INTRODUCTION TO INTERFEROMETRY

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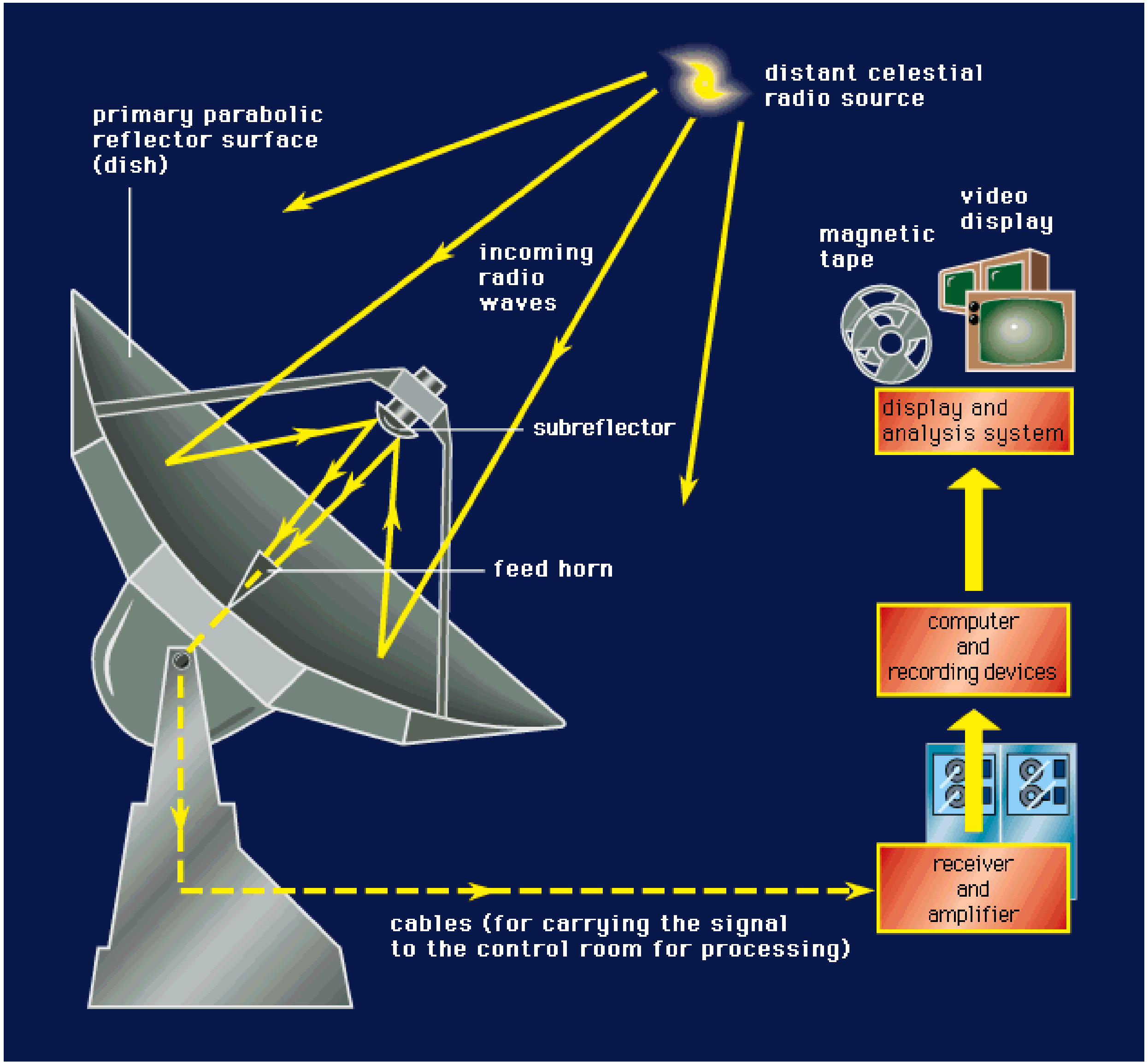
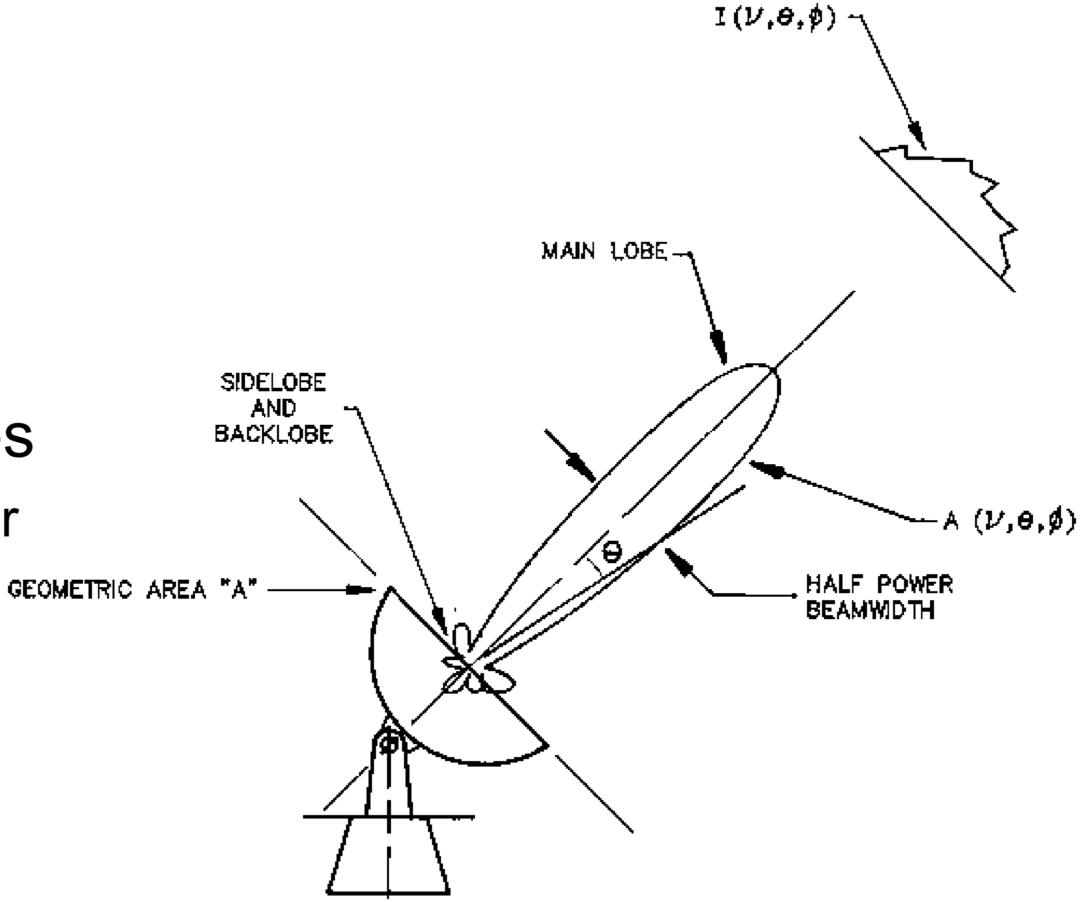
A typical radio telescope



Beam size and resolution

- Size of the main lobe in radians ~λ/D
  - λ is the wavelength
  - D is the diameter
- Better resolution requires
  - Shorter wavelength (higher frequency)
  - Bigger telescopes

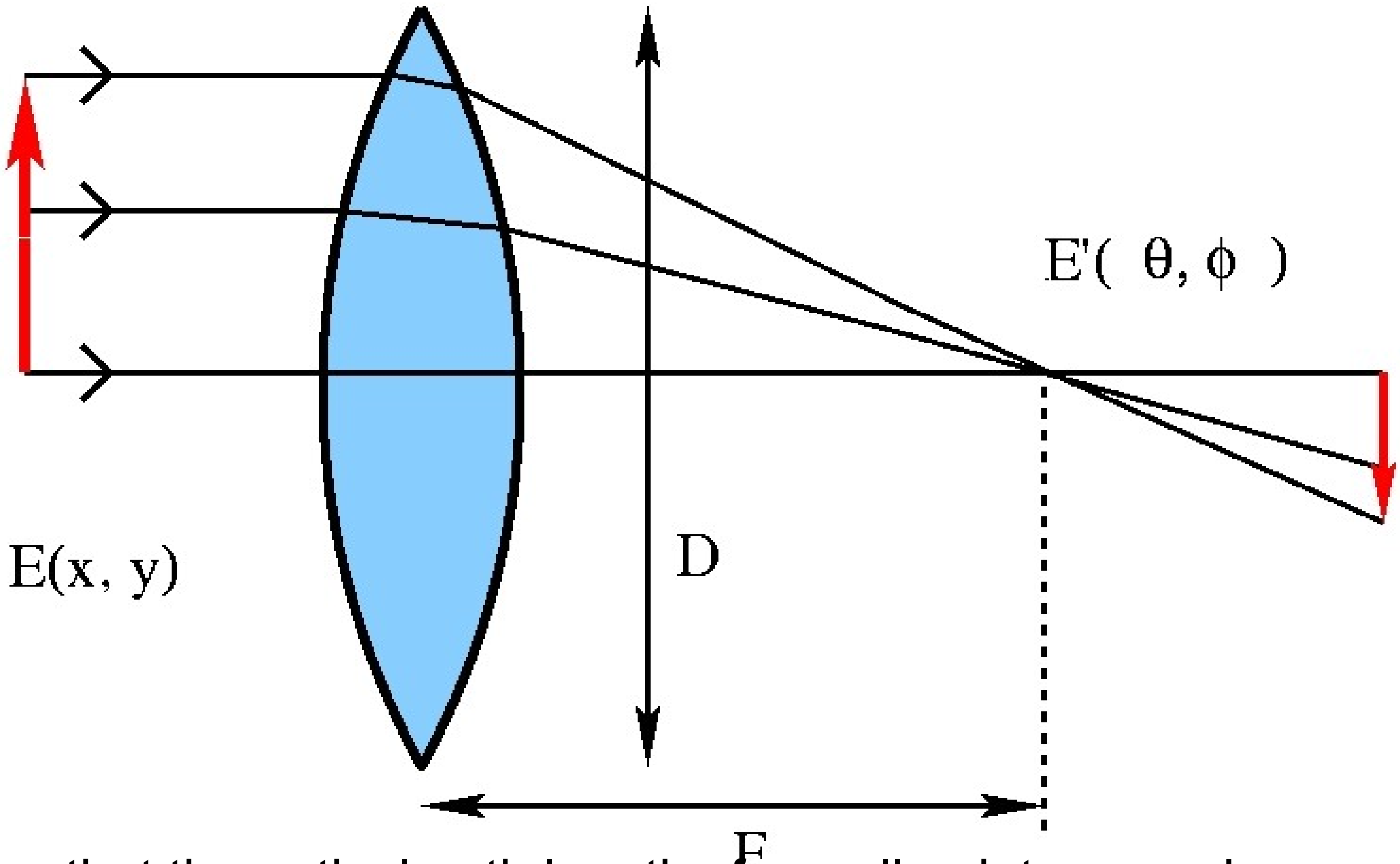


Why Interferometry?

- Resolution ~ (1.22) λ/D
  - λ - wavelength of observation
  - D - size of aperture (diameter of lens/mirror)
- An angular resolution of 1 arcsec requires D ~$2x10^5$ λ
  - A 2m optical telescope is ~$2.5x10^6$ λ (8000 Å)
  - In radio λ ranges from ~0.5 mm to ~10 km (600 GHz – 30 kHz)
  - 1 arc sec requires D ~100 m to ~$2x10^3$ km
- Impossible to build apertures of required dimensions and surface accuracy

- *Interferometry provides the solution - resolution corresponding to the separation between the elements (telescopes)*

Imaging with a lens (mirror)

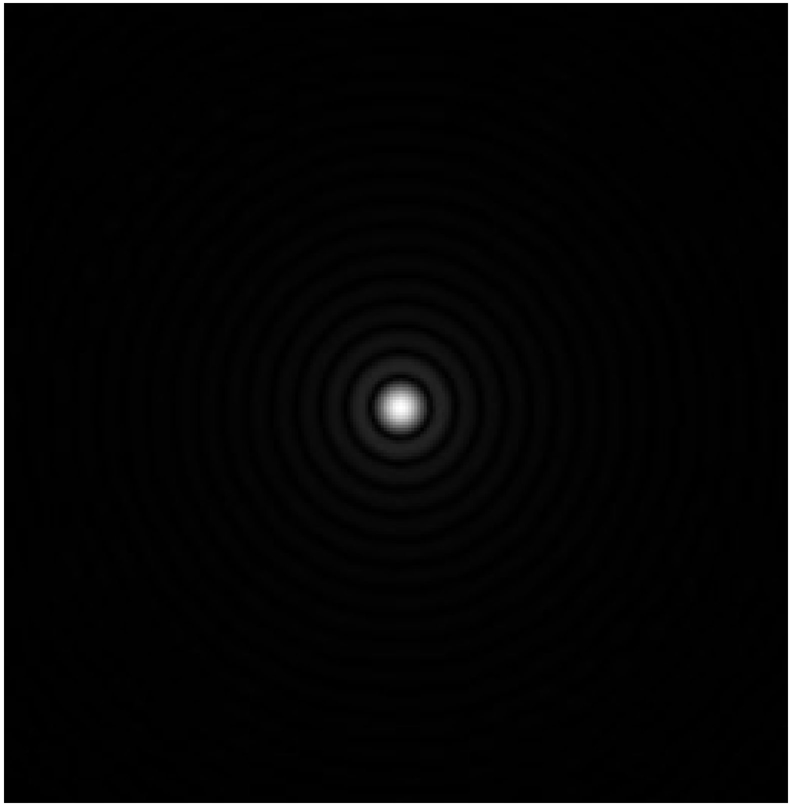


It ensures that the optical path lengths from all points on a plane wavefront (perpendicular to the optical axis) to the focal point are the same.

A more sophisticated perspective

Mathematically, a lens performs a Fourier Transform of the incident wavefront

$E(x,y) \leftrightarrow E'(\theta,\varphi)$



Some characteristics of optical imaging systems

- Transfer function / Point source response / Point spread function (PSF) - Airy pattern
- Resolution = $1.22\ \lambda/D$

The concept behind an interferometer

The important property of a parabolic dish is that it adds parallel light rays coherently

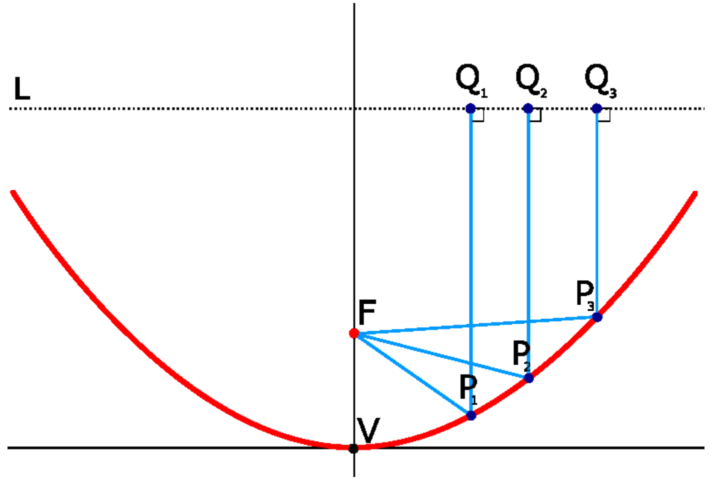
Parallel rays (from infinity) have equal path lengths to the focus, so they all arrive in phase

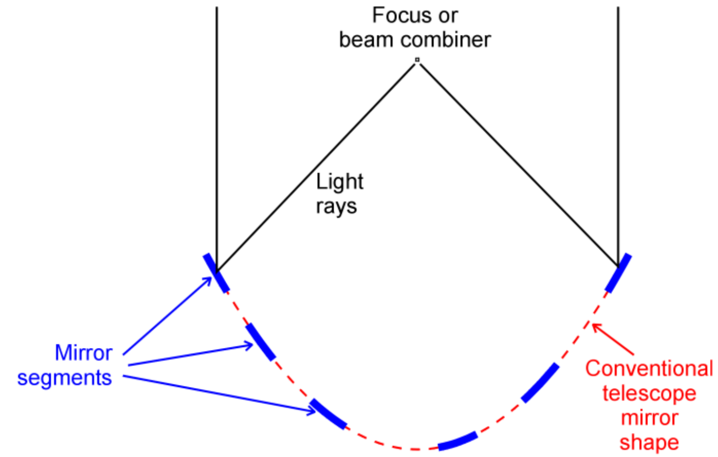
This is still true if we remove segments of the parabola – remaining rays still reach focus in phase

Now imagine moving the remaining segments of the dish off the surface of the paraboloid

So long as we know very precisely where the segments are located, we can delay their signals appropriately and still add them together coherently

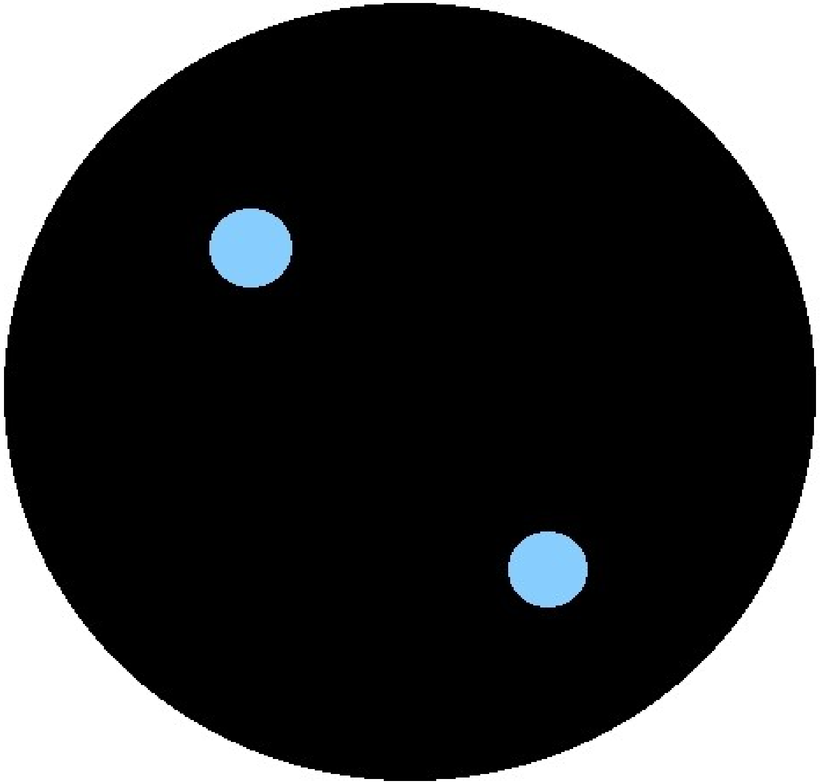
This, in essence, is what an interferometer does



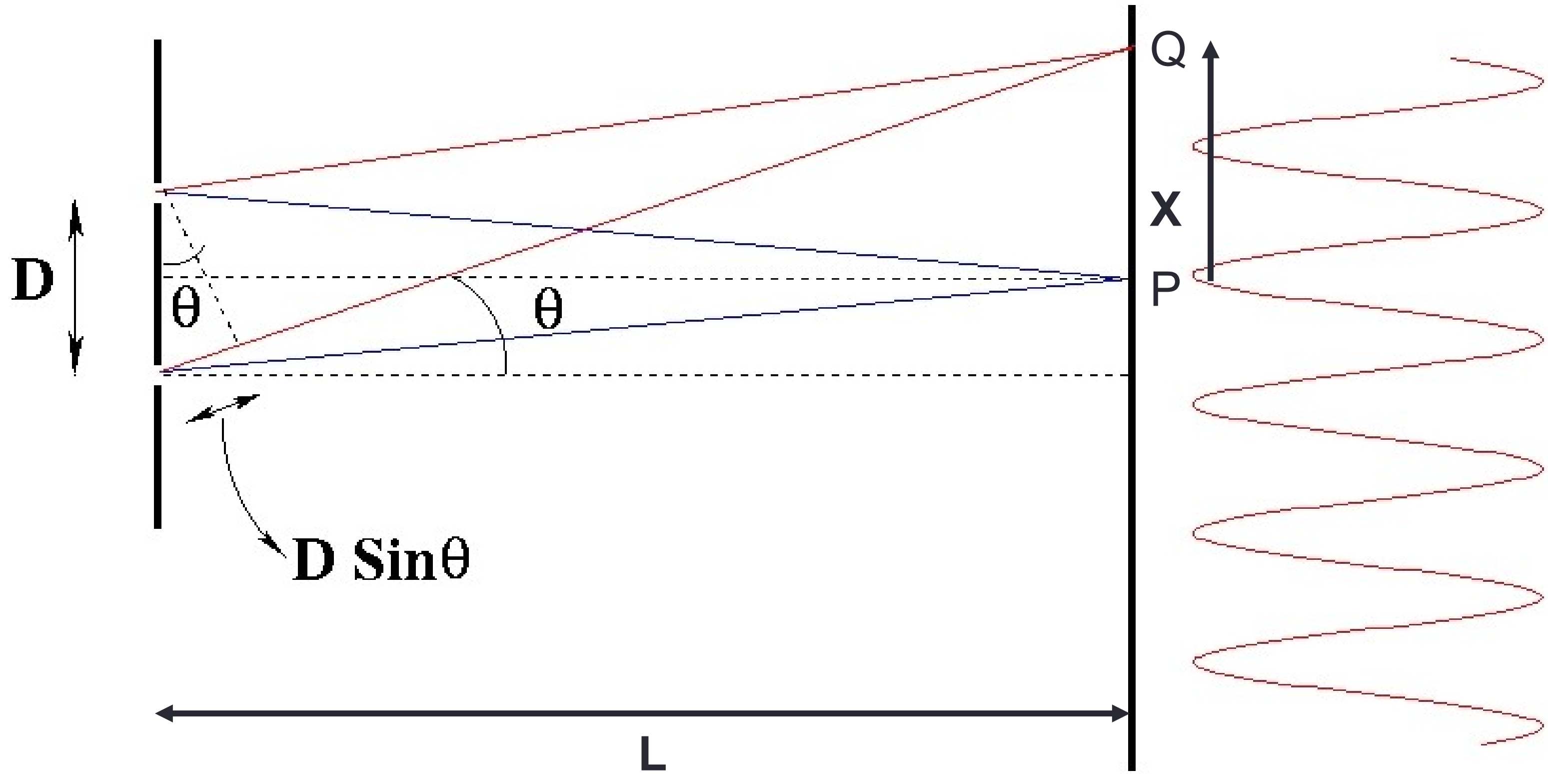


Images: wikipedia

Imaging with an *unfilled* aperture



Young’s double-slit experiment



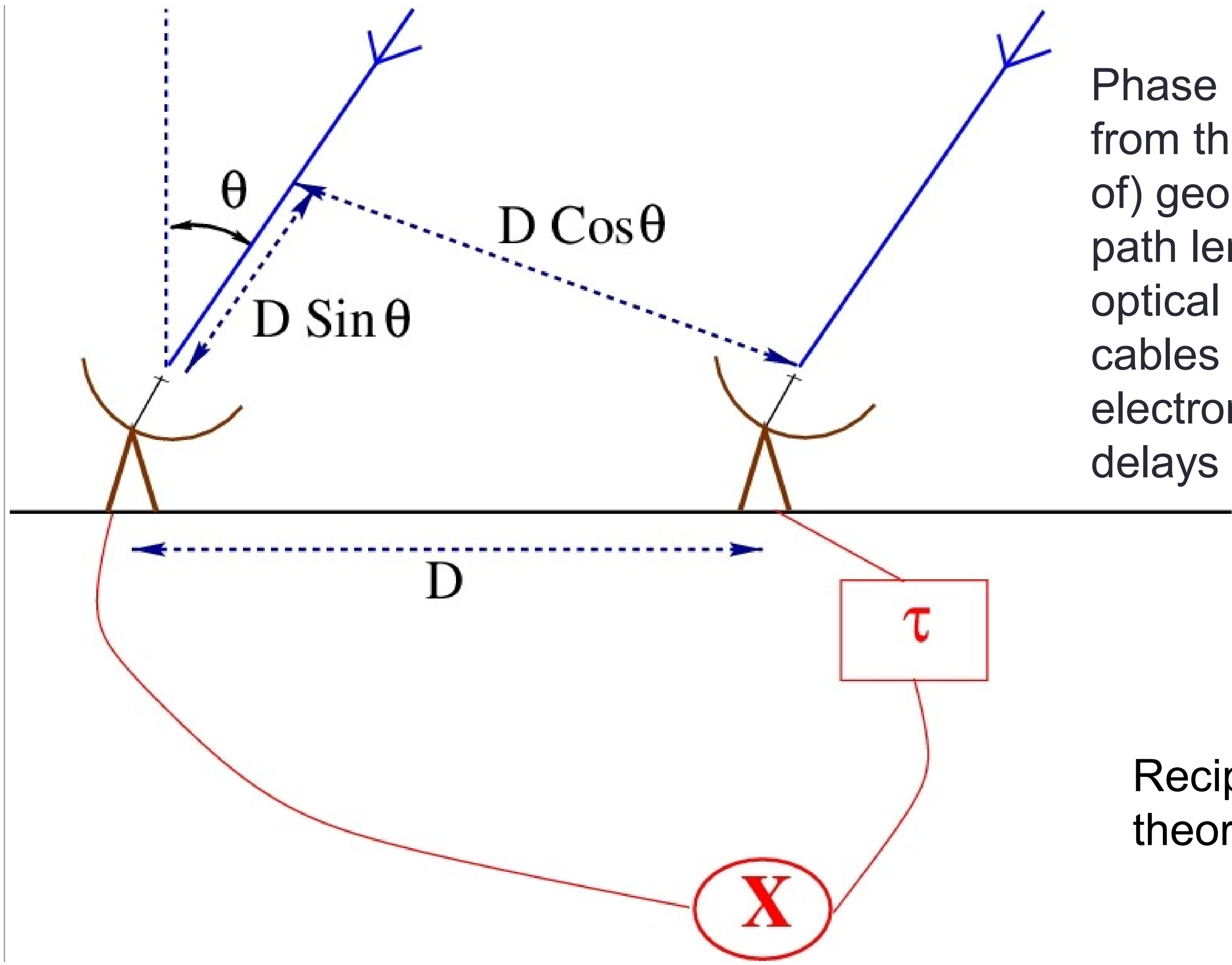
Path difference = $n\lambda \Rightarrow$ maxima $X = n\lambda\, L/D$

= $(n+1/2)\lambda \Rightarrow$ minima $X = (n+1/2)\,\lambda\, L/D$

$\lambda << D << L$

A two element interferometer

Slits → Telescopes

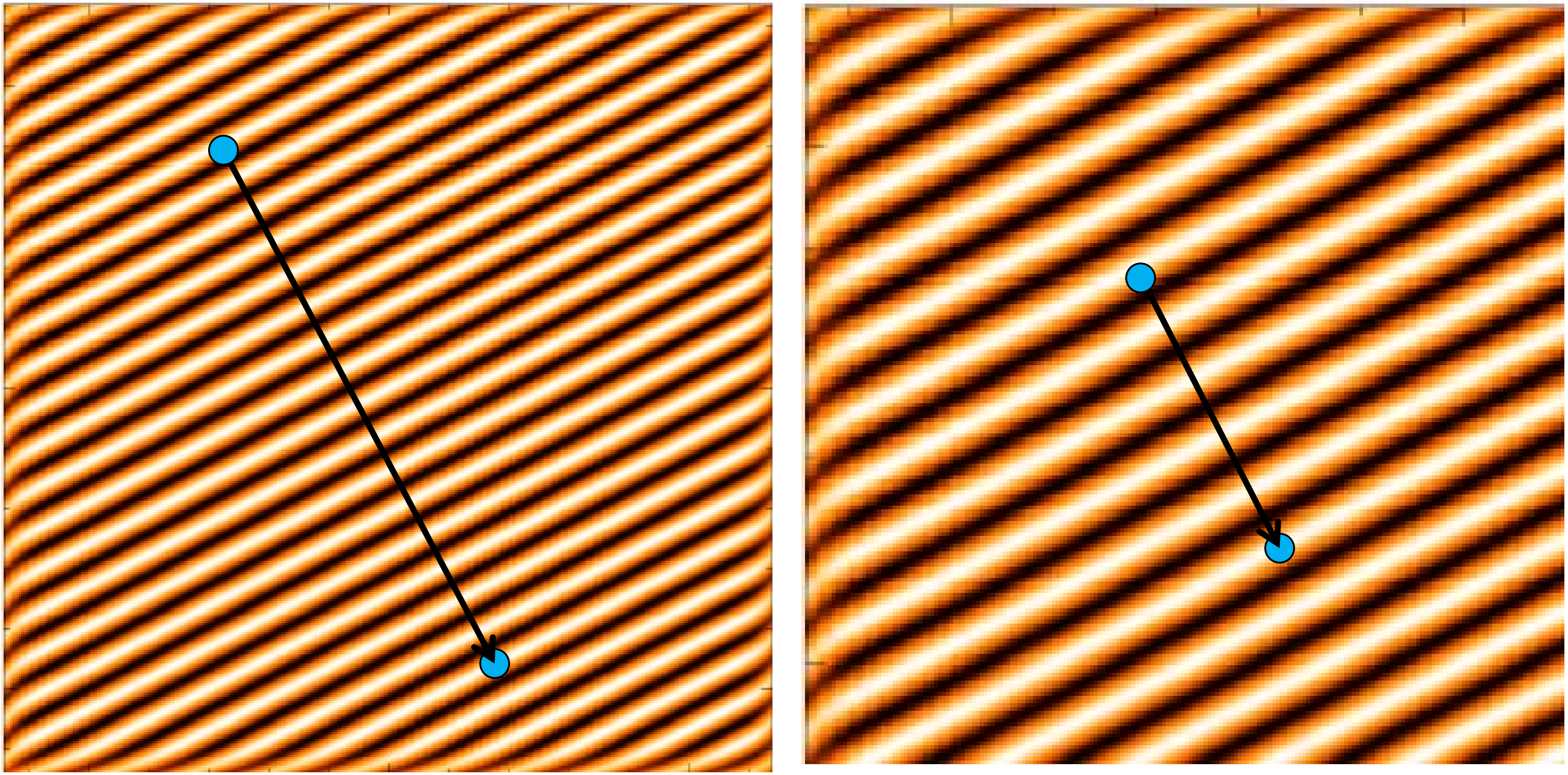


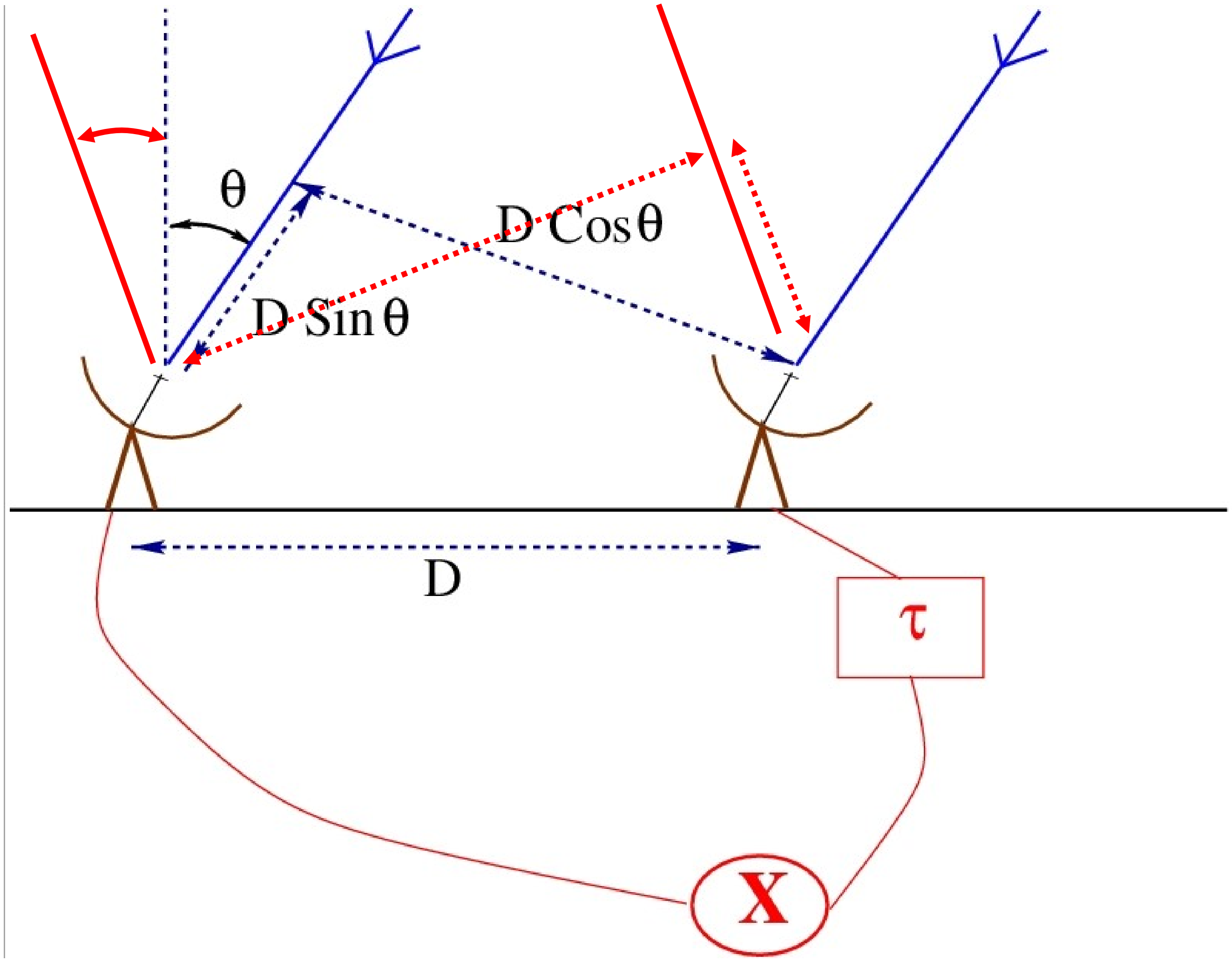
Phase comes from the (sum of) geometric path length, optical fiber, cables and electronic delays

Reciprocity theorem

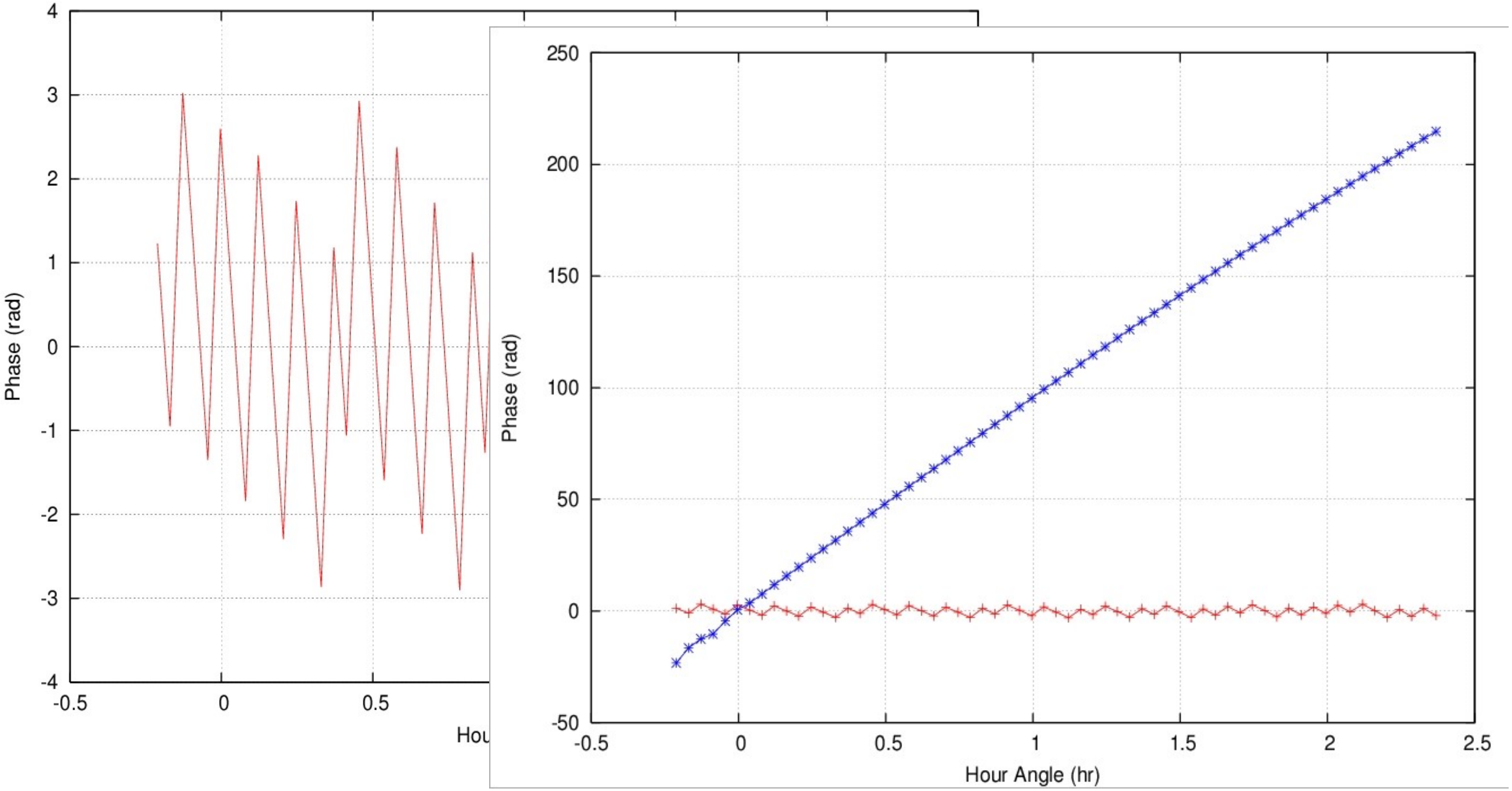
Screen/Detector → Correlator

Sky response of an individual baseline




θ
D Cos θ
D Sin θ
D
τ
X

Real life fringes



Sun @ 125 MHz, 26 Apr, 2005, Mileura, Western Australia

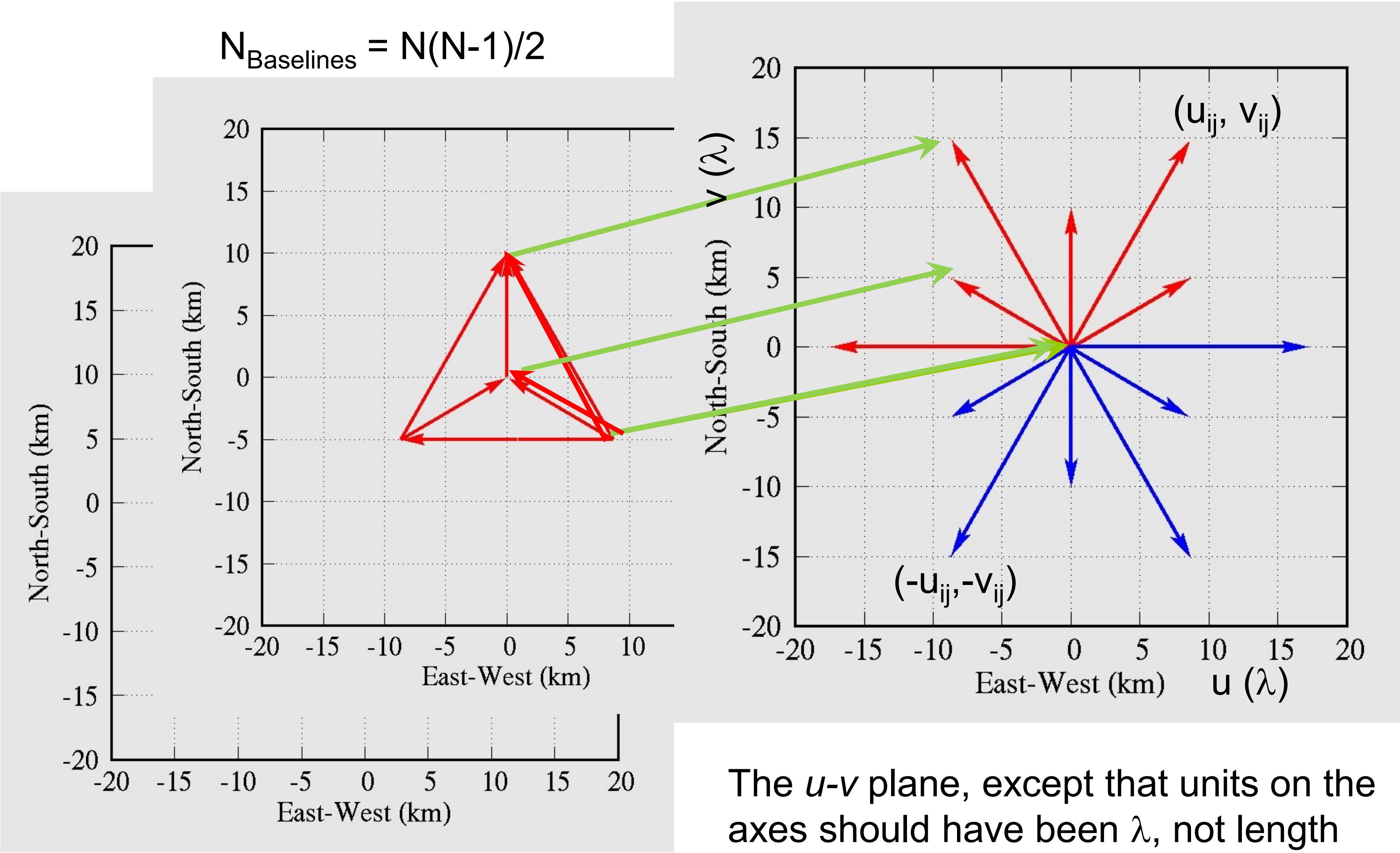
Murchison Widefield Array – Early Deployment effort, phase 2

What are these fringes?

- Young's double slit
  - Fringes are a function of position
  - Constant in time
- Astronomical fringes
  - Arise because the relative motion between the astronomical source and the interferometer changes the effective baseline (D Cosθ)
  - For a given baseline, function of time and direction
  - Assumption: source does not change during the course of the observation
  - *Fringestop – Usually this geometric phase is corrected for in the data, and you do not get to see it.*

Baselines and *u-v plane*

$N_{Baselines} = N(N-1)/2$



The *u-v* plane, except that units on the axes should have been λ, not length

Visibility V(u,v)

- The fundamental Radio Astronomy measurable

  $V_{ij}(u,v,t,\Delta t,\nu_0,\Delta\nu) = \langle V_i(\ldots) \times V_j^*(\ldots,t+\tau,\ldots)\rangle$

- van Cittert Zernike Theorem

  V(u,v) is 2D Fourier Transform of the sky Brightness distribution $B(\theta,\varphi)$

  (T(x,y) in the following slides)

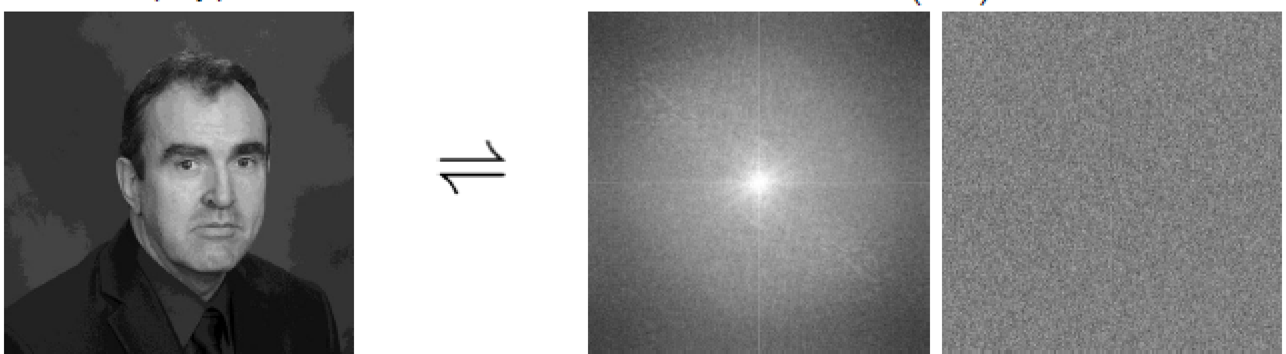
  - Incoherent source,
  - Small field of view
  - Far-field

Visibilities

- each V(u,v) contains information on T(x,y) *everywhere*, not just at a given (x,y) coordinate or within a given subregion

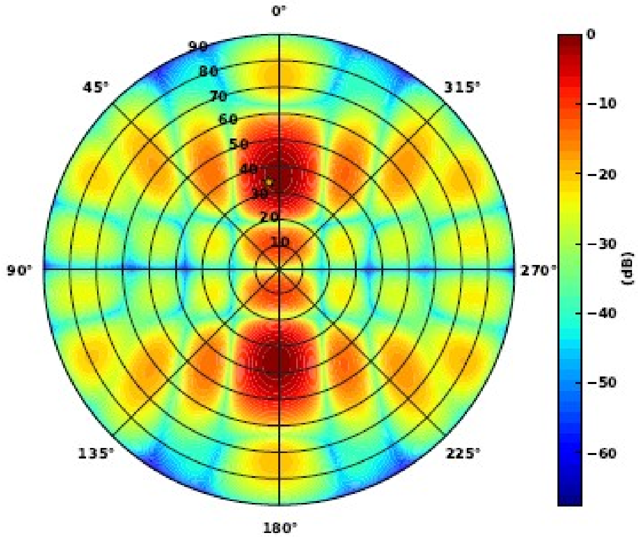
- V(u,v) is a complex quantity
  - visibility expressed as (real, imaginary) or (amplitude, phase)

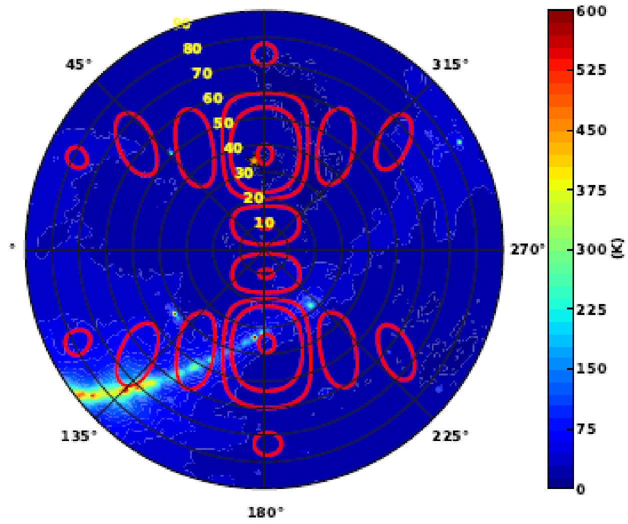
T(x,y) ⇌ V(u,v)

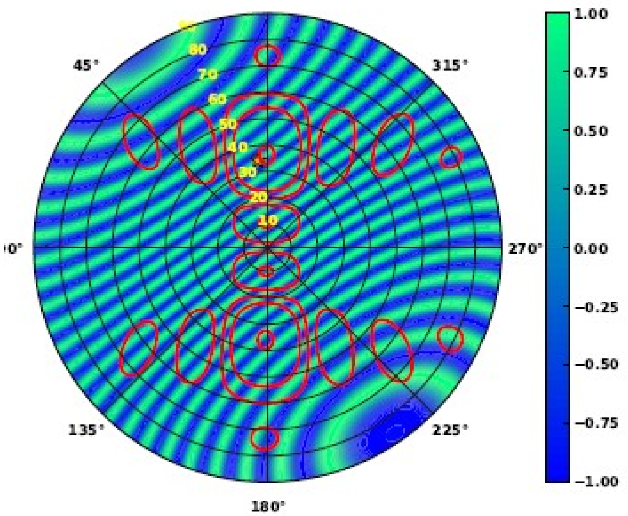


amplitude phase

Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA





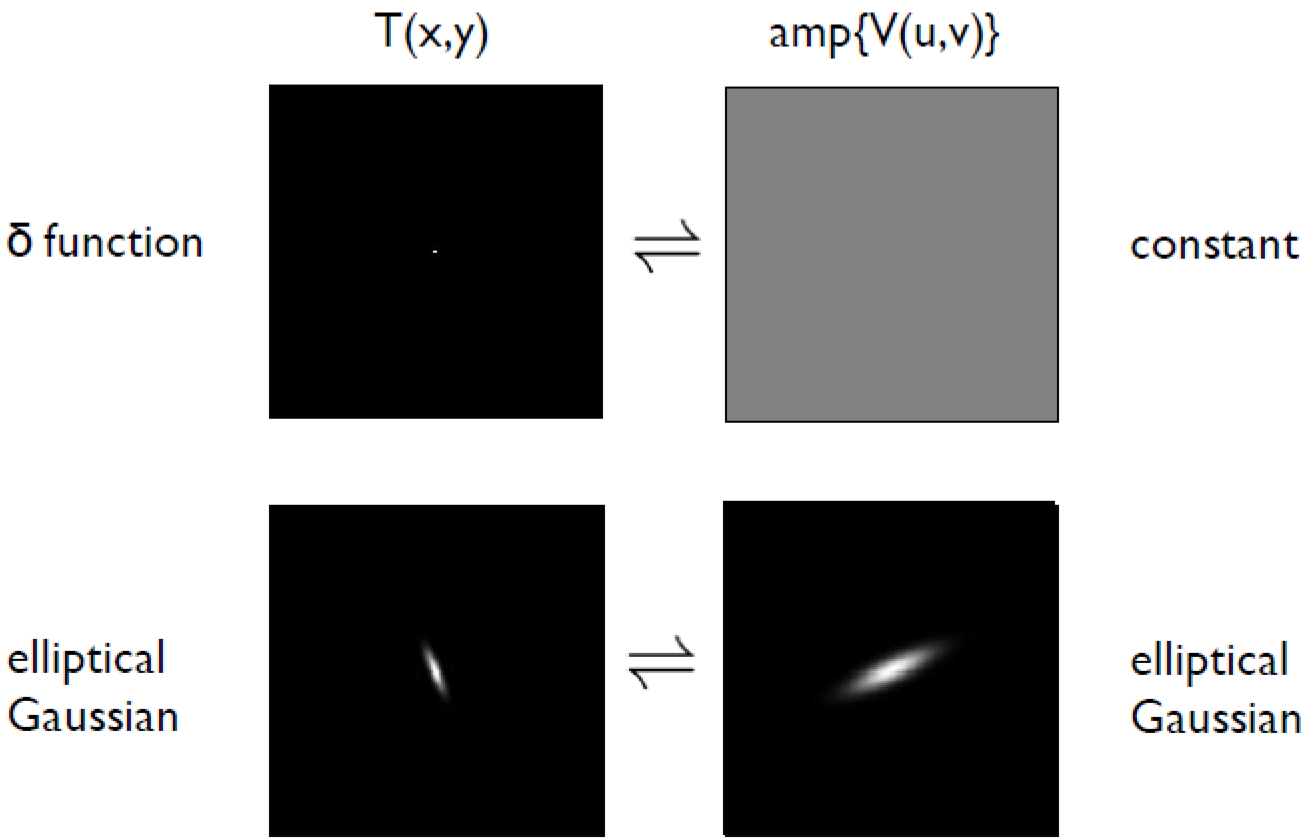


Beam
238 MHz

Sky

Baseline (cosine) fringe
u=7.68, v=6.19

Example 2D Fourier Transform Pairs



narrow features transform into wide features (and vice-versa)

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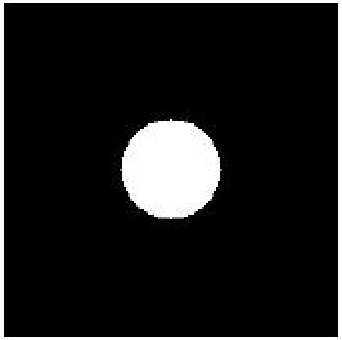
Example 2D Fourier Transform Pairs

T(x,y) amp{V(u,v)}

disk



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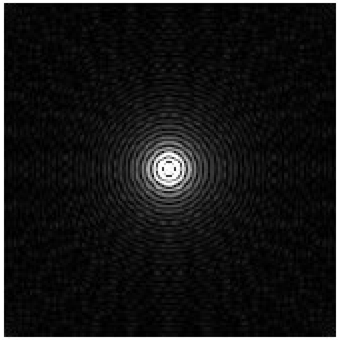
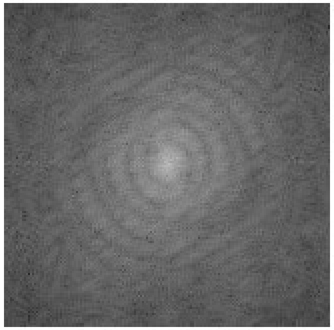


Bessel

⇌

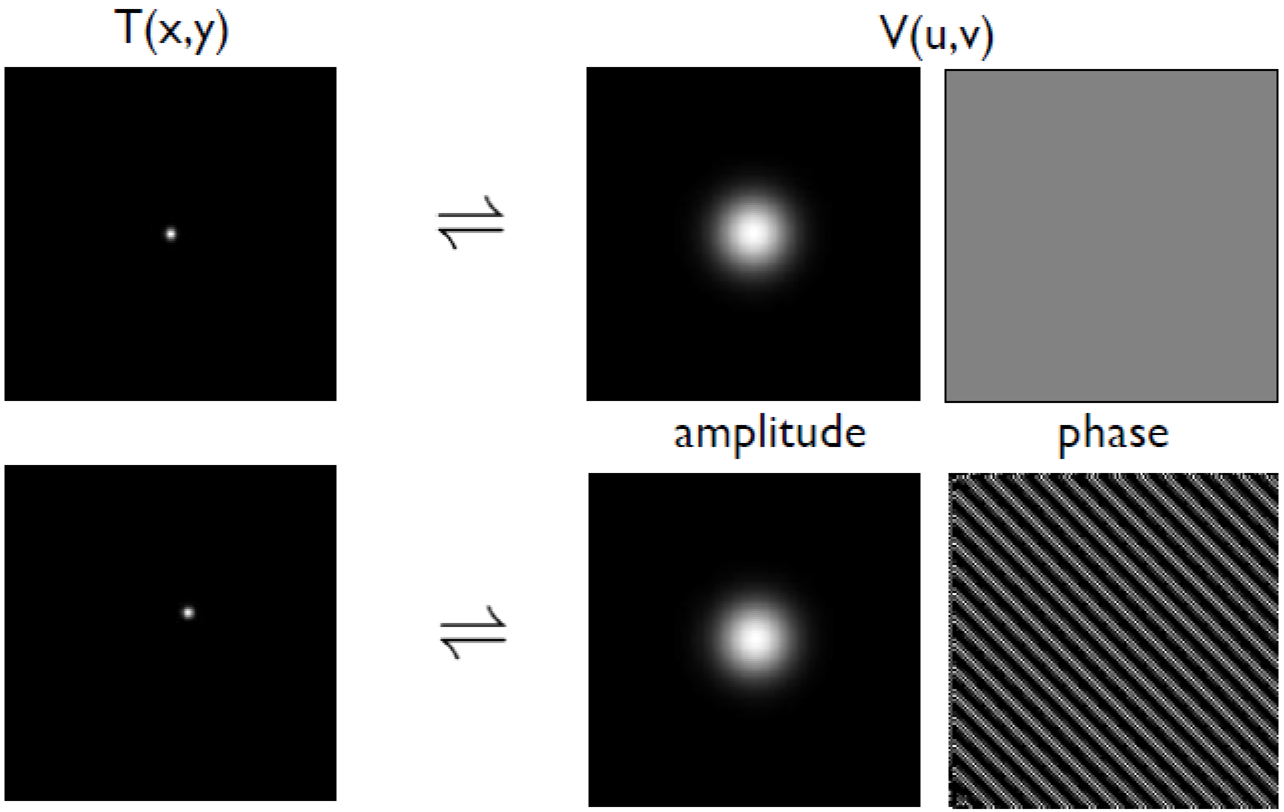


sharp edges result in many high spatial frequencies

Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Amplitude and Phase

- amplitude tells "how much" of a certain spatial frequency
- phase tells "where" this component is located



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The Visibility Concept

$$V(u,v) = \int\int T(x,y)e^{2\pi i(ux+vy)}dxdy$$

- visibility as a function of baseline coordinates (u,v) is the Fourier transform of the sky brightness distribution as a function of the sky coordinates (x,y)

- V(u=0,v=0) is the integral of T(x,y)dxdy = total flux

- since T(x,y) is real, V(u,v) is Hermitian: V(-u,-v) = V*(u,v)
  - get two visibilities for one measurement

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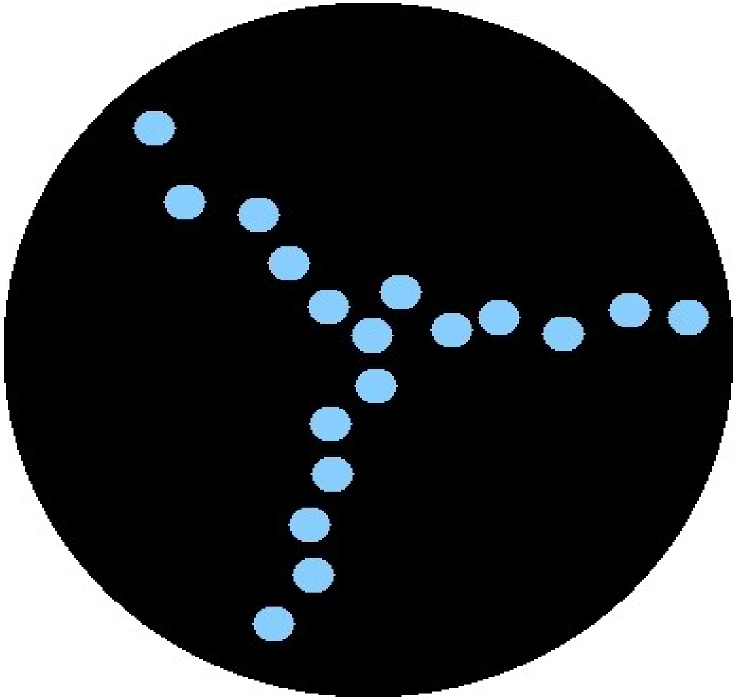
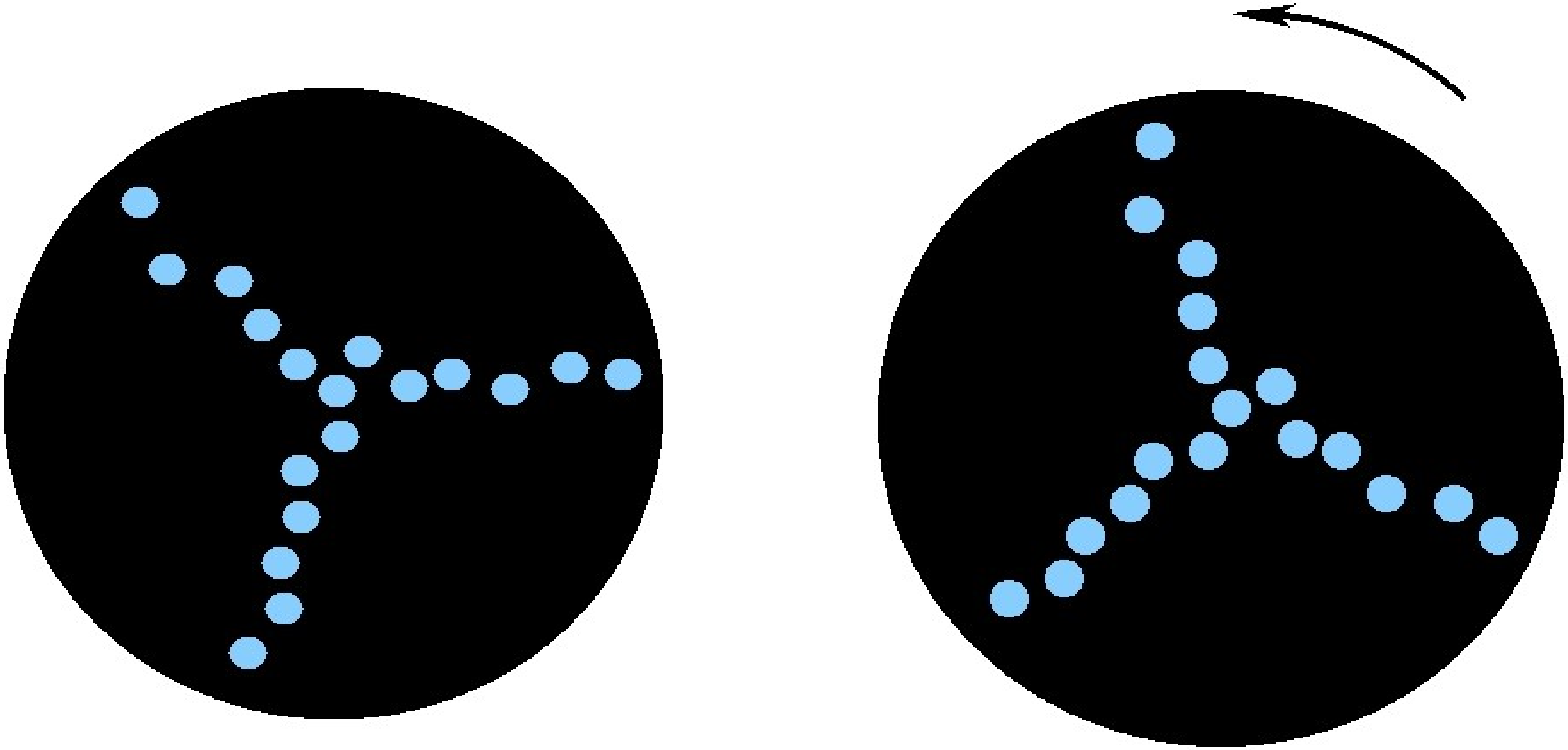
An N element interferometer

- ‘Baselines’ from N elements – N(N-1)/2
- Each of these will lead to a ‘fringe’ with different orientation and spacing
- The final response of the interferometer will be the superposition of fringes from all the baselines



Synthesis imaging



VLA - 27 antennas ⇒ 351 baselines

GMRT - 30 antennas ⇒ 435 baselines

MWA – 128 elements ⇒ 8,128 baselines

The mathematical basis

- Brightness distribution in the sky is Fourier transform of the Visibilities

  $B(\theta,\varphi) \leftrightarrow V(u,v)$

  V(u,v) – The quantity measured by a baseline (amplitude, phase / real, imaginary)

- In the uv-plane, we measure visibilities only at a few places i.e. we have a sampling function

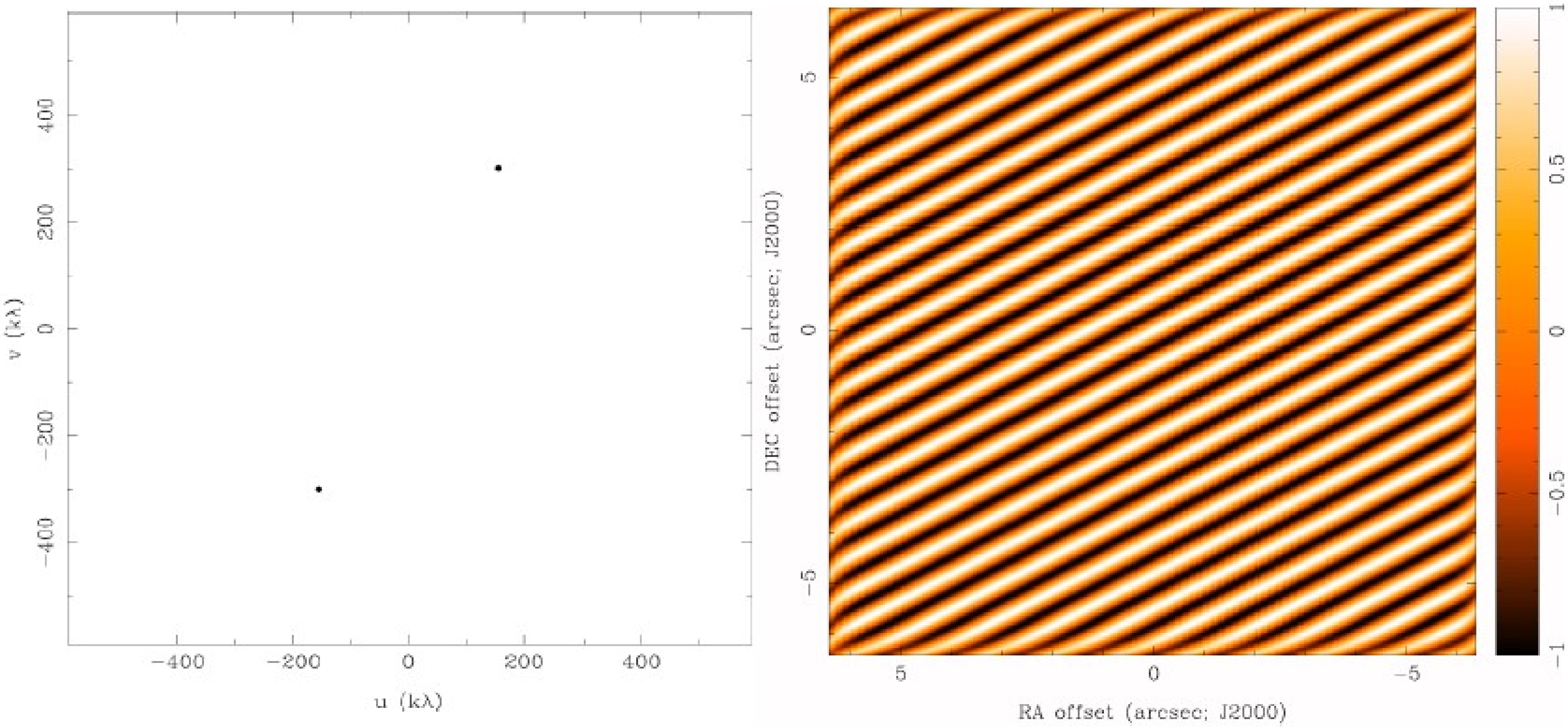
  $S(u,v) = \Sigma_k (u_k, v_k)$

- Point source response of an interferometer (PSF) is Fourier transform of S(u,v)

  $P(\theta,\varphi) \leftrightarrow S(u,v)$

Dirty Beam Shape and N Antennas

2 Antennas



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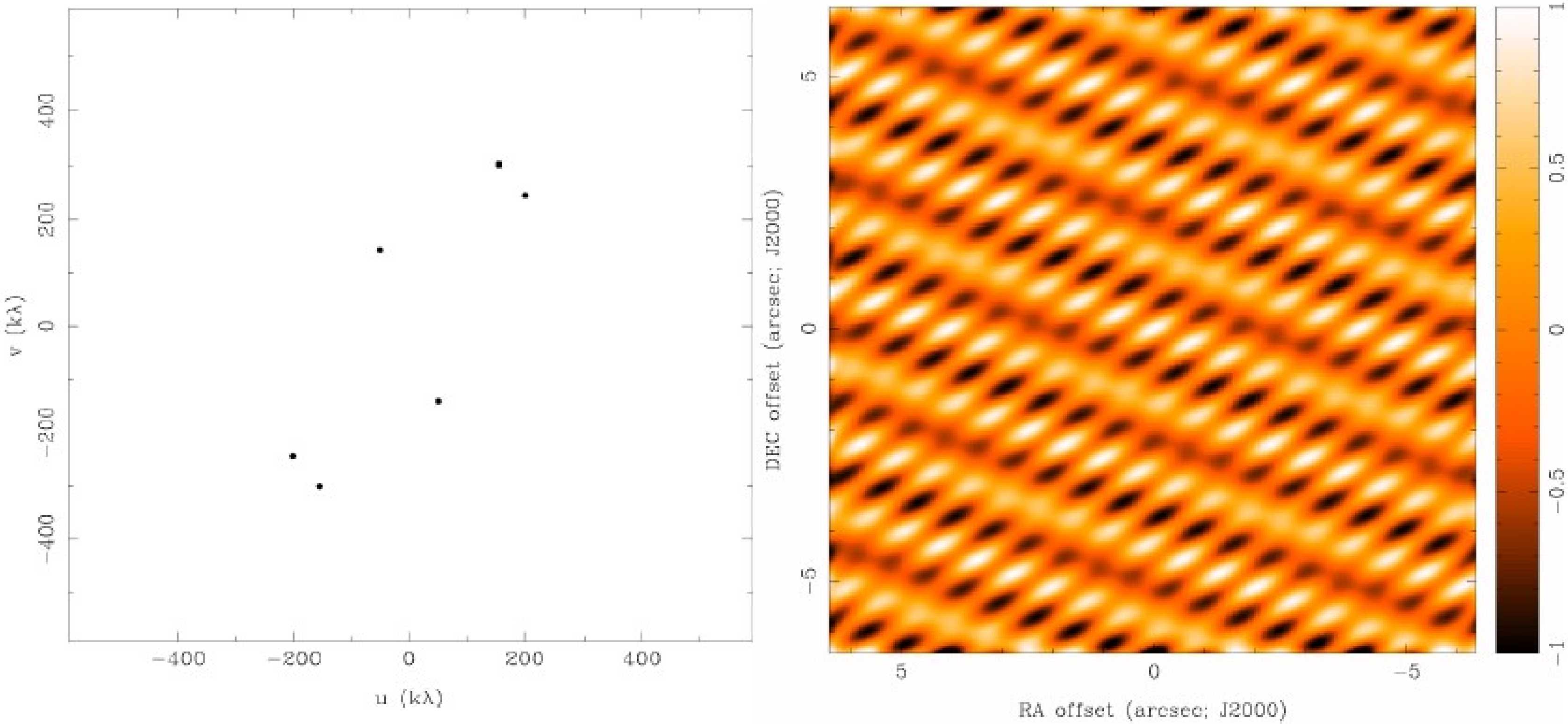
Dirty Beam Shape and N Antennas

3 Antennas



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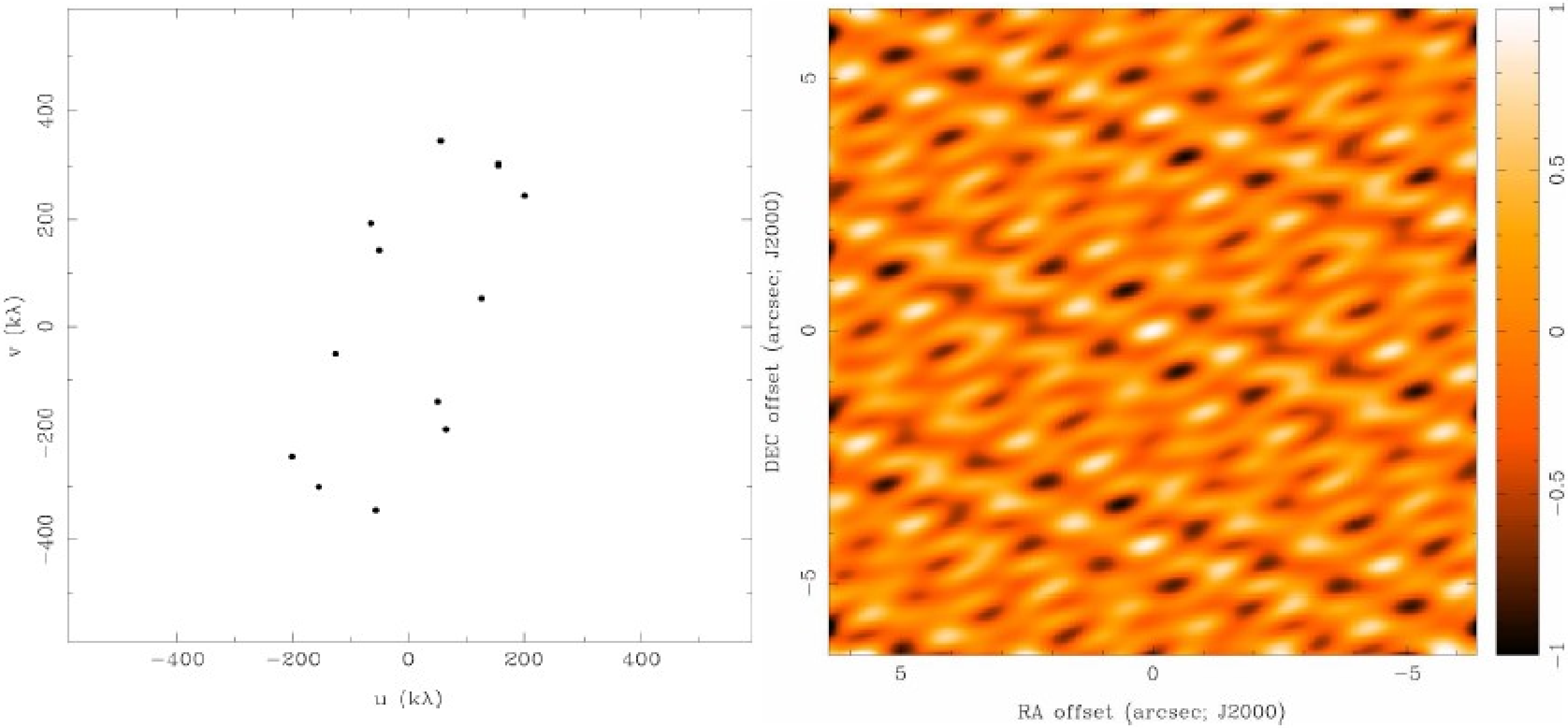
Dirty Beam Shape and N Antennas

4 Antennas



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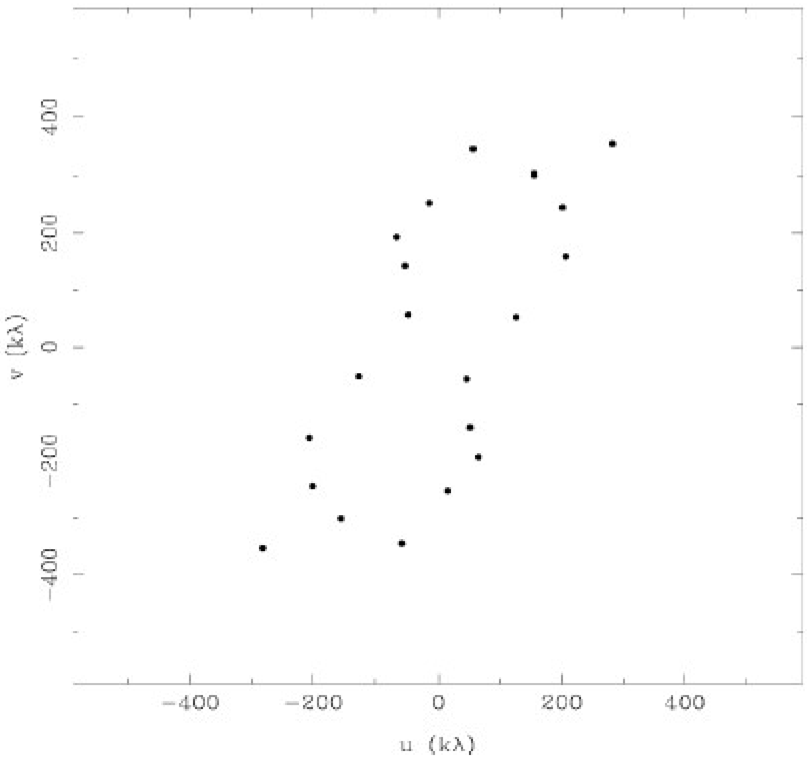
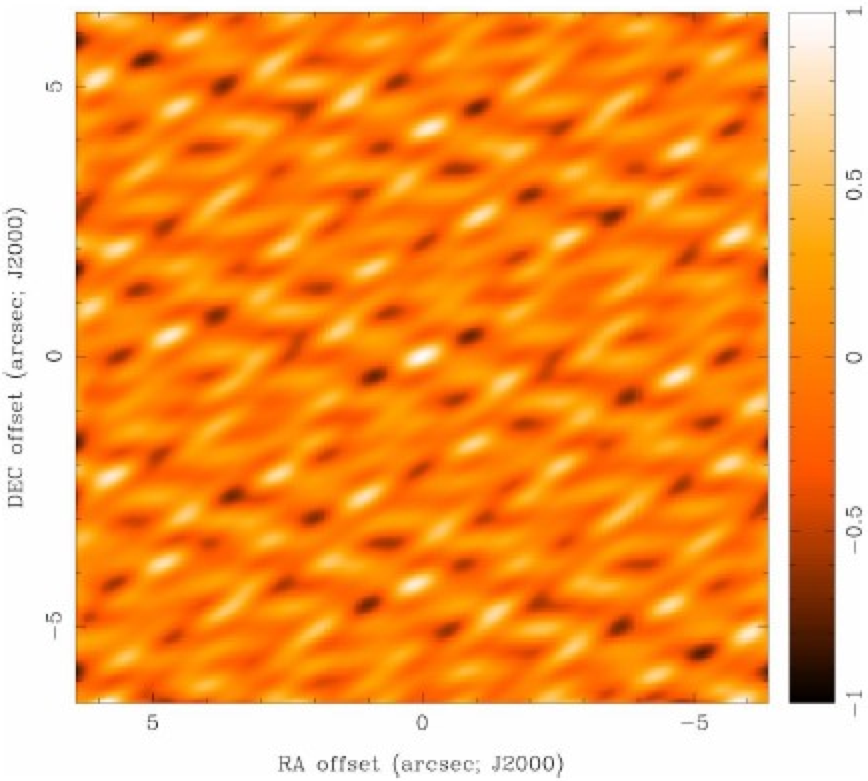
Dirty Beam Shape and N Antennas

5 Antennas





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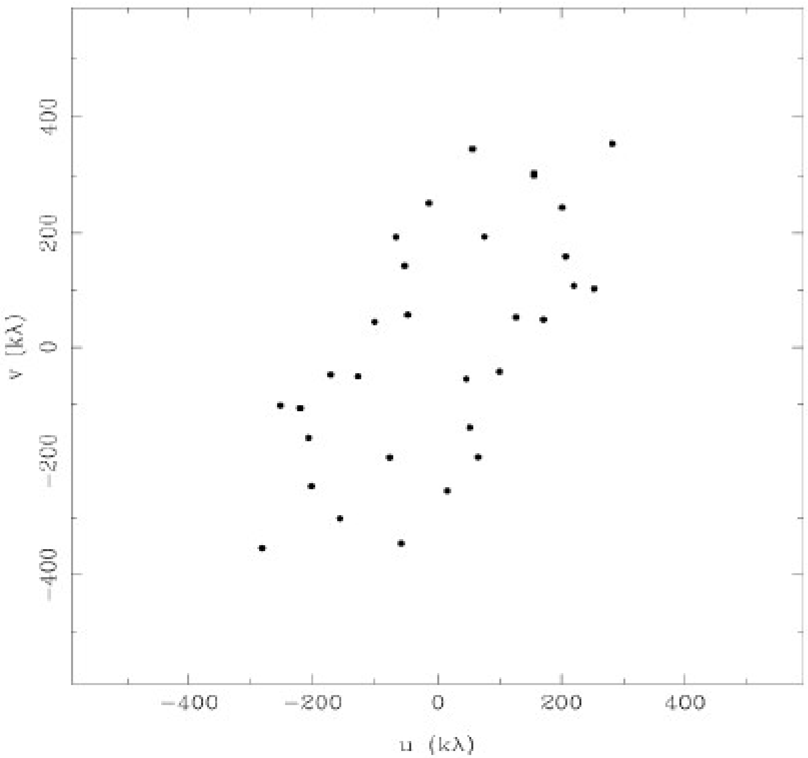
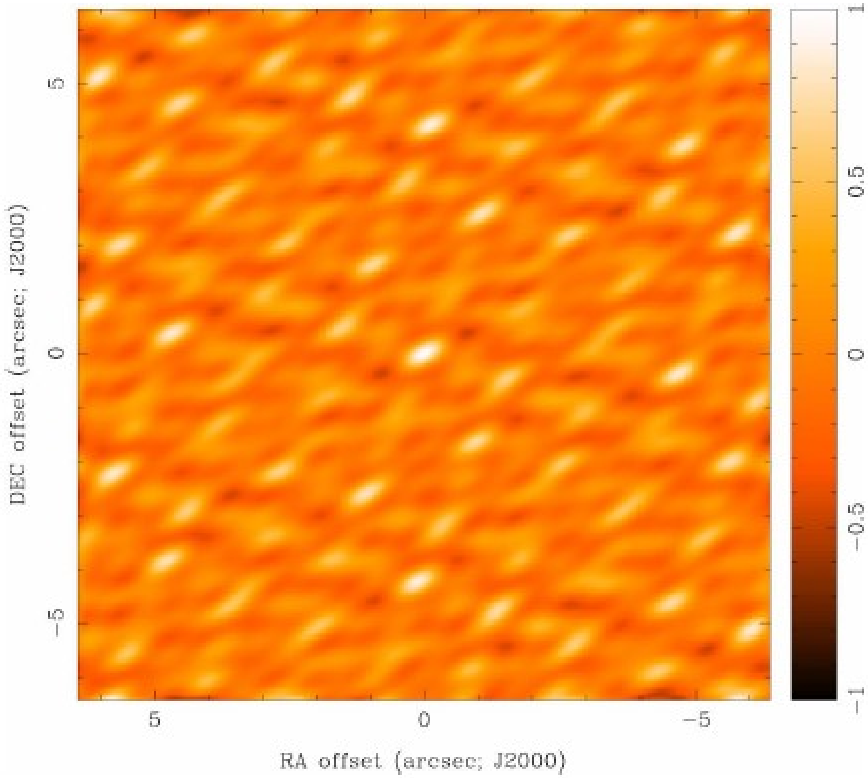
Dirty Beam Shape and N Antennas

6 Antennas





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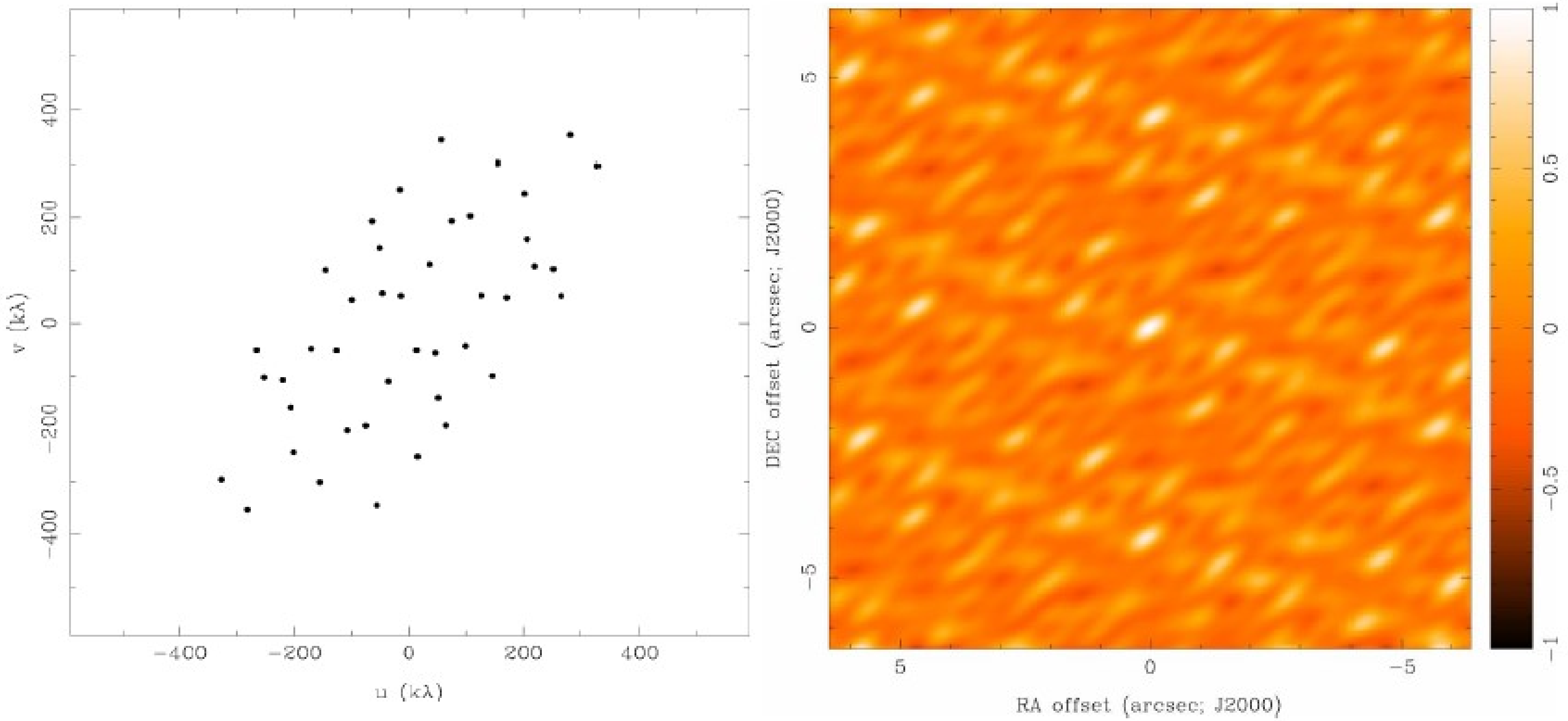
Dirty Beam Shape and N Antennas

7 Antennas



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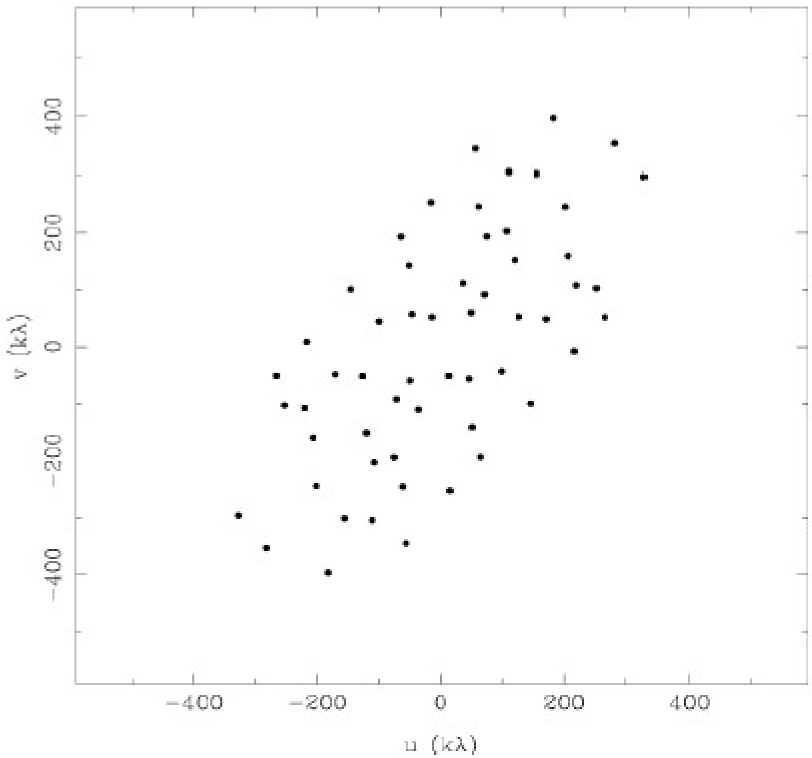
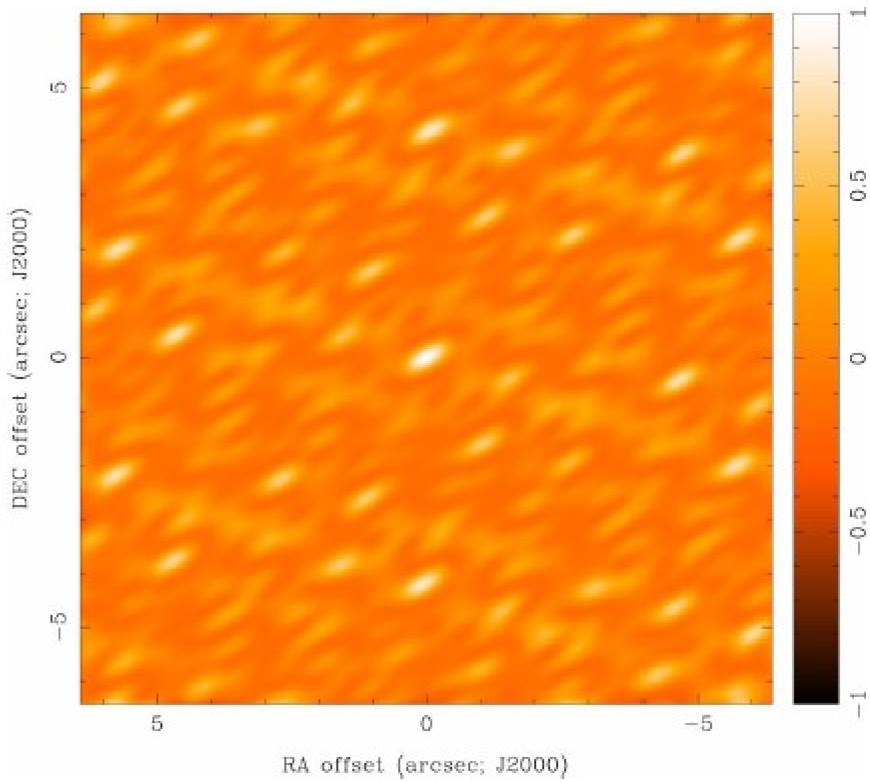
Dirty Beam Shape and N Antennas

8 Antennas





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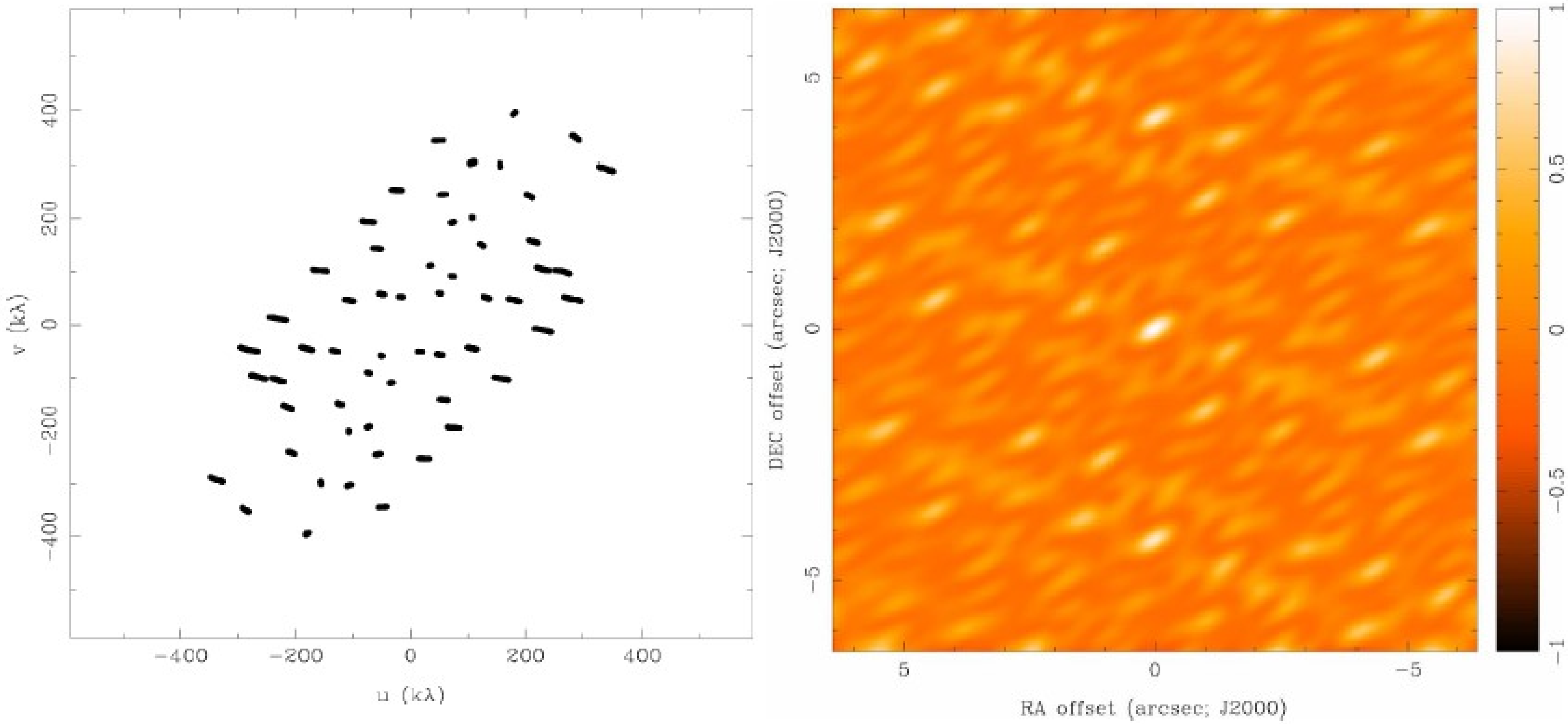
Dirty Beam Shape and N Antennas

8 Antennas x 30 samples



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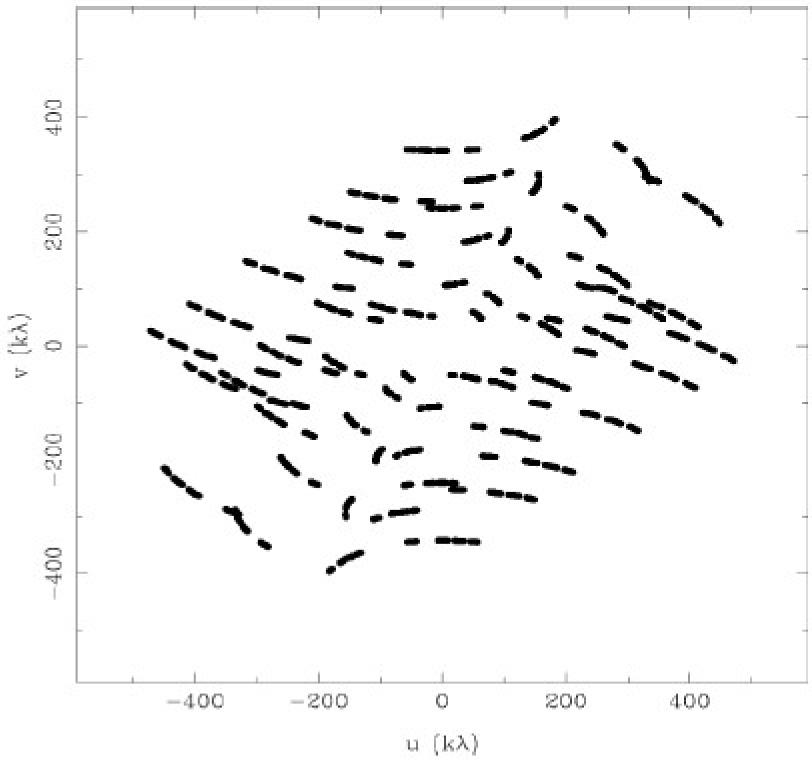
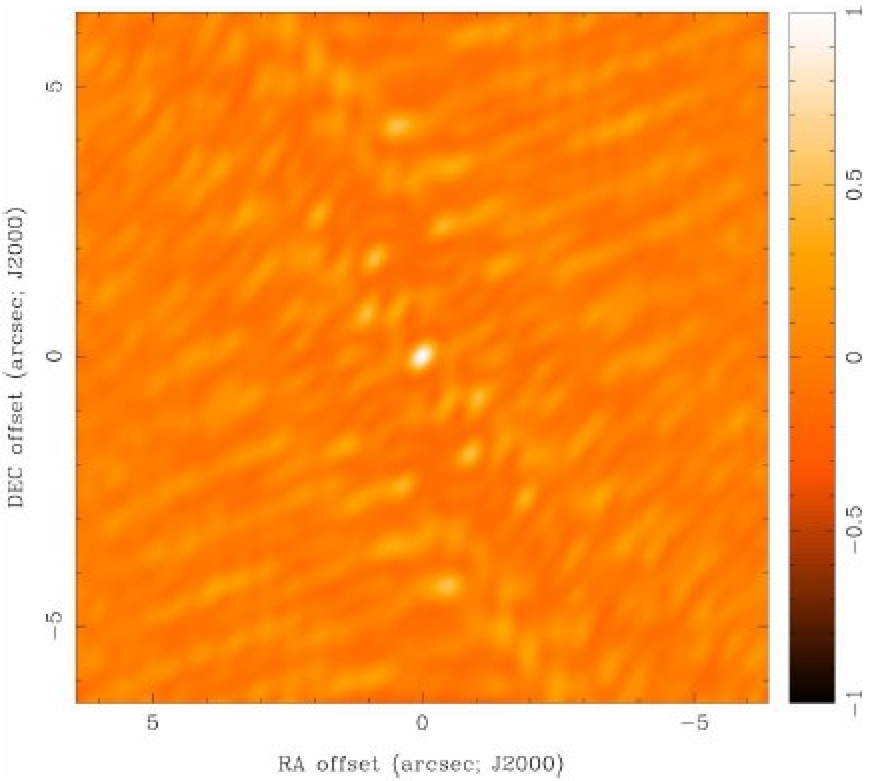
Dirty Beam Shape and N Antennas

8 Antennas x 120 samples





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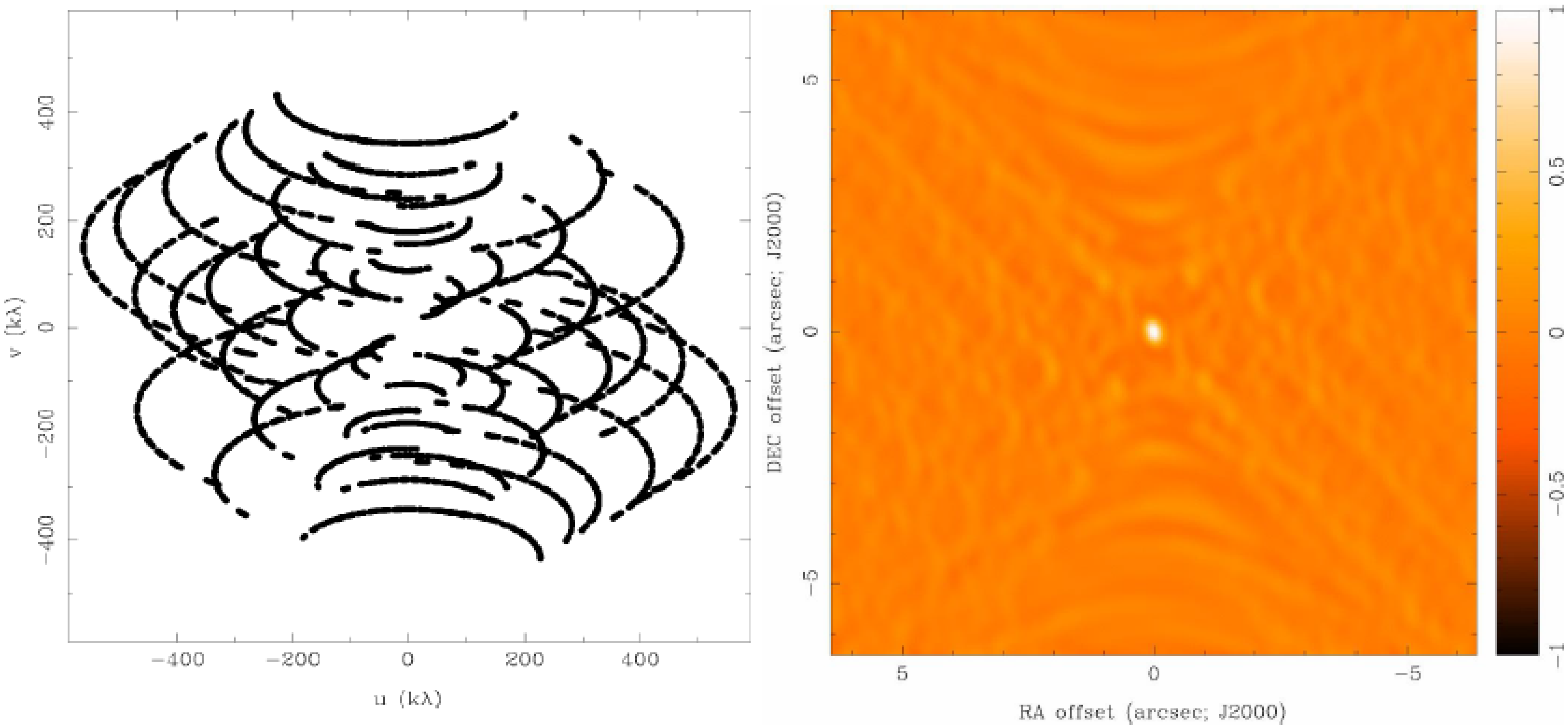
Dirty Beam Shape and N Antennas

8 Antennas x 480 samples



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So what do we finally have?

- $B^{True}(\theta,\varphi)$ = FT V(u,v)
- But we have measurements only at S(u,v)
- $B^{Obs}(\theta,\varphi)$ = FT(S(u,v) x V(u,v))
- Also PSF(θ,φ) = FT S(u,v)
- So from convolution theorem

$$B^{Obs}(\theta,\varphi) = PSF(\theta,\varphi) \otimes B^{True}(\theta,\varphi)$$

$\otimes$ - convolution

The FT of sampled visibilities gives the True sky Brightness distribution convolved with the Point Spread Function.

'***Dirty image***' is True image convolved with the '***Dirty beam***'.