Astronomical Techniques II
Lecture 3 - Noise, Temperature and SNR

Divya Oberoi

IUCAA NCRA Graduate School

div@ncra.tifr.res.in

March-May 2016

Airy Disc vs Beam Shape

- Airy Disc
  - Gives the Point Spread Function (PSF) for an imaging device
  - Independent of the Field-of-View (FoV), which is defined by other aspects (F ratio, magnification)
- Beam Shape
  - Defines the FoV
  - PSF for a non-imaging device
- In *Synthesis Imaging*, the analog of Airy disc is *synthesised beam*, which we will encounter later in this course.

Recap

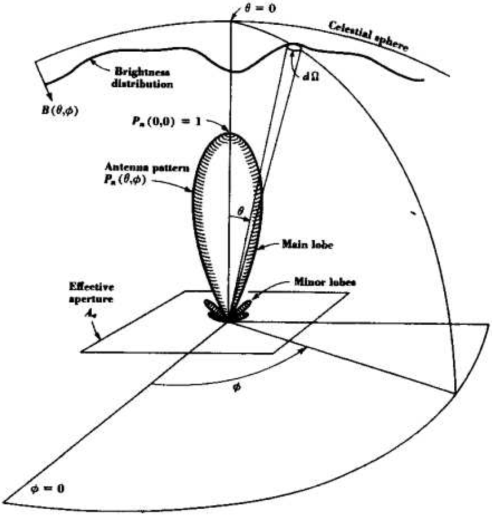


Fig. 3-2. Relation of antenna pattern to celestial sphere with associated coordinates.

Recap

$$W = \int_{\nu} \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \;\; d\nu \quad W$$

$$w_{\nu} = \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \quad W \;\; Hz^{-1}$$

$$w_{\nu} = \frac{1}{2} A_{eff} \int_{\Omega} B(\theta, \phi, \nu) \;\; P_n(\theta, \phi, \nu) \;\; d\Omega \quad W \;\; Hz^{-1}$$

For a uniform source of Brightness B_u, this becomes
$w_{\nu} = \frac{1}{2} \; A_{eff} \; B_u \; \Omega_A \quad W \; Hz^{-1}$

A question

Consider the following artificial scenario - a telescope has a beam width of 1° and uniform sidelobes -40 dB below the peak of the main lobe for the 2π sr centered on the main lobe and 0 in the remainin 2π. Assume the sky Brightness to be a constant all over the sky and the main lobe response to be a constant across the entire mainlobe.

- What fraction of the total power picked up by such a dish comes from the sidelobes.

Submit your answer in the next class!

Compact and extended sources

- The telescope measures an integral over the entire beam

$$S_\nu = \int_{Beam} B(\Omega, \nu)\ P_n(\Omega - \Omega_0, \nu)\ \ d\Omega \quad W\ m^{-2}\ Hz^{-1}$$

- Compact - much smaller than the main lobe
  - Assuming there is no other source in the beam, the S_ν equals the spectral flux density of the source
- Extended - comparable or larger than the main lobe
  - The measured S_ν underestimates the true spectral flux density of the source.
  - Correct for P_n
  - Use multiple pointings if needed

Spectral Power - Convolution form

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\theta,\phi,\nu) \;\; P_n(\theta,\phi,\nu) \;\; Sin\theta \;\; d\theta \;\; d\phi; \;\; W \; Hz^{-1}$$

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\theta,\phi,\nu) \;\; P_n(\theta-\theta_0,\phi-\phi_0,\nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Cross-correlation form:

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\Omega,\nu) \;\; P_n(\Omega-\Omega_0,\nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Convolution form:

$$w_\nu = \frac{1}{2} A_{eff} \int_\Omega B(\Omega,\nu) \;\; \tilde{P}_n(\Omega_0-\Omega,\nu) \;\; d\Omega \quad W \; Hz^{-1}$$

where $\tilde{P}_n(\Omega_0-\Omega,\nu) = P_n(\Omega-\Omega_0,\nu)$

Convolution implies *smoothing*

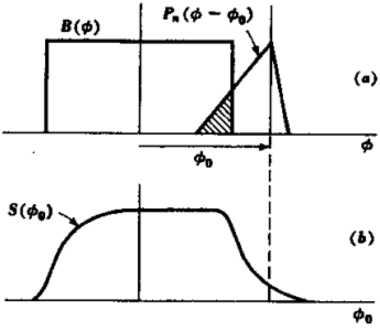
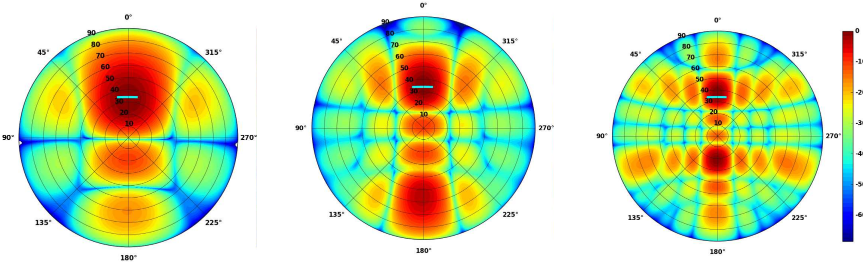


Fig. 3-7. Example of a uniform source distribution scanned by an antenna with an asymmetric pattern of triangular shape.

What will the sidelobes do?

What will the sidelobes do?



(a) 100 MHz (b) 186 MHz (c) 296 MHz

Figure: Beams of Murchison Widefield Array at Az = 0° and El = 54°.

A Blackbody and Planck's Law

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\ \ W\ m^{-2}\ sr^{-1}\ Hz^{-1}$$

$$B_\lambda = \frac{2hc^3}{\lambda^5}\frac{1}{e^{hc/kT\lambda}-1}\ \ W\ m^{-2}\ sr^{-1}\ m^{-1}$$

Planck's Law

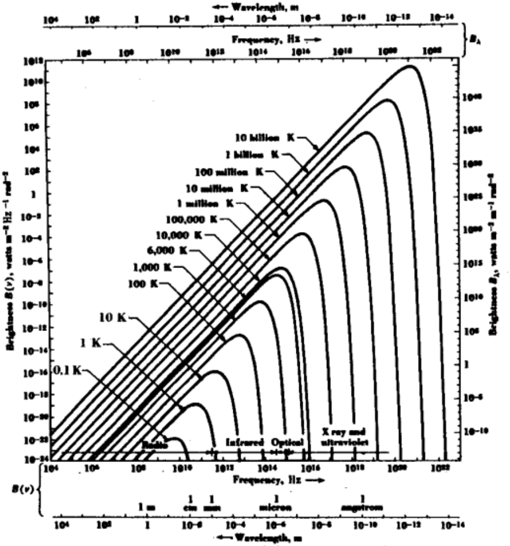


Fig. 3-14. Planck-radiation-law curves with frequency increasing to the right.

Planck's and Rayleigh-Jeans Law

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

$$B_\nu = \frac{2kT\nu^2}{c^2}\frac{\frac{h\nu}{kT}}{e^{h\nu/kT}-1}$$

Limit $h\nu << kT$
Rayleigh-Jeans Law

$$B_\nu = \frac{2kT}{\lambda^2} \;\; W \; m^{-2} \; sr^{-1} \; Hz^{-1}$$

Power received at a detector

$$dW = B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \;\; d\nu$$

dW - W
$B(\theta, \phi) - W \; m^{-2} \; sr^{-1} \; Hz^{-1}$

Practical quantitative definition

$$B(\theta, \phi, \nu) = \frac{dW}{d\Omega \; cos\theta dA \;\; d\nu} = \frac{2kT}{\lambda^2}$$

- Intrinsic property of the source
- Independent of the distance from the source (ONLY for a resolved object)
- Can be thought of as energy *received* at the detector OR as energy *emitted* by the source.

Spectral flux density and Temperature

$$S = \frac{2kT_a\Omega_s}{\lambda^2}$$

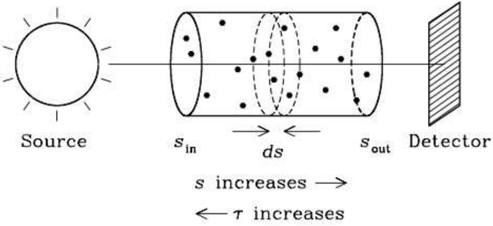
$$S_{True} = \frac{2k}{\lambda^2}\int_{\Omega_s} T(\Omega)\; d\Omega$$

$$S_{Measured} = \frac{2k}{\lambda^2}\int_{\Omega_{beam}} T(\Omega)\; \tilde{P}_n(\Omega_0 - \Omega)\; d\Omega$$

Brightness Temperature (T_B)

- Associates a unique temperature with the power received at any given frequency, or the Brightness of a source.
- A property of the source.
- Represents the temperature of a Blackbody which would have given out exactly as much radiation at that frequency
- For thermal radiation from an optically thick source - same as the physical temperature of the body emitting the radiation.
- For non-thermal radiation - eq. radiation temperature

Optical Depth and Radiative Transfer



$$T_{Observed} = T_{Source}e^{-\tau_M} + T_{Medium}(1 - e^{-\tau_M})$$

- $\tau_M = 0 \,; >> 1 \,; \sim 1$
- $T_{Source} = T_{Medium}$

Temperature and Noise

Spectral power density measured per unit bandwidth at the terminals of a resistance R at temperature T (Nyquist, 1928)
$w = kT \;\; W \; Hz^{-1}$

What does the spectrum of noise power look like?

The temperature of antenna radiation resistance

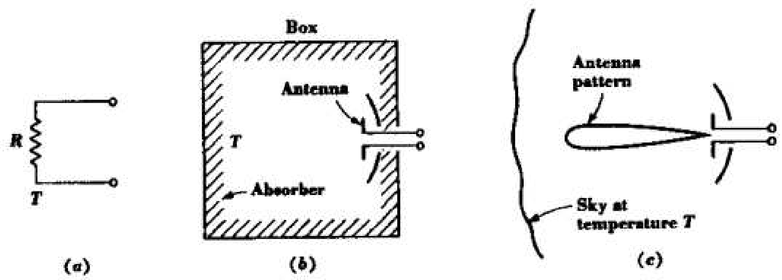


Fig. 3-24. (a) Resistor at temperature T; (b) antenna in an absorbing box at temperature T; and (c) antenna observing sky of temperature T. The same noise power is available at the terminals in all three cases.

Antenna Temperature (T_A)

load $\rightarrow$ lossless antenna of radiation resistance R, the impedence as seen at the terminals is unchanged.
The noise spectral power received by the antenna

$$w = \frac{1}{2} A_{eff} \int_\Omega B(\Omega)\, \tilde{P}_n(\Omega_0 - \Omega)\, d\Omega = kT_A$$

$$w = \frac{k\ A_{eff}}{\lambda^2} \int_\Omega T(\Omega)\, \tilde{P}_n(\Omega_0 - \Omega)\, d\Omega \ \ W\ Hz^{-1}$$

$$w = \frac{kA_{eff}}{\lambda^2}\ T\ d\Omega$$

But $\lambda^2 = A_{eff}\ d\Omega \implies w = kT \implies T_A = T$.

Antenna Temperature

$$T_A = \frac{A_{eff}}{\lambda^2} \int_\Omega T(\Omega)\ \tilde{P}_n(\Omega_0 - \Omega)\ d\Omega$$

$$T_A = \frac{1}{\Omega_A} \int_\Omega T(\Omega)\ \tilde{P}_n(\Omega_0 - \Omega)\ d\Omega$$

The compact source and extended source cases.

Noise and Signal

- Signal - T_{Ant} - what comes from the sky
- Noise - everything else
    - Receiver - T_{Rec}
    - Spillover - T_{Spill}
    - Leakage - T_{Leak}
    - Loss - T_{Loss}
    - Radio Frequency Interference (RFI)

$$T_{Sys} = T_{Ant} + T_{Rec} + T_{Spill} + T_{Leak} + T_{Loss}$$

- The *signal* has the same characteristics as *noise*
- One is looking for an increase of T_{Ant} over a background of T_{Sys}.

Minimum Detectable Signal

$$\Delta T_{min} = \frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

$$\Delta B_{min} = \frac{2k}{\lambda^2}\frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

$$\Delta S_{min} = \frac{2k}{A_{Eff}}\frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

- In theory, can be reduced to an arbitrarily small number by increasing the denominator.
- In practice:
  - Limited Δt - source evolution, source visibility, system stability, TAC, human effort
  - Limited $\Delta\nu$ - spectral signature (emission/absorption lines, EoR), source evolution, instrumental capability, technical challenges

References and Pre-requisites

- References
  - Kraus - Radio Astronomy (2nd Ed) - 3.5 – 3.19 + Exercises
- Pre-requisites
  - Fourier transforms and convolution