Astronomical Techniques II
Lecture 3 - Noise, Temperature and SNR

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Spectral Power

$$W = \int_{\nu} \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \; cos\theta dA \; d\Omega \; d\nu \quad W$$

$$w_{\nu} = \int_{aperture} \int_{\Omega} B(\theta, \phi, \nu) \; cos\theta dA \; d\Omega \quad W \; Hz^{-1}$$

$$w_{\nu} = A_{eff} \int_{\Omega} B(\theta, \phi, \nu) \; cos\theta \; d\Omega \quad W \; Hz^{-1}$$

$$w_{\nu} = A_{eff} \int_{\Omega} B(\theta, \phi, \nu) \; P(\theta, \phi, \nu) \; d\Omega \quad W \; Hz^{-1}$$

For a uniform source of Brightness B_u, this becomes
$w_{\nu} = \frac{1}{2} \; A_{eff} \; B_u \; \Omega_A \quad W \; Hz^{-1}$

Spectral Power

$$w_\nu = A_{eff} \int_\Omega B(\theta, \phi, \nu) \;\; P(\theta, \phi, \nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Cross-correlation form:

$$w_\nu = A_{eff} \int_\Omega B(\theta, \phi, \nu) \;\; P(\Omega - \Omega_0, \nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Convolution form:

$$w_\nu = A_{eff} \int_\Omega B(\theta, \phi, \nu) \;\; \tilde{P}(\Omega_0 - \Omega, \nu) \;\; d\Omega \quad W \; Hz^{-1}$$

Planck's Law

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \quad W\ m^{-2}\ sr^{-1}\ Hz^{-1}$$

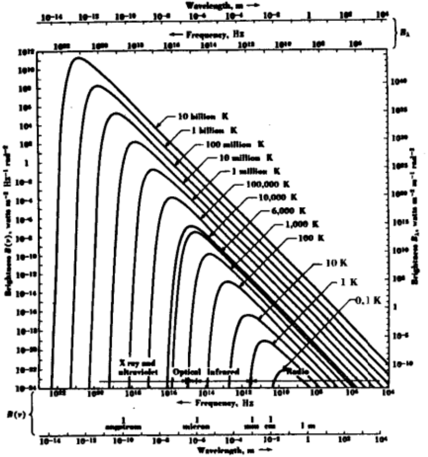

Fig. 3-13. Planck-law radiation curves to logarithmic scales with brightness expressed as a function of frequency $B(\nu)$ (left and bottom scales) and as a function of wavelength B_λ (right and top scales). Wavelength increases to the right.

Planck's and Rayleigh-Jeans Law

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

$$B_\nu = \frac{2kT\nu^2}{c^2}\frac{\frac{h\nu}{kT}}{e^{h\nu/kT}-1}$$

Limit $h\nu << kT$
Rayleigh-Jeans Law

$$B_\nu = \frac{2kT}{\lambda^2} \;\; W\ m^{-2}\ sr^{-1}\ Hz^{-1}$$

Power received at a detector

$$dW = B(\theta, \phi, \nu) \;\; cos\theta dA \;\; d\Omega \;\; d\nu$$

dW - W
$B(\theta, \phi) - W \; m^{-2} \; sr^{-1} \; Hz^{-1}$

Practical quantitative definition

$$B(\theta, \phi, \nu) = \frac{dW}{d\Omega \; cos\theta dA \;\; d\nu} = \frac{2kT}{\lambda^2}$$

- Intrinsic property of the source
- Independent of the distance from the source (ONLY for a resolved object)
- Can be thought of as energy *received* at the detector OR as energy *emitted* by the source.

Brightness and Temperature

$$S = \frac{2kT\Omega_s}{\lambda^2}$$

$$S = \frac{2k}{\lambda^2} \int_{\Omega_s} T(\theta, \phi)\ d\Omega$$

$$S = \frac{2k}{\lambda^2} \int_{\Omega_s} T(\theta, \phi)\ P(\theta, \phi)\ d\Omega$$

- Associate a unique temperature with the power received at any given frequency. **A property of the source.**
- Represents the temperature of a Blackbody which would have given out exactly as much radiation at that frequency ($W\ m^{-2}\ sr^{-1}\ Hz^{-1}$).
- For thermal radiation from an optically thick source - same as the physical temperature of the body emitting the radiation.
- For non-thermal radiation - eq. radiation temperature

Temperature and Noise

Noise power measured per unit bandwidth at the terminals of a resistance R at temperature T (Nyquist, 1928)
$w = kT \;\; W \; Hz^{-1}$

load $\rightarrow$ lossless antenna of radiation resistance R, the impedence as seen at the terminals is unchanged.
The noise spectral power received by the antenna

$$w = \frac{1}{2} A_{eff} \int_{\Omega} B(\theta, \phi) \; P(\theta, \phi) \; d\Omega$$

$$w = \frac{k \; A_{eff}}{\lambda^2} \int_{\Omega} T(\theta, \phi) \; P(\theta, \phi) \; d\Omega \;\; W \; Hz^{-1}$$

Antenna Temperature

The temperature of antenna radiation resistance

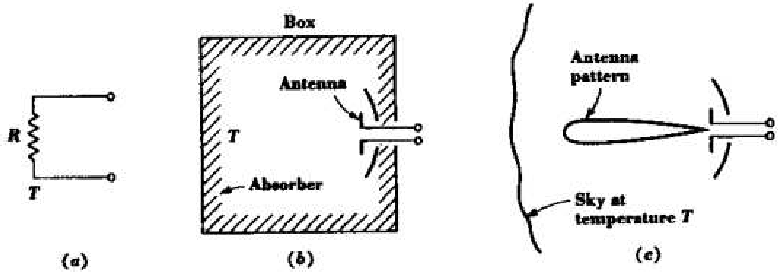

Fig. 3-24. (*a*) Resistor at temperature T; (*b*) antenna in an absorbing box at temperature T; and (*c*) antenna observing sky of temperature T. The same noise power is available at the terminals in all three cases.

$$w = \frac{1}{2}\, A_{eff} \int_{\Omega} B(\theta, \phi)\; P(\theta, \phi)\; d\Omega = kT$$

Recall that

$$S = \int_{\Omega} B(\theta, \phi)\; P(\theta, \phi)\; d\Omega$$

$$\implies S = \frac{2kT}{A_{eff}}$$

$$T_A = \frac{A_{eff}}{\lambda^2} \int_{\Omega} T_s(\theta, \phi)\; P(\theta, \phi)\; d\Omega$$

$$T_A = \frac{1}{\Omega_A} \int_{\Omega} T_s(\theta, \phi)\; P(\theta, \phi)\; d\Omega$$

Antenna Temperature

The compact source and extended source cases.

Noise and Signal

- Signal - T_{Ant} - what comes from the sky
- Noise - everything else
  - Receiver - T_{Rec}
  - Spillover - T_{Spill}
  - Leakage - T_{Leak}
  - Loss - T_{Loss}
  - Radio Frequency Interference (RFI)

$$T_{Sys} = T_{Ant} + T_{Rec} + T_{Spill} + T_{Leak} + T_{Loss}$$

- The *signal* has the same characteristics as *noise*
- One is looking for an increase of T_{Ant} over a background of T_{Sys}.

Minimum Detectable Signal

$$\Delta T_{min} = \frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

$$\Delta B_{min} = \frac{2k}{\lambda^2}\frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

$$\Delta S_{min} = \frac{2k}{A_{Eff}}\frac{T_{Sys}}{\sqrt{\Delta\nu\Delta\tau}}$$

- In theory, can be reduced to an arbitrarily small number by increasing the denominator.
- In practice:
  - Limited Δt - source evolution, source visibility, system stability, TAC, human effort
  - Limited $\Delta\nu$ - spectral signature (emission/absorption lines, EoR), source evolution, instrumental capability, technical challenges