

# Astronomical Techniques II

## Lecture 13 - Self-Calibration

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# Calibration errors - impact on the image

- 1 The origin of calibration errors
- 2 The impact of calibration errors
  - 1 Deconvolution equation - no longer valid
- 3 Sidelobes of the PSF  $\sim \frac{1}{\sqrt{N(N-1)}}$
- 4 Show up as increased RMS in the map

# Calibration approach

**1** Rely on frequent observations of radio sources of known structure, position and strength to determine the calibration solutions

**2**  $\tilde{V}_{i,j}(t) = g_i(t) g_j^*(t) G_{i,j}(t) V_{i,j}(t) + \mathcal{E}_{i,j}(t) + \epsilon_{i,j}(t)$

**3**  $\tilde{V}_{i,j}(t) = g_i(t) g_j^*(t) V_{i,j}(t) + \epsilon_{i,j}(t)$

**4**  $g_i(t)$

**1** Instrumental part - slowly varying

**2** Propagation part (troposphere and ionosphere) - faster varying

**3**  $\sigma_G = \frac{\sigma_V}{S \sqrt{(N-3)}}$ ; S - Flux density of the source

# Drawbacks of ordinary calibration

- 1  $g_i(t)$ s come from a time and direction different from that of interest!
- 2 Residual errors (frequency and baseline length dependent) remain
- 3 For stronger sources - dominate the error budget
- 4 Strength of the available calibrator
- 5 Presence of any resolved structure or other confusing sources in the field

# The idea of *self-calibration*

- 1 Allow the element gains to be free parameters in the imaging process
- 2 Impact of self-calibration
  - 1 Constraints = No. of measured visibilities =  $N(N - 1)/2$
  - 2 Instrumental DoF =  $N$
  - 3 Constraints available for the emission in the sky =  $N(N - 1)/2 - N$  (amplitude) and  $N(N - 1) - (N - 1)$  (phase)
  - 4 Loss of information of absolute position of the source
  - 5 Loss of information of absolute strength of the source

# Redundant Calibration

- 1 Consider a 1-D array of  $N$  elements with uniform spacing between antennas,  $d$  (Westerbork, Ooty)
- 2 Redundant measurements in the  $uv$  plane for all but the longest baseline
  - 1 Overdetermined - Of the  $N(N-1)/2$  measurements, only  $N-1$  are independent
- 3 Solve simultaneously for  $(N-1)$  visibilities and  $N$  complex gains

# Self-calibration

- 1 Basic premise - even after including the additional DoF of element gains, the job of estimation of an adequate model for  $I(l, m)$  is still overdetermined
- 2 Similar to Clean - use plausible assumptions about  $I(l, m)$  to interpret measured visibilities
- 3 Objective - Deduce  $\hat{I}$ , the FT of which,  $\hat{V}$ , after accounting for instrumental gains is consistent with the measured visibilities.

$$4 \quad S = \sum_k \sum_{i,j \ i \neq j} w_{i,j}(t_k) |\tilde{V}_{i,j}(t_k) - g_i(t_k)g_j^*(t_k)\hat{V}(i,j)(t_k)|^2$$

$$5 \quad S = \sum_k \sum_{i,j \ i \neq j} w_{i,j}(t_k) |\hat{V}_{i,j}(t_k)|^2 |X_{i,j}(t_k) - g_i(t_k)g_j^*(t_k)|^2,$$

where

$$X_{i,j}(t_k) = \frac{\tilde{V}_{i,j}(t_k)}{\hat{V}_{i,j}(t_k)}$$

# Self-calibration - practical implementation

- 1 Make an initial model of the source,  $\hat{l}$
- 2 Use the previous equation to convert it into a point source model
- 3 Solve for  $g_i$ s
- 4 Compute the corrected visibilities

$$V_{i,j,corr}(t) = \frac{\tilde{V}_{i,j}(t)}{g_i(t) g_j^*(t)}$$

- 5 Build a new model using  $V_{i,j,corr}(t)$
- 6 Iterate till satisfied



# Closure Phase and Amplitude

1  $\tilde{\phi}_{i,j}(t) = \phi_{i,j}(t) + \theta_i(t) - \theta_j(t) + \text{noise}$ , where

$$\theta_i(t) = \arg g_i(t)$$

2  $\tilde{C}_{i,j,k} = \tilde{\phi}_{i,j}(t) + \tilde{\phi}_{j,k}(t) + \tilde{\phi}_{k,i}(t)$

3  $\tilde{C}_{i,j,k} = \phi_{i,j}(t) + \phi_{j,k}(t) + \phi_{k,i}(t) + \text{noise}$

4  $\tilde{C}_{i,j,k} = C_{i,j,k} + \text{noise}$

5 
$$\Gamma_{i,j,k,l} = \frac{|\tilde{V}_{i,j}(t)| |\tilde{V}_{k,l}(t)|}{|\tilde{V}_{i,k}(t)| |\tilde{V}_{j,l}(t)|}$$

6 Iterative least-squares techniques to make  $\hat{V}_{i,j}(t)$  consistent with  $\tilde{V}_{i,j}(t)$

7 It can formally be shown that *self-calibration* is equivalent to using closure quantities (Cornwell and Wilkinson, 1981)

- 1 Relationship with Adaptive Optics
- 2 Why does *self-cal* work?
  - 1 Most successful for dense *uv* coverages for arrays with largeish  $N$  (few tens) and good SNR
  - 2 Sources are simple and can be represented by a small number of DoF
  - 3 For a large  $N$  interferometer, it still remains a vastly over determined problem
  - 4 No formal proof of convergence of self-calibration is available

# Driving *self-cal*

- 1 Initial model - usual calibration and subsequent imaging - good enough
- 2 Model *must not* contain any features due to calibration errors
- 3 Images at near by frequencies, higher/lower resolutions useful
- 4 One can even start from a point source model and slowly move towards a detailed model of a source which is many many resolution elements across
- 5 Prudent to solve only for phases to begin with
- 6 Use of weighting schemes
- 7 Choice of averaging time

# Baseline based errors

- 1 Random time varying pointing errors (jitter)
- 2 Non-isoplaneticity in the ionosphere
- 3 Departures of the primary beam from the reference primary beam
- 4 Correlator problems (bias, incorrectly set sampling levels)
- 5 Local RFI