega t)}{c}$$

The fringe. Omega is the angular rotation speed of earth.

$$\phi_{LO}(t) = \phi_{LO}(t_0) + 2\pi\nu_{LO}\frac{b\Omega\cos(\Omega t_0)}{c}\Delta t$$

For a short time interval this is the approximation.

$\phi_{LO}(t)$ is the phase of an oscillator with frequency $\nu_{LO}\frac{b\Omega\cos(\Omega t_0)}{c}$

After a short time interval this has to be updated: digital implementation of this is called as the number controlled oscillator (NCO).

Complex correlator

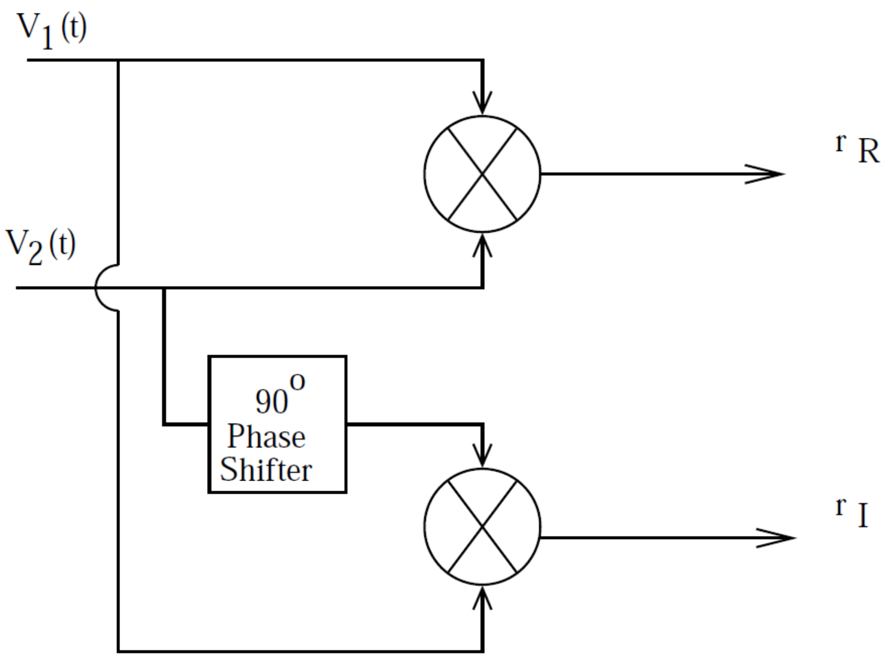
$$r_R = |\mathcal{V}| \cos(\Phi_{\mathcal{V}})$$

$$r_I(\tau_g) = |\mathcal{V}| \cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}} + \pi/2)$$

After compensating the delay,

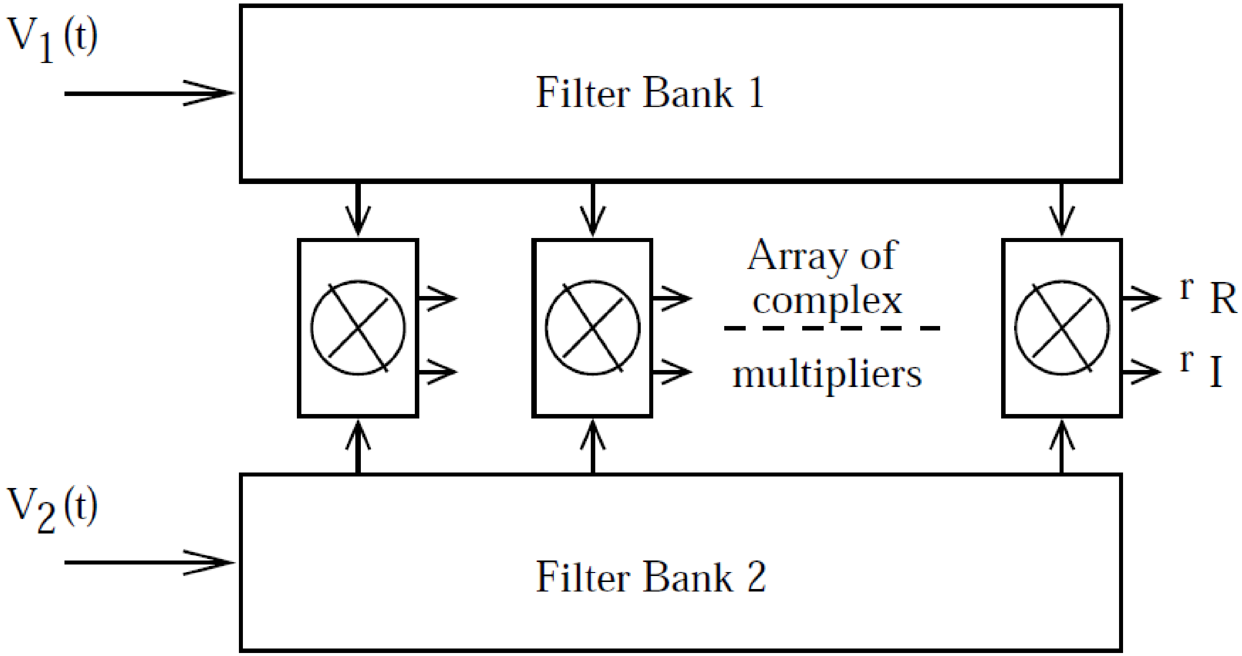
$$\begin{aligned} r_I &= |\mathcal{V}| \cos(\Phi_{\mathcal{V}} + \pi/2) \\ &= |\mathcal{V}| \sin(\Phi_{\mathcal{V}}) \end{aligned}$$

$$\begin{aligned} |\mathcal{V}| &= \sqrt{r_R^2 + r_I^2} \\ \Phi_{\mathcal{V}} &= \tan^{-1}(\frac{r_I}{r_R}) \end{aligned}$$



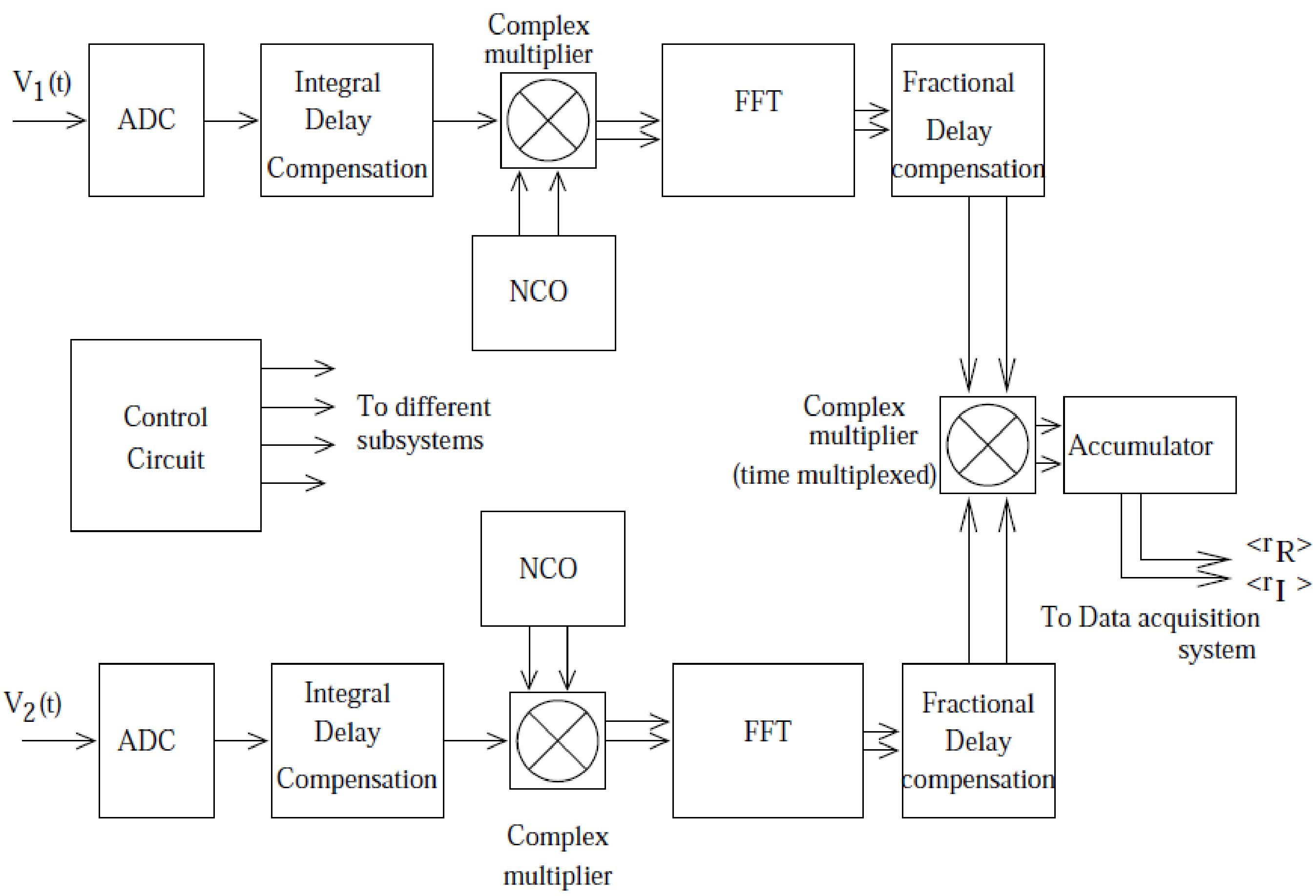
$$\mathcal{V} = r_R + j r_I$$

FX correlator (schematic)



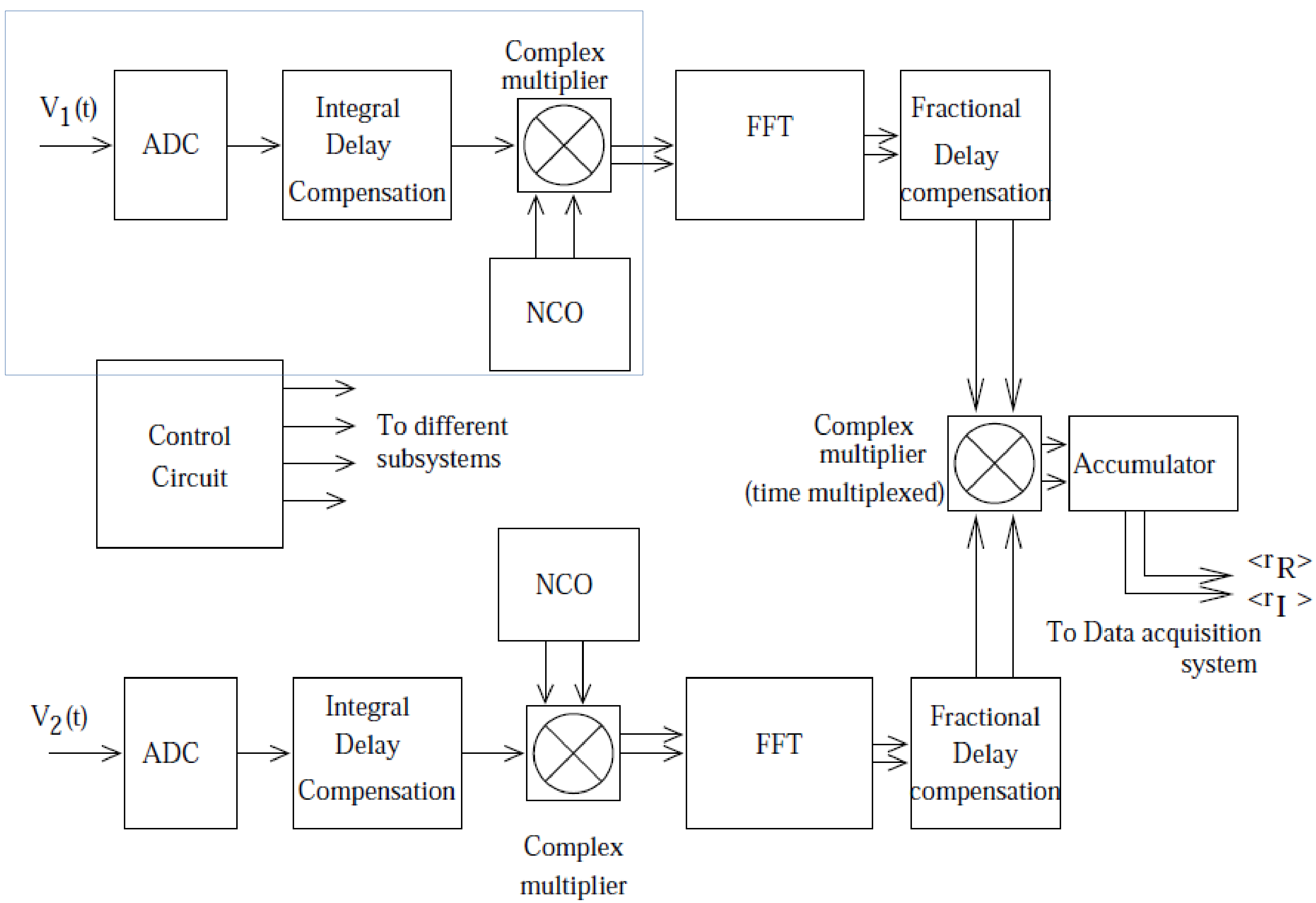
The band limited signal is decomposed into spectral components using a filter bank and each filter output is cross correlated with the corresponding one from the other using a complex multiplier.

FX correlator: implementation



Analog to digital convertor
Number Controlled Oscillator

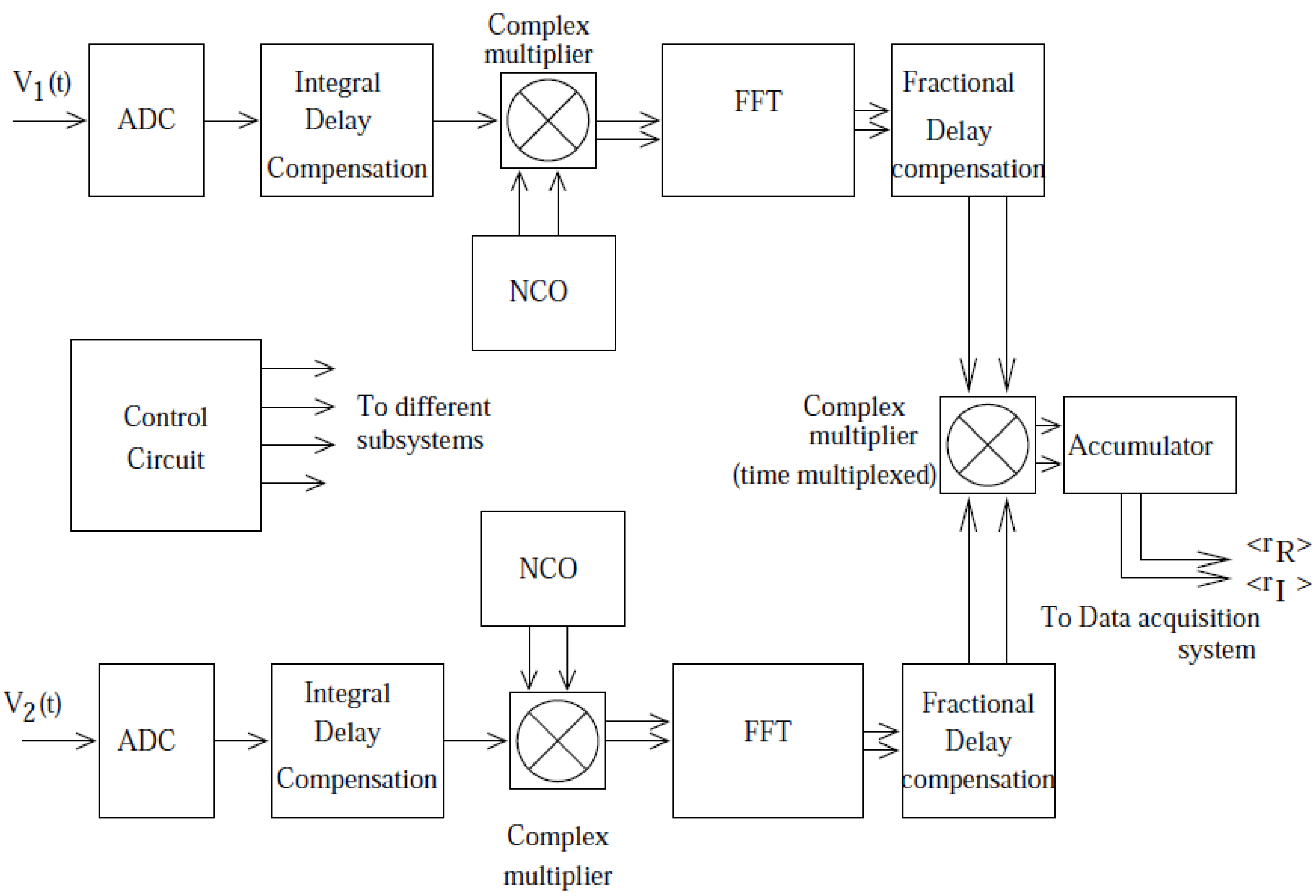
FX correlator: implementation



- Signal is digitized.
- Integral delays are compensated for.
- These then multiplied with NCO output for fringe stopping.

NCO: Number controlled oscillator

FX correlator: implementation



- Signal is digitized.
- Integral delays are compensated for.
- These then multiplied with NCO output for fringe stopping.
- Then passed through FFT block to realise a filter bank.
- Phase gradients applied for fractional delay compensation.
- Then multiplied with the similar output of second antenna.

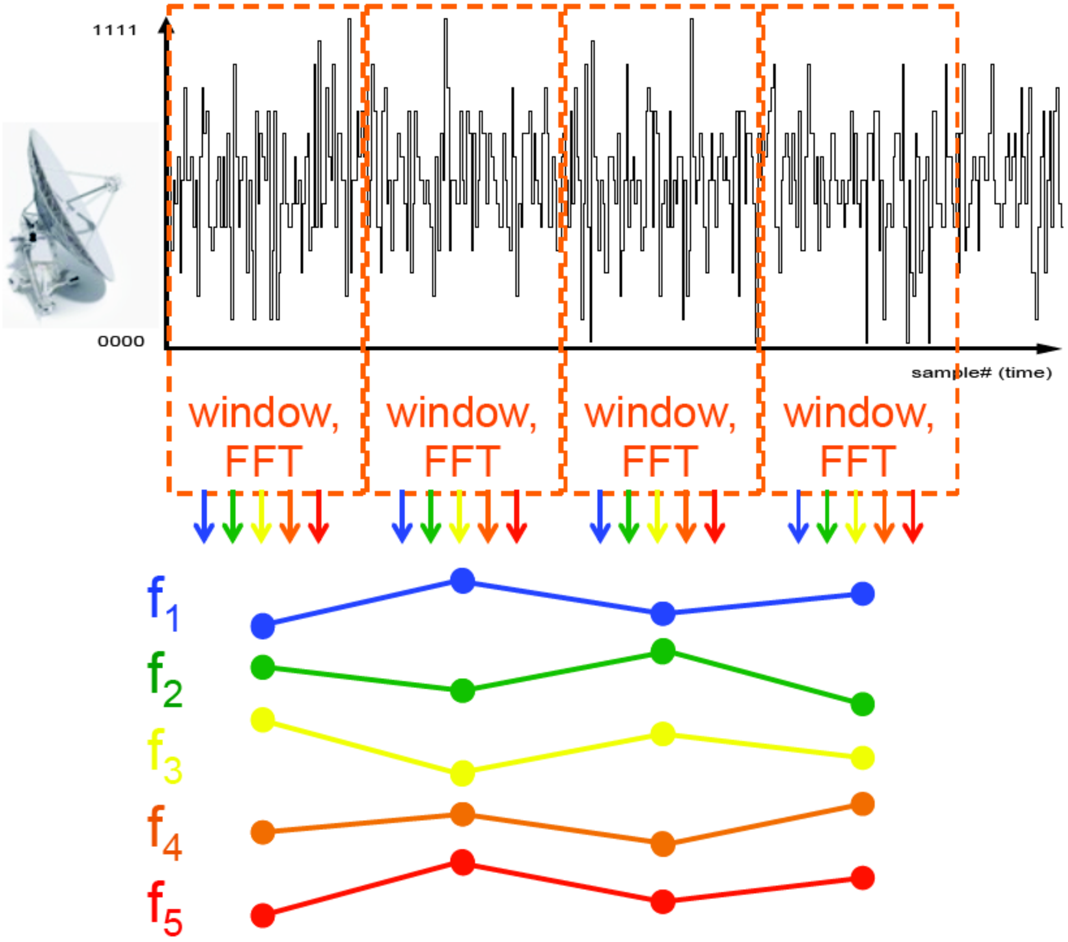
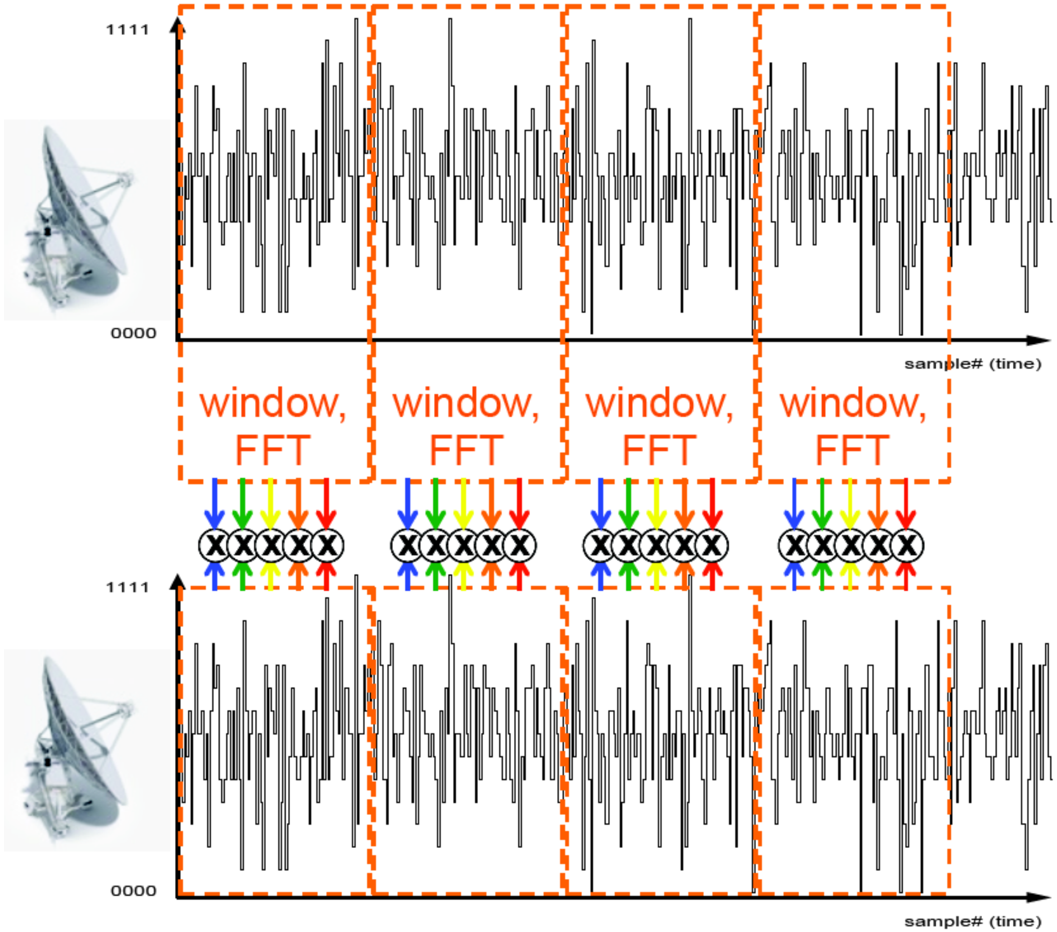


Figure credit: Adam Deller



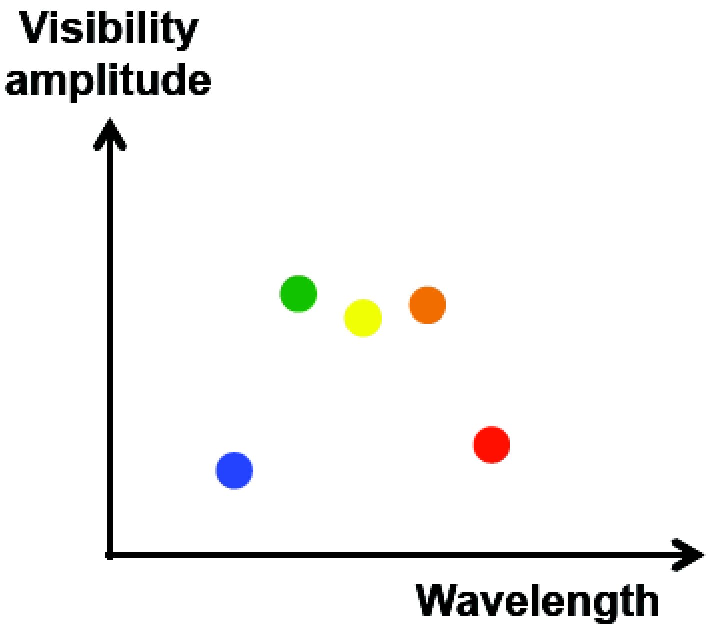
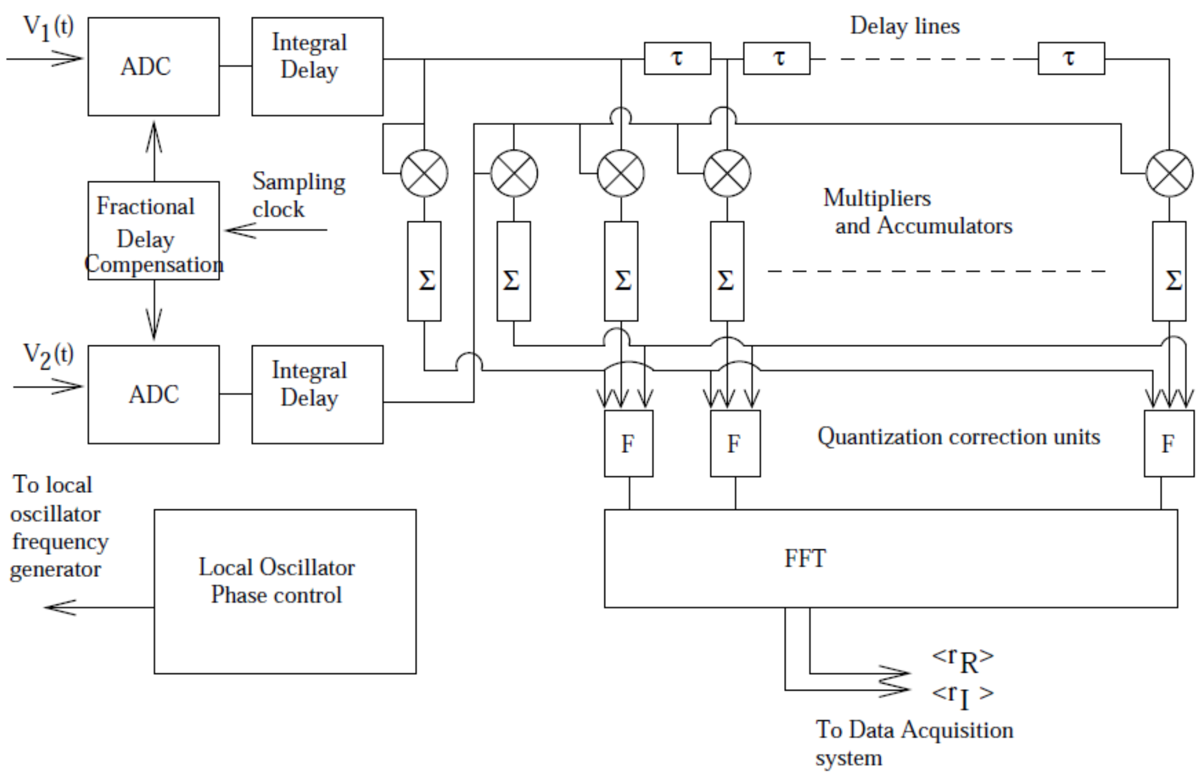


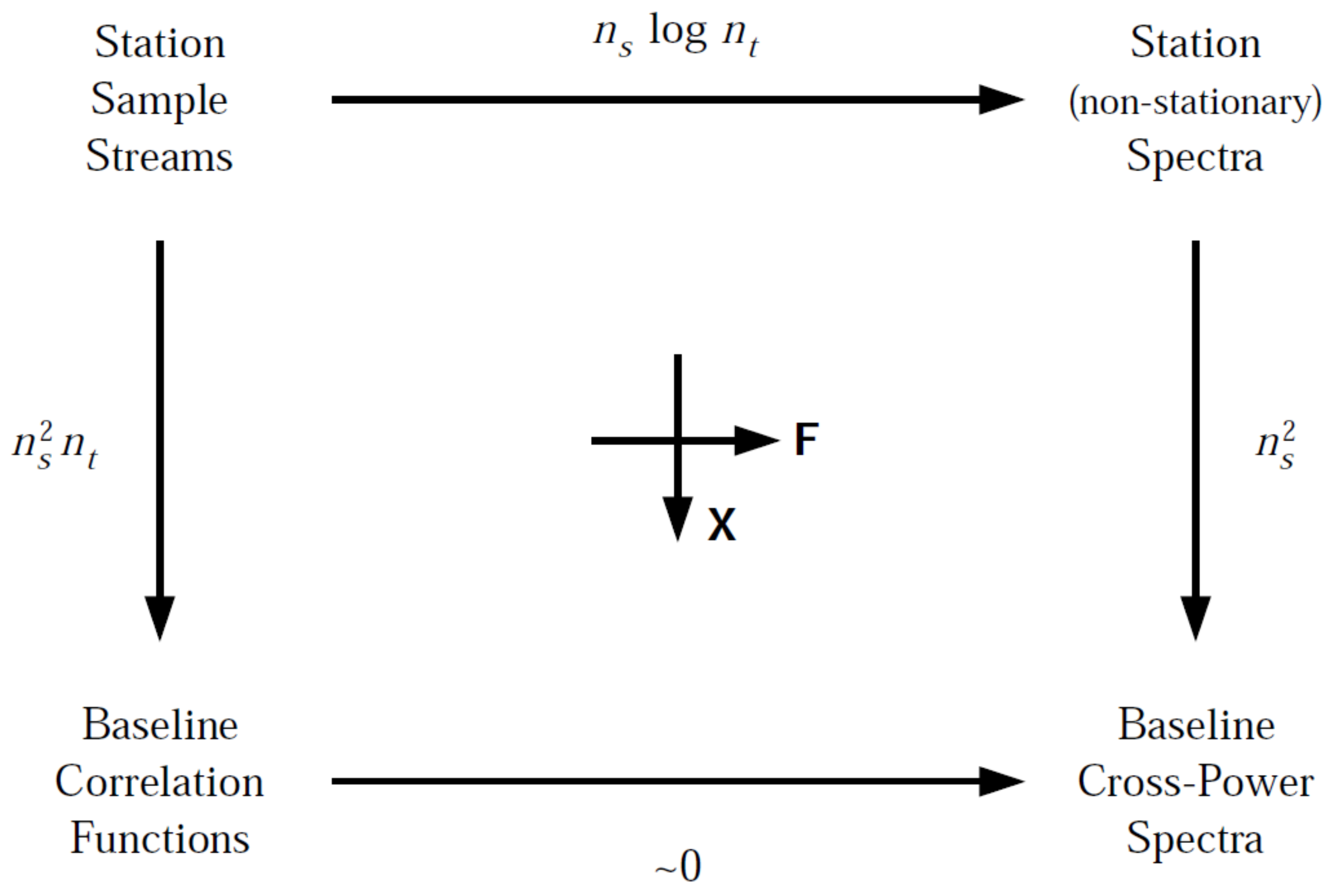
Figure credit: Adam Deller

XF correlator



- Signal is digitized.
- Integral delays are compensated for.
- Fractional delays corrected using sampling clock.
- Multipliers and delay lines.
- Quantization correction applied to normalied correlations.
- Cross correlation spectrum is obtained using DFT.

FX and XF data processing paths



n_s antennas/stations and n_t samples

Examples

GMRT, ALMA have an FX correlator.

VLA, IRAM have an XF correlator

IRAM: Institut de Radio Astronomie Millimetrique