THE INTERSTELLAR MEDIUM: VII
Tracers of Molecular gas : CO lines

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Outline

- Background.
- Radiative trapping: The CO J=1→0 line.
- Estimating the molecular gas mass.
- The CO-to-H_2 conversion factor.
- The large-scale distribution of molecular gas.

Background

- Cannot study molecular clouds directly in H_2 transitions.
- CO: second-most abundant molecule; J=1→0, 2→1 & 3→2 lines observable from the ground ⇨ Main molecular gas "tracer"!
- $T_B = (h\nu/k)\,(1-e^{-\tau_\nu})\,\{[e^{h\nu/kT_R} - 1]^{-1} - [e^{h\nu/kT_{CMB}} - 1]^{-1}\}$
  $T_R > T_{CMB}$ ⇨ Emission; $T_R = T_{CMB}$ ⇨ No line.
- Rotational lines: $n_c \gg n_{crit,u}$ ⇨ $T_R = T_K$; $n_c \ll n_{crit,u}$ ⇨ $T_R = T_{CMB}$.
- Emission lines only visible if density $\geqslant$ critical density.
  ⇨ Mere detection of lines provides information on density!
- n_{crit} (1→0): 1100 cm^{-3} (CO); 5×10^4 cm^{-3} (CS); 10^6 cm^{-3} (HCN).
- Critical density reduced at high opacities, due to stimulated emission, by $\beta \sim 1/(1+0.5\tau_0)$: $n_{crit} \sim 50$ cm^{-3} for CO (1→0) if $\tau \sim 50$.

Opacity Issues: Radiative Trapping

- For $\tau \gg 1$, stimulated emission critical in determining T_R.
- "Escape probability approximation": Assume that photons are emitted and absorbed at the *same* location. (Scoville & Solomon 1974)
- For a uniform medium, with level populations given by T_R.

$$\Rightarrow \quad I_\nu = I_\nu(0)\, e^{-\tau_\nu} + B_\nu(T_R)\,(1 - e^{-\tau_\nu})$$

- $n_\gamma \equiv (c^2/2h\nu^3)I_\nu \quad \Rightarrow \quad n_\gamma = n_\gamma(0)\, e^{-\tau_\nu} + [n_0 g_1/n_1 g_0 - 1]^{-1}(1-e^{-\tau_\nu})$.
- "Escape probability" $\beta_\nu \equiv e^{-\tau_\nu}$.
- Averaging over directions and integrating over the line profile

$$\Rightarrow \quad \langle n_\gamma \rangle = n_\gamma(0).\langle\beta\rangle + [n_0 g_1/n_1 g_0 - 1]^{-1}(1- \langle\beta\rangle).$$

- $(dn_1/dt) = n_0[n_c k_{01} + n_\gamma\, (g_1/g_0).A_{10}] - n_1[n_c k_{10} + (1 + n_\gamma).A_{10}]$.

$$= n_0\{n_c k_{01} + n_\gamma(0).(g_1/g_0).\langle\beta\rangle.A_{10}\} - n_1\{n_c k_{10} + [1 + n_\gamma(0)].\langle\beta\rangle.A_{10}\}.$$

OPACITY ISSUES: RADIATIVE TRAPPING

- For collisional and radiative excitation and de-excitation:

  $(dn_1/dt) = n_0[n_c k_{01} + n_\gamma\,(g_1/g_0)A_{10}] - n_1[n_c k_{10} + (1 + n_\gamma)A_{10}]$.

- Replacing $\langle n_\gamma \rangle = n_\gamma(0).\langle\beta\rangle + [n_0 g_1/n_1 g_0 - 1]^{-1}(1- \langle\beta\rangle)$ ⇨

  $(dn_1/dt) = n_0\{n_c k_{01} + n_\gamma(0).(g_1/g_0).\langle\beta\rangle.A_{10}\} - n_1\{n_c k_{10} + [1+ n_\gamma(0)].\langle\beta\rangle.A_{10}\}$

- This is the equation for (dn_1/dt) *if* (1) there's no internally produced radiation field, (2) the cloud is transparent to $I_\nu(0)$, and (3) the Einstein A-coefficient is $\langle\beta\rangle A_{10}$.

- $(n_1/n_0) = [n_c k_{01} + n_\gamma(0).(g_1/g_0).\langle\beta\rangle A_{10}] \,/\, \{n_c\, k_{10} + [1 + n_\gamma(0)].\langle\beta\rangle.A_{10}\}$.

- Critical density *reduced*: $n_{crit,u} = [\Sigma_{l<u}\,(1 + n_\gamma).\langle\beta\rangle.A_{ul}\,] \,/\, [\Sigma_{l<u}\, k_{ul}]$.

- $\langle\beta\rangle$ depends on the geometry and velocity structure. For a homogenous sphere and Gaussian velocities, $\langle\beta\rangle \approx (1 + 0.5\tau_0)^{-1}$.

Opacity Issues: The CO $J=1\rightarrow 0$ Line

- $\nu \sim 115.271$ GHz; $A_{10} \sim 7\times 10^{-8}\ s^{-1}$ ⇨ $n_{crit} \sim A_{10}/k_{10} \approx 1100\ cm^{-3}$.
- For a Gaussian velocity distribution, the attenuation coefficient:
  $\kappa_\nu = n_0.\ [1 - (n_1 g_0/n_0 g_1)]\ .(\lambda^2/8\pi).\ (g_1/g_0).\ A_{10}.\pi^{-1/2}.\ (\lambda/b).\ e^{-(\Delta v/b)^2}$
- The line-centre optical depth, $\tau_0 = \kappa_{\nu,0}\times R$ (Cloud radius $\equiv$ R).
  ⇨ $\tau_0 \approx 0.02\ (n_H/1000)\ (R/3\ pc)\ [n_0/n_H]\ (2/b)\ [1 - (n_1 g_0/n_0 g_1)]$.
- One needs to know T_R and $[n_0/n]$ to determine τ_0 !
- The fraction of CO in the J^{th} rotational level is
  $[n_J/n_{CO}] = (2J+1)e^{-B_0 J(J+1)/kT_R} / \Sigma_J\ (2J+1)\ e^{-B_0 J(J+1)/kT_R}$.
- The partition function $\Sigma_J\ (2J+1)\ e^{-B_0 J(J+1)/kT_R} \approx [1+(kT_R/B_0)^2]^{1/2}$.
- For $T_R \approx 8$ K, $\tau_0 \approx 46\ (n_H/1000)\ (R/3\ pc)\ [[n_{CO}/n_H]/(7\times 10^{-5})]\ (2/b)$
  ⇨ $\langle\beta\rangle \approx (1 + 0.5\tau_0)^{-1} \approx 0.04$ ⇨ $n_{crit} \sim \langle\beta\rangle A_{10}/k_{10} \approx 50\ cm^{-3}$!!!

THE CO COLUMN DENSITY

- The general expression for the attenuation coefficient, κ_ν :

  $\kappa_\nu = n_l . [1 - (n_u g_l / n_l g_u)] .(\lambda^2/8\pi). (g_u/g_l). A_{ul}.\phi_\nu$; $\int \phi_\nu d\nu = 1.$

- The column density in the lower level $N_l = \int n_l \, ds$

  $\Rightarrow N_l = (8\pi/\lambda^2) . (g_l/g_u) . (1/A_{ul}) . \int \tau_\nu/[1 - e^{h\nu/kT_R}] \, d\nu$

  $= 93.28(g_l/g_u).(\nu/\mathrm{GHz})^3.(1/A_{ul}).[1 - e^{h\nu/kT_R}]^{-1}. \int \tau_\nu \, (d\nu/\ \mathrm{km\ s^{-1}})$

- $A_{ul} = (64\pi^4/3hc^3) \, \nu^3 \, \mu^2 \, [J/(2J+1)] = 1.165 \times 10^{-11} \, \nu^3 \, \mu^2 \, [J/(2J+1)]$

  $\Rightarrow N_l = 8.0 \times 10^{12}. \mu^{-2}. [(2J-1)/J]. [1 - e^{h\nu/kT_R}]^{-1}. \int \tau_\nu \, d\nu$

- Measured $T_B = (h\nu/k) \, (1-e^{-\tau_\nu}) \, \{[e^{h\nu/kT_R} - 1]^{-1} - [e^{h\nu/kT_{CMB}} - 1]^{-1}\}$

- Problems: (1) Both τ_ν and T_R are unknown, and (2) $\tau_\nu \geq 1$!

The CO Column Density: LTE

- $N_l = 8.0 \times 10^{12} . \mu^{-2} . [(2J-1)/J] . [1 - e^{h\nu/kT_R}]^{-1} . \int \tau_v \, dv$
- Measured $T_B = (h\nu/k)(1-e^{-\tau_v}) \{[e^{h\nu/kT_R} - 1]^{-1} - [e^{h\nu/kT_{CMB}} - 1]^{-1}\}$
- LTE approximation: *Assume* $T_R = T_K$, for all rotational levels!
- *Assume* ^{13}CO and ^{12}CO have same T_R! Note $\tau_v(^{13}CO) \ll 1$!
- $\tau_v(^{12}CO) \gg 1 \Rightarrow T_R = 5.5/\ln\{1 + 5.5/(T_B + 0.82)\}$
- The fraction of ^{13}CO in the J^{th} rotational level is

  $[n_J/n_{CO}] = (2J+1)e^{-B_0 J(J+1)/kT_R} / \Sigma_J (2J+1) e^{-B_0 J(J+1)/kT_R}$

  $N(^{13}CO) = [\Sigma_J (2J+1) e^{-B_0 J(J+1)/kT_R}] . e^{+B_0 J(J+1)/kT_R} .N_J/(2J+1)]$

- Partition function $\Sigma_J (2J+1)e^{-B_0 J(J+1)/kT_R} \approx [1+(kT_R/B_0)^2]^{1/2} \approx T_R/2.76$

  $\Rightarrow N(^{13}CO) = 2.85 \times 10^{14} . T_R/(1 - e^{-5.3/T_R}) . \int \tau_v(^{13}CO) \, dv$

  Finally, $N(^{12}CO) \approx 90 \times N(^{13}CO)$ and $N(H_2) \approx 10^4 \times N(^{12}CO)$.

Caveat Emptor!!!

- CO is a "nice" molecule!!! A high line intensity, a low dipole moment (and hence a low A_{10} and a low critical density), an excitation close to LTE, and a large molecular abundance.
- Many isotopomers: At least one optically thin (^{13}CO, $C^{18}O$...).
- Correlation between measured $N(^{13}CO)$ or $N(C^{18}O)$ and E_{B-V}

$$N(H_2) = 4.4 \times 10^6 \ N(C^{18}O) \ cm^{-2} \qquad N(H_2) < 1.5 \times 10^{22} \ cm^{-2}.$$

$$N(H_2) = 3.8 \times 10^5 \ N(^{13}CO) \ cm^{-2} \qquad N(H_2) < 5 \times 10^{21} \ cm^{-2}.$$

- But.... ^{12}CO & ^{13}CO may not arise in the same gas ?

  High-J levels not thermalized: Errors in partition function ?

  Less abundant isotopes sub-thermally excited ($T_R < T_K$) ?
- Still one of the best ways of estimating CO and H_2 column densities! But probably only good to an order of magnitude.

Estimating the Molecular Gas Mass

- The CO line luminosity of a uniform cloud at a distance D is

$$L_{CO} = D^2 \int I_{CO}\, d\Omega \quad , \text{ where } I_{CO} = \int T_B\, dV.$$

$$\Rightarrow \; L_{CO} \approx \pi R^2 \; T_{CO}\, \Delta V \quad , \quad \Delta V \equiv \text{Line width}, \; R \equiv \text{cloud radius},$$

$$T_{CO} \equiv \text{Peak brightness temperature.}$$

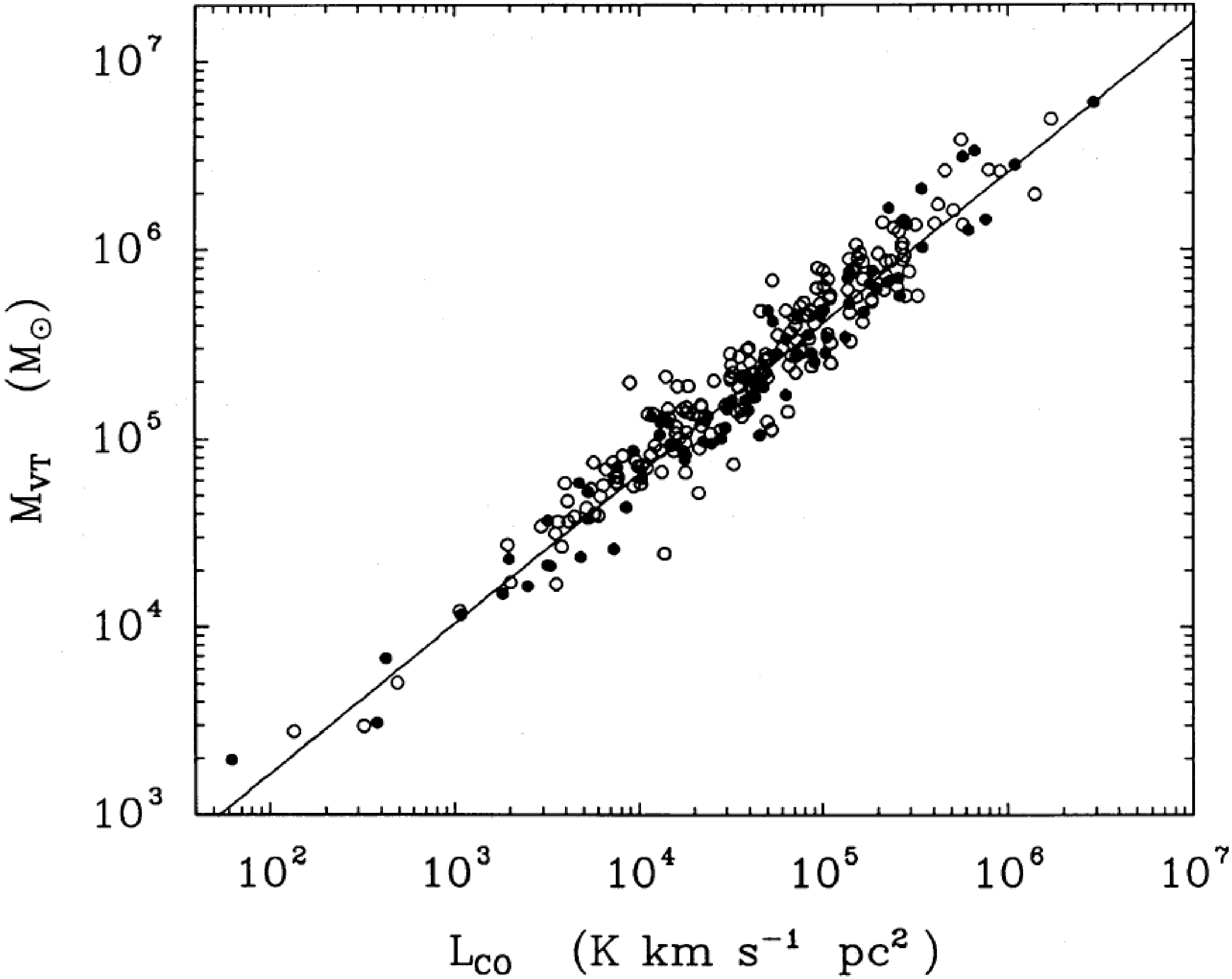
- For a spherical, virialized cloud of mass M, $\Delta V \approx (GM/R)^{1/2}$

$$\Rightarrow \; M = L_{CO} \,.\, (4\rho/3\pi G)^{1/2} \,.\, (1/T_{CO})$$

- *IF* the ratio $(\rho^{1/2}/T_{CO})$ doesn't vary (on the average) from one galaxy to another, and if different clouds don't overlap in velocity, the total mass is proportional to the line luminosity!

- Test this by inferring virial masses from ^{13}CO measurements of line width and cloud size ⇨ Compare with ^{12}CO intensity.

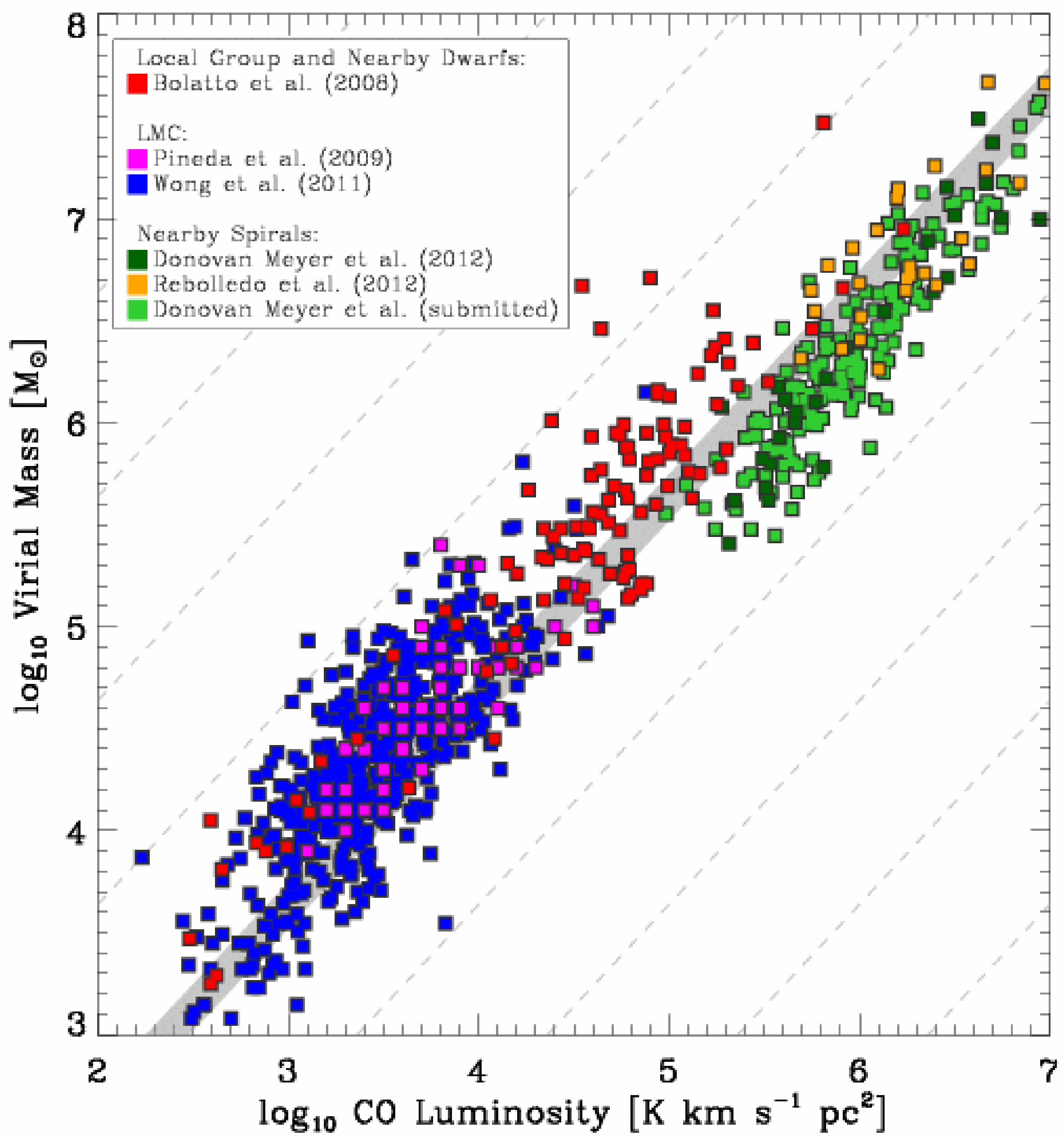
ESTIMATING THE MOLECULAR GAS MASS



Near-linear relation between CO line luminosity & virial mass!

(e.g. Solomon et al. 1987;
Scoville et al. 1987)

ESTIMATING THE MOLECULAR GAS MASS



Slope appears to depend on galaxy type.

(Bolatto et al. 2013)

THE CO-TO-H_2 CONVERSION FACTOR

- Conversion from total CO luminosity to molecular gas mass:

  $M_{MOL} = \alpha_{CO} L_{CO}$ $\alpha_{CO} \sim 4.3\ M_\odot\ (K\ km/s\ pc^2)^{-1}$

- Conversion from integrated CO line intensity to $N(H_2)$:

  $N(H_2) = X_{CO} I_{CO}$ $X_{CO} \sim 2 \times 10^{20}\ cm^{-2}\ (K\ km/s)^{-1}$

(e.g. Bolatto et al. 2013)

- Dependence on local conditions (e.g. density, T_R): factor of ~2.

- CO-to-H_2 factors lower in regions of strong star formation:

  e.g. $\alpha_{CO} \sim 0.8\ M_\odot\ (K\ km/s\ pc^2)^{-1}$ in LIRGs and ULIRGs.

(e.g. Downes & Solomon 1998; Papadopoulos et al. 2012)

- Metallicity critical! Lower dust content implies preferential dissociation of CO: e.g. $X_{CO} \sim (1 - 10) \times 10^{21}\ cm^{-2}\ (K\ km/s)^{-1}$.

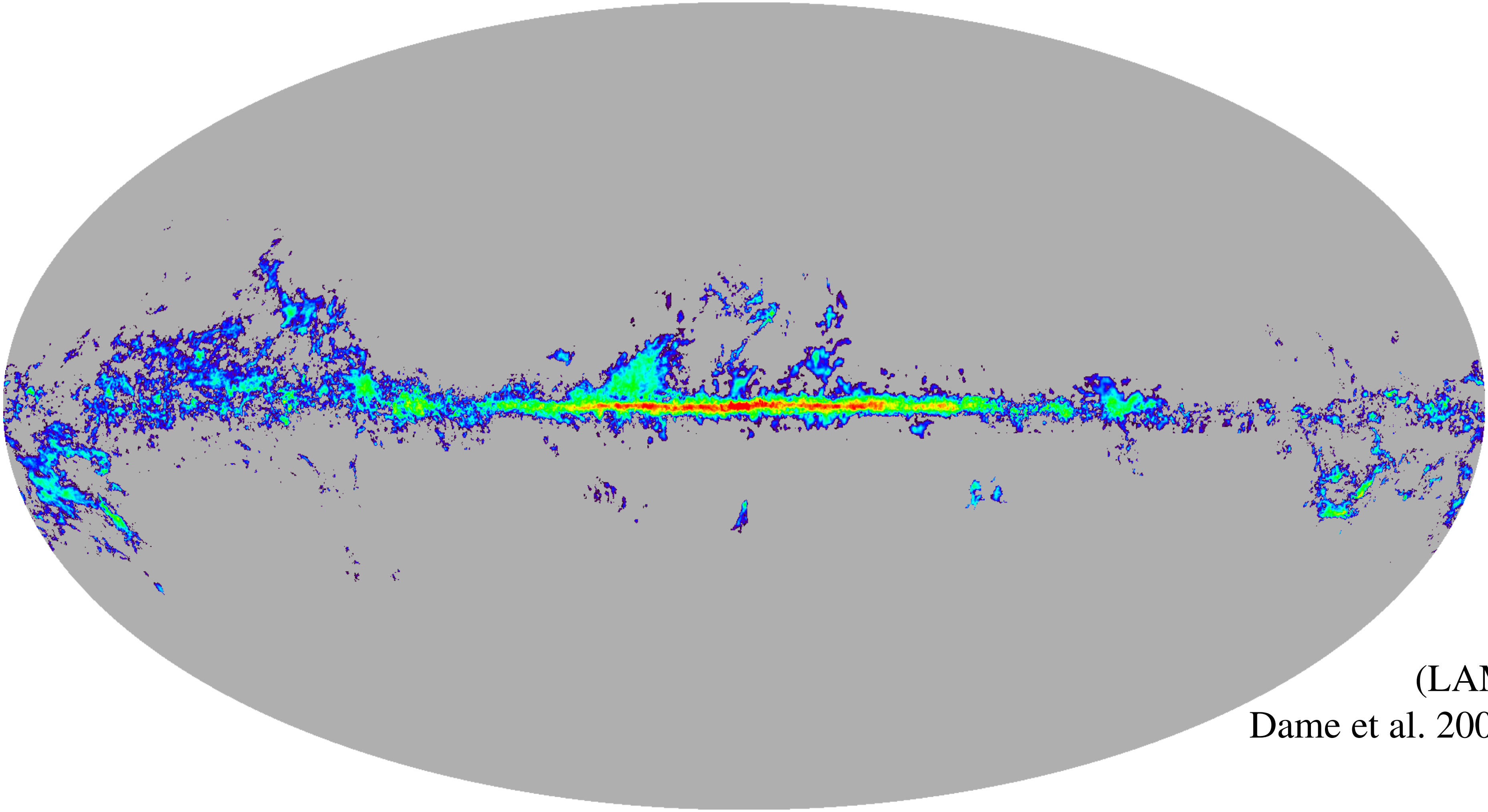
(e.g. Israel 1997; Leroy et al. 2011; Wolfire et al. 2010)

- Caution needed at high redshifts (B*z*Ks, SMGs, LAEs, DLAs!).

THE LARGE-SCALE DISTRIBUTION

- Difficult to detect H_2 ⇨ CO 1→0 line used as a tracer.

CO 1→0 all-sky map

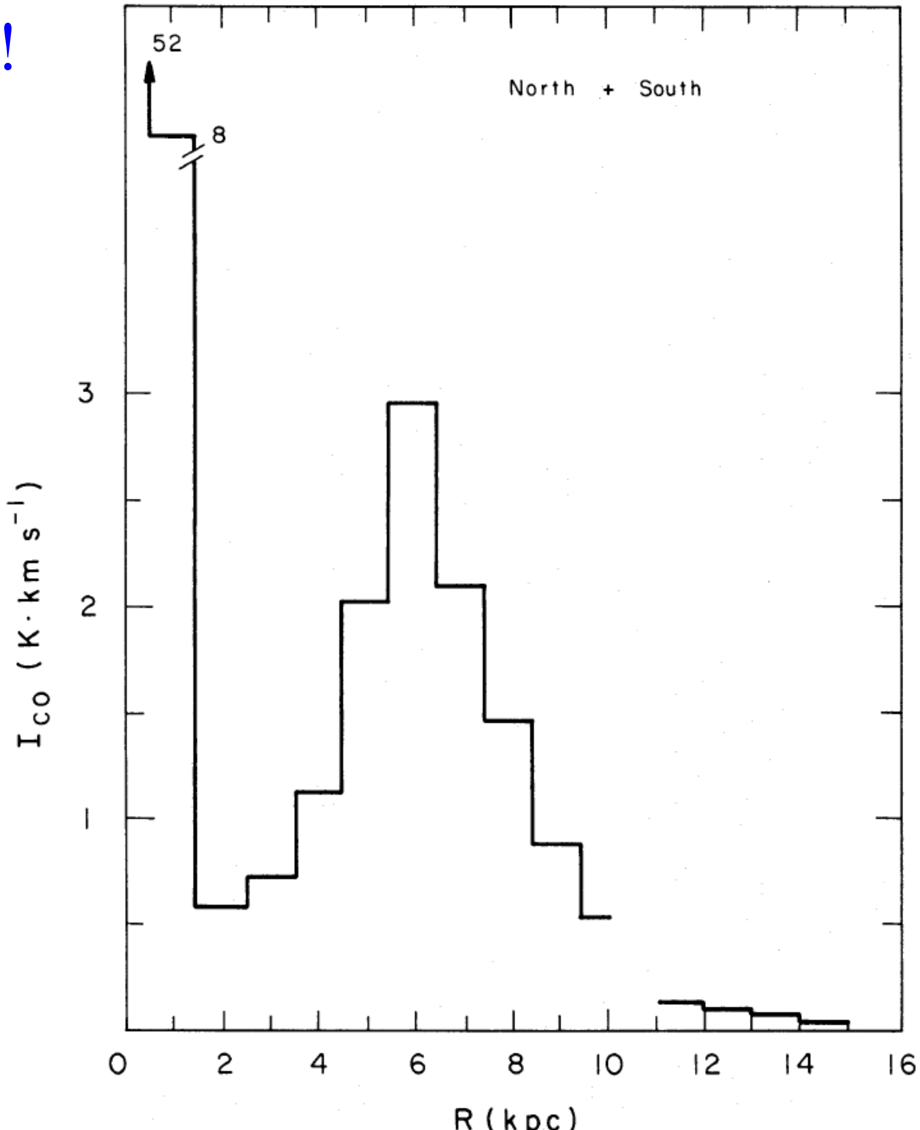


(LAMBDA;
Dame et al. 2001, ApJ)

- Most of the gas in Giant Molecular Clouds of size ~ 40 pc and mass > 10^5 $M_\odot$.

THE LARGE-SCALE DISTRIBUTION

- Velocity field: Galactic distribution!

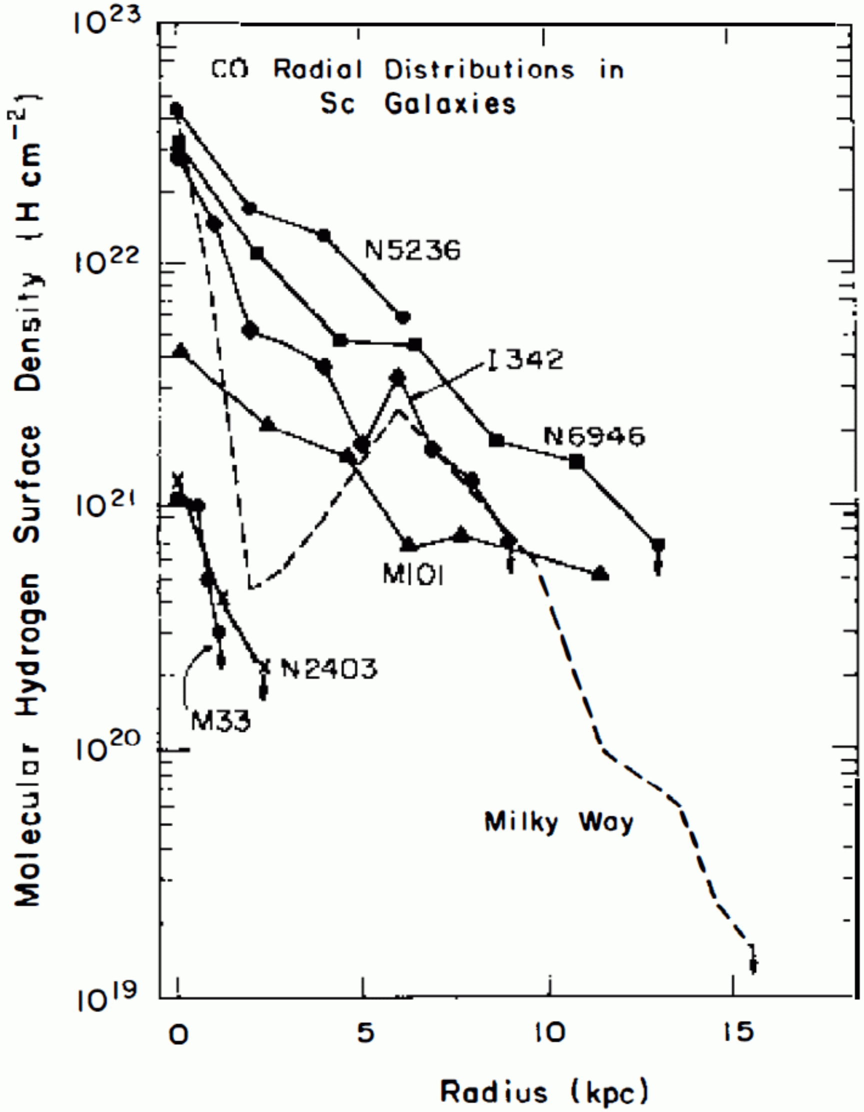


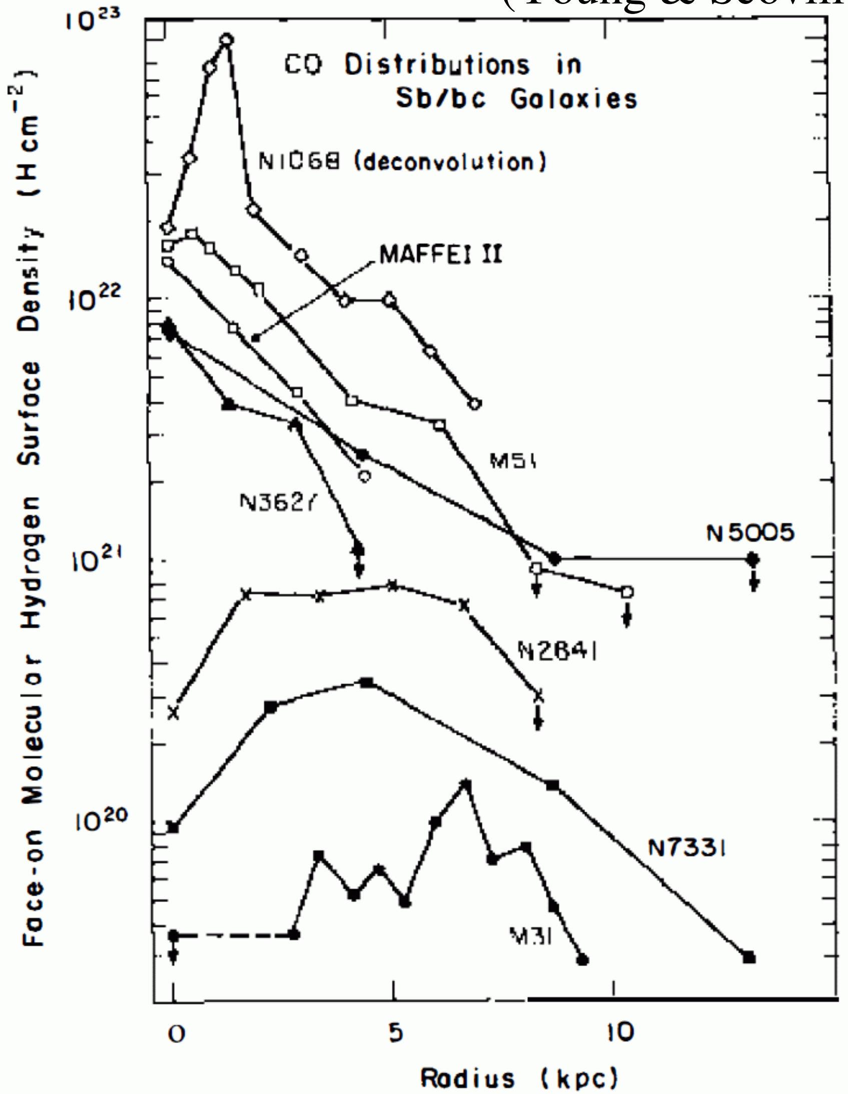
(Sanders et al. 1984)

- Lots of molecular gas in the central 1.5 kpc, "hole" at 1.5 – 3.5 kpc, massive "ring" at 4 – 8 kpc, steep decline beyond 8 kpc.

The Large-scale Distribution

(Young & Scoville 1991)





- "Hole" and "Ring" not seen in other spiral galaxies! The 5 kpc ring contains 70% of the molecular gas within the solar radius!

(e.g. Clemens et al. 1988; Jackson et al. 2006)

- Typical molecular gas extent in spirals ~ Half the optical radius.