Astronomical Techniques II
Lecture 11 - Imaging

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March-May 2016

Fourier Imaging

1. $\mathcal{A}(l,m)\ I(l,m) =$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} V(u,v)\ e^{2\pi i(ul+vm)} d\nu\ du\ dv$$
2. Assumptions
   1. $|\frac{\Delta\nu}{c}\vec{b}.(\vec{s}-\vec{s_0})| << 1$
   2. $|2\pi\ w(\sqrt{1-l^2-m^2}-1)| << 1$ or $|\pi w(l^2+m^2)| << 1$
3. $V(u,v)$ - $W\ m^{-2}\ Hz^{-1}$
4. $I(l,m)$ - $Jy\ beam^{-1}$
5. In reality we only have $V(u_k, v_k)$, how many?

Fourier Imaging

1. $$I^D(l,m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)\ V'(u,v)\ e^{-2\pi i(ul+vm)} du\ dv$$
   where

   $S(u,v)$ - Sampling Function

2. DFT - Direct Fourier Transform
   1. $I^D(l,m) = \frac{1}{M}\sum_{k=1}^{M} V'(u_k, v_k) e^{2\pi i(u_k l + v_k m)}$
   2. Computational cost for a $N \times N$ image - $O(M) \times O(N^2) \geq O(N^4)$

FFT Imaging

1. Requires regular gridding
2. Computational cost - $O(N) \times O(N\ log_2 N) \sim N^2\ log_2 N$

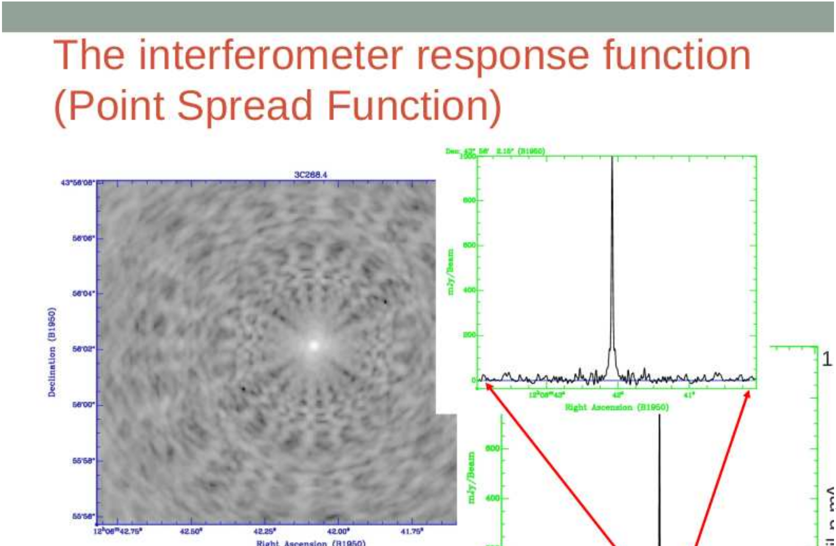
The sampling function

1. $S(u,v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)$
2. $V^S(u,v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)\ V'(u_k, v_k)$
3. $V^S = S\ V'$
4. $I^D = \mathcal{F} V^S$
5. $I^D = \mathcal{F} S \ast \mathcal{F} V'$

Weighting Functions - controlling the beam shape

1. $W(u, v) = \sum_{k=1}^{M} R_k \; T_k \; D_k \; \delta(u - u_k, v - v_k)$
2. $V^W(u, v) = \sum_{k=1}^{M} R_k \; T_k \; D_k \; \delta(u - u_k, v - v_k) \; V'(u_k, v_k)$
   1. R_k - Reliability $\sim \left(\frac{T_{sys}}{\sqrt{\Delta t \; \Delta\nu}} \right)^{-1}$
   2. T_k - Tapering function
   3. D_k - Density weighting function

Beam Shape Example



A desirable beam

1. No/low sidelobes
2. High resolution

3. High sensitivity - Competing requirement

The Tapering Function

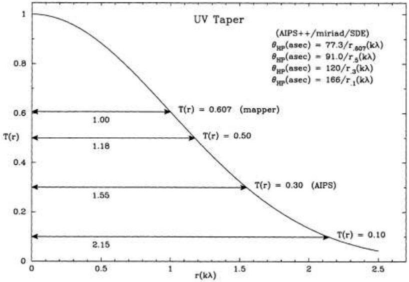


Figure 7–1. A Gaussian (u, v) taper with dispersion $\sigma = 1$ km.

1. Usually $T_k = T(u_k, v_k) = T(u_k)T(v_k) = T(r)$
2. Most useful when the relevant part of the (u, v) plane is densely sampled and is not truncated by the edge of the (u, v) plane
3. Inner (u, v) limit

Impact of tapering on the PSF

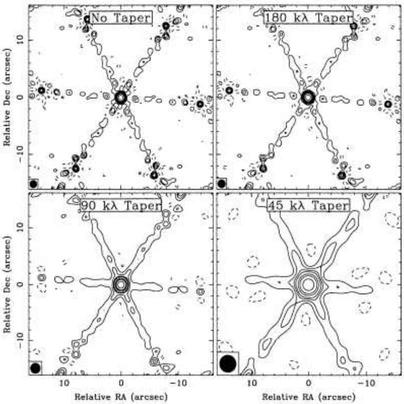
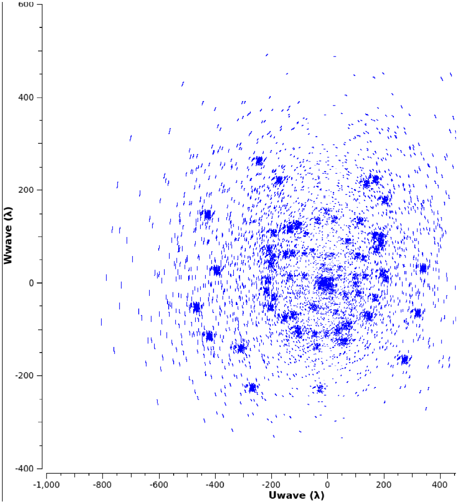


Figure 7–2. The effect of a Gaussian taper on the point source response of a VLA snapshot in the **A** configuration at 20-cm wavelength. As a narrower Gaussian taper (i.e., a heavier tapering) is applied, the half-power width of the point spread function increases and the inner sidelobes are reduced.

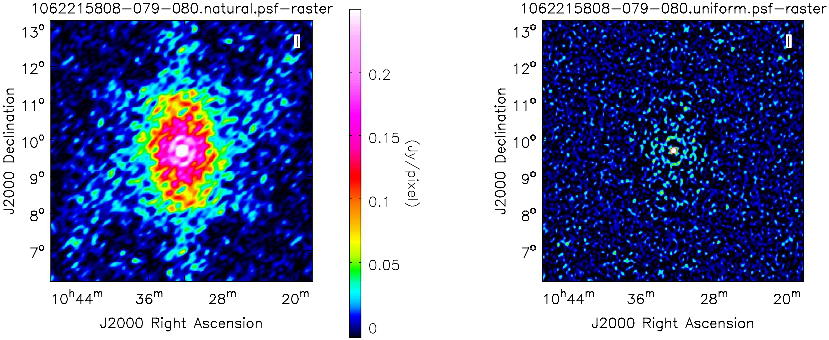
Density weighting function

1. Natural: $D_k = 1$
2. Uniform: $D_k = \dfrac{1}{N_s(k)}$
3. Robust:

PSF Weighting Motivation



PSF Weighting Example - MWA



PSF Weighting Example - VLA

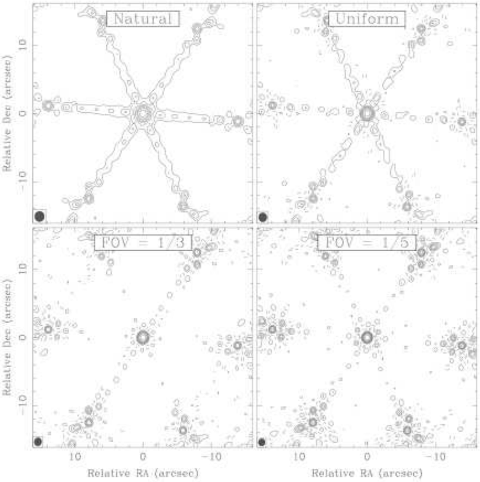


Figure 7–3. The effect of different weighting functions on a VLA 'snapshot' image of a point source.

Trade-offs involved

1. Sensitivity
2. High resolution
3. Good sidelobe performance (low sidelobe levels)

4. Match the PSF to the needs of the analysis

Gridding

1. Interpolation
2. Convolution
   1. Predictable impact on the images
   2. Convolve V^W with some C and then sample this convolution at centre of each cell of the *grid*
   3. $C = 0$, outside some small bounded region, A_C, support size.
   4. $V^R(u_c, v_c) = \sum_{k=1}^{M} \, C(u_c - u_k, v_c - v_k) \; V^W(u_k, v_k)$
   5. $V^R = R \left(C \bigstar V^W \right) = R(C \bigstar (W \;\; V'))$, where

      $R = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - \dfrac{u}{\Delta u}, \; k - \dfrac{v}{\Delta v})$