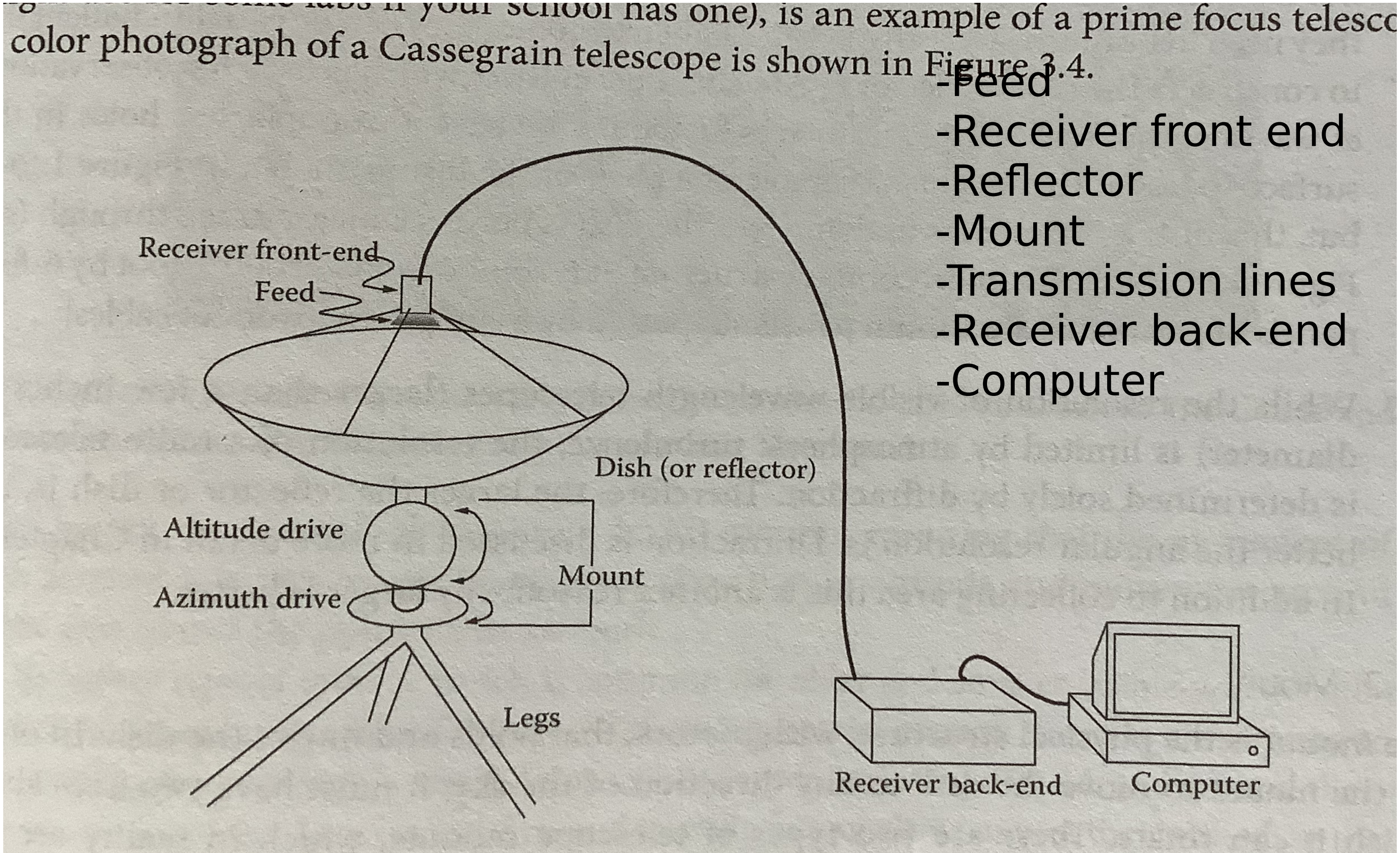
- Single dish radio telescopes

Astronomical Techniques II : Lecture 2

Ruta Kale

Essential Radio Astronomy (Chp 3)
Low Frequency Radio Astronomy (Chp. 3)

A basic radio telescope

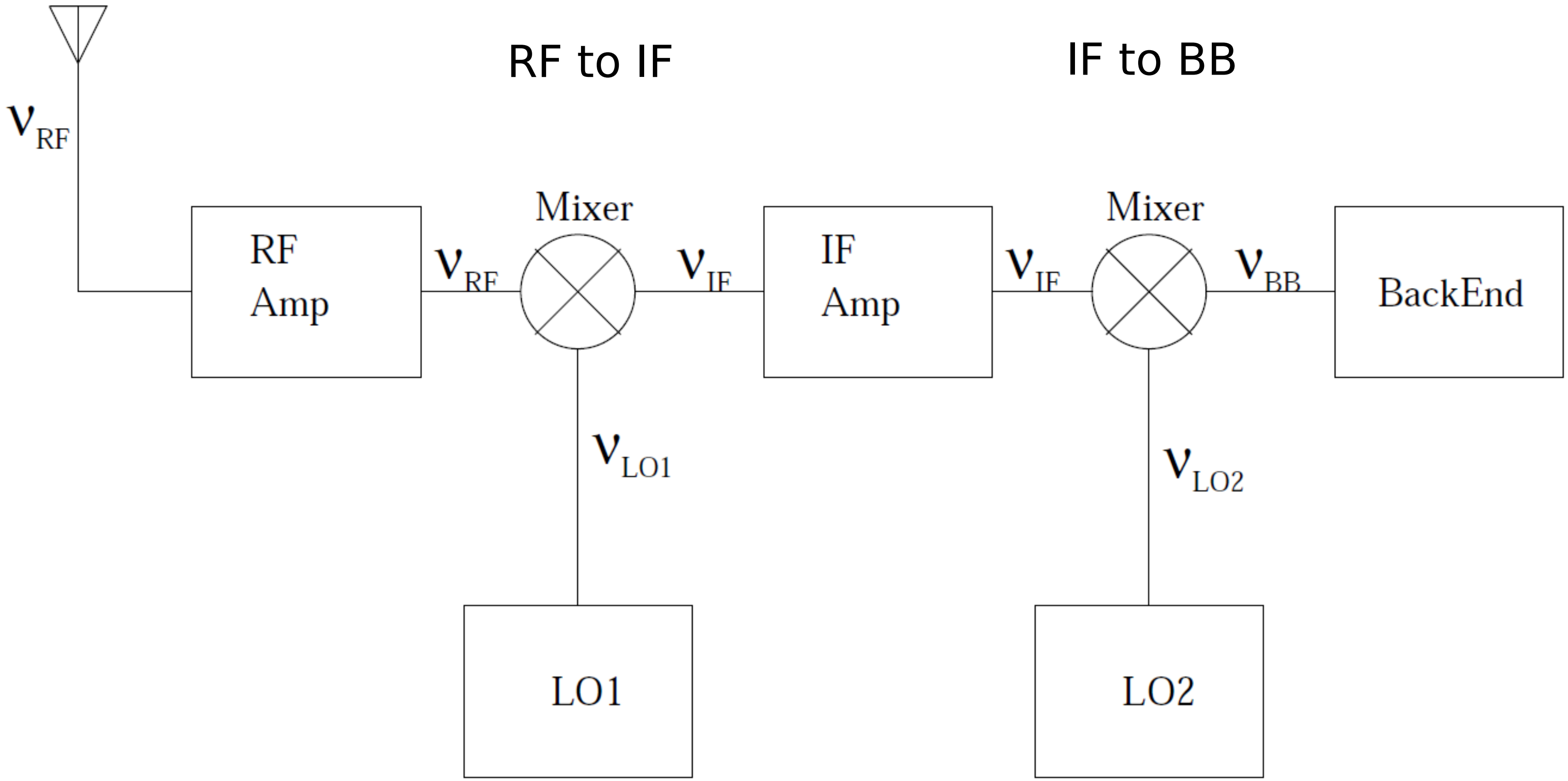


- Feed
- Receiver front end
- Reflector
- Mount
- Transmission lines
- Receiver back-end
- Computer

Radio telescope antennas

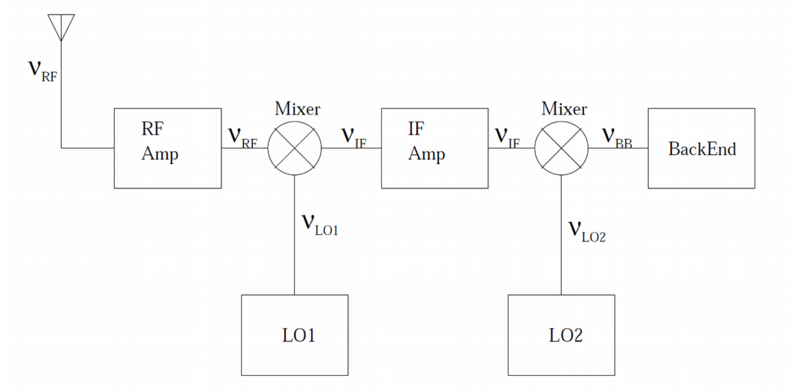
- The region of transition between a free space wave and a guided wave or vice-versa.
- For a radio telescope the antenna acts as a collector of radio waves.
- The response of an antenna as a function of direction is given by the antenna “pattern”. By *reciprocity* this pattern is the same for both receiving and transmitting.

Block diagram of a single dish radio telescope



Block diagram of a single dish radio telescope

- EM waves impinge on the antenna and create a fluctuating voltage – frequency is the same as of the incoming wave called *Radio frequency (RF)*.
- Needs *amplification*: Low noise amplifier (LNA) at the receiver front-end amplifies the signal.
- *Mixer*: changes the frequency of the incoming signal. Pure sine wave by tunable signal generator – Local oscillator (LO). Mixing – also called heterodyning.



GMRT LO 175 and 130 MHz IFs for two polarizations

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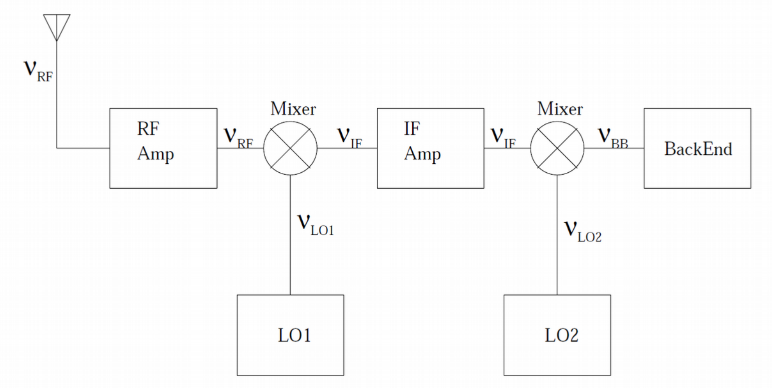

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Brings the signal to an *intermediate frequency (IF)*: allows the use of same backend for a number of different frequencies observed – economical. Sometimes more mixers: IFs that minimise transmission losses and those that are optimal for best amplification – super-heterodyne systems.

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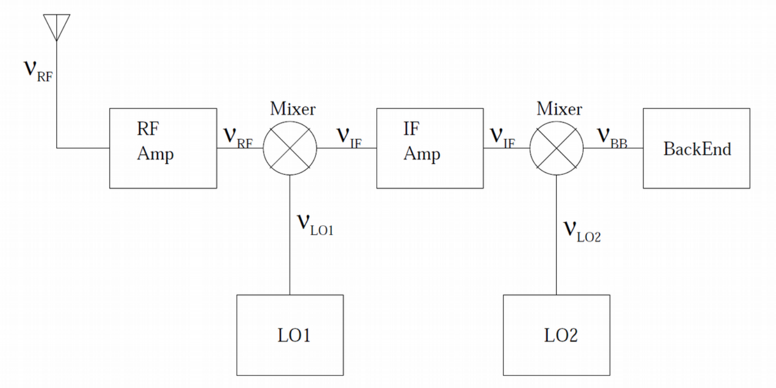


- Another stage of amplification followed by a mixer to convert the signal to *Baseband (BB)*.
- Passed to a backend: square-law detector/ correlation/ a pulsar backend

GMRT LO 175 and 130 MHz IFs for two polarizations

Effective aperture

Antenna's ability to absorb the waves that are incident on it is measured by the quantity "effective aperture", A_e.

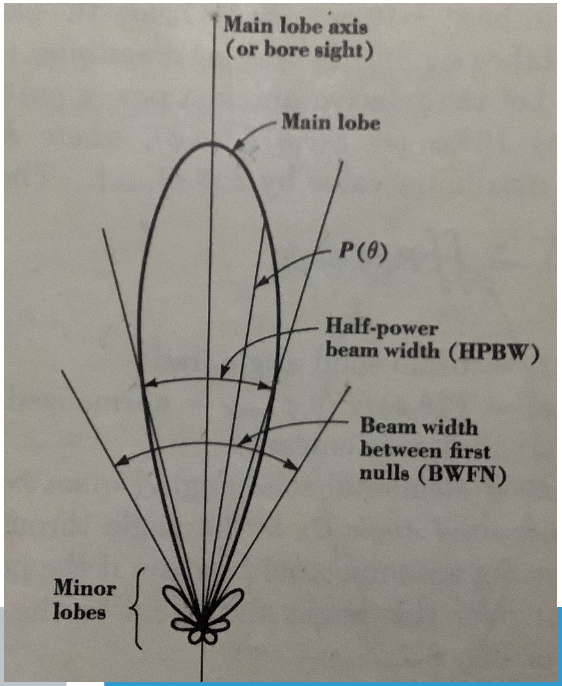
$$A_e = \frac{\text{Power density available at the antenna terminals}}{\text{Flux density of the wave incident on the antenna}} \qquad \frac{W/Hz}{W/m^2/Hz} = m^2$$

Also called effective area of the antenna. It is a function of direction, thus:

$$A_e \;=\; A_e(\theta,\phi)$$

The power pattern of the antenna describes the directional response of an antenna (normalized to unity at the maximum):

$$P(\theta,\phi) \;=\; \frac{A_e(\theta,\phi)}{A_e^{max}} \qquad \Theta_{HPBW} \sim \lambda/D$$



Directivity, gain and aperture efficiency

Another measure of the response of the antenna as a function of direction is described by "directivity":

$$D(\theta, \phi) = \frac{\text{Power emitted into } (\theta, \phi)}{(\text{Total power emitted})/4\pi} = \frac{4\pi P(\theta, \phi)}{\int P(\theta, \phi)\, d\Omega}$$

Aperture efficiency is the ratio of the maximum effective aperture and the geometric cross sectional area of the reflector:

$$\eta = \frac{A_e^{max}}{A_g}$$

Gain and directivity

$$G(\theta, \phi) = \frac{\text{Power emitted into } (\theta, \phi)}{(\text{Total power input})/4\pi} \ \eta$$

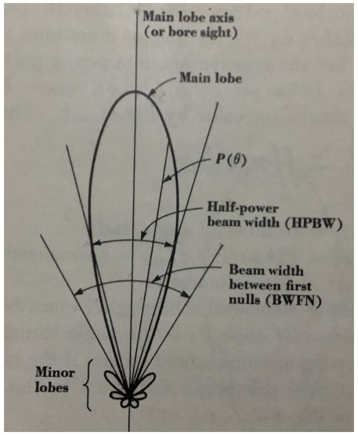
Gain is often given in decibels (dB) which is:

$G(dB) = 10\log_{10}G$

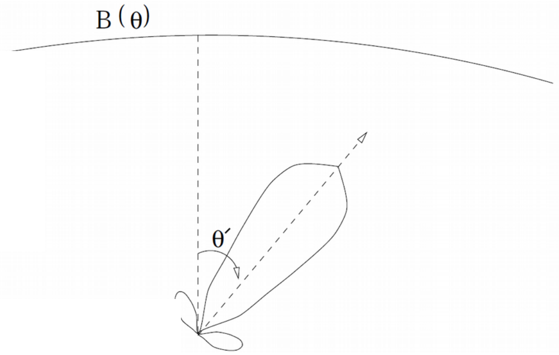
The convenience is that when there are amplifiers in succession the total gain is simply the addition.

Effect of the pattern on observed sky:

Consider observing a sky brightness distribution *B(θ)* with a telescope having a power pattern as shown. Then the power available at the antenna terminals is:



$$W(\theta^{'}) = \frac{1}{2}\int B(\theta)A_e(\theta-\theta^{'})d\theta$$



$$W(\theta^{'},\phi^{'}) = \frac{1}{2}\int B(\theta,\phi)A_e(\theta-\theta^{'},\phi-\phi^{'})\sin(\theta)d\theta d\phi$$

$$T_A(\theta^{'},\phi^{'}) = \frac{A_e^{max}}{\lambda^2}\int T_B(\theta,\phi)P(\theta-\theta^{'},\phi-\phi^{'})\sin(\theta)d\theta d\phi$$

Antenna temperature is the weighted average of the sky temperature – the weighting function is the power pattern of the antenna.

Relation between directivity and effective aperture

Consider an antenna terminated in a resistor and the entire setup placed in a blackbox at temperature T. At thermal equilibrium, the power flowing from resistor to antenna is:

$$P_{R \rightarrow A} = kT$$

And that flowing from the antenna to the resistor is:

$$P_{A \rightarrow R} = \left(\frac{A_e^{max} kT}{\lambda^2}\right) \int P(\theta, \phi) d\Omega$$

Since the net power is zero, we can equate the two and get:

$$A_e^{max} = \frac{\lambda^2}{\int P(\theta, \phi) d\Omega}$$

Maximum effective aperture is determined by the shape of the power pattern alone.

Relation between directivity and effective aperture

For a reflecting telescope,

$$\int P(\theta,\phi)d\Omega \sim \Theta^2_{HPBW} \sim \left(\frac{\lambda}{D}\right)^2$$

And thus,

$$A_e^{max} \sim D^2 \qquad\qquad A_e^{max} = \frac{\lambda^2}{\int P(\theta,\phi)d\Omega}$$

The max. effective aperture scales like the geometric area of the reflector. Also,

$$A_e = A_e^{max} P(\theta,\phi) = \frac{\lambda^2 P(\theta,\phi)}{\int P(\theta,\phi)d\Omega}$$

$$D(\theta,\phi) = \frac{4\pi}{\lambda^2} A_e(\theta,\phi)$$

Recall: $D(\theta,\phi) = \dfrac{4\pi P(\theta,\phi)}{\int P(\theta,\phi)\ d\Omega}$

Application: Finding power at one antenna from a signal transmitted from another

Consider sending information from antenna 1 with gain $G_1(\theta,\phi)$ and input power P_1 to antenna 2 with directivity $D_2(\theta,\phi)$ at a distance R away.
The flux density at antenna 2 is:

$$S = \frac{P_1}{4\pi R^2} G_1(\theta, \phi)$$

Factor G encodes that the power is not isotropically distributed

Power available at antenna 2 is :

$$W = A_{2e} S = \frac{P_1}{4\pi R^2} G_1(\theta, \phi) A_{2e}$$

Recall: $D(\theta, \phi) = \frac{4\pi}{\lambda^2} A_e(\theta, \phi)$

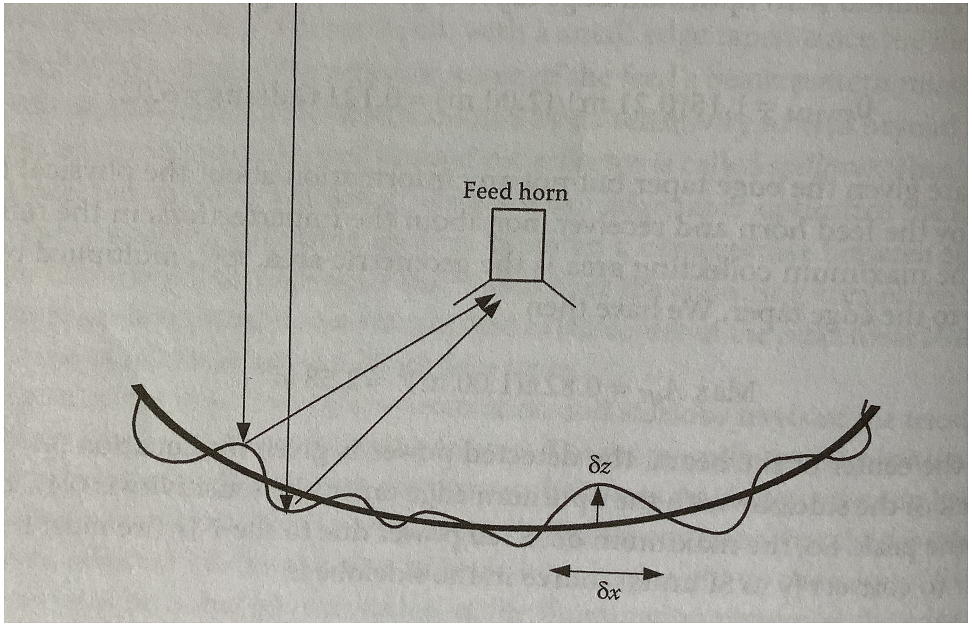
After substituting for the effective aperture,

$$W = (\frac{\lambda}{4\pi R})^2 P_1 G_1(\theta, \phi) D_2(\theta', \phi')$$

Friis transmission equation

Surface errors

- Imperfections in the reflecting surface
- Cause path length differences
- Reduce on-axis sensitivity of the telescope – loss in effective collecting area.
- Ruze equation
- *Follow 3.3.4 of ERA for derivation.*



$$\eta_s = \exp\left[-\left(\frac{4\pi\sigma}{\lambda}\right)^2\right]$$

Ratio of collecting area with and without errors

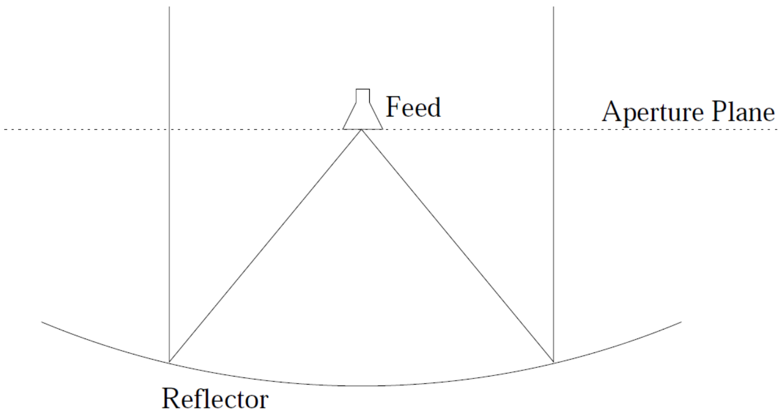
RMS of surface errors

Aperture illumination

The beam pattern of the feed determines the ilumination of the primary reflector.

Ideally would like to have uniform sensitivity from centre to the edge of the dish – but we do not want unwanted radiation from the ground to be picked up.

A quantity that describes how the feed's beam is distributed on the primary reflector is called *edge taper:* ratio of sensitivity at the centre to that at the edge.

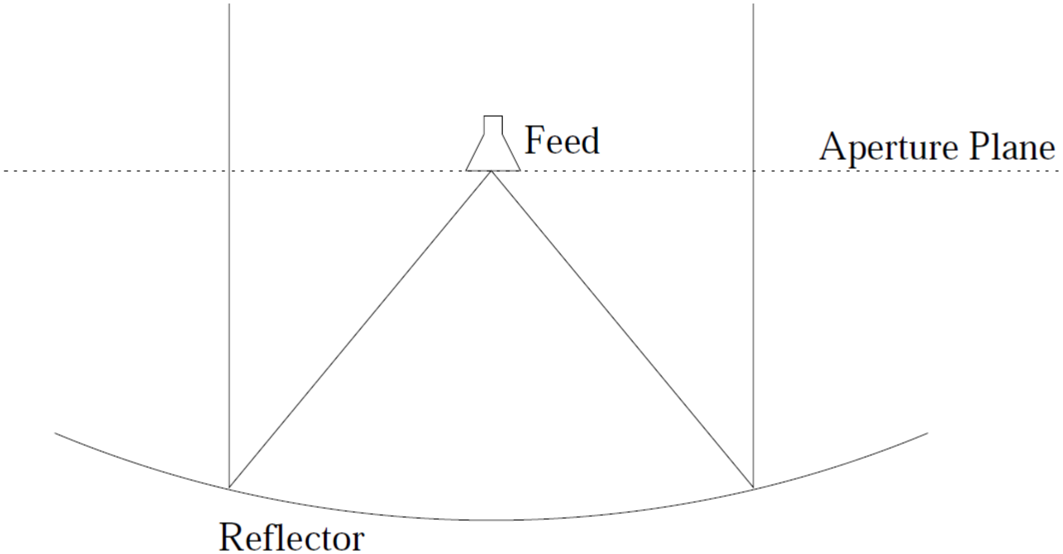


Illumination affects the angular resolution, sensitivity level in the sidelobes and effective collecting area.

A more tapered illumination will have a broader main beam or equivalently smaller effective aperture but also lower sidelobes than uniform illumination. If the illumination is high towards the edges there will be a lot of spillover.

Aperture illumination of a parabolic dish

Aperture of a reflector is the plane through with all the rays pass.
Beam pattern of an antenna is its power gain as a function of direction.

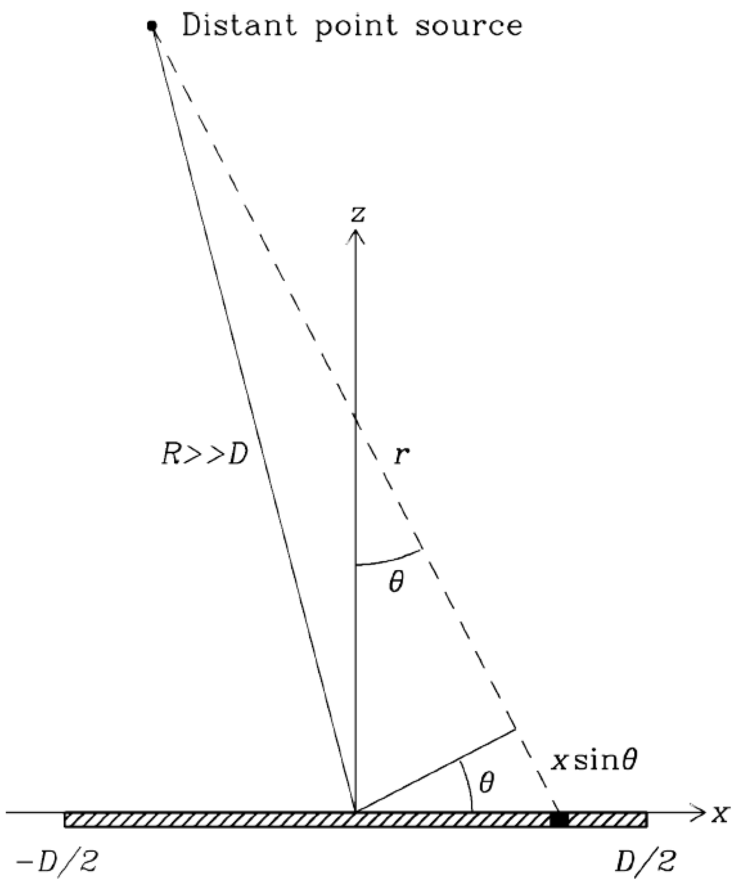


Finite size of the dish results in diffraction.

Huygen's principle:
Aperture can be treated as a collection of small elements acting individually.

Each point in a wave front can be regarded as an imaginary source – and the wave at at other point is the addition of the contribution of each of these point sources.

1D aperture of width D.



The electric field in the far field regime due to the element x to x + dx is:

$$df \propto g(x)\frac{\exp(-i2\pi r(x)/\lambda)}{r(x)}dx$$

$$r \approx R + x\sin\theta = R + xl \quad \text{where} \quad l \equiv \sin\theta$$

$$df \propto g(x)\exp(-i2\pi xl/\lambda)dx$$

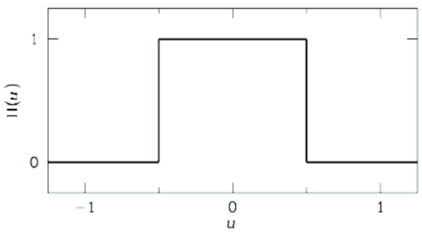
$u = x/\lambda$

$$f(l) = \int_{\text{aperture}} g(u)e^{-i2\pi lu}du$$

Fourier transform of the aperture field pattern.

1D uniformly illuminated aperture:

$$g(u) = \text{ constant}, \qquad -\frac{D}{2\lambda} < u < +\frac{D}{2\lambda}$$



$$f(l) = \int_{-1/2}^{+1/2} e^{-i2\pi l u} du \qquad l = \sin\theta$$

$$= \left.\frac{e^{-i2\pi l u}}{-i2\pi l}\right|_{-1/2}^{+1/2}$$

$$= \frac{e^{-i\pi l} - e^{i\pi l}}{-i2\pi l}$$

$$= \frac{\sin(\pi l)}{(\pi l)}$$

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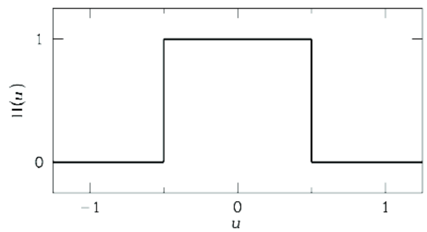

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$$\theta \ll 1 \text{ radian}, \; l = \sin\theta \approx \theta$$

$$= \left. \frac{e^{-i2\pi l u}}{-i2\pi l} \right|_{-1/2}^{+1/2}$$

$$f(\theta) = \frac{D}{\lambda}\,\mathrm{sinc}\left(\frac{\theta D}{\lambda}\right) \qquad P(\theta) = f^2(\theta) = \left(\frac{D}{\lambda}\right)^2 \mathrm{sinc}^2\left(\frac{\theta D}{\lambda}\right)$$

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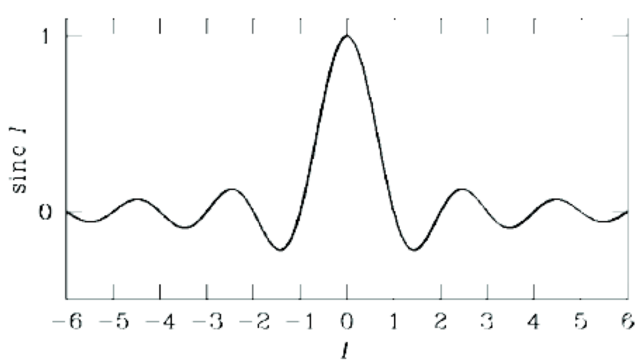

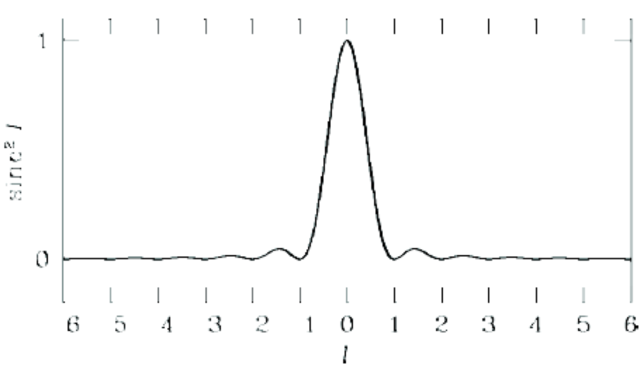

Aperture blockage

The feed is located above the reflector and thus blocks the aperture.
What is the effect of this on the antenna pattern ?

Now that we know the aperture plane and far field are related by a Fourier transform, we can find the estimate the effect of the aperture blockage using the properties of FT.

A uniform aperture with a width d - FT ?
A uniform aperture with with l – FT ?

Aperture blockage

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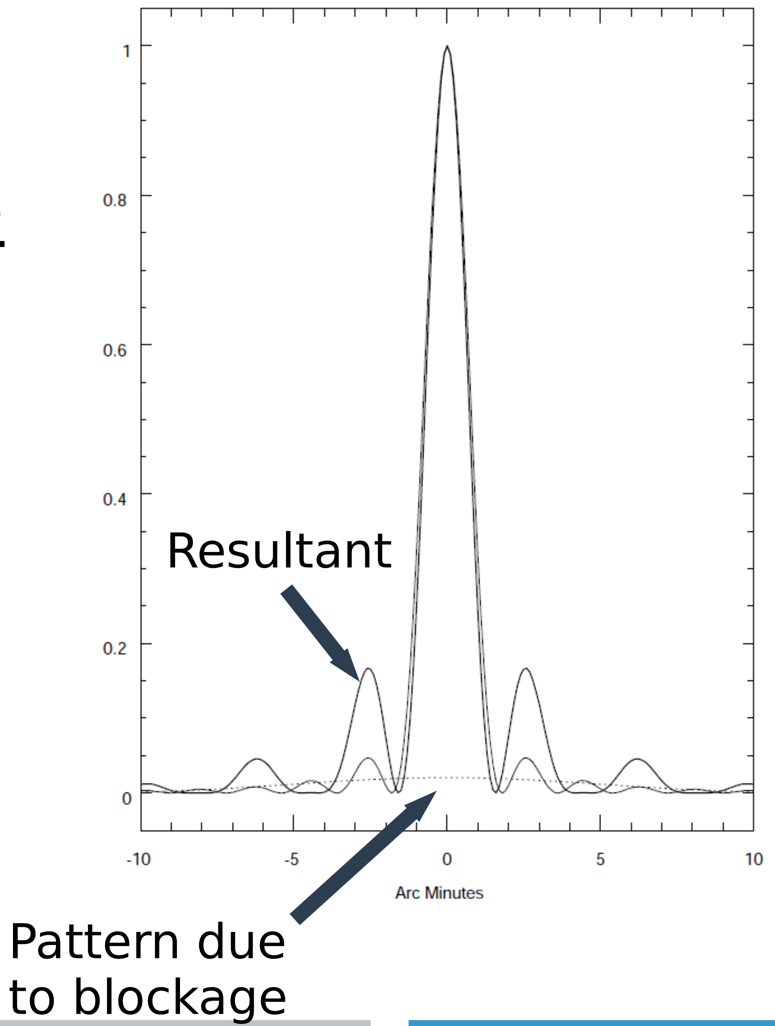
$$\mu = \lambda / l$$

$$E(\mu) \propto \frac{sin(\pi l \mu / \lambda)}{\pi \mu} - \frac{sin(\pi d \mu / \lambda)}{\pi \mu}$$

Should be minimised for a good beam.
Offset feeds to eliminate blockage



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