- Measurement equation
- Special topics

Astronomical Techniques II : **Lecture 14**

Ruta Kale

Hamaker, Bregman, Sault Measurement Equation

Jones matrices:

Electric field of a mono-chromatic wave

$$E_0 \cos(\omega t + \phi) \qquad\qquad E = E_0 \exp(i\phi)$$

If E_R and E_L are the complex amplitudes of right and left circularly polarized components of a wave respectively, the output polarization states are a linear combination of the input states:

$$\begin{pmatrix} E'_{\mathrm{R}} \\ E'_{\mathrm{L}} \end{pmatrix} = \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{} \begin{pmatrix} E_{\mathrm{R}} \\ E_{\mathrm{L}} \end{pmatrix}$$

↓

Jones matrix,
Jones 1941

$$\mathbf{J}_{\mathrm{overall}} = \mathbf{J}_1 \mathbf{J}_2 \mathbf{J}_3$$

In our context the Jones matrices are:

$$\mathbf{J}_{\mathrm{gain}} = \begin{pmatrix} g_{\mathrm{R}} & 0 \\ 0 & g_{\mathrm{L}} \end{pmatrix}$$

$$\mathbf{J}_{\mathrm{leakage}} = \begin{pmatrix} 1 & D_{\mathrm{R}} \\ -D_{\mathrm{L}} & 1 \end{pmatrix}$$

Jones matrix will vary from antenna to antenna and will be function of time and frequency.

Jones matrices are combined multiplicatively, even complicated systems can be handled.

In a non-polarimetric observation the measured visibility is related to the true visibility as,

$$V'_{ij} = g_i g_j^* V_{ij}$$

The outer product (A⊗B) of two matrices **A** and **B** is defined as a new matrix in which each element is a_{ij}B

It can be shown that: $$(\mathbf{A}_i\mathbf{B}_i) \otimes (\mathbf{A}_j\mathbf{B}_j) = (\mathbf{A}_i \otimes \mathbf{A}_j)(\mathbf{B}_i \otimes \mathbf{B}_j)$$

The input to the correlator is $E'_i = \mathbf{J}_i E_i$ $\quad E'_j = \mathbf{J}_j E_j$

$$\begin{aligned} E'_i \otimes E'^*_j &= (\mathbf{J}_i E_i) \otimes (\mathbf{J}_j E_j)^* \\ &= (\mathbf{J}_i \otimes \mathbf{J}_j^*)(E_i \otimes E_j^*) \end{aligned}$$

$$E_i \otimes E_j^* = \begin{pmatrix} E_{\mathrm{R},i}\, E^*_{\mathrm{R},j} \\ E_{\mathrm{R},i}\, E^*_{\mathrm{L},j} \\ E_{\mathrm{L},i}\, E^*_{\mathrm{R},j} \\ E_{\mathrm{L},i}\, E^*_{\mathrm{L},j} \end{pmatrix}$$

$$E_i \otimes E_j^* = \begin{pmatrix} E_{\mathrm{R},i}\, E_{\mathrm{R},j}^* \\ E_{\mathrm{R},i}\, E_{\mathrm{L},j}^* \\ E_{\mathrm{L},i}\, E_{\mathrm{R},j}^* \\ E_{\mathrm{L},i}\, E_{\mathrm{L},j}^* \end{pmatrix}$$

After integration one gets:

$$< E_i \otimes E_j^* > = \begin{pmatrix} V_{\mathrm{RR},ij} \\ V_{\mathrm{RL},ij} \\ V_{\mathrm{LR},ij} \\ V_{\mathrm{LL},ij} \end{pmatrix}$$

Coherency vector

The measured coherency vector is related to the true coherency vector as:

$$V'_{ij} = (\mathbf{J}_i \otimes \mathbf{J}_j^*) V_{ij}$$

Calibration will mean estimation of the various Jones matrices. Inverting them will give us nominally perfect data.

Elegant formulation for polarization and direction dependent effects: widely used now.

Application (CASA Documentation)

$$\vec{V}_{ij} = J_{ij}\,\vec{V}_{ij}^{\,\mathrm{IDEAL}}$$

Most of the effects are antenna based and thus:

$$J_{ij} = J_i \otimes J_j^*.$$

The effects represented by matrices are as follows:

$$\vec{V}_{ij} = M_{ij}\,B_{ij}\,G_{ij}\,D_{ij}\,E_{ij}\,P_{ij}\,T_{ij}\,\vec{V}_{ij}^{\,\mathrm{IDEAL}}$$

T_{ij} = Polarization-independent multiplicative effects due to troposphere
P_{ij} = Parallactic angle (antenna mount)
E_{ij} = Effects such as elevation dependent collecting area
D_{ij} = Instrumental polarization response
G_{ij} = Electronic gain along signal path – also polarization dependent effects
B_{ij} = Frequency dependent response – bandpass
M_{ij} = Baseline based errors – those not factorable into antenna-based parts (used with caution!)

Mosaicking

Large sources:

- Sources with angular sizes larger than that sampled with the shortest baseline of the interferometer.

$$\theta > \lambda / b_{\min}$$

- Sources larger than the size of the primary beam

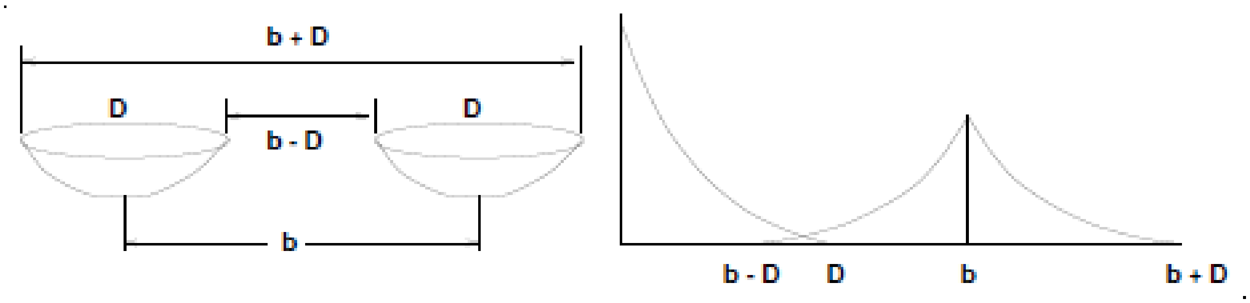
$$\theta > \lambda / D$$

Observe multiple pointing make individual images and then combine – linear mosaic.

Joint deconvolution (Cornwell 1985)

Ekers and Rots (1979)

SIRA Chp 20



Radio Frequency Interference

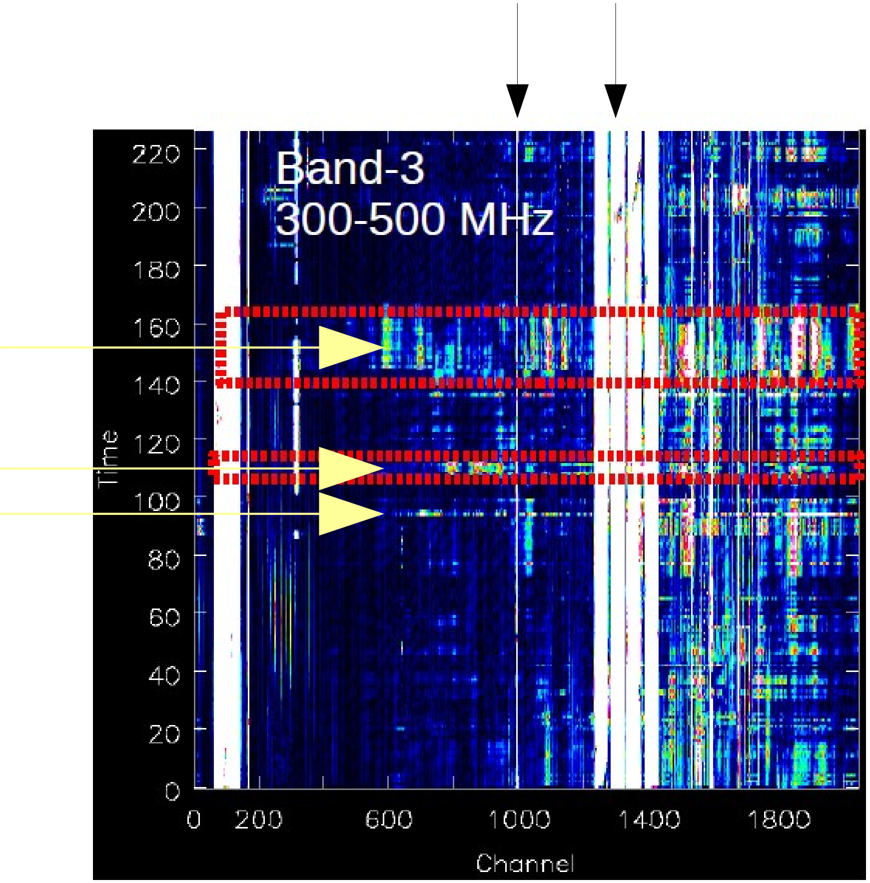
Mitigation of RFI in itself is a topic of research.

RFI is called *narrow band* when it affects data at certain frequencies or narrow bands of frequencies.

RFI is called *broadband* when it affects the entire spectrum – this is the effect of spikes in time that show up as broadband features in the spectrum.

At uGMRT notch filters are used e.g. Band-2 to arrest the effect of narrowband RFI.

For broadband RFI a realtime RFI excision system has been implemented – this is one of the first of its kind and is being tested.



Special topics

- Spectral line studies

HI studies and in general spectral lines – bandpass calibration and continuum subtraction are important aspects.

- Multi-frequency synthesis (MFS): SIRA Chp 21

Important for wideband studies such as with the uGMRT.

- Very Long Baseline Interferometry: SIRA Chp 22

Interferometry with antennas having no data links. Allows milliarcsec resolution – high resolution probes for extra-galactic sources.

- Direction dependent effects

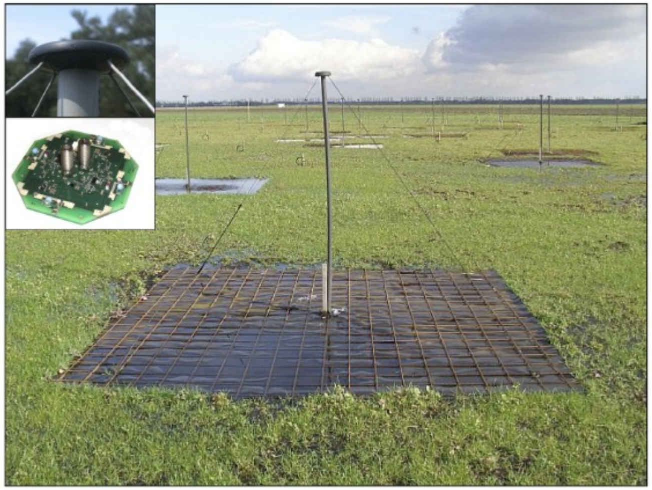
E.g. asymmetric sidelobes of the primary beam, non-uniform ionosphere over the array.
Refences: LOFAR imaging, CASA AW-projection

LOFAR

Low Frequency Array in The Netherlands (van Haarlem et al 2013). Operational and producing fantastic images at low frequencies.

High band Antennas (110 – 240 MHz)

Low Band Antennas (10 – 90 MHz)



Additional references

NRAO Synthesis Imaging School online lectures:
http://www.cvent.com/events/virtual-17th-synthesis-imaging-workshop/agenda-0d59eb6cd1474978bce811194b2ff961.aspx

Synthesis Imaging in Radio Astronomy II
Chp. 33 Noise and Interferometry by Radhakrishnan

NRAO CASA documentation
https://casa.nrao.edu/casadocs/casa-6.1.0/usingcasa

Observatory websites have some useful resources: uGMRT, JVLA, LOFAR, MeerKAT, ASKAP, SKA

A look at real dataset

Data from the uGMRT Band-4.

Flagging
Calibration (sometimes further flagging is done and data are recalibrated.)
- delay
- complex gain
- flux calibration
- bandpass calibration

Imaging
Self-calibration:
- Phase only
- Amplitude and phase

The number of iterations depend on how much the image improves.

CASA tutorial
http://www.ncra.tifr.res.in/~ruta/ras-tutorials/CASA-tutorial.html