Astronomical Techniques II
Lecture 2 - Single Dish Astronomy

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Brightness

Assumption

- No absorption, emission, scattering or any other propagation effect along the path, or propagation through empty space
- Geometric optics

Brightness - $B(\theta, \phi, \nu, t)$

- Units - $W\ m^{-2}\ sr^{-1}\ Hz^{-1}$
- AKA Surface Brightness, Specific Intensity or Spectral Radiance
- Conserved along a ray in empty space

Power received at a detector

$$dW = B(\theta, \phi, \nu) \ \ cos\theta dA \ \ d\Omega \ \ d\nu$$

dW - W
$B(\theta, \phi) - W \ m^{-2} \ sr^{-1} \ Hz^{-1}$

Practical quantitative definition

$$B(\theta, \phi, \nu) = \frac{dW}{d\Omega \ cos\theta dA \ \ d\nu}$$

- Intrinsic property of the source
- Independent of the distance from the source (ONLY for a resolved object)
- Can be thought of as energy *received* at the detector OR as energy *emitted* by the source.

Total Intensity

Total Intensity - Specific Intensity integrated over frequency

Conservation of Brightness applies here as well

Example: Looking through a telescope

Flux Density, S_ν

- Total spectral power received from a source by a detector of unit area.
- $S_\nu = \int_{Source} B(\theta, \phi, \nu)\ cos\theta\ d\Omega$
- For a source with a well defined solid angle
- Unit - $W\ m^{-2}\ Hz^{-1}$
- 1 $Jansky\ (Jy) = 10^{-26}\ W\ m^{-2}\ Hz^{-1}$

Flux Density, S_ν

- Not an intrinsic property of the source - dependent on the distance to the source
- The $cos\theta$ is $\sim$1.0 if angular size $<<$ 1 rad
- Useful for compact (unresolved) sources

Luminosity

Spectral Luminosity

- Total power radiated by the source per unit bandwidth at ν
- $L_\nu = 4\pi \, d^2 \, S_\nu$
- Property of the source
- Involves d, the distance to the source!

Bolometric Luminosity

- Total power radiated by the source integrated over the entire spectrum
- $L_{bol} = \int_0^\infty L_\nu \, d\nu$

A Quick Application

Assume the Sun to be blackbody at 5800 K. What is the specific intensity of the Sun at $\nu = 10 GHz$? What is the flux density of the Sun measured at Earth

1. Verify if Rayleigh Jeans law is applicable
2. Use it to compute B_ν
3. To get S_ν, compute the angular size of the Sun. Assume the Sun to be a disc of uniform *Brightness* and integrate over it.

How will B_ν and S_ν change if they are measured from Mars, rather than the Earth?

Submit your solution in the next class!

Gain of an Antenna, $G(\theta, \phi)$

$$G(\theta,\phi) = \frac{\text{Power transmitted towards } (\theta,\phi) \text{ (per unit solid angle)}}{\text{Power transmitted by an isotropic antenna (per unit solid angle)}}$$

- Dimensionless
- Measure of how *directional* an antenna is
  - Gain of an isotropic antenna is 1.0
- Usually expressed in dB, i.e. $G(dB) = 10 \times log_{10} G$
- For a lossless antenna, same as the *Directivity* as well.

Gain of an Antenna, $G(\theta, \phi)$

- Conservation of Energy (for a lossless antenna)

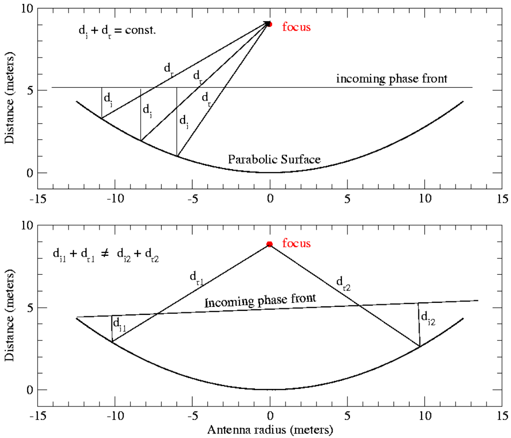
$$\implies < G > = \frac{\int_{sphere} G(\theta, \phi)\, d\Omega}{\int_{sphere} d\Omega} = 1$$

- $\int_{sphere} d\Omega = 4\pi$

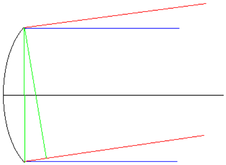
$$\implies \int_{sphere} G(\theta, \phi)\, d\Omega = 4\pi$$

- $\Delta\Omega \sim \frac{4\pi}{G_{max}}$

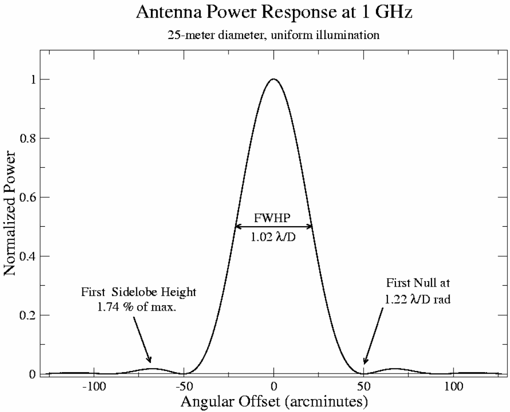
Directivity of a Parabolic Dish



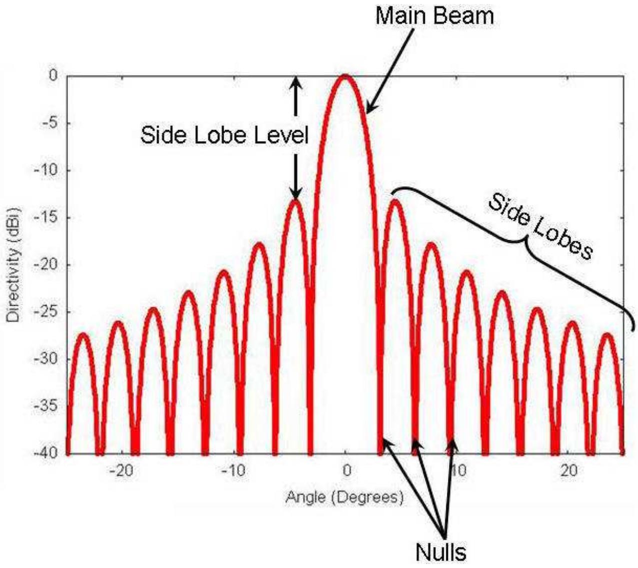
Directivity of a Parabolic Dish



Directivity of a Parabolic Dish



Directivity of a Parabolic Dish



A Measured Antenna Pattern (ATA)

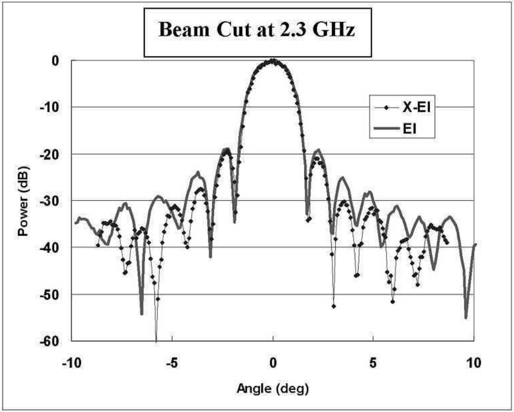
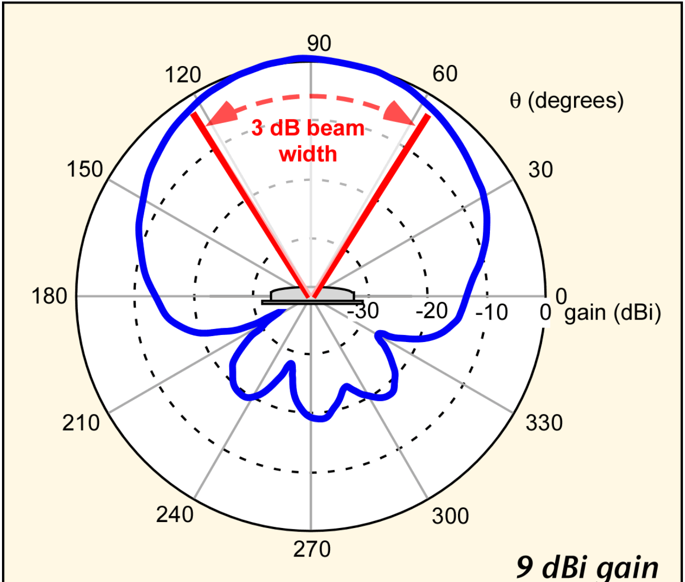


Figure 1: Two cuts through the primary beam pattern of one of the ATA dishes.

Antenna Pattern of a Patch Antenna



Beam shape of Arecibo Antenna

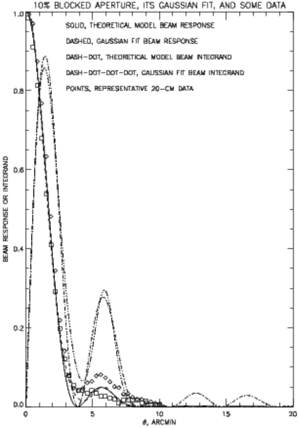


Fig. 4.— Normalized power pattern P_n, and also the integrand θP_n, for **(1)** the standard model of a uniformly illuminated 10% blocked aperture (solid, dash-dot lines), and **(2)** its Gaussian-fit counterpart (dash, dash-dot-dot-dot lines). The squares and diamonds are representative data points for P_n from the LBW feed at 1415 MHz, obtained by averaging different cuts in one single observing pattern.

A more sophisticated perspective

$E(\psi, \eta)$ - Aperture illumination (electric field distribution across the aperture)
ψ and η - aperture coordinates

$U(\alpha, \beta)$ - Far field electric field (diffraction pattern)
α and β - directions relative to the optical axis of the telescope

$E(\psi, \eta)$ and $U(\alpha, \beta)$ form a Fourier transform pair

Normalised Antenna Power Pattern, $P(\theta, \phi, \nu)$

$$P(\theta, \phi, \nu) = \frac{G(\theta, \phi, \nu)}{G(\theta_0, \phi_0, \nu)}$$

where θ_0 and ϕ_0 define the optical axis of the aperture.

- $\int_{sphere} P(\theta, \phi, \nu)\, d\Omega = \Omega_A$
- $\lambda^2 = \Omega_A \times A_{eff}$
- For an isotropic antenna $A_{eff} = \frac{\lambda^2}{4\pi}$

Gain and Aperture

$$G = \frac{4\pi\ A_{eff}}{\lambda^2},\ A_{eff} -\ \text{Effective collecting area}$$

$$A_{eff} = \eta\ A_{geom}$$

- η typically in the range 0.35 – 0.7
- GMRT: $\eta \sim$ 0.65–0.60 in the range 150 – 610 MHz, and $\sim$ 0.4 at 1400 MHz.
- J-VLA: η peaks at 3 GHz at $\sim$0.62, and drops to $\sim$0.45 at 1.4 GHz and $\sim$0.34 at 45 GHz
- ALMA: $\eta \sim$ 0.75–0.45 in the range 35 – 850 GHz

Spectral Power

$$W = \int_{\nu} \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \;\; d\nu \quad W$$

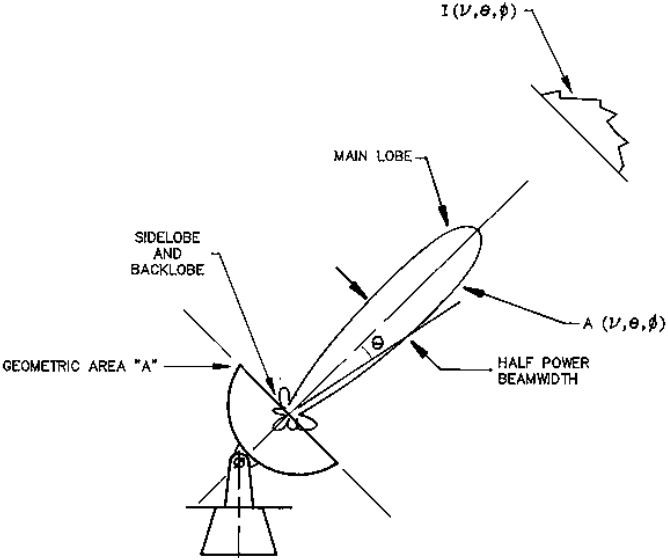
$$w_{\nu} = \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \quad W \; Hz^{-1}$$

$$w_{\nu} = A_{eff} \int_{\Omega} B(\theta, \phi, \nu) \;\; cos\theta \;\; d\Omega \quad W \; Hz^{-1}$$

$$w_{\nu} = A_{eff} \int_{\Omega} B(\theta, \phi, \nu) \;\; P(\theta, \phi, \nu) \;\; d\Omega \quad W \; Hz^{-1}$$

For a uniform source of Brightness B_u, this becomes
$w_{\nu} = \frac{1}{2} \; A_{eff} \; B_u \; \Omega_A \quad W \; Hz^{-1}$

The image to keep in mind



References and Pre-requisites

- References:
  - Kraus - Radio Astronomy (2nd ed): Sec 3.1–3.5
- Pre-requisites:
  - Concepts of blackbody radiation, Planck's law, Rayleigh-Jeans law
  - Concepts of random variables and statistics