- Calibration
- Closure phase and self-calibration

Astronomical Techniques II : **Lecture 12**

Ruta Kale

Low Frequency Radio Astronomy (Chp. 5)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy

Synthesis imaging in radio astronomy II, Chp 5

Interferometry and synthesis in radio astronomy (Chp 10)

Purpose of calibration

To remove the effects of
a) *instrumental* factors and b) *atmospheric* factors in the measurements.

- Such factors largely depend on individual antennas or pairs of antennas. Thus these should be removed before performing the imaging.

Calibration

- Role of calibrators:

In order to measure the effect of instrumental factors, one needs to observe something for which one can predict the visibility.

Data taken towards calibrator sources are useful here. Calibrators are usually *bright, unresolved* sources that dominate the field of view – and chosen to be located close to the position of the target. The point source response of interferometer is expected to be constant across baselines and thus is predictable.

Instrumental factors I (long term)

Among the instrumental factors there are some that vary only on timescales of several weeks or months. These include:

1. *Antenna position coordinates*
2. *Antenna pointing corrections* resulting from axis misalignments or other mechanical tolerances.
3. *Zero-point settings for the delays* – settings for which the delays from the antennas to the correlator inputs are equal.

Such parameters only vary with major changes such as antenna location change (e. g. movable antennas such as of VLA). Usually observatories make these corrections – individual observations do not need measurements.

Instrumental factors II (short term)

There are also instrument parameters that *vary over the course of a single observation* – on timescales of a few to several minutes or hours. Either these are predictable changes or need continuous monitoring.

Predictable changes:

- *atmospheric attenuation* as a function of zenith angle (~)
- *variation of antenna gain as a function of elevation*
- *shadowing* of one antenna by a neighboring antenna at low elevation angles. Generally data where shadowing affects the observation are discarded as the correction can be difficult due to effects of diffraction.

Instrumental factors III (short term)

There are also instrument parameters that vary over the course of a single observation – on timescales of a few to several minutes or hours.

Changes that need to be monitored during an observation:

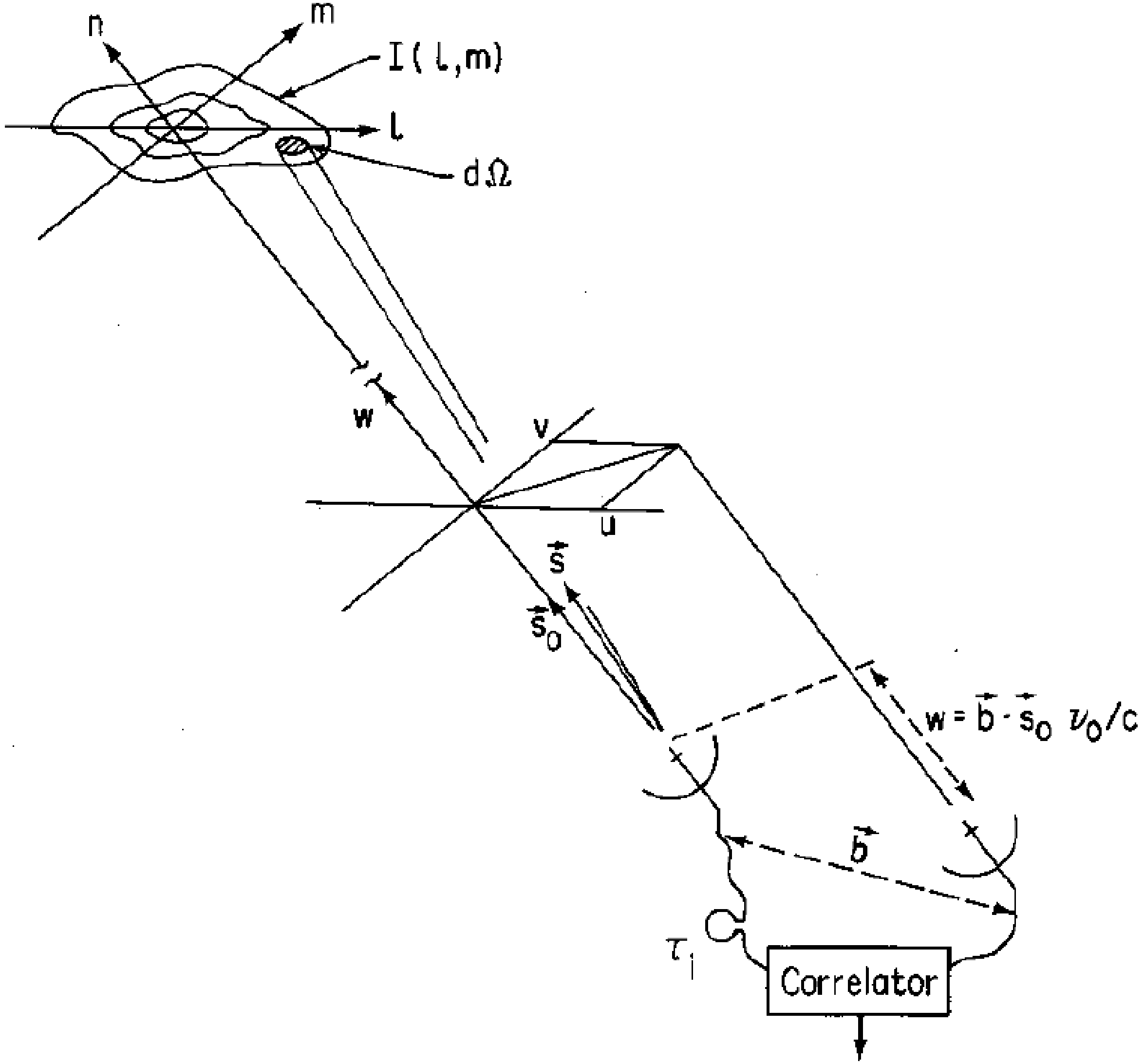
- *variable atmospheric attenuation* as a function of zenith angle
- phase *variation in the local oscillator system*
- *variation of system noise temperature* due to changing ground pickup in the sidelobes.

Background

Visibility sampled at each antenna pair i, j:

$$V_{ij}(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{A}_\nu(l,m)\ I_\nu(l,m) e^{-2\pi i (u_{ij}(t) l + v_{ij}(t) m)}\, dl\, dm$$

Geometric phase difference produced by the differential path length between the radiation from a source located at (l,m) to each antenna, compared with a fictitious source at the phase tracking center.


n
m
I(l,m)
l
dΩ
w
v
u
s
s0
w = b · s0 ν0/c
b
τi
Correlator

Visibility sampled at each antenna pair i, j:

$$V_{ij}(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_\nu(l,m)\; I_\nu(l,m) e^{-2\pi i(u_{ij}(t)l + v_{ij}(t)m)}\, dl\, dm$$

Geometric phase difference produced by the differential path length between the radiation from a source located at (l,m) to each antenna, compared with a fictitious source at the phase tracking center.

The total geometric phase difference:

$$\phi_g = 2\pi \mathrm{w}$$

Visibility sampled at each antenna pair i, j:

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Recall:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0 \cos H_0 & \sin\delta_0 \sin H_0 & \cos\delta_0 \\ \cos\delta_0 \cos H_0 & -\cos\delta_0 \sin H_0 & \sin\delta_0 \end{pmatrix} \begin{pmatrix} L_X \\ L_Y \\ L_Z \end{pmatrix}$$

The phase due to geometric delay is $2\pi w$

$$\phi_g = \frac{2\pi}{\lambda}(L_x \cos H \cos\delta - L_y \sin H \cos\delta + L_z \sin\delta)$$

$$\phi_g = 2\pi\nu\tau_g = \frac{2\pi}{\lambda}(L_x \cos H \cos\delta - L_y \sin H \cos\delta + L_z \sin\delta)$$

The differential geometric delay when the baselines differ slightly from the positions that are assumed ?

$$\phi_g = 2\pi\nu\tau_g = \frac{2\pi}{\lambda}(L_x \cos H \cos\delta - L_y \sin H \cos\delta + L_z \sin\delta)$$

$$\begin{aligned}\Delta\phi_g = 2\pi\nu\Delta\tau_g \quad &= \quad \frac{2\pi}{\lambda}(\Delta L_x \cos H \cos\delta - \Delta L_y \sin H \cos\delta + \Delta L_z \sin\delta \\ &+ \quad \Delta\alpha \cos\delta (L_x \sin H + L_y \cos H) \\ &+ \quad \Delta\delta(-L_x \cos H \sin\delta + L_y \sin H \sin\delta + L_z \cos\delta))\end{aligned}$$

$\Delta\tau_g$ Differential geometric delay

(L_x, L_y, L_z) Assumed baseline separation for antennas i, j

$$\phi_g = 2\pi\nu\tau_g = \frac{2\pi}{\lambda}(L_x \cos H \cos\delta - L_y \sin H \cos\delta + L_z \sin\delta)$$

$$\begin{aligned}\Delta\phi_g = 2\pi\nu\Delta\tau_g \quad &= \quad \frac{2\pi}{\lambda}(\Delta L_x \cos H \cos\delta - \Delta L_y \sin H \cos\delta + \Delta L_z \sin\delta \\ &+ \quad \Delta\alpha \cos\delta (L_x \sin H + L_y \cos H) \\ &+ \quad \Delta\delta(-L_x \cos H \sin\delta + L_y \sin H \sin\delta + L_z \cos\delta))\end{aligned}$$

$\Delta\tau_g$	Differential geometric delay
(L_x, L_y, L_z)	Assumed baseline separation for antennas i, j
$(\Delta L_x, \Delta L_y, \Delta L_z)$	True – assumed separation
(α, δ)	True source position
$(\Delta\alpha, \Delta\delta)$	True – minus assumed position

$$\phi_g = 2\pi\nu\tau_g = \frac{2\pi}{\lambda}(L_x \cos H \cos\delta - L_y \sin H \cos\delta + L_z \sin\delta)$$

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$(\Delta L_x, \Delta L_y, \Delta L_z)$ True – assumed separation

(α, δ) True source position

$(\Delta\alpha, \Delta\delta)$ True – minus assumed position

Phase delays result in error in positions in the image

Calibration formalism

Relation between the observed visibilities and the true visibilities:

$$\widetilde{V}_{ij}(t) = \mathcal{G}_{ij}(t)V_{ij}(t) + \epsilon_{ij}(t) + \eta_{ij}(t)$$

t	Time of observation
$\mathcal{G}_{ij}(t)$	Baseline-based complex gain
$\epsilon_{ij}(t)$	Baseline based complex offset
$\eta_{ij}(t)$	Stochastic complex noise

If we assume that most of the data corruption occurs before the signal pairs are correlated, we can separate the complex gain into a product of antenna based complex gains.

Calibration formalism

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Can be separated into antenna based amplitude and phases

$$\mathcal{G}_{ij}(t) = g_i(t)g_j^*(t) = a_i(t)a_j(t)e^{i(\phi_i(t)-\phi_j(t))}$$

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Observations of calibrators provide N(N-1)/2 measurements of G_{ij} and thus we can solve for the N values of $g_i(t)$.

Antenna pointing error

- Difference between the actual pointing position and the desired position.

Antenna pointing errors can be result of :
Mis-alignment of the polar or elevation axes, gravitational deformation of the structure, atmospheric refraction, time dependent deformation due to heating of the antenna, wind loading of the antenna.

Measuring this requires observations of positions well distributed across the sky.
Antenna based pointing solutions are found and applied. (Done by the observatories: ref. Reports by Subhashis Roy for the GMRT)

These are to be applied during or before the observations begin.

Delay calibration

Mono-chromatic visibilities:

$$V_{ij}(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{A}_\nu(l,m)\ I_\nu(l,m) e^{-2\pi i (u_{ij}(t) l + v_{ij}(t) m)}\, dl\, dm$$

Visibilities measured over a finite bandwidth are given by:

$$V_{ij}(t) = \int_{0}^{\infty} \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{A}_\nu(l,m) I_\nu(l,m) e^{-2\pi i \nu \Delta\tau_g}\, dl\, dm \right) e^{2\pi i \nu \Delta\tau_r} \mathcal{G}_{ij}(\nu)\, d\nu$$

Delay calibration

Visibilities measured over a finite bandwidth are given by:

$$V_{ij}(t) = \int_0^{\infty} \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{A}_\nu(l,m) I_\nu(l,m) e^{-2\pi i \nu \Delta\tau_g} \, dl \, dm \right) e^{2\pi i \nu \Delta\tau_r} \mathcal{G}_{ij}(\nu) \, d\nu$$

(i, j)	Antenna pair
ν	Frequency
$V_{ij}(t)$	Visibility integrated over a bandwidth
$\mathcal{G}_{ij}(\nu)$	Complex gain as a function of frequency
$\Delta\tau_g$	Differential geometric delay
$\Delta\tau_r$	Residual instrumental delay – the error in the inserted delay relative to the delay tracking center.

Delay calibration

Visibilities measured over a finite bandwidth are given by:

$$V_{ij}(t) = \int_0^{\infty} \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{A}_\nu(l,m) I_\nu(l,m) e^{-2\pi i \nu \Delta\tau_g} \, dl \, dm \right) e^{2\pi i \nu \Delta\tau_r} \mathcal{G}_{ij}(\nu) \, d\nu$$

The net phase difference between the ends of the band that results from a net residual delay is given by:

$$\Delta\phi = 2\pi\Delta\nu(\Delta\tau_g - \Delta\tau_r)$$

~ a radian

Loss of coherence if this is too high

Delay calibration

Visibilities measured over a finite bandwidth are given by:

$$V_{ij}(t) = \int_0^{\infty} \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{A}_\nu(l,m) I_\nu(l,m) e^{-2\pi i \nu \Delta\tau_g} \, dl \, dm \right) e^{2\pi i \nu \Delta\tau_r} \mathcal{G}_{ij}(\nu) \, d\nu$$

The net phase difference between the ends of the band that results from a net residual delay is given by:

$$\Delta\phi = 2\pi\Delta\nu(\Delta\tau_g - \Delta\tau_r)$$ ~ a radian

Loss of coherence if this is too high

- Observation of a strong calibrator are used – vary the delays by small amounts to find the maximum coherence and then correct them.
- Such residual delays are removed by fitting and removing the phase slopes across the frequency band in the data.
- Delay between the two orthogonal polarization channels also needs correction or there is a loss in the cross-hand response.

Atmospheric effects

- Propagation of radio signal through the atmosphere introduces modification of the phase of the signal due to refraction.

- Components of the atmosphere are modeled and the delays corrected.

- At mm waves water vapour in the atmosphere is of concern.

- At low frequencies (<a couple of GHz), the ionospheric effects become important.
- Ionosphere is a magnetized plasma region in the atmosphere at 60 – 2000 km above the Earth's surface – consists of charged particles that result in refraction of the waves and rotation of plane of polarization.

- Could introduce a systematic shift in source positions as a function of time if all antennas are affected by the same column of plasma – but can introduce more complex effects if different parts of the array pass through patches having distinct properties.

- Can be corrected by obtaining total electron content (TEC) measurements.

Bandpass calibration

- Compensating for the change of gain as a function of frequency is called bandpass calibration.
- Strong source observed and the response across the band is determined.
- Usually this response varies over several hours and thus observation of a strong source every few hours is sufficient.
- The baseline based gain is factored into antenna based gains and determined and the visibilities are corrected for.

Polarization calibration

Polarimetry: The polarization of a quasi-monochromatic wave is described by the 2x2 matrix of correlations between two orthogonal components. For right circular and left circular polarizations as orthogonal modes, the polarization is described by:

$$\begin{pmatrix} RR^* & RL^* \\ LR^* & LL^* \end{pmatrix}$$

Stokes' parameters: intensity I, two linear polarization parameters Q and U and a circular polarization parameter V
The above matrix represented in linear combinations of Stokes' parameters is:

$$\begin{pmatrix} I+V & Q+iU \\ Q-iU & I-V \end{pmatrix}$$

Polarization calibration

Stokes' parameters: intensity I, two linear polarization parameters Q and U and a circular polarization parameter V

These parameters fully describe the polarization state of the radiation. Corresponding to each parameter there are visibilities and a corresponding image. The visibilities are measured as a linear combination of the 4 correlations produced by an interferometer. For R and L polarized feeds:

$$\begin{aligned} V[R \star R] &= V_I + V_V\ , \\ V[L \star L] &= V_I - V_V\ , \\ V[R \star L] &= (V_Q + iV_U)\ e^{-2i\chi} \\ V[L \star R] &= (V_Q - iV_U)\ e^{2i\chi} \end{aligned}$$

χ is parallactic angle which determines the orientation of the feed with respect to the sky.

Polarization calibration

Leakage terms

The feeds are not exactly orthogonal and this results in “leakage” of RR polarization into LL and vice versa.
If v_R and v_L are voltages from the two polarizations and the true electric fields are E_R and E_L, then,

$$v_R = E_R e^{-i\chi} + D_R E_L e^{i\chi}$$
$$v_L = E_L e^{i\chi} + D_L E_R e^{-i\chi}$$

The leakage terms are also called as “D-terms”.

The visibilities are given by:

$$\begin{aligned} V[R\star R] &= V_I + V_V \\ V[L\star L] &= V_I - V_V \\ V[R\star L] &= e^{-2i\chi}(V_Q + iV_U) + (D_{R1} + D^*_{L2})V_I \\ V[L\star R] &= e^{2i\chi}(V_Q - iV_U) + (D_{L1} + D^*_{R2})V_I \end{aligned}$$

Delay between R and L:

$$e^{\pm i(\phi_R - \phi_L)}$$

The phase term that multiplies with the last two equations

Polarization calibration

For polarization calibration one requires to find the leakages or “D-terms”
And the delay between R and L.

For measuring D-terms observation of a bright unpolarized calibrator is used.
Q and U terms are zero and thus the cross hand visibility is the sum of the leakages.

For the delay, a strongly polarized source with known position angle is needed. For equatorial mounts parallactic angle is constant but for alt-az mounts it needs to be calculated.

The visibilities are given by:

$$
\begin{aligned}
V[R\star R] &= V_I + V_V \\
V[L\star L] &= V_I - V_V \\
V[R\star L] &= e^{-2i\chi}(V_Q + iV_U) + (D_{R1} + D^*_{L2})V_I \\
V[L\star R] &= e^{2i\chi}(V_Q - iV_U) + (D_{L1} + D^*_{R2})V_I
\end{aligned}
$$

Delay between R and L:

$$e^{\pm i(\phi_R - \phi_L)}$$

Primary beam correction

$$V_{ij}(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{A}_\nu(l,m)\ I_\nu(l,m) e^{-2\pi i(u_{ij}(t)l+v_{ij}(t)m)}\ dl\, dm$$

The effect of A(l,m) must be removed from the image.

Needs to be measured in all four polarization correlations.

1. *Beam squint between polarizations*: a slight difference in the electrical pointing centers of right and left circular polarizations due to off-axis feed placement is called beam squint. The primary beam correction is done separately for each pol.
2. *Polarization characteristics across the beam*: D-terms substantially change outside 50% of the beam – thus polarization measurements outside this are not reliable.
3. *Alt-azimuth mount*: Since the beam rotates w. r. t. the source, any asymmetry in the beam will lead to effect in the cross hand correlations. Thus correction has to be done for short intervals when the change is small and then added together.
4. *Different antenna elements*: If antenna characteristics differ, polarization only at the center can be relied upon.

Calibration in practice (GMRT)

A typical dataset contains:

Primary calibrator : absolute flux calibrator
Bandpass calibrator : usually the flux calibrator serves as a bandpass calibrator
Secondary calibrator or phase calibrator
Target source

Flux cal – Phase cal – (Target source – Phase cal) – Flux cal
Loop over

-Delay calibration
-Absolute flux calibration
-Phase calibration
-Bandpass calibration
-Application of the calibration to the target

http://www.ncra.tifr.res.in/~ruta/ras-tutorials/CASA-tutorial.html

Calibration in practice (GMRT)

A typical dataset contains:

Primary calibrator : absolute flux calibrator   3C48, 3C286, 3C147
Bandpass calibrator : usually the flux calibrator serves as a bandpass calibrator
Secondary calibrator or phase calibrator (compact *isolated* sources chosen to lie within a few to 20 degrees of the target)
Target source

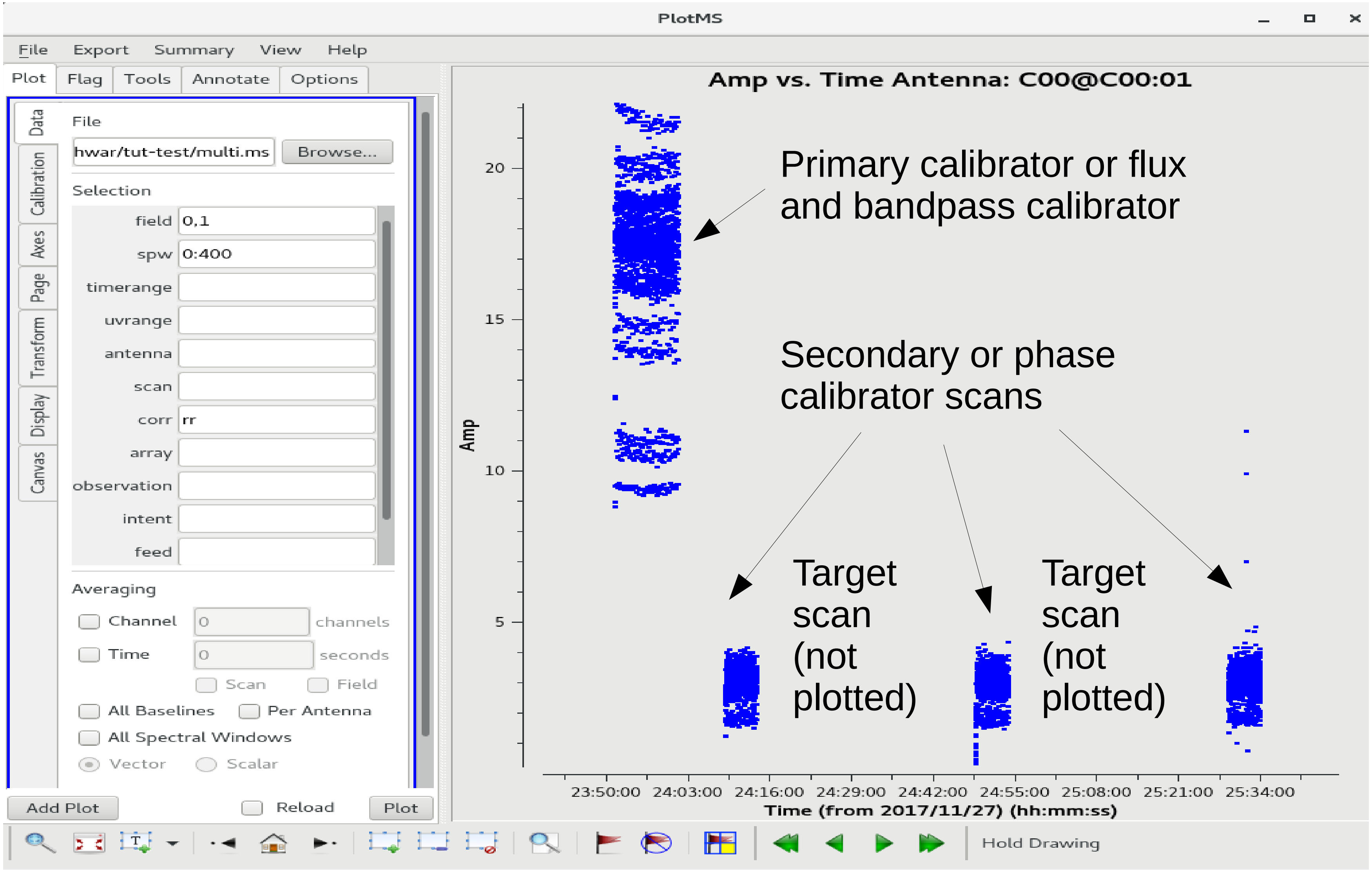
Flux cal – Phase cal – (Target source – Phase cal) – Flux cal
Loop over

-Delay calibration
-Absolute flux calibration
-Phase calibration
-Bandpass calibration
-Application of the calibration to the target

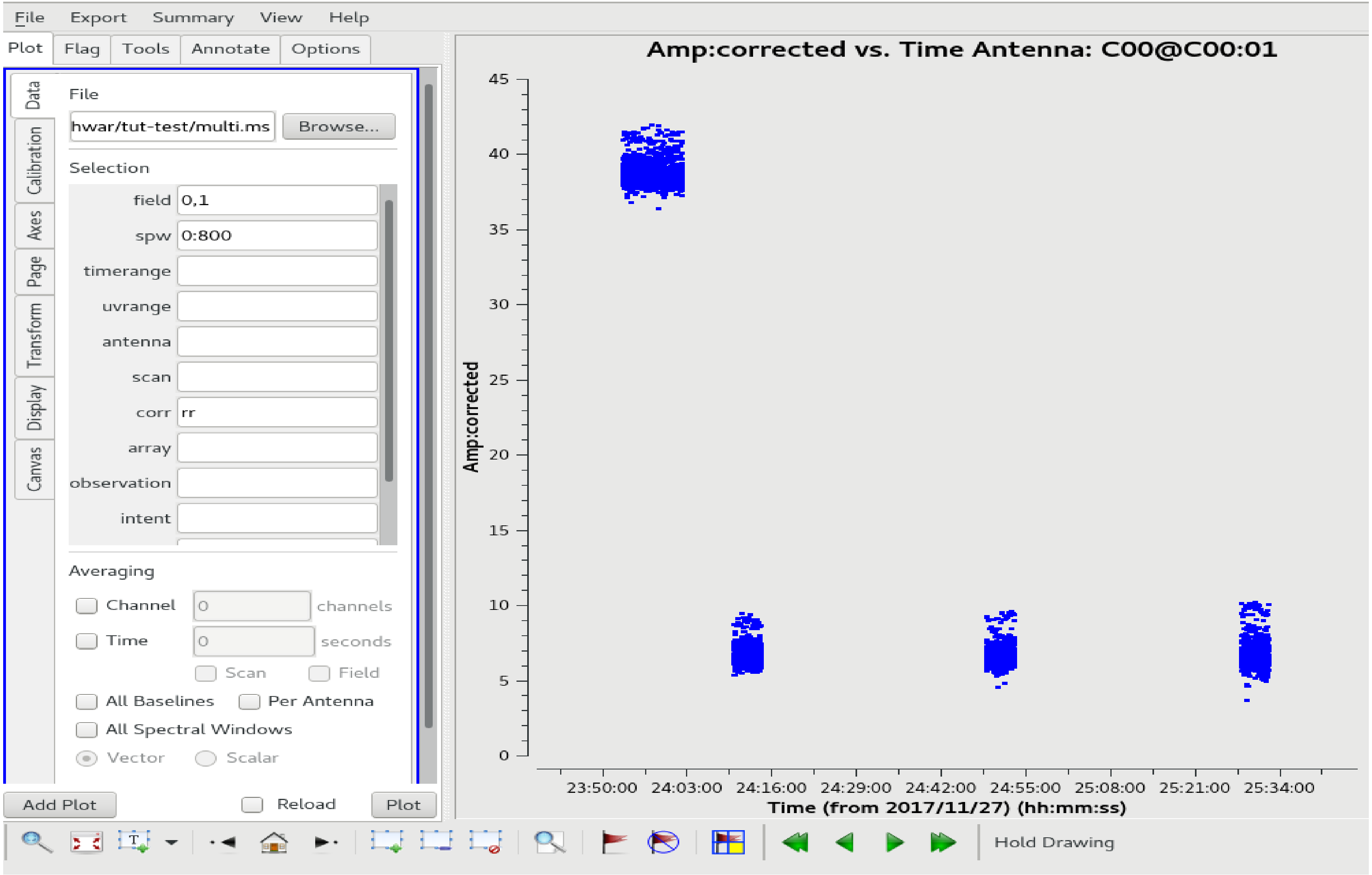
Example : MS file header

```
…:summary+    Observed from   27-Nov-2017/23:53:01.1   to   28-Nov-2017/01:32:52.6 (TAI)
…::summary
…:summary+    ObservationID = 0          ArrayID = 0
…:summary+   Date              Timerange (TAI)          Scan  FldId FieldName            nRows     SpwIds   Average Interval
…:summary+   27-Nov-2017/23:53:01.1 - 00:02:49.0      1      0 3C147                 31755  [0]   [8.05]
…:summary+   28-Nov-2017/00:10:03.8 - 00:14:53.7      2      1 1248-199              15660  [0]   [8.05]
…:summary+               00:17:59.0 - 00:48:02.8      3      2 SGRB                  97440  [0]   [8.05]
…:summary+               00:48:51.2 - 00:53:49.1      4      1 1248-199              16095  [0]   [8.05]
…:summary+               00:57:02.4 - 01:27:06.3      5      2 SGRB                  97440  [0]   [8.05]
…:summary+               01:27:54.6 - 01:32:52.6      6      1 1248-199              16095  [0]   [8.05]
…::summary           (nRows = Total number of rows per scan)
…::summary Fields: 3
…:summary+   ID   Code Name               RA               Decl            Epoch   SrcId      nRows
…:summary+   0    C    3C147              05:42:36.129706 +49.51.07.19954 J2000   0          31755
…:summary+   1    C    1248-199           12:48:23.899892 -19.59.18.69778 J2000   1          47850
…:summary+   2    C    SGRB               13:09:48.089888 -23.22.53.34805 J2000   2         194880
…::summary Spectral Windows:  (1 unique spectral windows and 1 unique polarization setups)
…:summary+   SpwID  Name   #Chans   Frame    Ch0(MHz)  ChanWid(kHz)  TotBW(kHz) CtrFreq(MHz)  Corrs
…:summary+   0       none    2048   TOPO     550.049        97.656    200000.0     650.0000   RR  LL
…::summary Sources: 3
…:summary+   ID   Name                  SpwId RestFreq(MHz)  SysVel(km/s)
…:summary+   0    3C147                 0     0              1.03605565933e-320
…:summary+   1    1248-199              0     0              1.03605565933e-320
…:summary+   2    SGRB                  0     0              1.03605565933e-320
…::summary Antennas: 30:
```

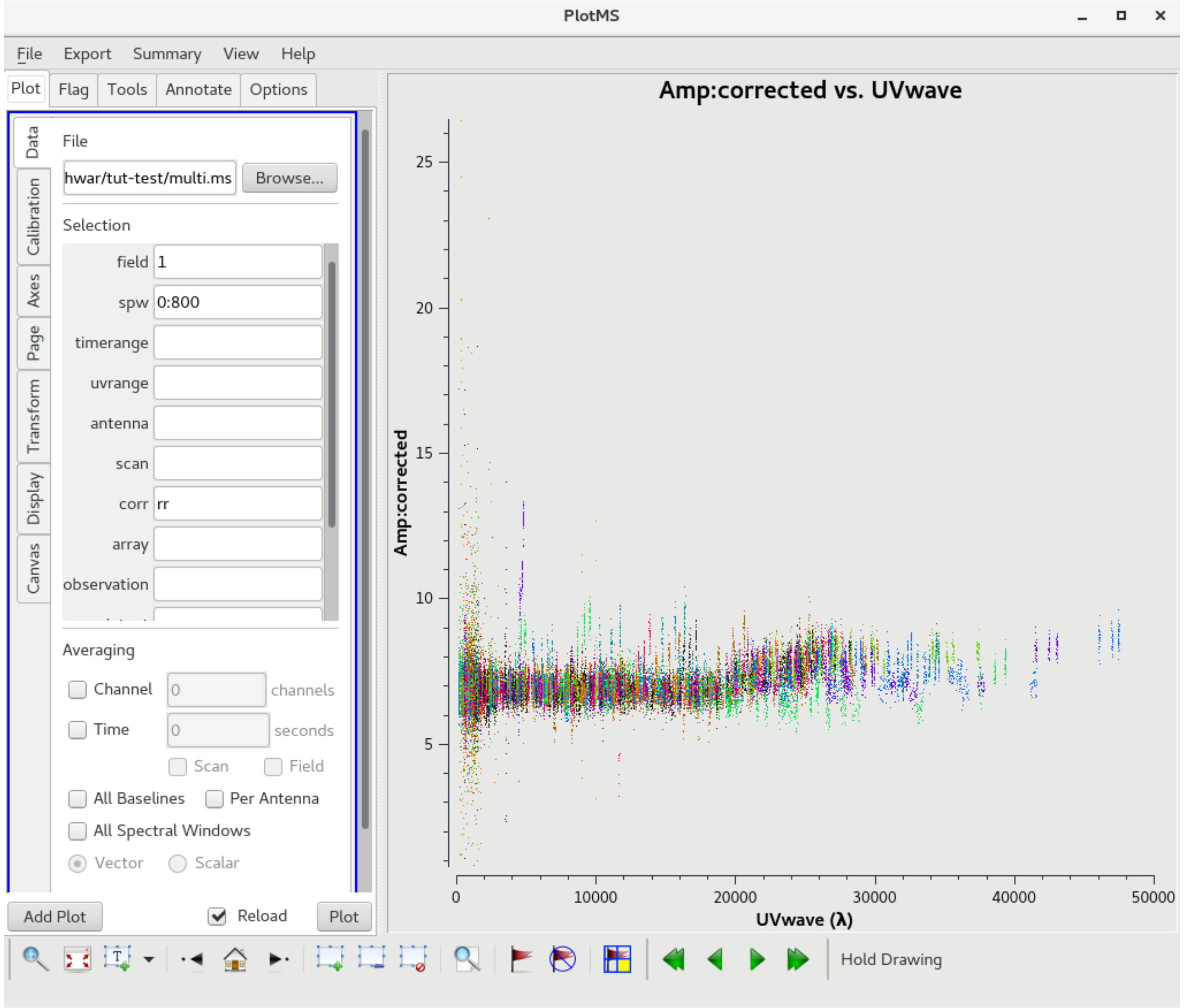
Uncalibrated data on calibrators



Calibrated data on calibrators



Calibrated amplitude Vs uv



Calibrator phases (point source)