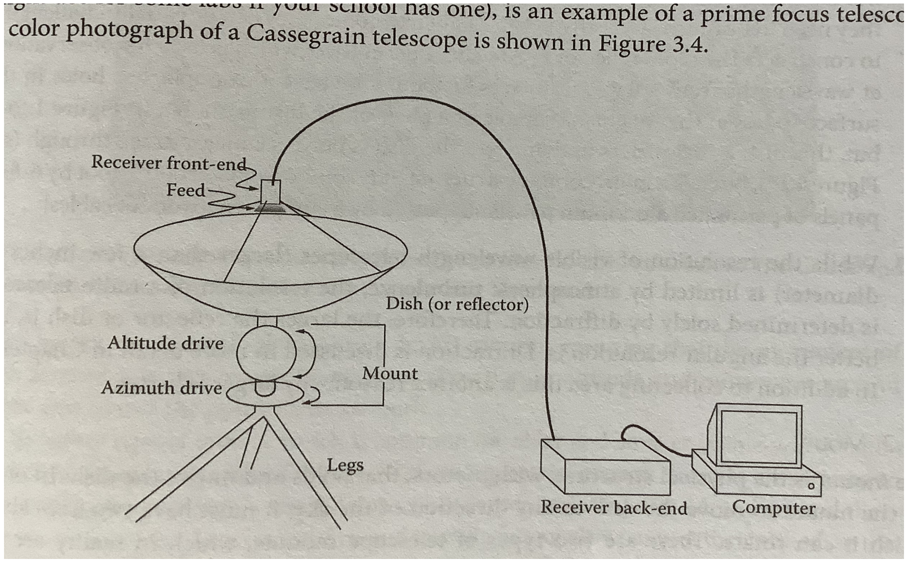
- Single dish radio telescope
- Antenna surface
- Antenna pattern

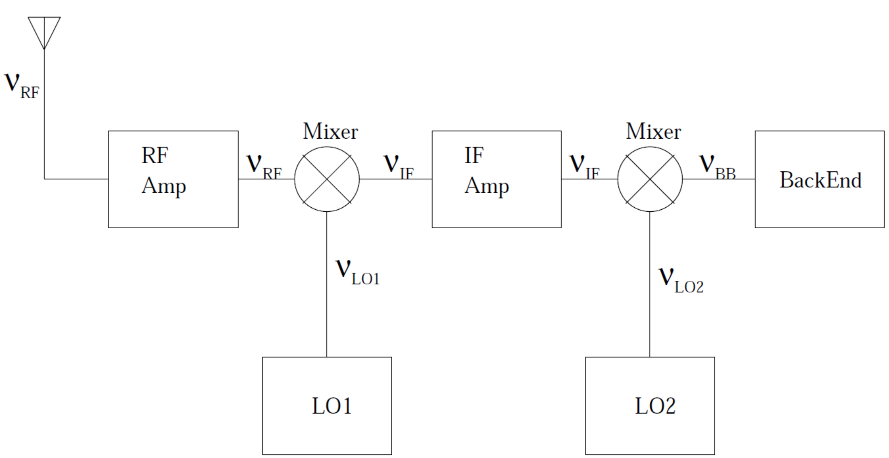
Astronomical Techniques II : Lecture 3

Ruta Kale

Low Frequency Radio Astronomy (Chp. 1, 2)
Synthesis imaging in radio astronomy II, Chp 1
Essential Radio Astronomy
Antenna Theory

A basic radio telescope





-Feed
-Receiver front end
-Reflector
-Mount
-Transmission lines
-Receiver back-end
-Computer

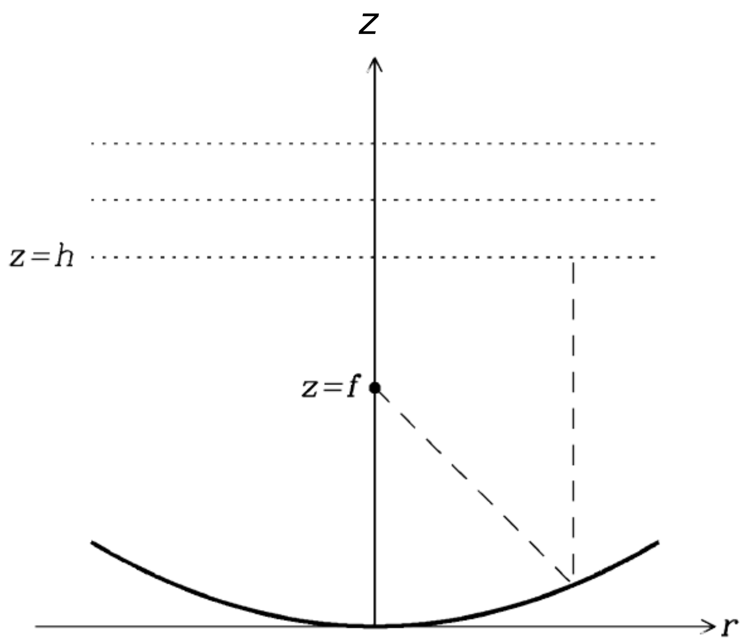
- Brightness temperature, Antenna temperature
- Antenna parameters (directivity, gain, surface errors)
- Illumination

Reflector antennas

The most common reflector shape is a paraboloid.

The reflector must keep all parts of an on-axis plane wavefront in phase at its focal point.

$$z = \frac{r^2}{4f}$$

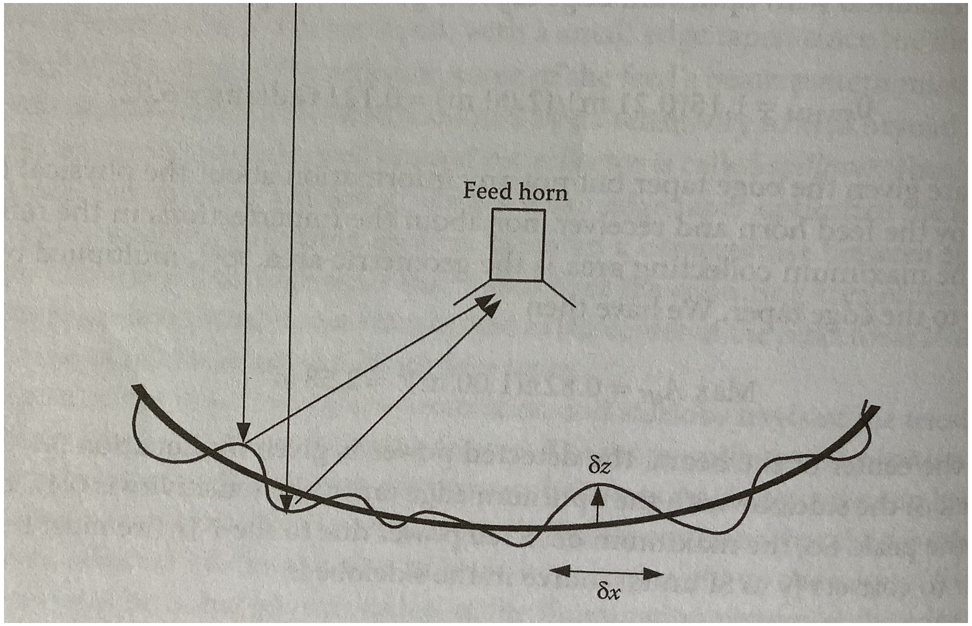


Focal ratio = focal length / diameter

Typically, f/D ~ 0.4 is used.

Surface errors

- Imperfections in the reflecting surface
- Cause path length differences
- Reduce on-axis sensitivity of the telescope – loss in effective collecting area.



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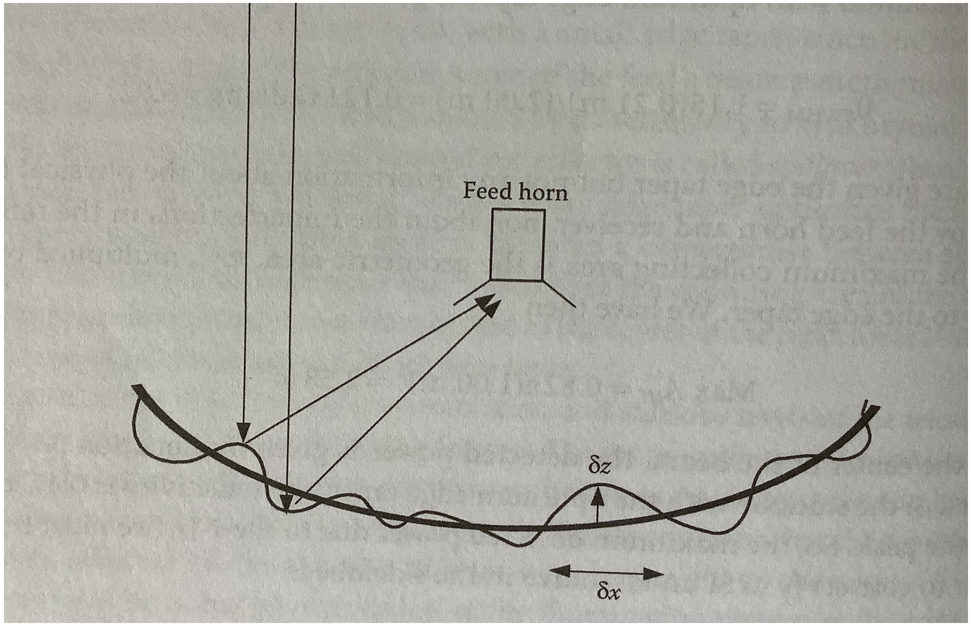
- *The **reflector surface efficiency** η_s is defined as the power gain of the actual reflector divided by the power gain of a perfect paraboloidal reflector with the same size and illumination.*



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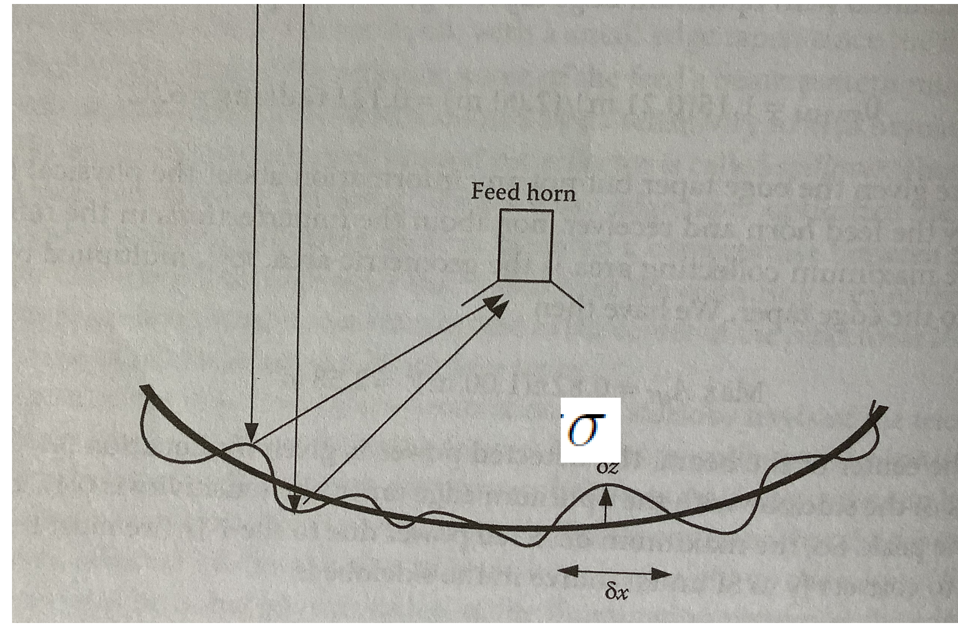
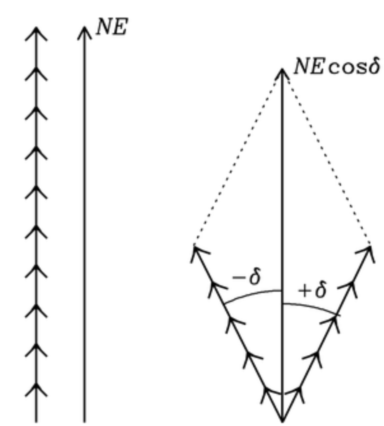


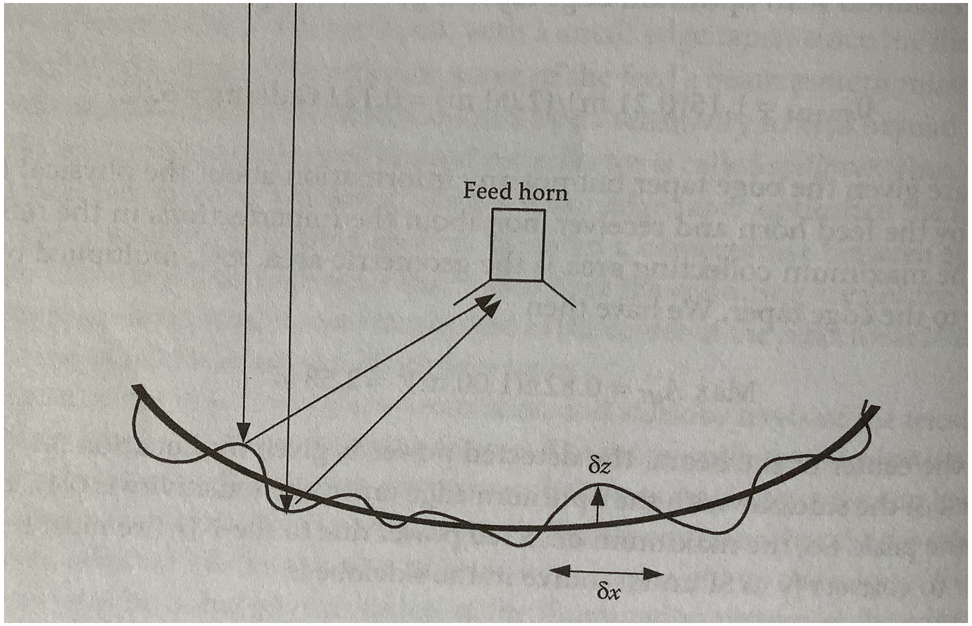
Vector sum of N electric fields E:

$$\delta = \pm \frac{4\pi\sigma}{\lambda}$$



Surface errors

- Imperfections in the reflecting surface
- Cause path length differences
- Reduce on-axis sensitivity of the telescope – loss in effective collecting area.
- Ruze equation
- *Follow 3.3.4 of ERA for derivation.*



$$\eta_s = \exp\left[-\left(\frac{4\pi\sigma}{\lambda}\right)^2\right]$$

Ratio of collecting area with and without errors

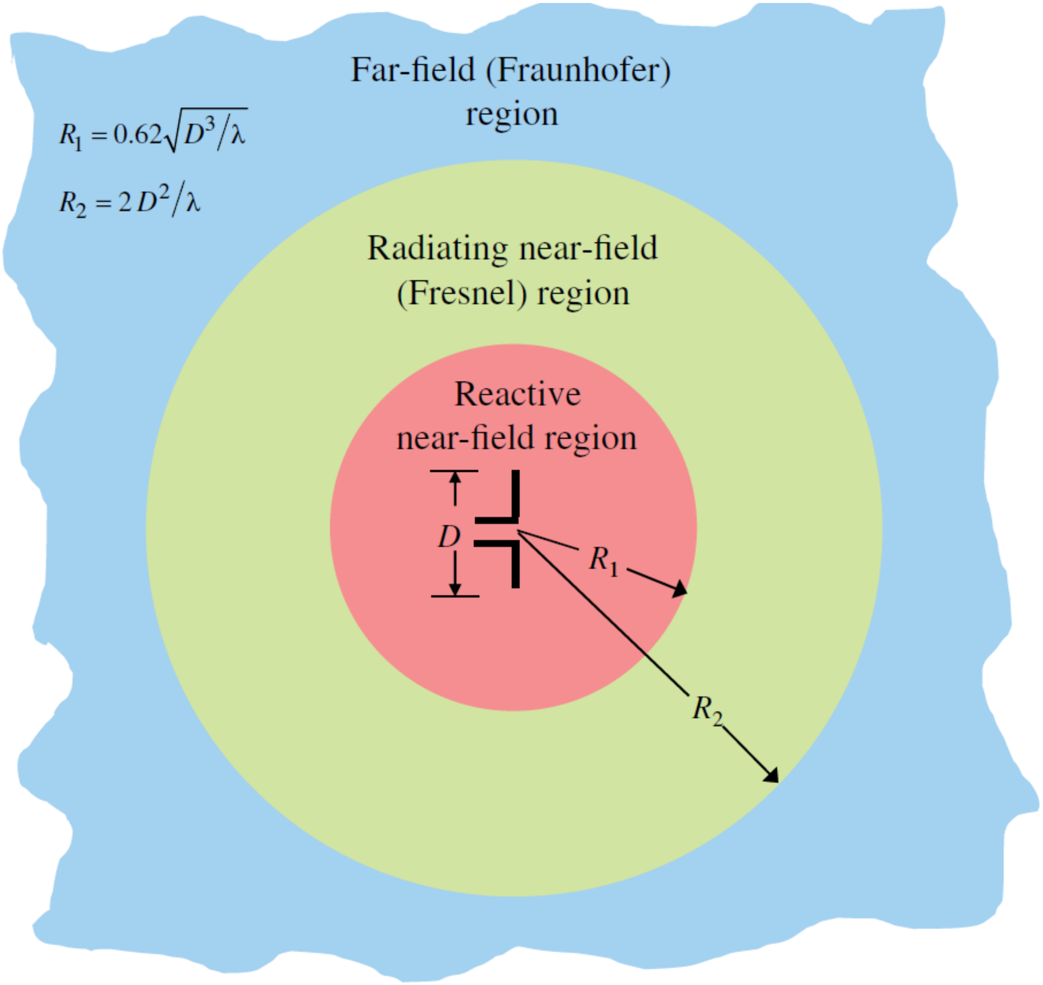
RMS of surface errors

Field regions around an antenna

Near-field: region adjacent to the antenna

Radiating near field (Fresnel region): angular field distribution depends on the distance from the antenna

Far field (Fraunhofer):angular field distribution is independent of the distance from the antenna

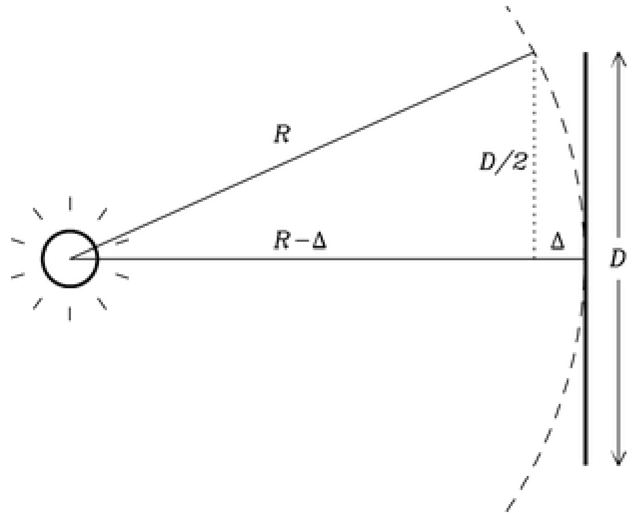


The far field distance

How far away must a point source be for the received waves to satisfy the assumption that they are nearly planar across the reflector?

Wavelength and reflector diameter are both important.

Reflector



$$R^2 = (R-\Delta)^2 + \left(\frac{D}{2}\right)^2$$

$$R = \frac{\Delta}{2} + \frac{D^2}{8\,\Delta}$$

$$\Delta < \lambda/16$$

In the limit, $\Delta \ll D$ $\quad \Delta/2 \ll D^2/(8\,\Delta)$

$$R \approx \frac{D^2}{8\,\Delta}$$

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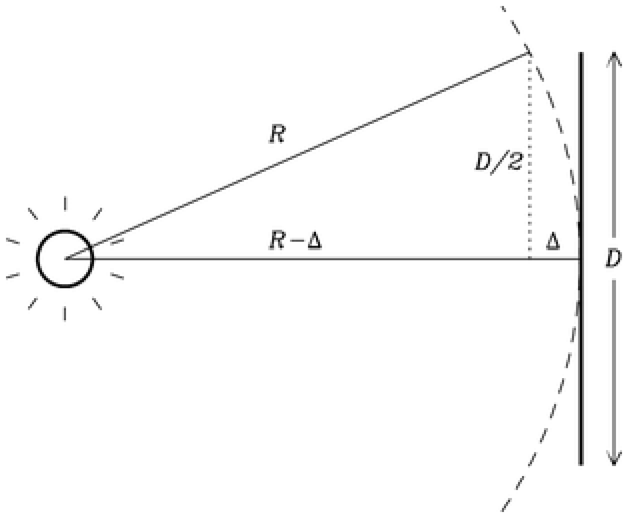

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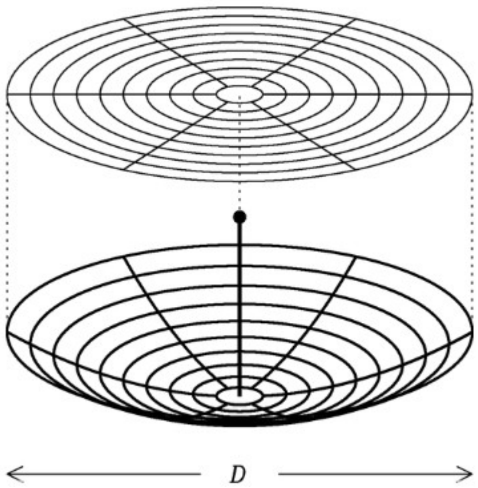
$$\Delta < \lambda/16$$

$$\boxed{R_{\rm ff} \approx \frac{2D^2}{\lambda}.}$$

Aperture plane

The aperture of a paraboloidal reflector antenna would be the plane circle, normal to the rays from a distant point source, that just covers the paraboloid.



The phase of the plane wave from a distant point source would be constant across the aperture plane when the aperture is perpendicular to the line of sight.

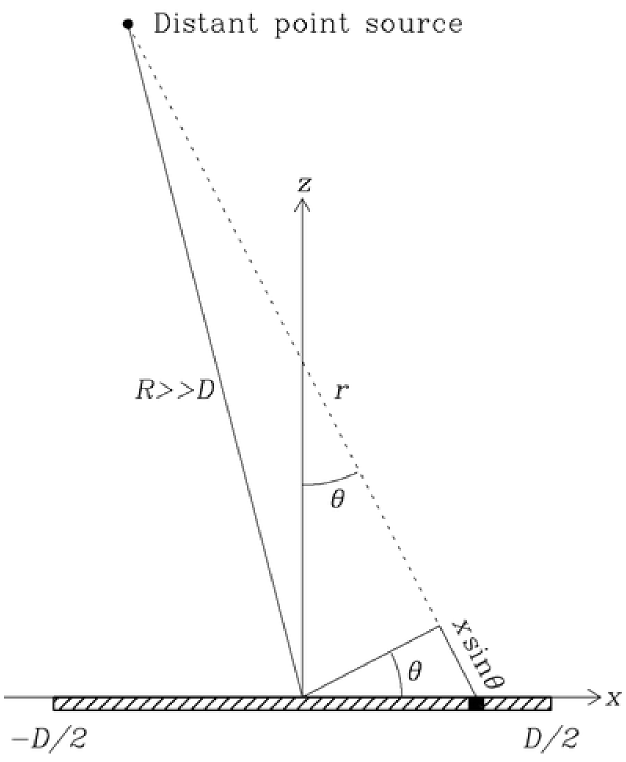
Beam pattern or power gain of an aperture antenna

How to calculate the beam pattern, or power gain as a function of direction, of an aperture antenna?

Consider a 1-d aperture of width D and a “distant” point source ($R>>R_{ff}$).

In a transmitting antenna, the feed can illuminate the aperture antenna with a sine wave of fixed frequency $\nu=\omega/(2\pi)$ and electric field strength g(x) that varies across the aperture. The illumination induces currents in the reflector. The currents will vary with both position and time:

$$I \propto g(x)\exp(-i\omega t)$$



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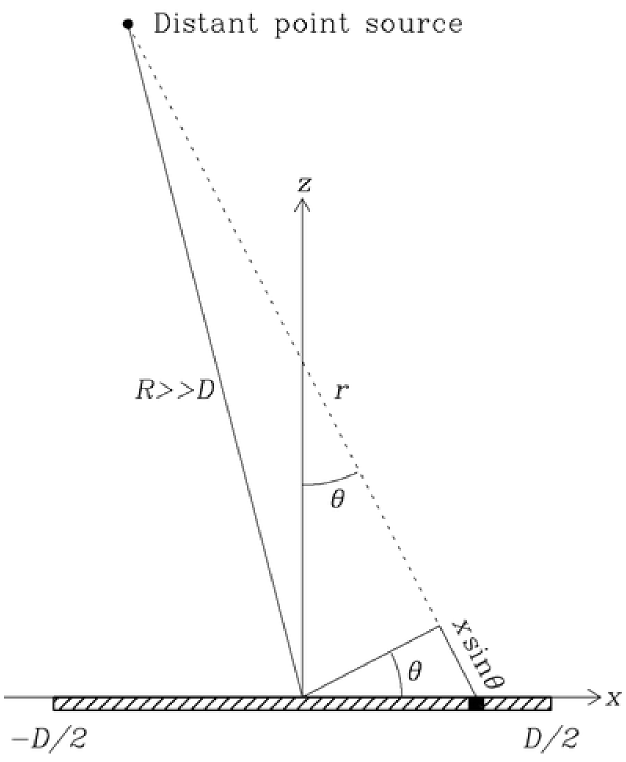
Huygens's principle asserts that the aperture can be treated as a collection of small elements which act individually as small antennas. The electric field produced by the whole aperture at large distances is just the vector sum of the elemental electric fields from these small antennas.



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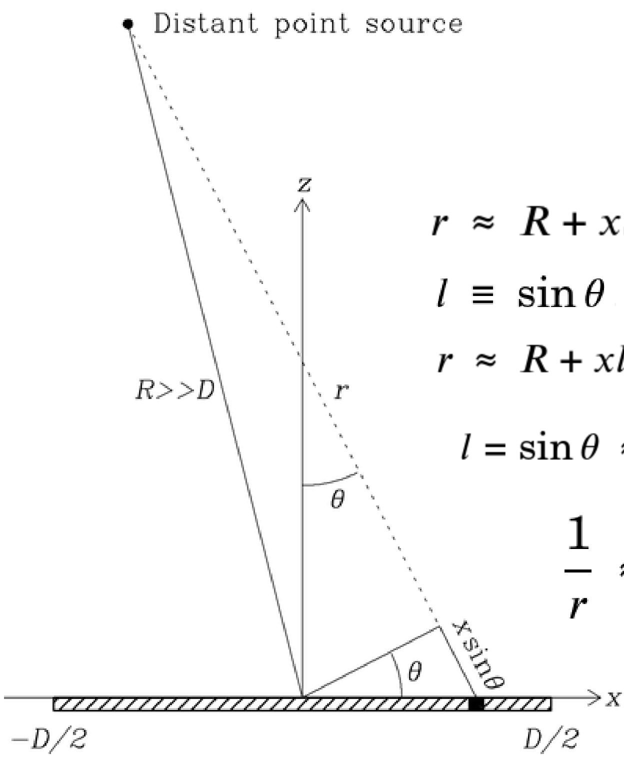
The field from each element extending from x to x+dx is,

$$df \propto g(x) \frac{\exp\left(-i2\pi r(x)/\lambda\right)}{r(x)} dx$$



$$r \approx R + x\sin\theta$$

$$l \equiv \sin\theta$$

$$r \approx R + xl$$

$$l = \sin\theta \approx \theta.$$

$$\frac{1}{r} \approx \frac{1}{R}$$

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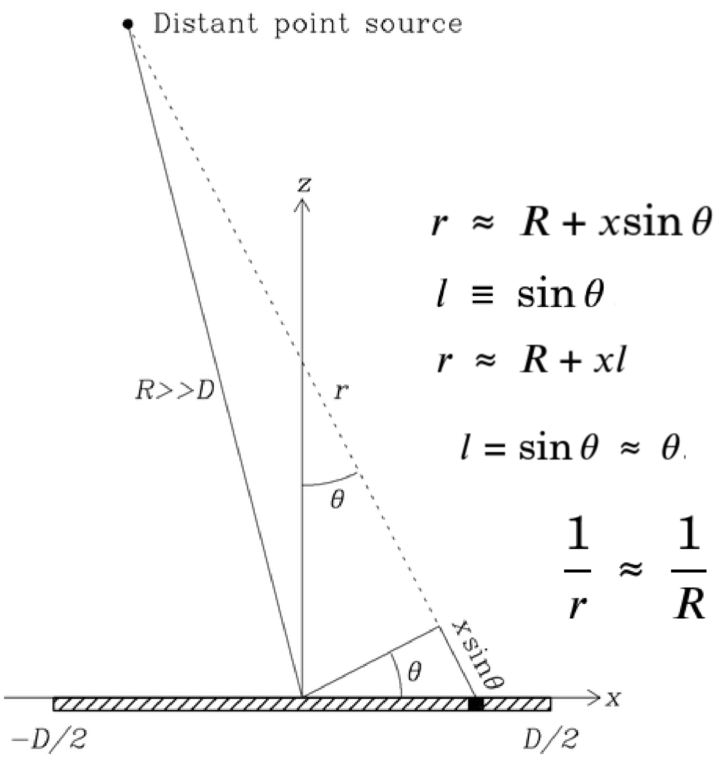

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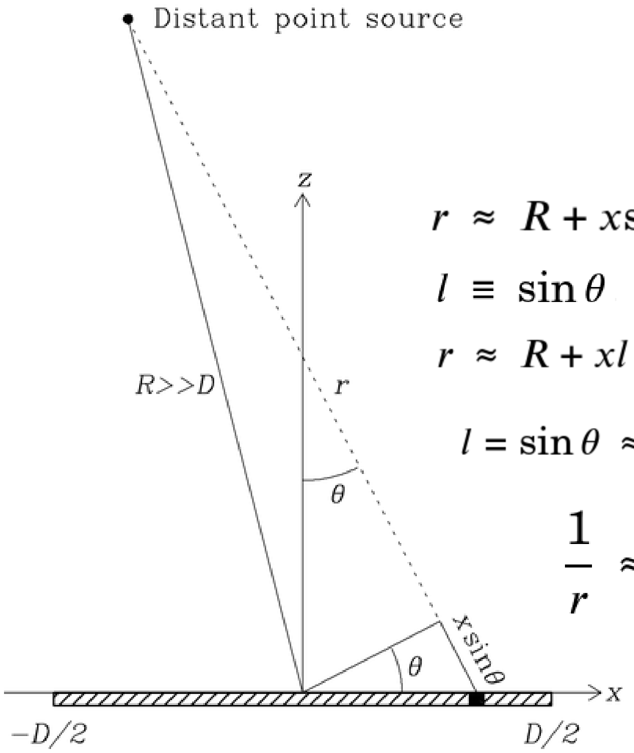
Variable part of r(x) in the denominator can be ignored.

$$df \propto g(x)\exp(-i2\pi xl/\lambda)dx$$

When θ≠0 the phase 2πxl/λ≈2πxsinθ/λ varies linearly across the aperture, and different parts of the aperture add constructively or destructively to the total electric field f(l). Defining

$$u \equiv \frac{x}{\lambda}$$

$$f(l) = \int_{\text{aperture}} g(u)e^{-i2\pi lu}du$$



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Beam pattern or power gain of an aperture antenna

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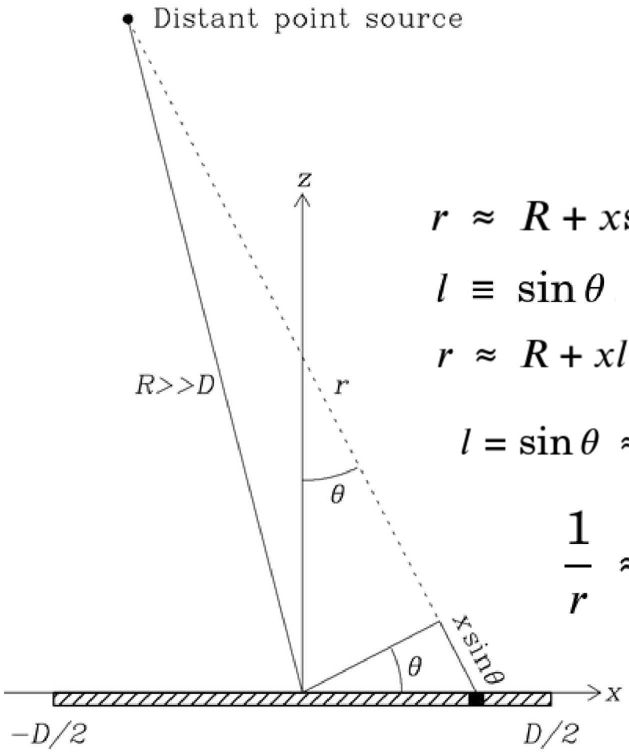
In the far field, the electric-field pattern f(l) of an aperture antenna is the Fourier transform of the electric field distribution g(u) illuminating that aperture.



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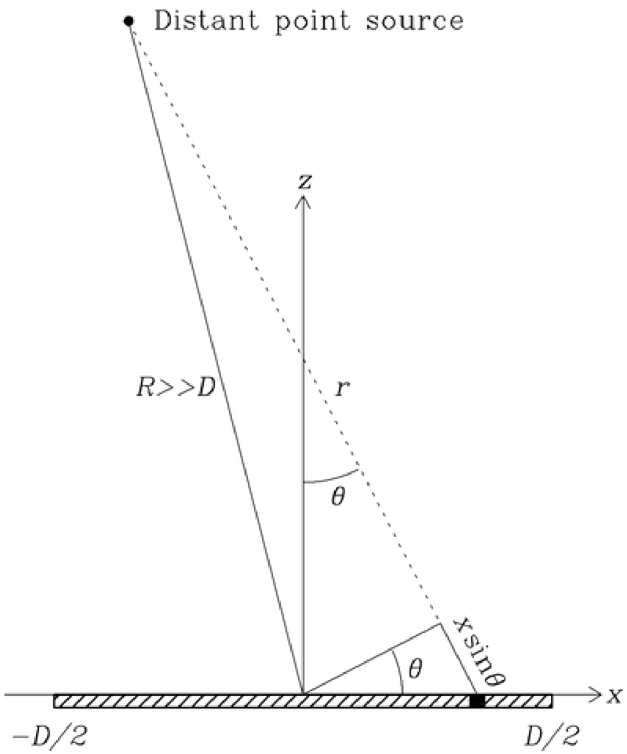
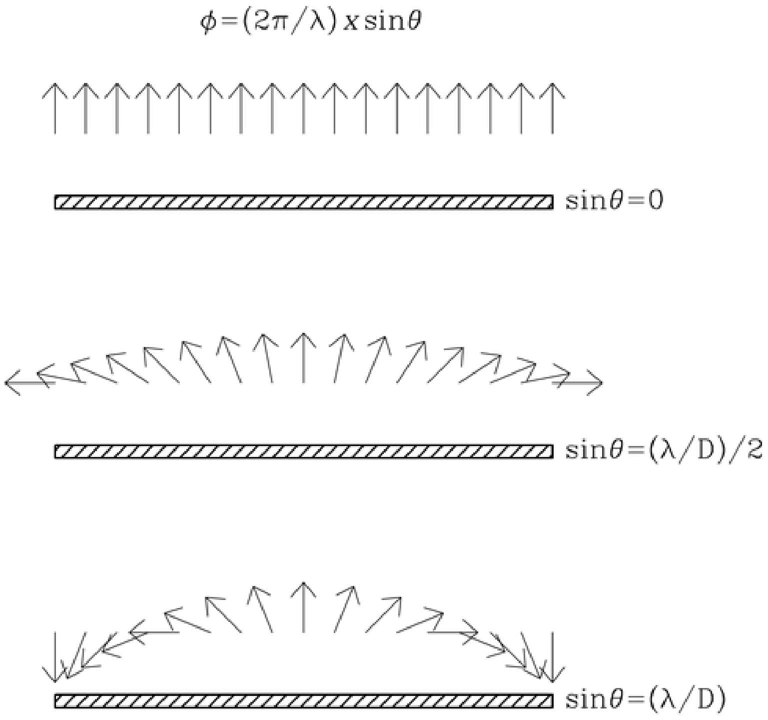
Beam pattern or power gain of an aperture antenna

Why the variable part in the phase term, xl, cannot be ignored ?





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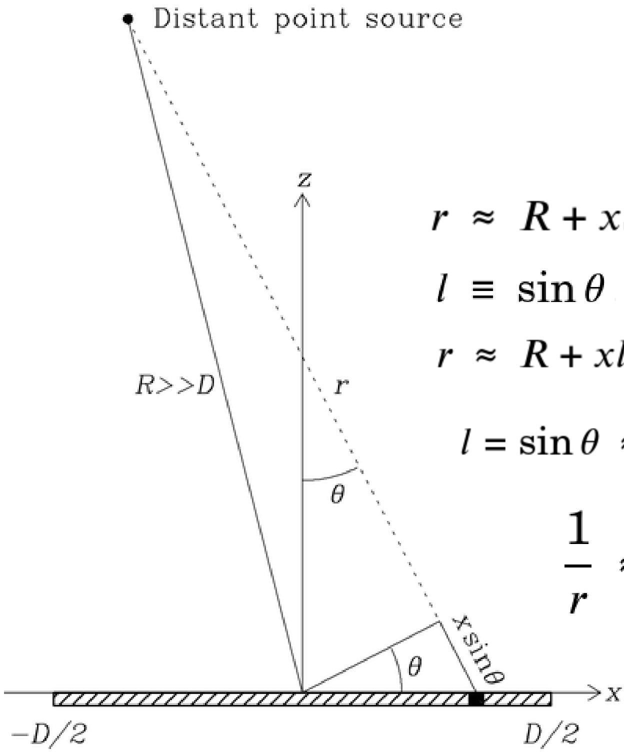

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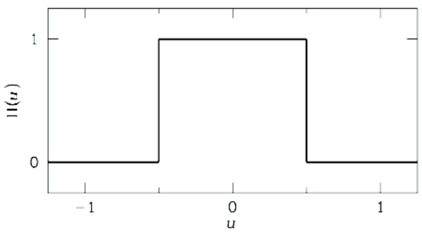
1D uniformly illuminated aperture:

$$g(u) = \text{ constant}, \qquad -\frac{D}{2\lambda} < u < +\frac{D}{2\lambda}$$



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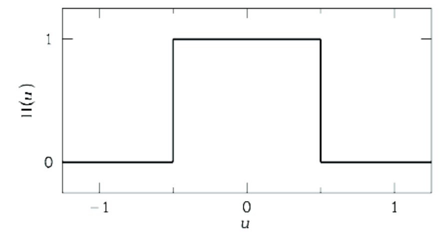

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$$= \frac{\sin(\pi l)}{(\pi l)}$$

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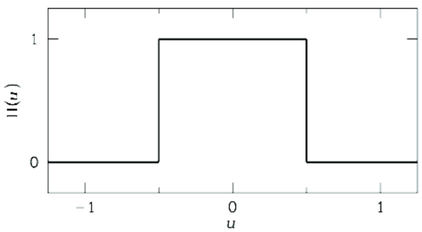

$$f(l) = \int_{-1/2}^{+1/2} e^{-i2\pi l u} du \qquad l = \sin\theta \qquad \theta \ll 1 \text{ radian}, \, l = \sin\theta \approx \theta$$

$$= \left. \frac{e^{-i2\pi l u}}{-i2\pi l} \right|_{-1/2}^{+1/2} \qquad f(\theta) = \frac{D}{\lambda} \operatorname{sinc}\left(\frac{\theta D}{\lambda}\right)$$

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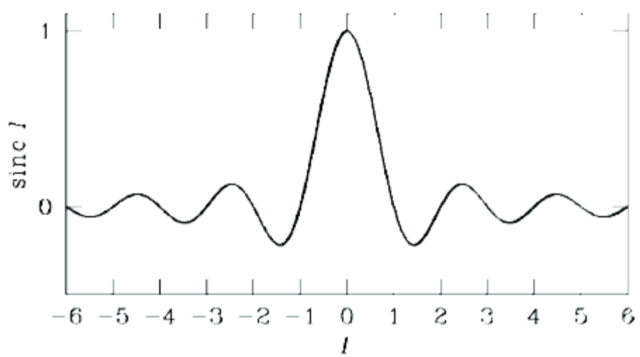

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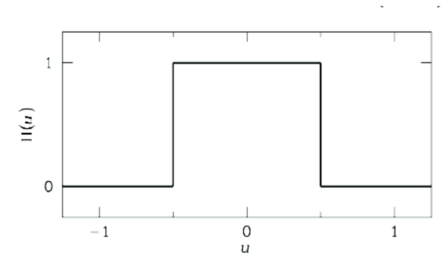

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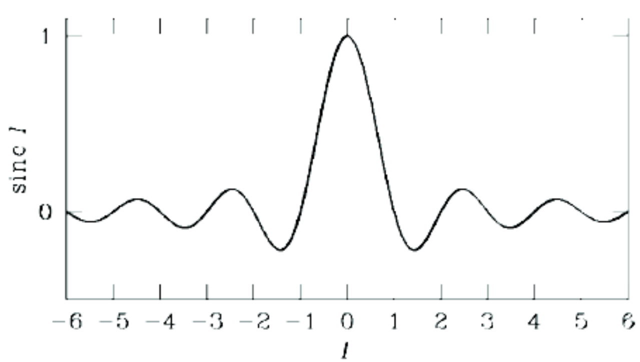
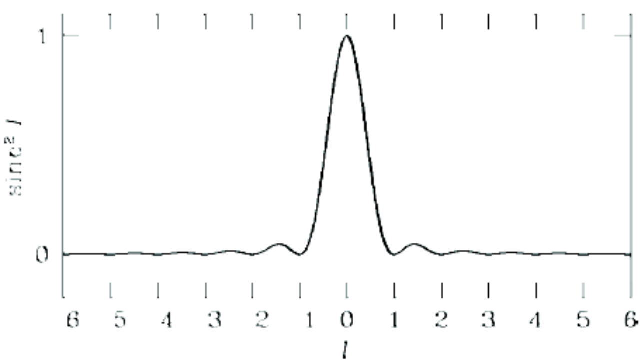
Power pattern:

$$P(\theta) = f^2(\theta) = \left(\frac{D}{\lambda}\right)^2 \operatorname{sinc}^2\left(\frac{\theta D}{\lambda}\right)$$





Half power beam width

The narrow beamwidth $\theta_{HPBW} \ll 1$ rad of a large ($D \gg \lambda$) one-dimensional uniformly illuminated aperture satisfies

$$P(\theta_{\mathrm{HPBW}}/2) = \frac{1}{2} = \mathrm{sinc}^2\left(\frac{\theta_{\mathrm{HPBW}} D}{2\lambda}\right)$$

Show that:

$$\theta_{\mathrm{HPBW}} \propto \frac{\lambda}{D}$$

Diffraction-limited resolution of a uniformly illuminated aperture antenna.

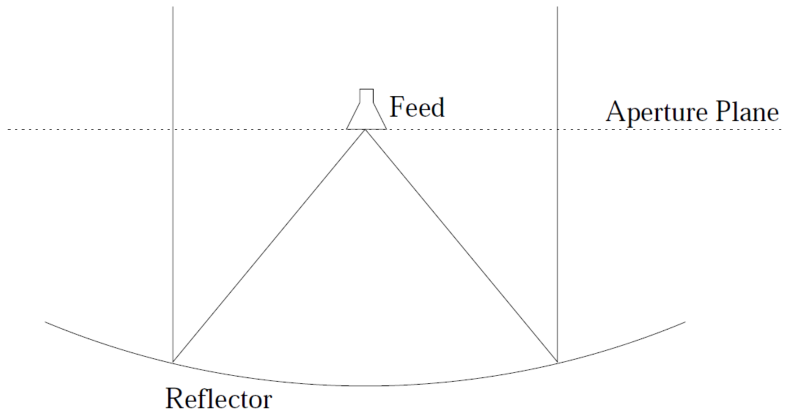
In receiving terms, the analog of the power pattern is called the point-source response. For a uniformly illuminated aperture, scanning a radio telescope beam in angle θ across a point source will cause the antenna temperature to vary as $\mathrm{sinc}^2(\theta)$, and the width of the half-power response will equal the transmitting HPBW.

Aperture illumination

The beam pattern of the feed determines the illumination of the primary reflector.

Ideally would like to have uniform sensitivity from centre to the edge of the dish – but we do not want unwanted radiation from the ground to be picked up.

A quantity that describes how the feed's beam is distributed on the primary reflector is called *edge taper:* ratio of sensitivity at the centre to that at the edge.



Illumination affects the angular resolution, sensitivity level in the sidelobes and effective collecting area.

A more tapered illumination will have a broader main beam or equivalently smaller effective aperture but also lower sidelobes than uniform illumination. If the illumination is high towards the edges there will be a lot of spillover.

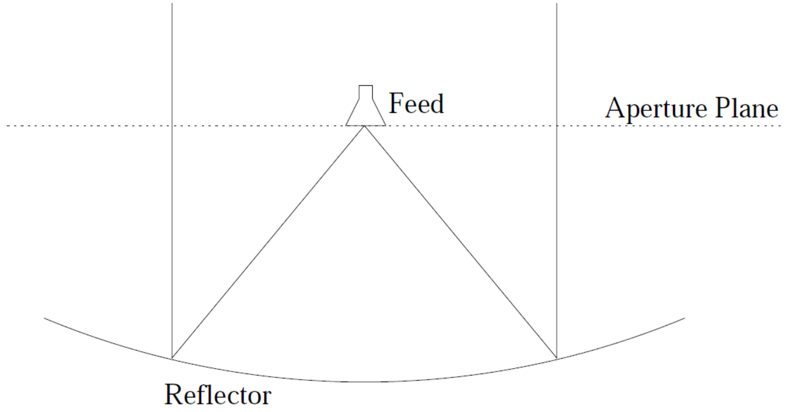
Aperture blockage

The feed is located above the reflector and thus blocks the aperture.
What is the effect of this on the antenna pattern ?

Now that we know the aperture plane and far field are related by a Fourier transform, we can find the estimate the effect of the aperture blockage using the properties of FT.

A uniform aperture with a width d

A uniform aperture with with l



Aperture blockage

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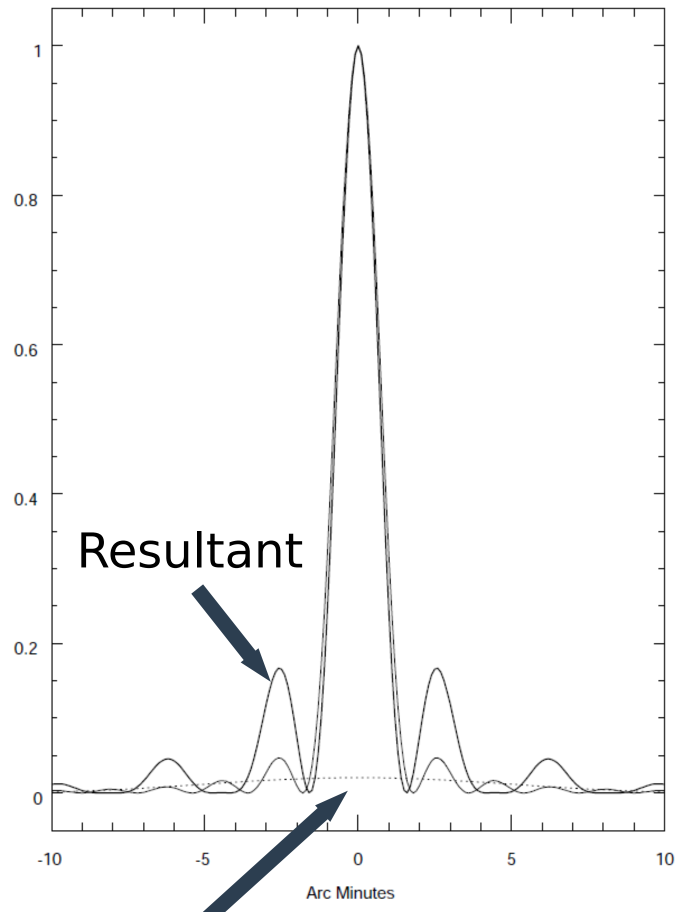
$$\mu = \lambda / l$$

$$E(\mu) \propto \frac{sin(\pi l \mu / \lambda)}{\pi \mu} - \frac{sin(\pi d \mu / \lambda)}{\pi \mu}$$

Should be minimised for a good beam.
Offset feeds to eliminate blockage.



The Wiener-Khinchin Theorem

Consider a random process x(t). The auto-correlation of x is defined as

$$r_{xx}(t,\tau) = \langle x(t)x(t+\tau)\rangle$$

for stationary signals

$$r_{xx}(\tau) = \langle x(t)x(t+\tau)\rangle$$

where angular brackets indicate taking the mean value.

The Fourier transform S(ν) of the auto-correlation function is the power spectrum:

$$S(\nu) = \int_{-\infty}^{\infty} r_{xx}(\tau) e^{-i2\pi\tau\nu} d\tau$$

and

$$r_{xx}(\tau) = \int_{-\infty}^{\infty} S(\nu) e^{i2\pi\tau\nu} d\nu$$

The auto-correlation function is the Fourier transform of the power spectrum.
- Weiner-Khinchin theorem

The Wiener-Khinchin Theorem

Example: A process whose auto-correlation function is a delta function has a power spectrum that is flat – "white noise".

In radio astronomy we usually have *band-limited signals* - in this case auto-correlation is a sinc function with a width ~1/$\Delta\nu$.

This width is also called the "coherence time" of the signal.

$$S(\nu) = \int_{-\infty}^{\infty} r_{xx}(\tau) e^{-i2\pi\tau\nu} d\tau \quad \text{and} \quad r_{xx}(\tau) = \int_{-\infty}^{\infty} S(\nu) e^{i2\pi\tau\nu} d\nu$$

The auto-correlation function is the Fourier transform of the power spectrum.
- Weiner-Khinchin theorem

Temporal and spatial correlations

In the previous example we had random processes that are a function of time alone. But the signal received from a distant cosmic source is in general a function of both time and receiver location. One can also define spatial correlation functions.

Consider the signal *E(r)* at a particular instant in the observer's plane, then the spatial correlation function is:

$$V(x) = \langle E(r)E^*(r+x)\rangle$$

This function V is referred to as the visibility and is central to the topic of interferometry.