Astronomical Techniques II
Lecture 9 - Calibration and Imaging

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Calibration Framework

1. $\tilde{V}_{i,j}(t) = \mathcal{G}_{i,j}(t) \, V_{i,j}(t) + \epsilon_{i,j}(t) + \eta_{i,j}(t)$

   1. $\mathcal{G}_{i,j}(t)$ - baseline based complex gain
   2. $\epsilon_{i,j}(t)$ - baseline based complex offset
   3. $\eta_{i,j}(t)$ - Gaussian random complex noise

Antenna based calibration

1. $\mathcal{G}_{i,j}(t) = g_i(t)\, g_j^*(t) = a_i(t)a_j(t)e^{i(\phi_i(t)-\phi_j(t))}$

2. No. of constraints $\sim 2 \times N(N-1)/2$
3. No. of independent DoF $\sim 2N$
4. Vastly over determined problem

5. Assumptions
   1. All/most Data corruption happens before correlation
   2. *Linear* regime $\implies$ $g_i(t)$s are independent of $V_{i,j}(t)$
   3. One $g_i(t)$ specifies an antenna $\implies$ *direction independent*
   4. Availability of a *perfect* calibrator

Linearity of GMRT 327 MHz front-end
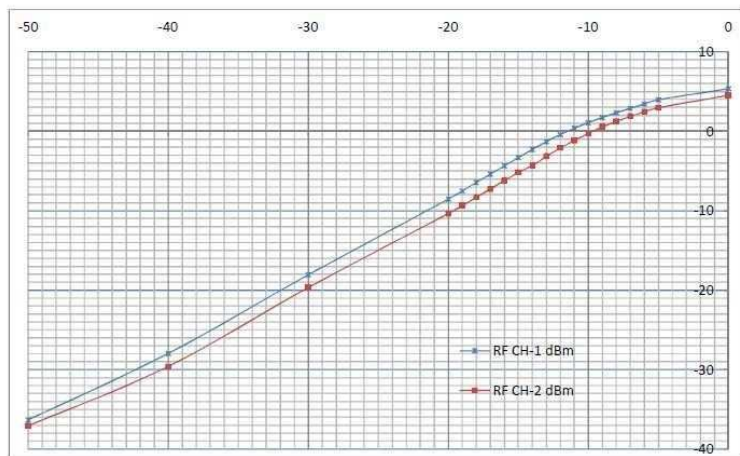
Delay Calibration

1. $V_{i,j}(t) =$
$$\int_0^\infty \left(\int_{-\infty}^\infty \int_{-\infty}^\infty A_\nu(l,m) \, B_\nu(l,m) e^{-2\pi i \nu \tau_g} \, dl \, dm \right)$$
$$e^{2\pi i \nu \Delta \tau_r} \mathcal{G}_{i,j}(t,\nu) \, d\nu$$

2. Residual delay gives rise to a phase ramp
$\Delta\phi = 2\pi\Delta\nu(\tau_g - \Delta\tau_r)$

3. $\Delta\phi$ has contributions from
   1. the geometric delay (can be corrected precisely only one direction - the *phase center*)
   2. the *fixed delay* - due to cables and electronics etc.

Time and Antenna locations

1. $\phi_g = 2\pi\nu\tau_g = 2\pi w =$
   $\dfrac{2\pi}{\lambda} \left(L_x\, cosH\, cos\delta - L_y\, sinH\, cos\delta + L_z\, sin\delta \right)$

2. $\Delta\phi_g = \dfrac{2\pi}{\lambda}(\Delta L_x\, cosH\, cos\delta - \Delta L_y sinH\, cos\delta + \Delta L_z\, sin\delta$
   $+\Delta\alpha\, cos\delta(L_x\, sinH + L_y\, cosH)$
   $+\Delta\delta(-L_x\, cosH\, sin\delta + L_y\, sinH\, sin\delta + L_z\, cos\delta))$

3. Reduce $\Delta\phi_g(t)$ to (much) less than 1 radian over a characteristic time scale

Time scales

1. Time scale for calibration
   1. $<$ than the time scale over which ϕ_{sys} varies significantly
2. Time scale for integration over $V_{i,j}(t)$
   1. $<$ than the time scale over which ϕ_{source} varies significantly
   2. Includes contributions from propagation effects

Propagation effects due to the atmosphere

1. Ionosphere at low frequencies, and Troposphere at high frequencies
2. Absorption (Emission) by the medium - attenuation (distortion) in amplitude
3. Distortion of the phase
4. Variability $\mathcal{P}(x, y, \theta, \phi, t)$
   1. Large scale changes (e.g. diurnal variation)
   2. Small scale changes (e.g. turbulence)
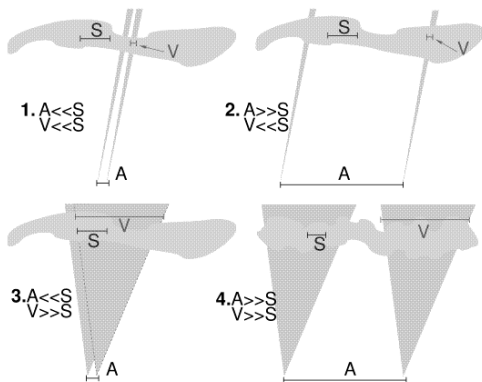
Ionospheric Calibration regimes

Figure 1. Illustration of the 4 ionospheric calibration regimes into which a low-frequency array can fall. The quantities A, V and S are, respectively, the array size on the ground, the field of view at ionospheric heights, and the scale size of ionospheric irregularities. Regimes 1 and 2 (top), featuring narrow fields of view, are readily dealt with using conventional self-calibration. Regime 3 can be handled by modelling the distortions as position-dependent refractive shifts, a technique currently in use for VLA 74 MHz data reduction. Regime 4, which will be inhabited by the LWA, requires a new approach to calibration, which in turn demands careful attention to array design and configuration.

Direction Dependent Effects

1. Instrumental - Departures of individual primary beams from the reference model
   1. Mechanical deficiencies
   2. Residual pointing offsets
   3. usually stable or change in a predictable manner
2. Natural - Effects of propagation through the atmosphere - ionosphere, troposphere ($\mathcal{F}(x, y, \theta, \phi, t)$)
3. Image plane effects

# Direction Dependent Effects...

1. Taking DDEs into account
   1. Account for differences between beams of different antennas
   2. Need to be taken into account on a per visibility basis
   3. Current approach - perturbation theory based
   4. Enromous increase in computational complexity
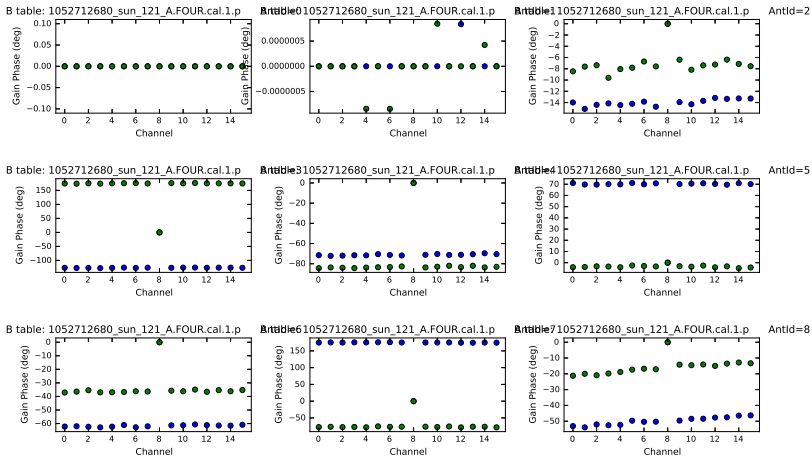   5. Possible in research labs, but not in practise yet

Bandpass calibration

1. Delay calibration - removes any signal path (delay) differences across individual signal paths - pure phase ramps
2. Take into account changes in antenna gain with frequency
$\mathcal{G}_{i,j}(\nu) = g_i(\nu)\, g_j^*(\nu)$

Bandpass Amplitudes
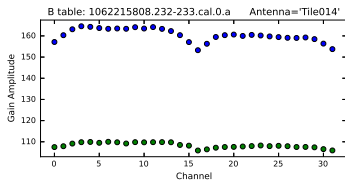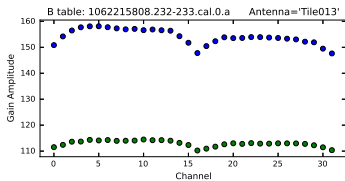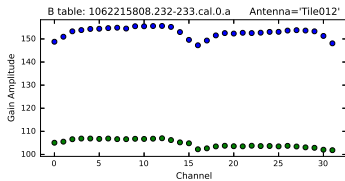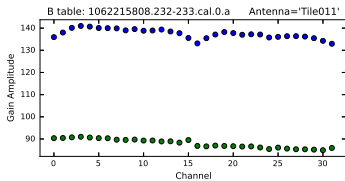
Bandpass Phases

Calibration Model

1. $g_i(\nu, t) = g_i(\nu)\, g_i(t)$
2. $g_i(\nu)$ - **Bandpass** varies slowly - needs to be calibrated infrequently - typically at the start and end of an observing run
3. $g_i(t)$ - **Gain** varies faster - needs to be calibrated more frequently - typically every 30 min to an hour
   1. Flux calibration (amplitude) - stable and usually done at the start and end of an observation
   2. Phase calibration - variable,

Test signals in the sky

1. Accurately known positions in the sky
2. Not significantly variable
3. Known and simple spectra
4. Lie in comparatively 'empty' fields, no strong 'confusing' sources nearby
5. Firm prediction of $V_{i,j}(\nu, t)$

1. Flux calibration
    1. Stable, regularly monitored fluxes with accurate radiometers
    2. Unresolved, or a good model for the source
    3. Usually quite a strong source
    4. Primary flux calibrators - 3C48, 3C147, 3C286 and 3C295
2. Bandpass calibration
    1. Strong source
    2. Source structure is less important
3. Phase calibration
    1. Unresolved source close to the target source

Closure Quantities - How good is your calibration?

1. $\mathcal{G}_{i,j}(t) = g_i(t)\, g_j^*(t)\, g_{i,j}(t)$

   1. $\mathcal{A}_{i,j}(t) = a_i(t)\, a_j(t)\, a_{i,j}(t)$

   2. $\tilde{\phi}_{i,j}(t) = \phi_i(t) - \phi_j(t) \, + \phi_{i,j}(t)$

2. Closure Amplitude (for a point source of flux S)
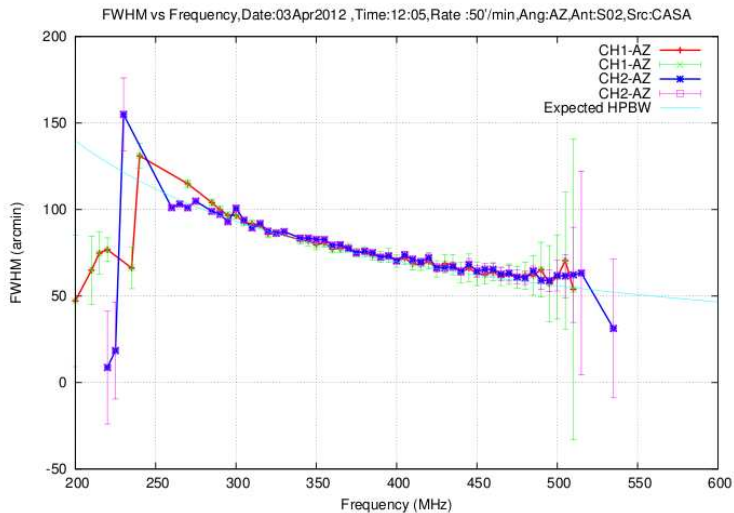   1. $\tilde{A}_{i,j} = a_i\, a_j\, a_{i,j}\, S$
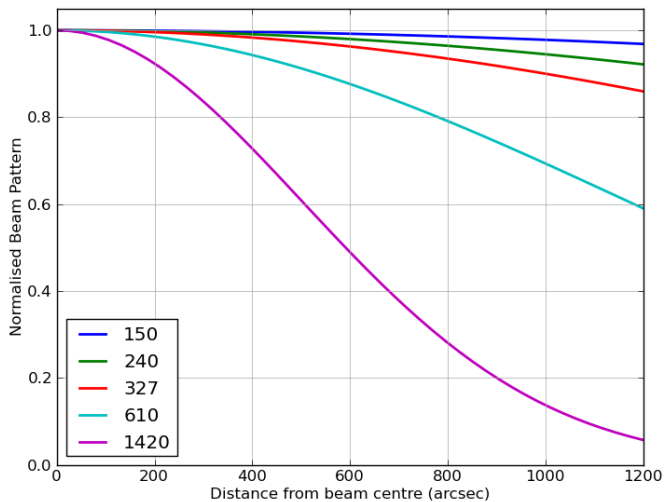   2. $a_{i,j} = \dfrac{\tilde{A}_{i,j}}{a_i\, a_j\, S}$

3. Closure Phase
   1. $\tilde{\phi}_{i,j} = \phi_i - \phi_j + \phi_{i,j}$
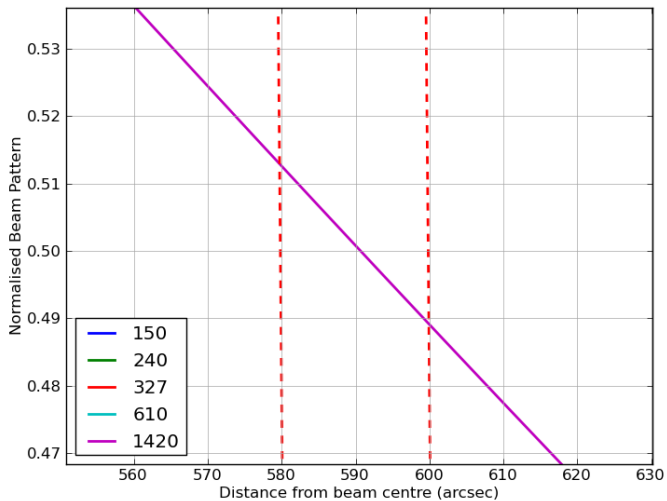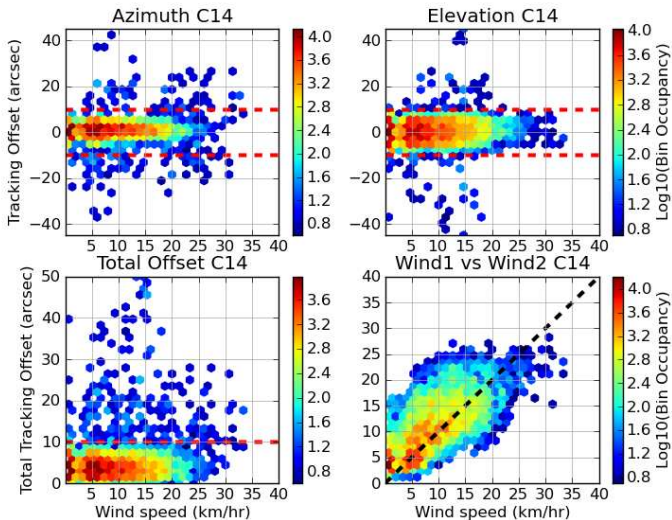   2. $\phi_{i,j}(t) = \tilde{\phi}_{i,j}(t) - \phi_i(t) + \phi_j(t)$

Beamwidth vs Freq.

FWHM vs Frequency,Date:03Apr2012 ,Time:12:05,Rate :50'/min,Ang:AZ,Ant:S02,Src:CASA

Impact of Tracking errors

Impact of Tracking errors

GMRT Tracking errors

Perytons at Parkes - 27 yr old Mircowaves!

Table 1. Properties of the perytons from January 2015

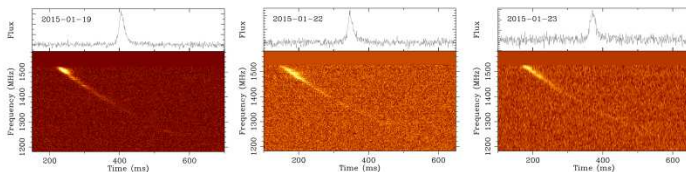| Date (dd-mm-yy)	Time (UTC)	DM (pc cm^{-3})	DM error	S/N (beam 01)	Width (ms)	Telescope Azimuth (deg)	Telescope Elevation (deg)
2015-01-19	00:39:05	386.6	1.7	24.8	18.5	10.7	75.3
2015-01-22	00:28:33	413.8	1.1	42.5	18.5	73.9	36.2
2015-01-23	03:48:31	407.4	1.4	10.6	18.5	323.2	40.2

Figure 1. The time-frequency structure of the three January perytons. In the case of events on 2015-01-19 and 2015-01-23 the summed 13-beam data is shown. For 2015-01-22 only beam 01 is plotted as the outer beam data was not recorded to disk.

Petroff et al., 2015