- Closure phase and self-calibration
- Sensitivity

Astronomical Techniques II : **Lecture 11**

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Low Frequency Radio Astronomy (Chp. 4)
http://www.ncra.tifr.res.in/ncra/gmrt/gmrt-users/low-frequency-radio-astronomy

Synthesis imaging in radio astronomy II, Chp 5

Interferometry and synthesis in radio astronomy (Chp 10)

CASA tutorial
http://www.ncra.tifr.res.in/~ruta/ras-tutorials/CASA-tutorial.html

Self-calibration

Aim is to produce a model of the intensity distribution, the Fourier transform of which when corrected by gain factors will reproduce the measured visibilities within the noise level.

A convenient method due to Schwab 1980 is to *minimize* the sum of squares of residuals by varying complex gains g_i, g_j and the model sky,

$$\mathcal{S} = \sum_k \sum_{\substack{i,j \\ i \neq j}} w_{ij}(t_k) \left| \widetilde{V}_{ij}(t_k) - g_i(t_k) g_j^*(t_k) \widehat{V}_{ij}(t_k) \right|^2$$

The w_{ij} are weights. The time over which gains are assumed to be constant depend on the effects that govern their variation.

In most cases we have small number of degrees of freedom (the gains to be determined) and a large number of measurements of visibilities.

Closure quantities and self-calibration

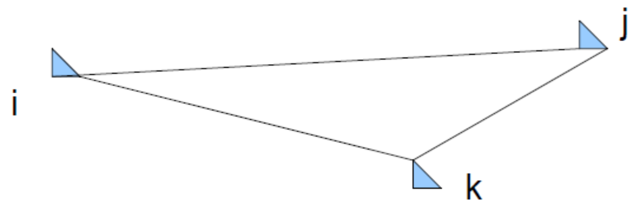
$$\widetilde{V}_{ij}(t) = g_i(t)g_j^*(t)V_{ij}(t) + \epsilon_{ij}(t)$$

Observed True

$$\theta_i(t) = \arg g_i(t)$$

$$\widetilde{\phi}_{ij}(t) = \phi_{ij}(t) + \theta_i(t) - \theta_j(t) + \text{noise term}$$

Considering a loop of three antennas, the observed "closure phase" is,



$$\begin{aligned}\widetilde{C}_{ijk}(t) &= \widetilde{\phi}_{ij}(t) + \widetilde{\phi}_{jk}(t) + \widetilde{\phi}_{ki}(t) \\ &= \phi_{ij}(t) + \phi_{jk}(t) + \phi_{ki}(t) + \text{noise term} \\ &= C_{ijk}(t) + \text{noise term}\end{aligned}$$

Notice that this is independent of the individual errors, thus can be used.

A closure amplitude can be formed for a loop of 4 antennas:

$$\Gamma_{ijkl}(t) = \frac{|\widetilde{V}_{ij}(t)|\,|\widetilde{V}_{kl}(t)|}{|\widetilde{V}_{ik}(t)|\,|\widetilde{V}_{jl}(t)|}$$

Used in self-calibration by Readhead and Wilkinson 1978.

1. Make a model image
2. for all independent closure phases, use the model to provide estimates of the true phases on two baselines and derive the phase on the other baseline in the loop from the observed phase.
3. form a new model using CLEAN from the observed visibility amplitudes and the predicted phases.
4. Repeat 2 until the model is satisfactory.

Cotton 1979 revised this scheme.

Sensitivity

Sensitivity is a measure of the weakest source of emission that can be detected.

In radio astronomy power is written in terms of an equivalent temperature, T, of a matched termination on the input of the receiver:

$$P = k_{\mathrm{B}}\, T\, \Delta\nu$$

The power entering the feed is amplified by g in voltage and thus in power,

$$P_{\mathrm{a}} = g^2\, k_{\mathrm{B}}\, T_{\mathrm{a}}\, \Delta\nu$$

$$P_{\mathrm{N}} = g^2\, k_{\mathrm{B}}\, T_{\mathrm{sys}}\, \Delta\nu$$

T_a is the antenna temperature and
T_{sys} is the system temperature: includes receiver noise, feed losses, spillover, atmospheric emission, galactic background and cosmic background.

The power from the source can be related to the flux density S, the area of the antenna A, the antenna efficiency η_a as

$$P_{\mathrm{a}} = \frac{1}{2}\, g^2\, \eta_{\mathrm{a}}\, A\, S\, \Delta\nu$$
$$= g^2\, k_{\mathrm{B}}\, K\, S\, \Delta\nu$$

Radiation received only from one channel; half for an unpolarized source.

Where $K = (\eta_{\mathrm{a}}\, A)\, /\, (2\, k_{\mathrm{B}})$

K is a measure of antenna performance.

System equivalent flux density (SEFD) is also often used:

$$SEFD = \frac{T_{\mathrm{sys}}}{K}$$

Sensitivity of an interferometer

Consider an interferometer with a single output which is the product of the voltages from the two elements. The voltage from antenna i is the sum of source voltage and noise voltage, the power from antenna i is given by, (factor a includes gain) the expectation value of the square of the voltage:

$$
\begin{aligned}
\langle P_{\rm i} \rangle &= a_{\rm i} \left\langle (s_{\rm i} + n_{\rm i})^2 \right\rangle \\
&= a_{\rm i} \left[\langle s_{\rm i}^2 \rangle + \langle n_{\rm i}^2 \rangle \right] \\
&= g_{\rm i}^2 \, k_{\rm B} \, (T_{\rm ai} + T_{\rm sysi}) \, \Delta\nu \\
&= g_{\rm i}^2 \, k_{\rm B} \, (K_{\rm i} S_{\rm T} + T_{\rm sysi}) \, \Delta\nu
\end{aligned}
$$

Cross terms taken to be zero.

S_T is the total flux density seen by the antenna and we assume it is same for all.

The power after cross multiplication in the correlator can be obtained as (i and j are the antennas):

$$
\begin{aligned}
\langle P_{\rm ij} \rangle &= \frac{\sqrt{a_{\rm i} a_{\rm j}}}{\eta_{\rm s}} \Big\langle \, (s_{\rm i} + n_{\rm i})(s_{\rm j} + n_{\rm j}) \Big\rangle \\
&= \frac{\sqrt{a_{\rm i} a_{\rm j}}}{\eta_{\rm s}} \langle s_{\rm i} s_{\rm j} \rangle \\
&= \frac{g_{\rm i} \, g_{\rm j}}{\eta_{\rm s}} \sqrt{K_{\rm i} \, K_{\rm j}} \, k_{\rm B} \, \Delta\nu \, S_{\rm c}
\end{aligned}
$$

Efficiency factor

S_c is correlated flux density

Crane and Napier 1989, Walker 1995

To obtain SNR we look at the RMS fluctuations of the correlator output. We use the fact that the square of RMS fluctuation of a Gaussian random variable (product from the correlator) is equal to the expectation value of the variable squared minus the square of the mean.

$$\sigma^2(P_{\mathrm{ij}}) = \frac{a_{\mathrm{i}}a_{\mathrm{j}}}{\eta_{\mathrm{s}}^2}\left\langle \left[(s_{\mathrm{i}}+n_{\mathrm{i}})(s_{\mathrm{j}}+n_{\mathrm{j}})\right]^2 \right\rangle - \frac{g_{\mathrm{i}}^2\, g_{\mathrm{j}}^2}{\eta_{\mathrm{s}}^2} K_{\mathrm{i}}\, K_{\mathrm{j}}\, k_{\mathrm{B}}^2\, S_{\mathrm{c}}^2\, \Delta\nu^2$$

$$\langle P_{\mathrm{ij}}\rangle \;=\; \frac{\sqrt{a_{\mathrm{i}}a_{\mathrm{j}}}}{\eta_{\mathrm{s}}}\left\langle (s_{\mathrm{i}}+n_{\mathrm{i}})(s_{\mathrm{j}}+n_{\mathrm{j}})\right\rangle \;=\; \frac{g_{\mathrm{i}}\, g_{\mathrm{j}}}{\eta_{\mathrm{s}}}\sqrt{K_{\mathrm{i}}\, K_{\mathrm{j}}}\, k_{\mathrm{B}}\, \Delta\nu\, S_{\mathrm{c}}$$

To obtain SNR we look at the RMS fluctuations of the correlator output. We use the fact that the square of RMS fluctuation of a Gaussian random variable (product from the correlator) is equal to the expectation value of the variable squared minus the square of the mean.

$$\sigma^2(P_{\mathrm{ij}}) = \frac{a_{\mathrm{i}} a_{\mathrm{j}}}{\eta_{\mathrm{s}}^2} \left\langle \left[(s_{\mathrm{i}} + n_{\mathrm{i}})(s_{\mathrm{j}} + n_{\mathrm{j}}) \right]^2 \right\rangle - \frac{g_{\mathrm{i}}^2\, g_{\mathrm{j}}^2}{\eta_{\mathrm{s}}^2} K_{\mathrm{i}}\, K_{\mathrm{j}}\, k_{\mathrm{B}}^2\, S_{\mathrm{c}}^2\, \Delta\nu^2$$

$$\downarrow$$

$$\frac{a_{\mathrm{i}} a_{\mathrm{j}}}{\eta_{\mathrm{s}}^2} \left[2 \left\langle (s_{\mathrm{i}} + n_{\mathrm{i}})(s_{\mathrm{j}} + n_{\mathrm{j}}) \right\rangle^2 + \left\langle (s_{\mathrm{i}} + n_{\mathrm{i}})^2 \right\rangle \left\langle (s_{\mathrm{j}} + n_{\mathrm{j}})^2 \right\rangle \right]$$

Assuming all the processes involved are Gaussian processes then we can use the properties of these processes that are already known.
If x1, x2, x3 and x4 have a joint Gaussian distribution then,

$$\langle x_1 x_2 x_3 x_4 \rangle = \langle x_1 x_2 \rangle \langle x_3 x_4 \rangle + \langle x_1 x_3 \rangle \langle x_2 x_4 \rangle + \langle x_1 x_4 \rangle \langle x_2 x_3 \rangle$$

$$\left\langle \left[(s_\mathrm{i} + n_\mathrm{i})(s_\mathrm{j} + n_\mathrm{j}) \right]^2 \right\rangle$$

→

$$\left[2 \left\langle (s_\mathrm{i} + n_\mathrm{i})(s_\mathrm{j} + n_\mathrm{j}) \right\rangle^2 + \left\langle (s_\mathrm{i} + n_\mathrm{i})^2 \right\rangle \left\langle (s_\mathrm{j} + n_\mathrm{j})^2 \right\rangle \right]$$

See chp 5 of LRFA.

To obtain SNR we look at the RMS fluctuations of the correlator output. We use the fact that the square of RMS fluctuation of a Gaussian random variable (product from the correlator) is equal to the expectation value of the variable squared minus the square of the mean.

$$\sigma^2(P_{\rm ij}) = \frac{a_{\rm i}a_{\rm j}}{\eta_{\rm s}^2}\left\langle \left[(s_{\rm i}+n_{\rm i})(s_{\rm j}+n_{\rm j})\right]^2 \right\rangle - \frac{g_{\rm i}^2\, g_{\rm j}^2}{\eta_{\rm s}^2} K_{\rm i}\, K_{\rm j}\, k_{\rm B}^2\, S_{\rm c}^2\, \Delta\nu^2$$

$$\begin{aligned}
\sigma^2(P_{\rm ij}) &= \frac{a_{\rm i}a_{\rm j}}{\eta_{\rm s}^2}\left[2\left\langle(s_{\rm i}+n_{\rm i})(s_{\rm j}+n_{\rm j})\right\rangle^2 + \left\langle(s_{\rm i}+n_{\rm i})^2\right\rangle\left\langle(s_{\rm j}+n_{\rm j})^2\right\rangle\right] \\
&\quad - \frac{g_{\rm i}^2\, g_{\rm j}^2}{\eta_{\rm s}^2} K_{\rm i}\, K_{\rm j}\, k_{\rm B}^2\, \Delta\nu^2\, S_{\rm c}^2 \\
&= 2\frac{g_{\rm i}^2 g_{\rm j}^2}{\eta_{\rm s}^2} K_{\rm i}\, K_{\rm j}\, (k_{\rm B}\, \Delta\nu\, S_{\rm T})^2 \\
&\quad + \frac{g_{\rm i}^2\, g_{\rm j}^2}{\eta_{\rm s}^2} (k_{\rm B}\, \Delta\nu)^2 (K_{\rm i}\, S_{\rm T} + T_{\rm sysi})(K_{\rm j}\, S_{\rm T} + T_{\rm sysj}) \\
&\quad - \frac{g_{\rm i}^2\, g_{\rm j}^2}{\eta_{\rm s}^2} K_{\rm i}\, K_{\rm j}\, k_{\rm B}^2\, \Delta\nu^2\, S_{\rm c}^2
\end{aligned}$$

$$\begin{aligned}
\langle P_{\rm i}\rangle &= a_{\rm i}\left\langle(s_{\rm i}+n_{\rm i})^2\right\rangle \\
&= g_{\rm i}^2\, k_{\rm B}\, (K_{\rm i} S_{\rm T} + T_{\rm sysi})\, \Delta\nu
\end{aligned}$$

To obtain SNR we look at the RMS fluctuations of the correlator output. We use the fact that the square of RMS fluctuation of a Gaussian random variable (product from the correlator) is equal to the expectation value of the variable squared minus the square of the mean.

$$\sigma^2(P_{ij}) = \frac{a_i a_j}{\eta_s^2}\left\langle \left[(s_i + n_i)(s_j + n_j)\right]^2 \right\rangle - \frac{g_i^2\, g_j^2}{\eta_s^2} K_i\, K_j\, k_B^2\, S_c^2\, \Delta\nu^2$$

$$\begin{aligned}
\sigma^2(P_{ij}) &= \frac{a_i a_j}{\eta_s^2}\left[2\left\langle (s_i + n_i)(s_j + n_j)\right\rangle^2 + \left\langle (s_i + n_i)^2\right\rangle\left\langle (s_j + n_j)^2\right\rangle\right] \\
&\quad - \frac{g_i^2\, g_j^2}{\eta_s^2} K_i\, K_j\, k_B^2\, \Delta\nu^2\, S_c^2 \\
&= 2\frac{g_i^2 g_j^2}{\eta_s^2} K_i\, K_j\, (k_B\, \Delta\nu\; S_c)^2 \\
&\quad + \frac{g_i^2\, g_j^2}{\eta_s^2} (k_B\, \Delta\nu)^2 (K_i\, S_T + T_{sysi})(K_j\, S_T + T_{sysj}) \\
&\quad - \frac{g_i^2\, g_j^2}{\eta_s^2} K_i\, K_j\, k_B^2\, \Delta\nu^2\, S_c^2
\end{aligned}$$

$$\begin{aligned}
\langle P_{ij} \rangle &= \frac{\sqrt{a_i a_j}}{\eta_s}\left\langle (s_i + n_i)(s_j + n_j)\right\rangle \\
&= \frac{g_i\, g_j}{\eta_s}\sqrt{K_i\, K_j}\, k_B\, \Delta\nu\, S_c
\end{aligned}$$

$$\sigma^2(P_{\rm ij}) = k_{\rm B}^2\,\Delta\nu^2\frac{g_{\rm i}^2\,g_{\rm j}^2}{\eta_{\rm s}^2}\Big(K_{\rm i}\,K_{\rm j}\,S_{\rm c}^2 + K_{\rm i}\,K_{\rm j}\,S_{\rm T}^2 + K_{\rm i}\,S_{\rm T}\,T_{\rm sysi} + K_{\rm j}\,S_{\rm T}\,T_{\rm sysj} + T_{\rm sysi}\,T_{\rm sysj}\Big)$$

To obtain the noise level in units of flux density, a) divide by $g_{\rm i}\,g_{\rm j}\sqrt{K_{\rm i}\,K_{\rm j}}\,k_{\rm B}\,\Delta\nu$

and b) $\sqrt{2\,\Delta\nu\,\tau_{\rm acc}}$

$$\Delta S_{\rm ij} = \frac{1}{\eta_{\rm s}\sqrt{2\,\Delta\nu\,\tau_{\rm acc}}}\sqrt{S_{\rm c}^2 + S_{\rm T}^2 + S_{\rm T}\left(\frac{T_{\rm sysi}}{K_{\rm i}} + \frac{T_{\rm sysj}}{K_{\rm j}}\right) + \frac{T_{\rm sysi}\,T_{\rm sysj}}{K_{\rm i}\,K_{\rm j}}}$$

Crane and Napier 1989

$$\Delta S_{\mathrm{ij}} = \frac{1}{\eta_{\mathrm{s}} \sqrt{2\, \Delta\nu\, \tau_{\mathrm{acc}}}} \sqrt{S_{\mathrm{c}}^2 + S_{\mathrm{T}}^2 + S_{\mathrm{T}} \left(\frac{T_{\mathrm{sysi}}}{K_{\mathrm{i}}} + \frac{T_{\mathrm{sysj}}}{K_{\mathrm{j}}} \right) + \frac{T_{\mathrm{sysi}}\, T_{\mathrm{sysj}}}{K_{\mathrm{i}}\, K_{\mathrm{j}}}}$$

For a non flat bandpass: $\int_0^\infty g_{\mathrm{i}}(\nu)\, g_{\mathrm{j}}(\nu)\, d\nu$

$$\Delta S_{\mathrm{ij}} = \frac{\sqrt{\int_0^\infty g_{\mathrm{i}}^2(\nu)\, g_{\mathrm{j}}^2(\nu) \left[S_{\mathrm{c}}^2 + S_{\mathrm{T}}^2 + S_{\mathrm{T}} \left(\frac{T_{\mathrm{sysi}}}{K_{\mathrm{i}}} + \frac{T_{\mathrm{sysj}}}{K_{\mathrm{j}}} \right) + \frac{T_{\mathrm{sysi}}\, T_{\mathrm{sysj}}}{K_{\mathrm{i}}\, K_{\mathrm{j}}} \right] d\nu}}{\eta_{\mathrm{s}} \sqrt{2\, \tau_{\mathrm{acc}} \int_0^\infty g_{\mathrm{i}}(\nu)\, g_{\mathrm{j}}(\nu)\, d\nu}}$$

Special cases assuming flat bandpass

$$\Delta S_{\mathrm{ij}} = \frac{1}{\eta_{\mathrm{s}}\sqrt{2\,\Delta\nu\,\tau_{\mathrm{acc}}}}\sqrt{S_{\mathrm{c}}^2 + S_{\mathrm{T}}^2 + S_{\mathrm{T}}\left(\frac{T_{\mathrm{sysi}}}{K_{\mathrm{i}}} + \frac{T_{\mathrm{sysj}}}{K_{\mathrm{j}}}\right) + \frac{T_{\mathrm{sysi}}\,T_{\mathrm{sysj}}}{K_{\mathrm{i}}\,K_{\mathrm{j}}}}$$

Case of strong source:

$$S_{\mathrm{T}} \gg S_{\mathrm{c}}.$$

$$\Delta S_{\mathrm{ij}} = \frac{S_{\mathrm{T}}}{\eta_{\mathrm{s}}\sqrt{2\,\Delta\nu\,\tau_{\mathrm{acc}}}}$$

Typically source is weak and thus ignoring terms involving S:

$$\Delta S_{\mathrm{ij}} = \frac{1}{\eta_{\mathrm{s}}}\sqrt{\frac{T_{\mathrm{sysi}}\,T_{\mathrm{sysj}}}{2\,\Delta\nu\,\tau_{\mathrm{acc}}\,K_{\mathrm{i}}\,K_{\mathrm{j}}}}$$

Special cases assuming flat bandpass

$$\Delta S_{\rm ij} = \frac{1}{\eta_{\rm s}\sqrt{2\,\Delta\nu\,\tau_{\rm acc}}}\sqrt{S_{\rm c}^2 + S_{\rm T}^2 + S_{\rm T}\left(\frac{T_{\rm sysi}}{K_{\rm i}} + \frac{T_{\rm sysj}}{K_{\rm j}}\right) + \frac{T_{\rm sysi}\,T_{\rm sysj}}{K_{\rm i}\,K_{\rm j}}}$$

Case of strong source:

$$S_{\rm T} \gg S_{\rm c}.$$

$$\Delta S_{\rm ij} = \frac{S_{\rm T}}{\eta_{\rm s}\sqrt{2\,\Delta\nu\,\tau_{\rm acc}}}$$

Typically source is weak and thus ignoring terms involving S:

$$\Delta S_{\rm ij} = \frac{1}{\eta_{\rm s}}\sqrt{\frac{T_{\rm sysi}\,T_{\rm sysj}}{2\,\Delta\nu\,\tau_{\rm acc}\,K_{\rm i}\,K_{\rm j}}}$$

In terms of SEFD:

$$SEFD = \frac{T_{\rm sys}}{K}$$

$$\Delta S_{\rm ij} = \frac{1}{\eta_{\rm s}}\sqrt{\frac{SEFD_{\rm i}\,SEFD_{\rm j}}{2\,\Delta\nu\,\tau_{\rm acc}}}$$

If SEFD same for two antennas:

$$\Delta S_{\rm ij} = \frac{1}{\eta_{\rm s}}\frac{SEFD}{\sqrt{2\,\Delta\nu\,\tau_{\rm acc}}}$$

Noise in amplitudes and phases

Real and imaginary signals:

$$S_{\rm m} = \sqrt{S_{\rm R}^2 + S_{\rm i}^2}$$

$$\phi_{\rm m} = \tan^{-1}(S_{\rm i}/S_{\rm R})$$

$$P(S_{\rm m}) = \frac{S_{\rm m}}{\Delta S^2} I_0\left(\frac{S_{\rm m}\, S}{\Delta S^2}\right) \exp\frac{-(S_{\rm m}^2 + S^2)}{2\,\Delta S^2}$$

Probability distribution of amplitude and phase

$$P(\phi - \phi_{\rm m}) = \frac{1}{2\pi}\exp\left(\frac{-S^2}{2\,\Delta S^2}\right)\left(1 + G\sqrt{\pi}e^{G^2}(1 + {\rm erf}\, G)\right)$$

Crane and Napier 1989

Sensitivity of an image

$$I_{\mathrm{m}}(l,m) = C \sum_{\mathrm{k}=1}^{2L} T_{\mathrm{k}}\, W_{\mathrm{k}}\, w_{\mathrm{k}}\, V_{\mathrm{k}}\, e^{2\pi i(u_{\mathrm{k}} l + v_{\mathrm{k}} m)}$$

Normalisation constant

Taper

Weights

Weight based on S/N of individual point

Consider only the centre of the image:

$$I_{\mathrm{m}}(0,0) = 2\,C \sum_{\mathrm{k}=1}^{L} T_{\mathrm{k}}\, W_{\mathrm{k}}\, w_{\mathrm{k}}\, S_{\mathrm{Rk}}$$

Error in this image point is due to error ΔS_{k} ; thus variance in I is just sum of variances in the Fourier components:

$$\Delta I_{\mathrm{m}} = 2\,C \sqrt{\sum_{\mathrm{k}=1}^{L} T_{\mathrm{k}}^2\, W_{\mathrm{k}}^2\, w_{\mathrm{k}}^2\, \Delta S_{\mathrm{k}}^2}$$

C is set to: $1\ /\ (2 \sum_{\mathrm{k}=1}^{L} T_{\mathrm{k}}\, W_{\mathrm{k}}\, w_{\mathrm{k}})$

$$\Delta I_{\mathrm{m}} = 2\, C \sqrt{\sum_{\mathrm{k}=1}^{L} T_{\mathrm{k}}^2\, W_{\mathrm{k}}^2\, w_{\mathrm{k}}^2\, \Delta S_{\mathrm{k}}^2}$$

For no taper, natural weights, and all points of weight w,

$$\begin{aligned} \Delta I_{\mathrm{m}} &= 2\, C\, \Delta S\, w \sqrt{\sum_{\mathrm{k}=1}^{L} 1} \\ &= \frac{2\, \Delta S\, w \sqrt{\sum_{\mathrm{k}=1}^{L} 1}}{2\, w \sum_{\mathrm{k}=1}^{L} 1} \\ &= \Delta S \sqrt{L}/L \\ &= \Delta S/\sqrt{L} \end{aligned}$$

$$\Delta I_{\rm m} = 2\,C\,\sqrt{\sum_{\rm k=1}^{L} T_{\rm k}^2\,W_{\rm k}^2\,w_{\rm k}^2\,\Delta S_{\rm k}^2}$$

For no taper, natural weights, and all points of weight w,

$$\begin{aligned}
\Delta I_{\rm m} &= 2\,C\,\Delta S\,w\sqrt{\sum_{\rm k=1}^{L} 1} \\
&= \frac{2\,\Delta S\,w\sqrt{\sum_{\rm k=1}^{L} 1}}{2\,w\sum_{\rm k=1}^{L} 1} \\
&= \Delta S\sqrt{L}/L \\
&= \Delta S/\sqrt{L}
\end{aligned}$$

$$L = \frac{1}{2}\,N\,(N-1)\,(t_{\rm int}/\tau_{\rm acc})$$

$$\Delta I_{\rm m} = \frac{1}{\eta_{\rm s}}\frac{SEFD}{\sqrt{N\,(N-1)\,\Delta\nu\,t_{\rm int}}}$$

Sensitivity of a single polarization image formed from a homogenous array of N identical antennas:

$$\Delta I_{\rm m} = \frac{1}{\eta_{\rm s}} \frac{SEFD}{\sqrt{N\,(N-1)\,\Delta\nu\,t_{\rm int}}}$$

For full Stokes data:

$$\Delta I = \Delta Q = \Delta U = \Delta V = \frac{\Delta I_{\rm m}}{\sqrt{2}}$$

Sensitivity of a single polarization image formed from a homogenous array of N identical antennas:

$$\Delta I_{\mathrm{m}} = \frac{1}{\eta_{\mathrm{s}}} \frac{SEFD}{\sqrt{N\,(N-1)\,\Delta\nu\, t_{\mathrm{int}}}}$$

For full Stokes data:

$$\Delta I = \Delta Q = \Delta U = \Delta V = \frac{\Delta I_{\mathrm{m}}}{\sqrt{2}}$$

Noise in linear polarized flux density follows Rayleigh statistics and the position angle will follow uniform statistics.

$$P = \sqrt{Q^2 + U^2} \qquad \chi = \frac{1}{2}\tan^{-1}(U/Q)$$

See Interferometry and Synthesis in RA for more details.