THE INTERSTELLAR MEDIUM: II
On equilibrium and spectral lines

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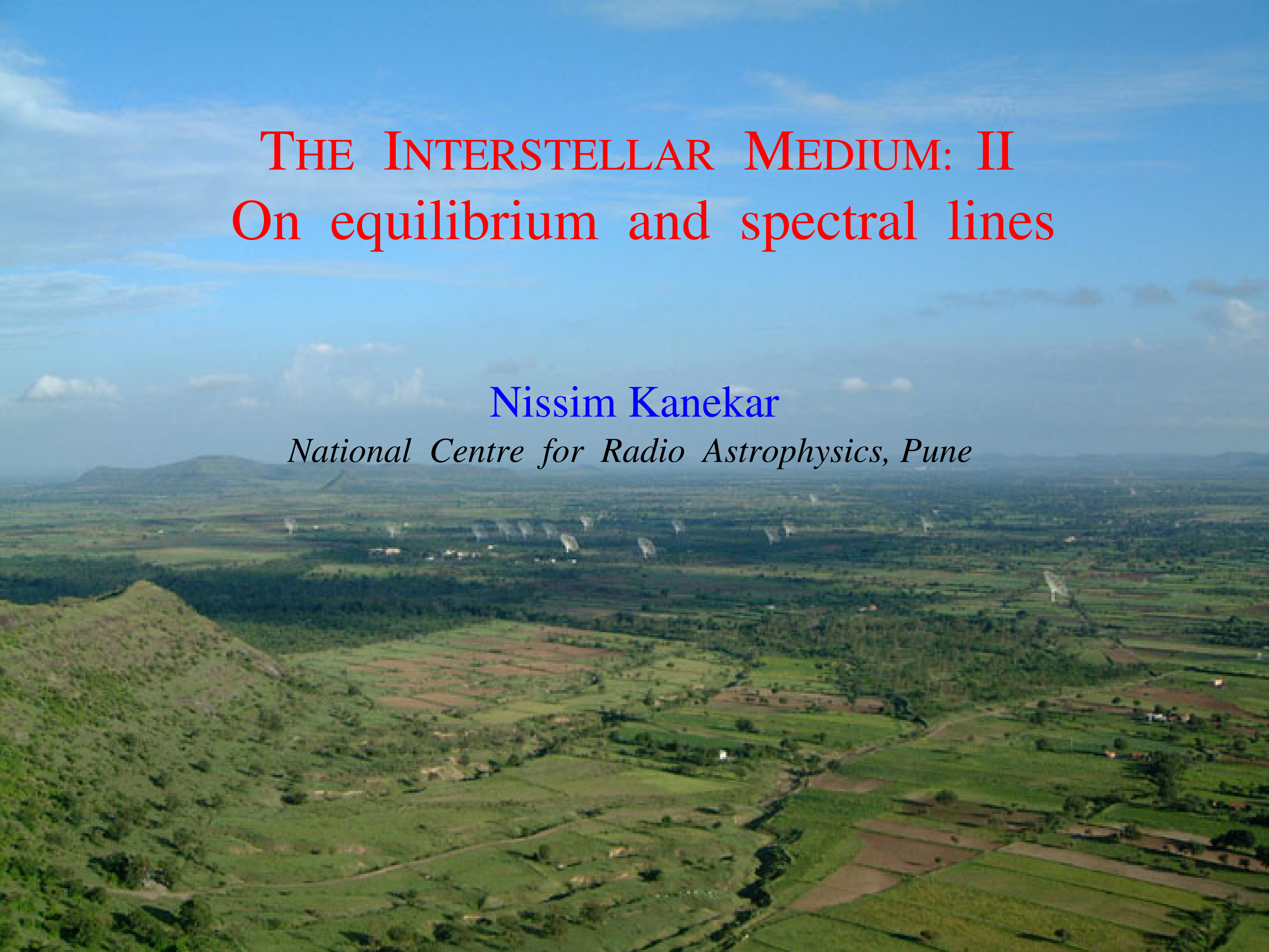


OUTLINE

- Equilibrium issues.
- Radiative processes.
- Line shapes and broadening mechanisms.
- The equation of radiative transfer.
- Absorption lines: The curve of growth.

EQUILIBRIUM ISSUES

- Thermodynamic equilibrium: Maxwell-Boltzmann velocity distribution, Boltzmann energy levels, Planck radiation field, Ionization equilibrium ⇨

- Velocity distribution : $f(v) = (m/2\pi kT)^{1/2}\, e^{-mv^2/2kT}$

- Level populations : $(n_u/n_l) = (g_u/g_l)\, e^{-h\nu/kT}$

- Radiation field : $B(\nu,T) = (2h\nu^3/c^2)\, [e^{h\nu/kT} - 1]^{-1}$

  Wien limit : $B(\nu,T) = (2h\nu^3/c^2)\, e^{-h\nu/kT}$

  Rayleigh-Jeans limit: $B(\lambda,T) = (2kT/\lambda^2)$

- Ionization and recombination rates must be equal.

- Critical aspect: *A single temperature!*

Species	Density cm^{-3}	Temperature K	Pressure P/k $cm^{-3}K$	Mass $10^9\ M_\odot$
HI (CNM)	30	80	~2500	2.8
HI (WNM)	0.3	8000	~2500	2.2
HII (WIM)	0.3	8000	~2500	1.0
H_2	>1000	10	$>10^4$	1.3
HII (HIM)	0.003	10^6	~3000	< 1 ?
Dust,PAHs	-	-	-	0.01

(e.g. Draine 2011)

The ISM is NOT in thermodynamic equilibrium!

Equilibrium Issues

- For typical ISM densities, the timescale for thermalization in any phase is short ⇨ Phases have a well-defined kinetic temperature.

  ⇨ Velocity distribution : $f(v) = (m/2\pi kT_K)^{1/2}\, e^{-mv^2/2kT_K}$

- But, low ISM pressure ⇨ Mixing of phases is very slow.

  ⇨ Multiple phases at different temperatures.

- For very different pressures, gas motions which would tend to equalize the pressures ⇨ Pressure equilibrium ?

  Used by Spitzer to argue that there should be a Hot Ionized Medium!

(Spitzer 1956, ApJ)

Equilibrium Issues

- Radiative timescales different from collisional timescales ⇨ Level populations *not* determined by the kinetic temperature.

- *Define* the Excitation Temperature T_X of a transition by

$$(n_u/n_l) = (g_u/g_l)\, e^{-h\nu/kT_X}$$

- In general, $T_X \neq T_K$, except for very high densities.

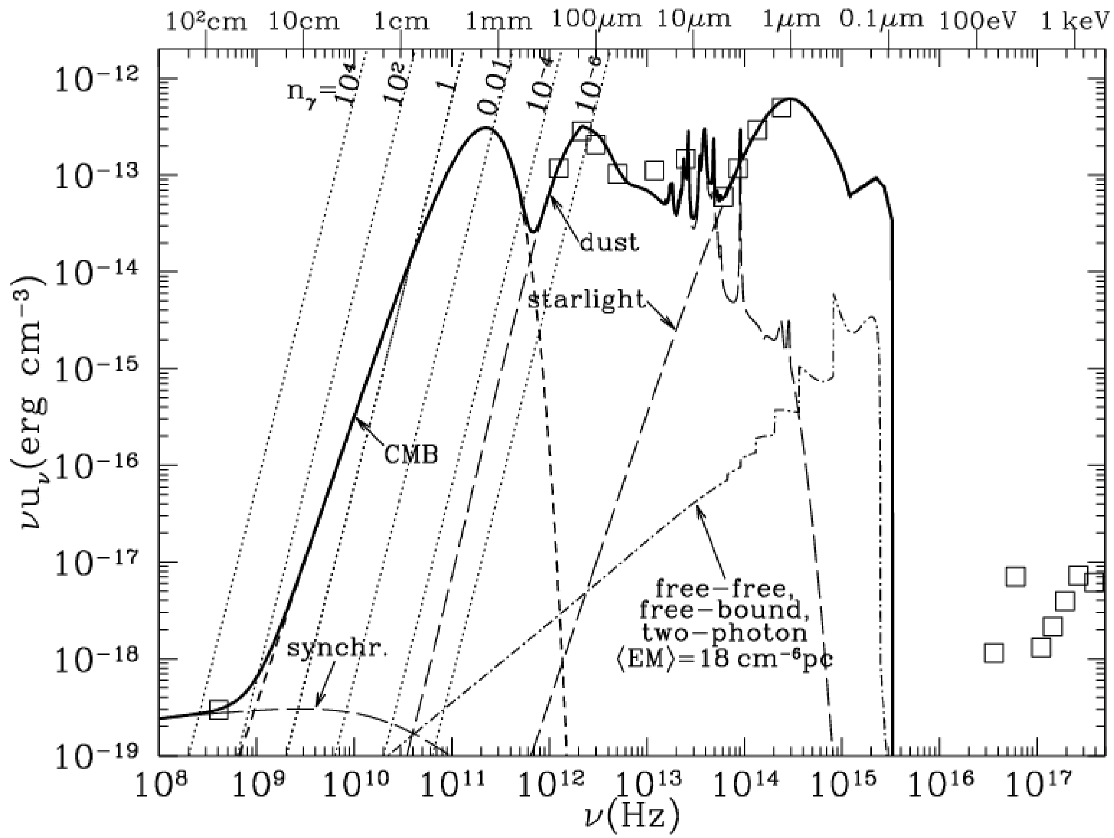
- T_X depends on the kinetic temperature, the local radiation field at the line frequency, and the radiation field at the frequencies of transitions connected to the levels in question.

- NOTE: T_X is *NOT* a physical temperature !

EQUILIBRIUM ISSUES

(Draine 2011)

- Interstellar radiation field very different from the radiation field of a black body!



- Power per unit area between ν and ν+dν, in solid angle dΩ:

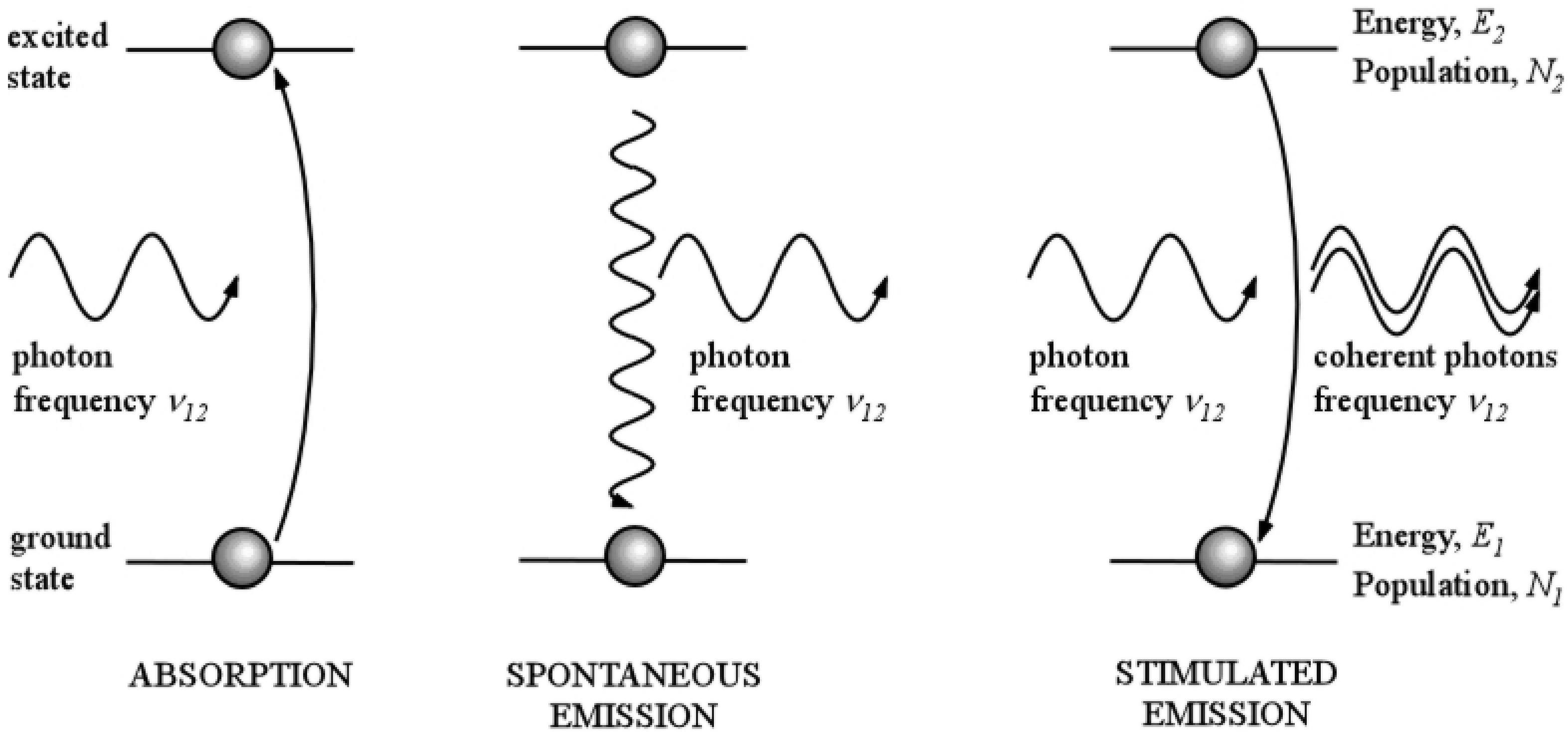
$$I_\nu(\nu, \mathbf{n}, \mathbf{r}, t)\, d\nu\, d\Omega \qquad (I_\nu \equiv \text{Intensity})$$

- At low (radio) frequencies, *define* the Brightness Temperature by

$$I_\nu = (2kT_B/\lambda^2)$$

- In general, $T_B \neq T_K$. Note: T_B is *NOT* a physical temperature!

RADIATIVE PROCESSES



- Spontaneous emission from levels "u" to "l" : $dn_l/dt = n_u A_{ul}$

  Emission stimulated by the radiation field : $dn_l/dt = n_u B_{ul} u_\nu$

  Absorption from the radiation field : $dn_l/dt = -n_l B_{lu} u_\nu$

- These are related quantities: $A_{ul} = (8\pi h\nu^3/c^3)B_{ul}$; $g_l B_{lu} = g_u B_{ul}$

- Einstein A-coefficient: Transition rate per unit time.

Radiative Processes

- $A_{ul} = (64\pi^4 e^2 \nu^3 / 3hc^3)(g_u/g_l)\,|D_{ul}|^2 \;\Rightarrow\; A_{ul} \propto \nu^3$.
  ⇨ For a species, optical transitions more favoured than radio.
  e.g. $A_{Ly\text{-}\alpha} \sim 10^8\ s^{-1}$, but $A_{21cm} \sim 10^{-15}\ s^{-1}$.

- Selection rules for strong radiative transitions:
  (1) Parity must change.
  (2) $\Delta J = 0, \pm 1$ (but $J=0 \rightarrow J=0$ forbidden).
  (3) Only one electron wave function nl changes, $\Delta l = \pm 1$.
  (4) $\Delta L = \pm 1$ (but $L=0 \rightarrow L=0$ forbidden).
  (5) $\Delta S = 0$.

- Semi-forbidden: $\Delta S \neq 0$. Lines weaker by $\sim\alpha^2$. E.g. N II] λ2143.

- Forbidden: One of the top 4 rules violated. Lines weaker by α^4.
  E.g. [NII] λ5756. Such states can only decay via collisions!
  Low ISM number density ⇨ Can observe forbidden transitions!

RADIATIVE PROCESSES

- The absorption rate from level "l" to level "u" is given by

  $dn_u/dt = cn_l \int \sigma_{lu}(\nu)\,(u_\nu/h\nu)\,d\nu \approx cn_l\,(u_\nu/h\nu_0) \int \sigma_{lu}(\nu)\,d\nu$

  $\sigma_{lu}(\nu)$ is the absorption cross-section for photons of frequency ν.

- However, $dn_u/dt = n_l\, u_\nu\, B_{lu}$ ⇨ $B_{lu} = (c/h\nu_0) \int \sigma_{lu}(\nu)\,d\nu$

  ⇨ $\sigma_{lu}(\nu) = (g_u/g_l)\,(c^2/8\pi\nu_0{}^2)\,A_{ul}\,\phi(\nu)$

  where $\phi(\nu)$ is the line shape function, with $\int \phi(\nu)\,d\nu = 1$.

- For "classical" absorption of radiation by an atom, the total absorption cross-section $\int \sigma_{lu}(\nu)\,d\nu = (\pi e^2/m_e c)$.

- For real atoms, the oscillator strength $f_{lu} = (m_e c/\pi e^2) \int \sigma_{lu}(\nu)\,d\nu$.

  i.e. the true absorption cross-section = $f_{lu} \times$ classical cross-section.

- Absorption lines characterized by f_{lu}, emission lines by A_{ul}.

The Line Profile: Natural Broadening

- Uncertainty principle ⇨ Absorption lines have *intrinsic* widths.

- Intrinsic profile: $\sigma_{int}(\nu) = (\pi e^2/m_e c)\, f_{lu}\, \phi_{int}(\nu)$

  Lorentzian shape: $\phi_{int}(\nu) \approx 4\,\gamma_{ul}\,[16\pi^2(\nu - \nu_{ul})^2 + \gamma^2_{ul}]^{-1}$

  Line FWHM $= (\gamma_{ul}/2\pi)$.

- De-excitation by spontaneous decay: $\gamma_{ul} = \Sigma_{j<u} A_{uj} + \Sigma_{j<l} A_{lj}$.

  For resonance lines ($l = 0$): $\gamma_{ul} = \Sigma_{j<u} A_{uj}$.

  For Lyman-α : $\gamma_{Ly\text{-}\alpha} = A_{Ly\text{-}\alpha}$ ⇨ $\Delta\lambda_{FWHM} \approx 0.00012$ Å.

- Natural broadening important for very strong lines (e.g. Lyman-α).

The Line Shape: Voigt Profiles

• Natural broadening ⇨ Lorentzian line shape.

• Maxwell-Boltzmann velocity distribution ⇨ Gaussian line shape.

• The net line profile is a convolution of a Gaussian and a Lorentzian: a Voigt profile.

$$\phi(\nu) = (1/\pi)^{1/2} \int (dv/b)\ e^{-v^2/b^2}\ 4\gamma_{ul}\ [16\pi^2[\nu - (1-(v/c)\nu_{ul})]^2 + \gamma^2_{ul}]^{-1}$$

• Near the line centre, the Gaussian dominates:
Line profile: $\sigma \approx \pi^{1/2} (e^2/m_e c)(f_{lu}\lambda_{ul}/b)\ e^{-v^2/b^2}$.

• Far away from the line centre, the damping wings dominate:
Line profile: $\sigma \approx \pi^{1/2} (e^2/m_e c)(f_{lu}\lambda_{ul}/b)\ [1/4\pi]^{3/2} (\gamma_{ul}\lambda_{ul}/b)(b^2/v^2)$.

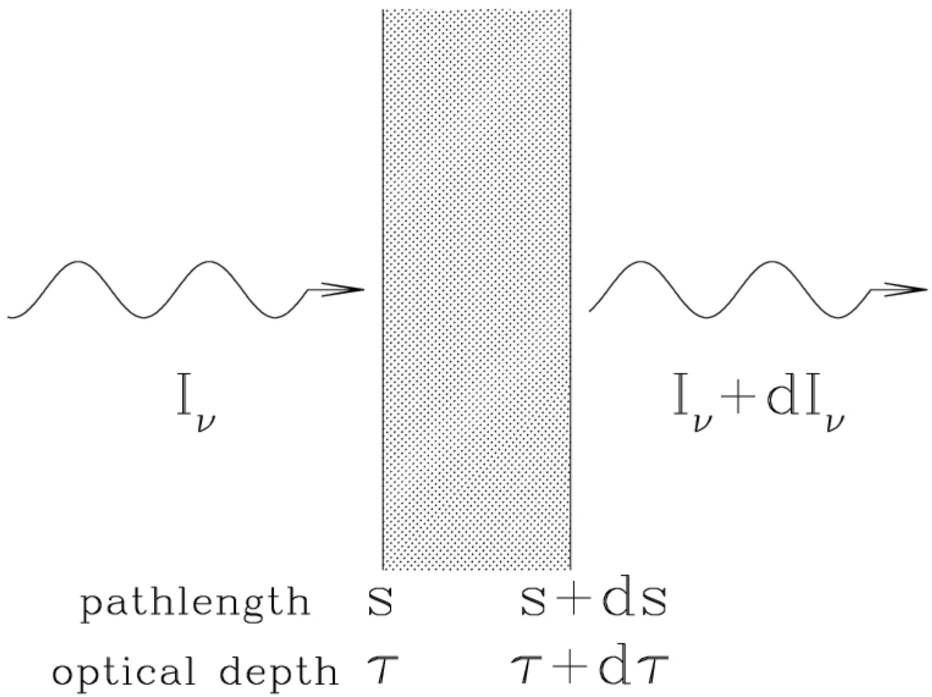
RADIATIVE TRANSFER

- Power per unit area between ν and $\nu+d\nu$, in solid angle $d\Omega$:

  $I_\nu(\nu, \mathbf{n}, \mathbf{r}, t)\, d\nu\, d\Omega$ ($I_\nu \equiv$ Intensity)

- Equation of radiative transfer:

  $$dI_\nu = -I_\nu \kappa_\nu ds + j_\nu ds$$



(Draine 2011)

- $j_\nu \equiv$ Emissivity.

  $\kappa_\nu \equiv$ Attenuation coefficient.

  $\kappa_\nu = n_l \sigma_{lu}(\nu) - n_u \sigma_{ul}(\nu)$.

- Define optical depth, $d\tau_\nu = \kappa_\nu\, ds$

  $\Rightarrow$ $dI_\nu = -I_\nu\, d\tau_\nu + (j_\nu/\kappa_\nu)\, d\tau_\nu$

- $I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau)} S_\nu\, d\tau$, where $S_\nu \equiv (j_\nu/\kappa_\nu)$.

Radiative Transfer

- $I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + {}_0\int^{\tau_\nu} e^{-(\tau_\nu - \tau)}\, S_\nu\, d\tau$, where $S_\nu \equiv (j_\nu/\kappa_\nu)$.

- For a uniform medium, with level populations given by T_X

  Kirchhoff's law: $S_\nu \equiv (j_\nu/\kappa_\nu) = B_\nu(T_X)$.

  $\Rightarrow\ I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + {}_0\int^{\tau_\nu} e^{-(\tau_\nu - \tau)}\, B_\nu(T_X)\, d\tau$

  $\Rightarrow\ I_\nu = I_\nu(0)\, e^{-\tau_\nu} + B_\nu(T_X)\,(1 - e^{-\tau_\nu})$

- In the radio regime, $I_\nu = (2k\nu^2 T_B/c^2)$

  $\Rightarrow\ T_B = T_B(0)\, e^{-\tau_\nu} + (h\nu/k)\,[e^{h\nu/kT_X} - 1]^{-1}\,(1 - e^{-\tau_\nu})$.

- If $(h\nu/kT_X) << 1$ $\Rightarrow$ $T_B = T_B(0)\, e^{-\tau_\nu} + T_X\,(1 - e^{-\tau_\nu})$.

The Curve of Growth

- Column density, $N = \int n \, ds$.

- At optical wavelengths, emission from the ISM is negligible.
  ⇨ For optical absorption, the observed intensity $I_\nu = I_\nu(0)\, e^{-\tau_\nu}$.

- Equivalent Width $W = \int (d\nu/\nu_0)\, [1 - I_\nu/I_\nu(0)] = \int (d\nu/\nu_0)\, [1 - e^{-\tau_\nu}]$.

- Optical depth $\tau(\nu) = \int \kappa_\nu ds = \int n_l \sigma_{lu}(\nu)\, ds = (\pi e^2/m_e c)\, f_{lu}\, N_l \phi(\nu)$.

- Optical depth at line centre, $\tau_0 = \pi^{1/2}(e^2/m_e c)\, (f_{lu} N_l\, \lambda_{lu}/b)$.
  ⇨ $\tau_0 = 0.7580\, (N_l/10^{13}\ \mathrm{cm}^{-2})\, (f_{lu}/0.4164)\, (\lambda_{lu}/1215.7)\, (10\ \mathrm{km/s}\ /\ b)$.

THE CURVE OF GROWTH

(Draine 2011)

- Optical depth at line centre, $\tau_0 = \pi^{1/2}(e^2/m_e c)\,(f_{lu} N_l\, \lambda_{lu}/b)$

  ⇨ $\tau_0 = 0.7580\,(N_l/10^{13}\ \mathrm{cm}^{-2})\,(f_{lu}/0.4164)\,(\lambda_{lu}/1215.7)\,(10\ \mathrm{km/s}\ /\ b)$

- Optically-thin, $\tau_0 \leqslant 1$.

  $N_l = 1.130 \times 10^{12}\,(W/f_{lu}\lambda_{lu})$.

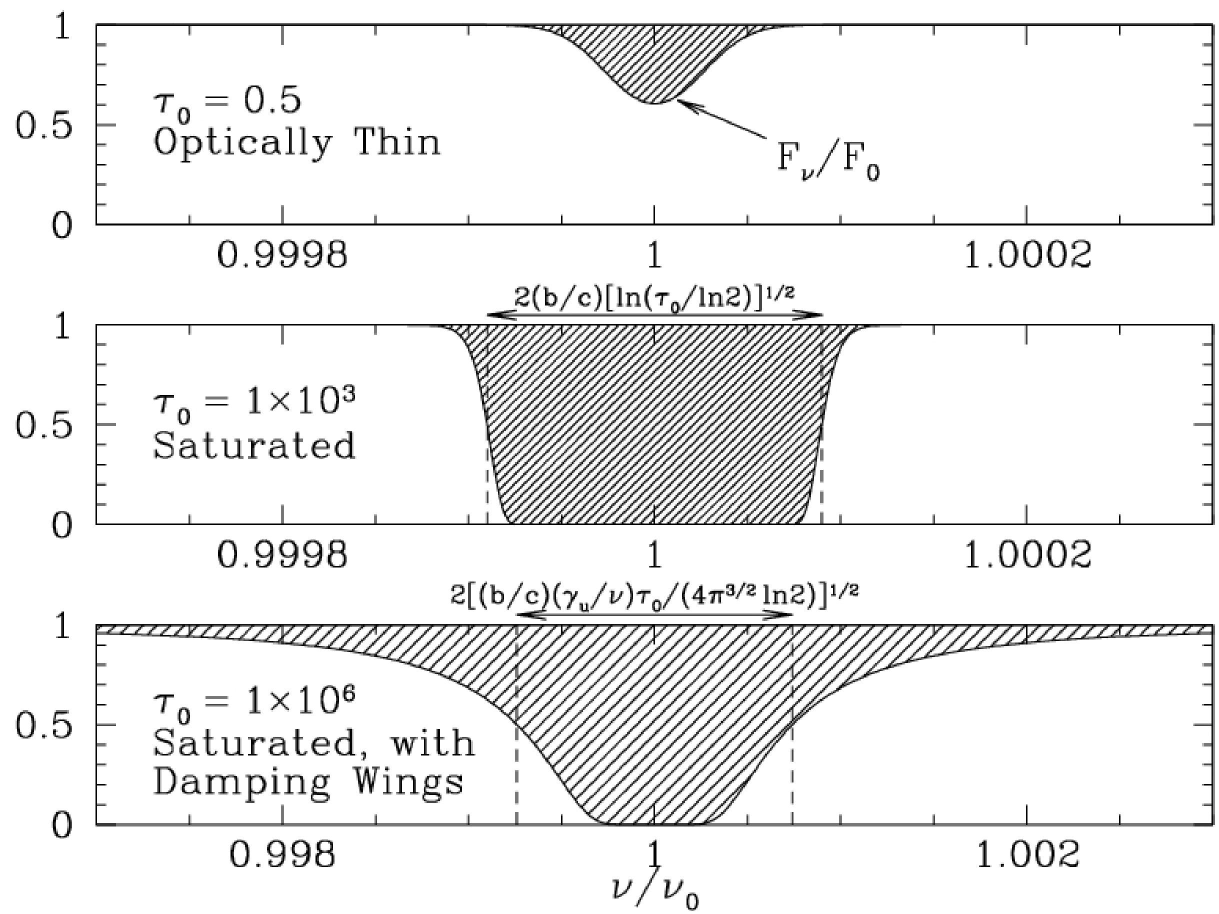
- Flat, $10 \leqslant \tau_0 \leqslant \tau_D$.

  $W = (2b/c)\,[\ln[\tau_0/\ln(2)]]^{1/2}$.

  $N_l \propto \exp(\,[cW/2b]^2\,)$.

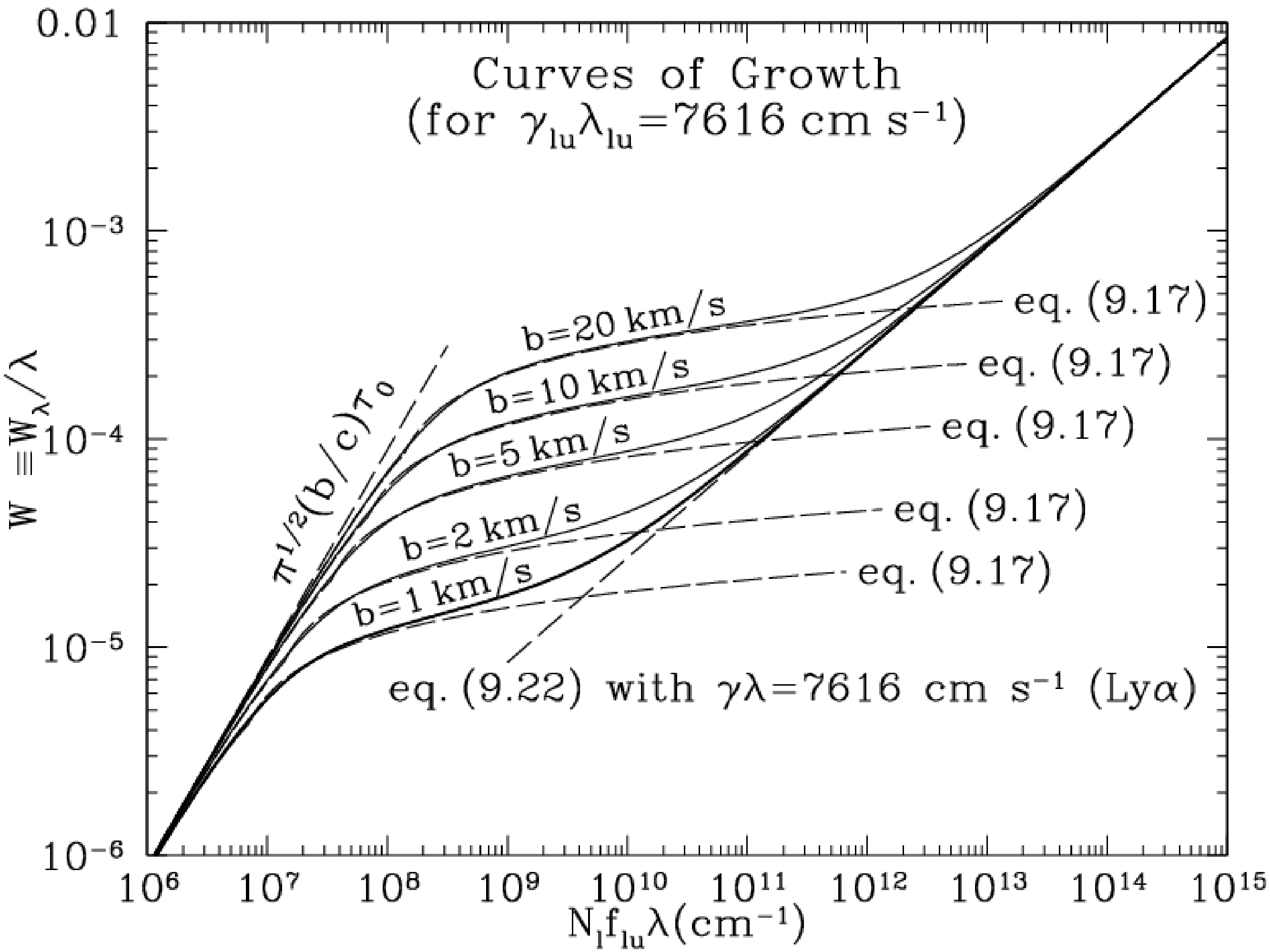
- Damped, $\tau_0 \geqslant \tau_D$.

  $N_l = (m_e c^3/e^2)(W^2/f_{lu}\gamma_{lu}\lambda^2_{lu})$.



$= 2.759 \times 10^{24}\ \mathrm{cm}^{-2} \times W^2\,(0.4164/f_{lu})\,(7616\ \mathrm{cm/s}\ /\gamma_{lu}\lambda_{lu})\,(\lambda_{lu}/1215.7)$.

The Curve of Growth



(Draine 2011)

- Lyman-α absorption lines: Measure the HI column density!
- UV metal absorption lines: Determine metal abundances.
- Deuterium Lyman-α lines: Determine [D/H].