Astronomical Techniques II
Lecture 8 - Correlators and Calibration Framework

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Wiener-Khinchin Relation

- $r(\tau) = V_1(t) \bigstar V_2(t) = lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} V_1(t)\ V_2^{\star}(t-\tau)\ dt$
- $V_1(t) \bigstar V_2(t) = lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} V_1(t)\ V_{2-}^{\star}(\tau - t)\ dt$

  where $V_{2-}(t) = V_2(-t)$
- Now $V_1(t) \rightleftharpoons \hat{V}_1(\nu)$; $V_2(t) \rightleftharpoons \hat{V}_2(\nu)$; $V_{2-}^{\star}(t) \rightleftharpoons \hat{V_2}^{\star}(\nu)$;
- From Convolution theorem

  $V_1(t) \bigstar V_2(t) \rightleftharpoons \hat{V}_1(\nu)\ \hat{V_2}^{\star}(\nu)$
- When $V_2(t) = V_1(t)$, it becomes the Wiener-Khinchin relation

Wiener-Khinchin Relation

- Power (density) spectrum of a signal is the FT of its auto-correlation.
- $|V(\nu)|^2 = \int_{-\infty}^{\infty} r(\tau) e^{-i2\pi\,\nu\tau} d\tau$

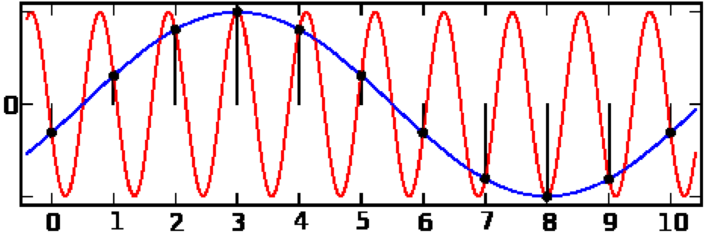
Correlators

- Devices to measure the *mutual coherence function*
- Measuring the cross-correlation function of voltage signals from each of the antennas
- Digital correlators - require sampling and quantization

Sampling

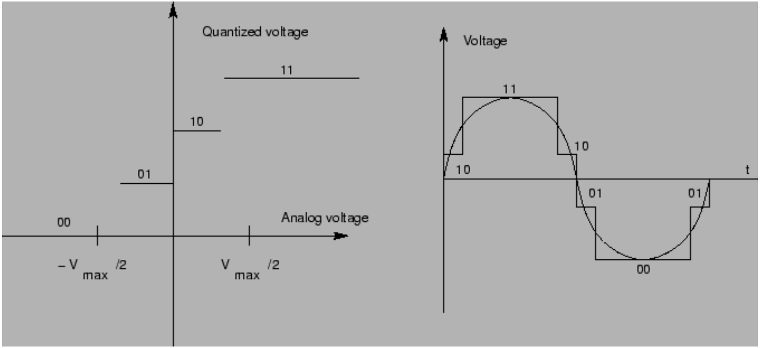
- Band limited signal - information limited to a finite bandwidth $\Delta\nu$
- Baseband signal - Mixed down RF signal such that it lies between 0 and $\Delta\nu$
- Minimum sampling frequency $= 2\Delta\nu$ (Nyquist criterion)
- Undersampling and Oversampling
- Aliasing

Aliasing



- $\nu_{blue} = 0.1Hz$; $\nu_{red} = 0.9Hz$; $\nu_{sampling} = 1Hz$
- Samples indistinguishable from a signal at frequency $\nu - N \times \nu_{sampling}$, where N is an integer
- $N \neq 0$ - images or aliases of ν
- Nyquist sampling (sampling at $2\Delta\nu$) prevents aliasing

Quantization



1. $V_{Error} = V_{True} - V_{Quantized}$

Quantization

1. Quantization distorts both the amplitude and the spectrum of the input signal
2. Spectrum of quantized signal extends beyond the original $\Delta\nu$ of V_{True}, implies aliasing
3. Largest value which can be expressed (within the error of $\pm q/2$) depends on the no. of bits (M) is $q(2^M - 1)$
4. $V_{Max} = 4.42\sigma$, probability of exceeding V_{Max} is 10^{-5}.

Correlator...

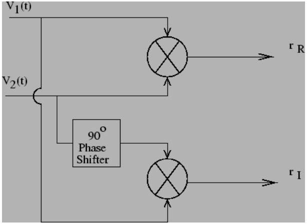
1. Dynamic Range - minimum change in signal which can be expressed is q
2. Discreet Fourier Transform (vs. Continuous Fourier Transform)
   1. Windowing, Sampling and Filtering
3. Digital delays
   1. Discreet delays in units of sampling time
   2. A delay of τ correponds to a $\phi = 2\pi\nu\tau$. So delays smaller than the sampling time are corrected by applying phase gradients to the sampled data.

Correlators...

1. $R(m) = \frac{1}{N}\sum_{n=0}^{N-1} v_1(n)v_2(n+m) \;\; 0 \leq m \leq M$
2. Correlation measured by a digital correlator differs from that measured by an ideal device with infinite precision ($R_c(m)$).
3. Deviation depends on the value of correlation and the number of correlator bits - Van Vleck Correction
4. Monotonic and approx linear for small correlation values, linearity imroves with the number of bits.

Correlators...

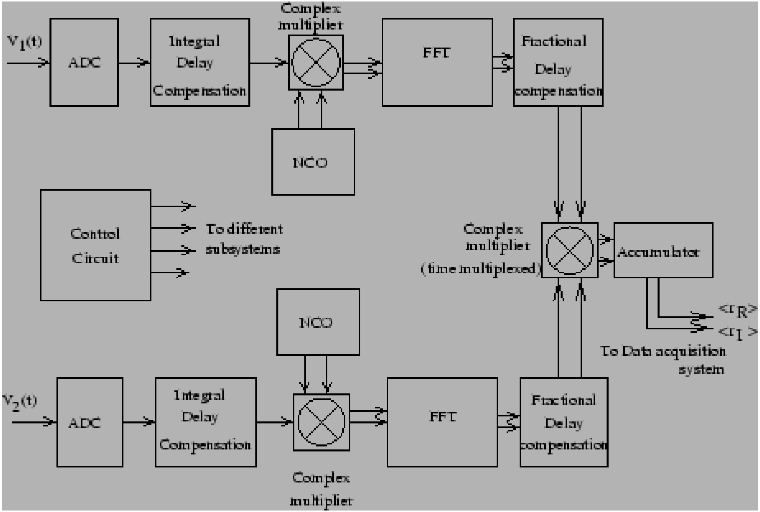
1. $r_R(\tau_g) = Re[v_1(\nu, t)\ v_2^*(\nu, t)] = |\mathcal{V}|\ cos(2\pi\nu\tau_g - \phi_{\mathcal{V}})$



1. $r_I(\tau_g) = |\mathcal{V}|\ cos(2\pi\nu\tau_g - \phi_{\mathcal{V}} + \pi/2)$
2. $|\mathcal{V}| = sqrt r_R^2 + r_I^2; \quad \phi_{\mathcal{V}} = tan^{-1}\frac{r_I}{r_R}$

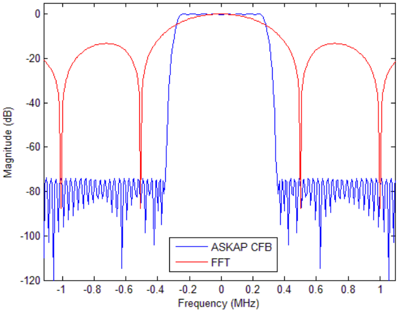
Spectral Correlators

1. FX correlators



Present/New Generation Correlators

1. FFT $\rightarrow$ Polyphase filters
2. Real time sample level statistics and data flagging
3. Multiple modes - time resolution vs spectral resolution



Calibration Framework

1. $V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_\nu(l,m)\, B_\nu(l,m) e^{-2\pi i(ul+vm)} \frac{dl\, dm}{\sqrt{1-l^2-m^2}}$
2. $V_{i,j}(u,v,\mathbf{t},\nu) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_\nu(l,m)\, B_\nu(l,m) e^{-2\pi i(u_{i,j}(t)l+v_{i,j}(t)m)} \frac{dl\, dm}{\sqrt{1-l^2-m^2}}$
3. $r_{i,j}(\vec{D}_\lambda,\ \vec{s_0}) = A_0\ \Delta\nu\ |\mathcal{V}_{i,j}| cos(2\pi \vec{D}_{\lambda,i,j} \cdot \vec{\sigma} - \phi_V)$
4. $\phi_{g,i,j} = 2\pi\nu\tau_{g,i,j} = 2\pi w_{i,j} = \frac{2\pi}{\lambda}(L_{x,i,j}\ cosH\ cos\delta - L_{y,i,j}\ sinH\ cos\delta + L_{z,i,j}\ sin\delta)$
5. Need to know - L_x, L_y, L_z, time, α, δ.

Calibration Framework

1. $\tilde{V}_{i,j}(t) = \mathcal{G}_{i,j}(t)\ V_{i,j}(t) + \epsilon_{i,j}(t) + \eta_{i,j}(t)$
    1. $\mathcal{G}_{i,j}(t)$ - baseline based complex gain
    2. $\epsilon_{i,j}(t)$ - baseline based complex offset
    3. $\eta_{i,j}(t)$ - gaussian random complex noise

Editing and Flagging

1. Getting rid of data *known* to be bad
2. Getting rid of data *ascertained* to be bad

Calibration Methods

1. Direct Calibration
2. Sky based calibration (calibrator sources)
3. Self-calibration

Antenna based calibration

1. $\mathcal{G}_{i,j}(t) = g_i(t)\, g_j^*(t) = a_i(t)a_j(t)e^{i(\phi_i(t)-\phi_j(t))}$
2. No. of constraints $\sim N(N-1)/2$
3. No. of independent DoF $\sim N$
4. Vastly over determined problem

Antenna Pointing and Gain

1. Determining pointing offsets $(\Delta Az, \Delta El)$
2. For each antenna and feed
3. Sources of error ($\sim$ 10" for GMRT)
    1. Tracking errors - Servo system feed back loop
    2. Distorting of the dish due to gravity - Pointing model
    3. Wind buffeting