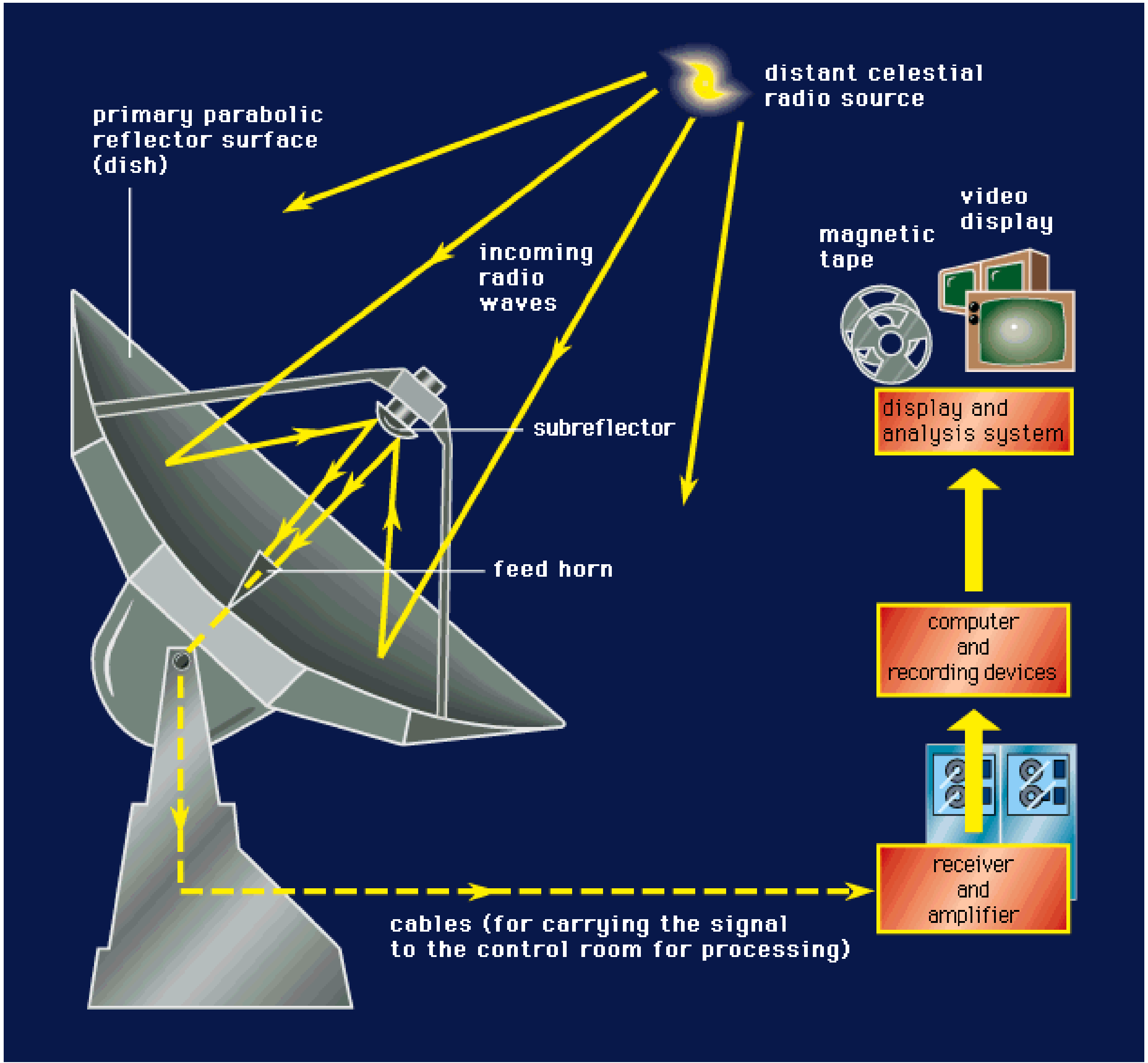
INTRODUCTION TO INTERFEROMETRY

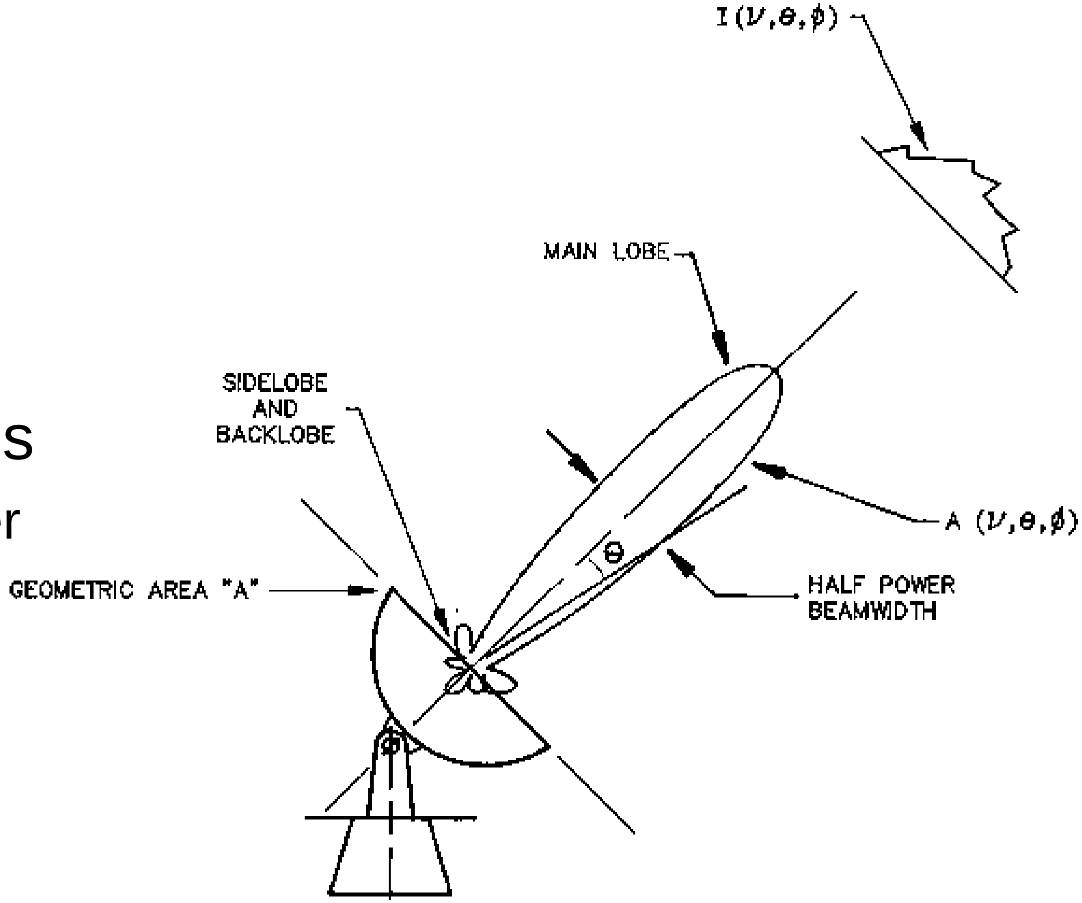
Divya Oberoi div@ncra.tifr.res.in
National Centre for Radio Astrophysics
Tata Institute of Fundamental Research
Pune

A typical radio telescope



Beam size and resolution

- Size of the main lobe in radians ~λ/D
  - λ is the wavelength
  - D is the diameter
- Better resolution requires
  - Shorter wavelength (higher frequency)
  - Bigger telescopes



Why Interferometry?

- Resolution ~ λ/D

  λ - wavelength of observation

  D - size of aperture (diameter of lens/mirror)
- A 4m optical telescope is ~$5x10^6$ λ (8000 Å)

  (1arc sec resolution requires D ~$2x10^5$ λ)
- In radio λ ranges from ~0.5 mm to ~10 km

  (1 arc sec requires D ~100 m to ~$2x10^3$ km)
- Impossible to build apertures of required dimensions and surface accuracy
- *Interferometry provides the solution - resolutions corresponding to the separation between the elements (telescopes)*

The concept behind an interferometer

The important property of a parabolic dish is that it adds parallel light rays coherently

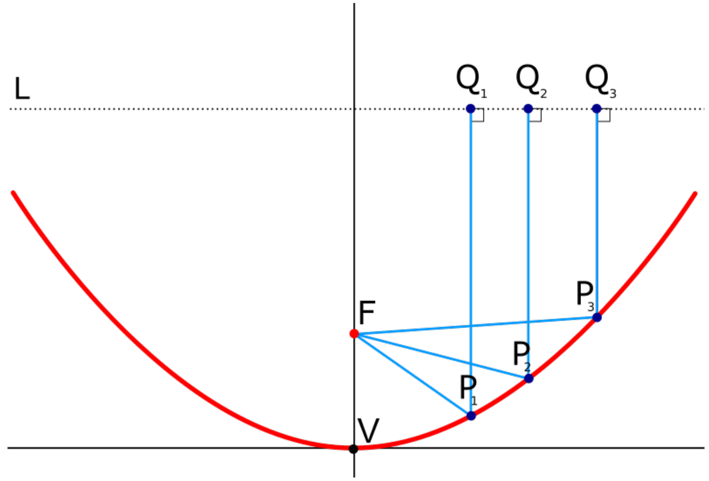
Parallel rays (from infinity) have equal path lengths to the focus, so they all arrive in phase

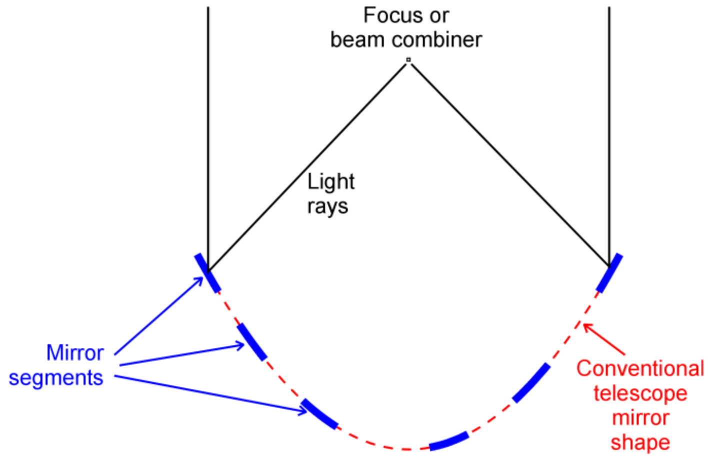
This is still true if we remove segments of the parabola – remaining rays still reach focus in phase

Now imagine moving the remaining segments of the dish off the surface of the paraboloid

So long as we know very precisely where the segments are located, we can delay their signals appropriately and still add them together coherently

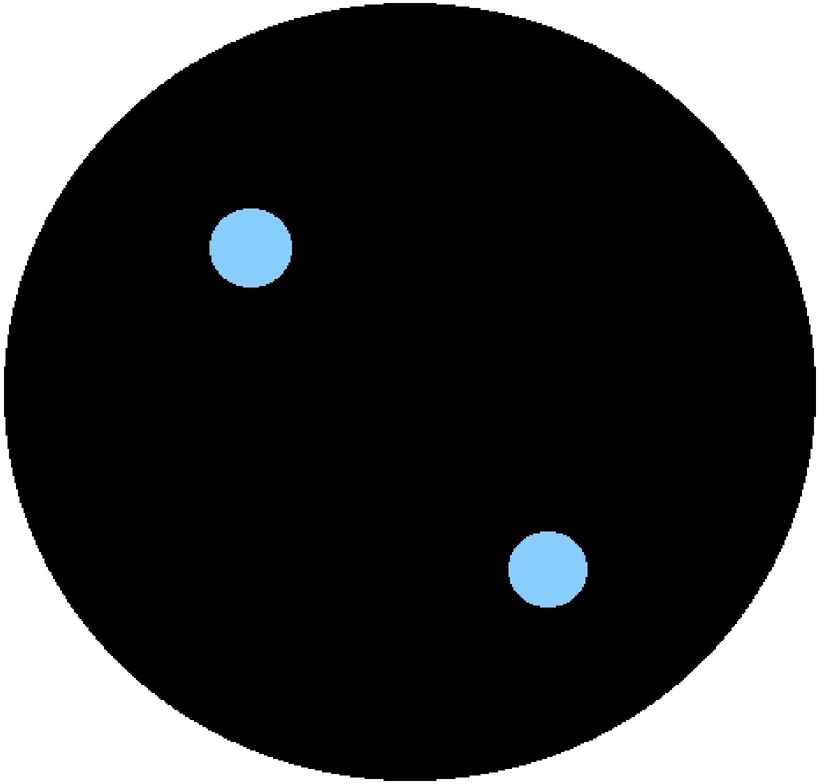
This, in essence, is what an interferometer does



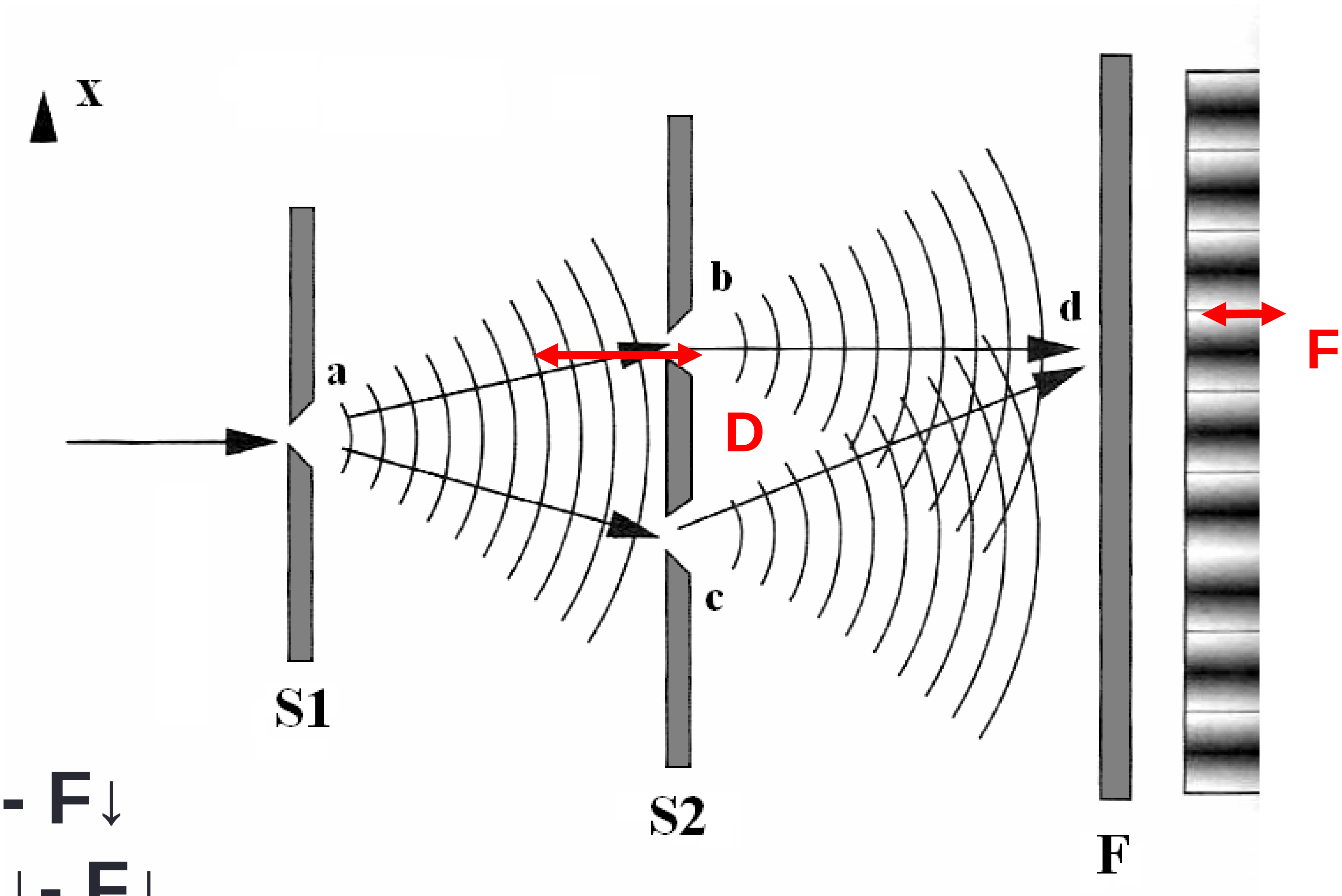


Images: wikipedia

Imaging with an *unfilled* aperture



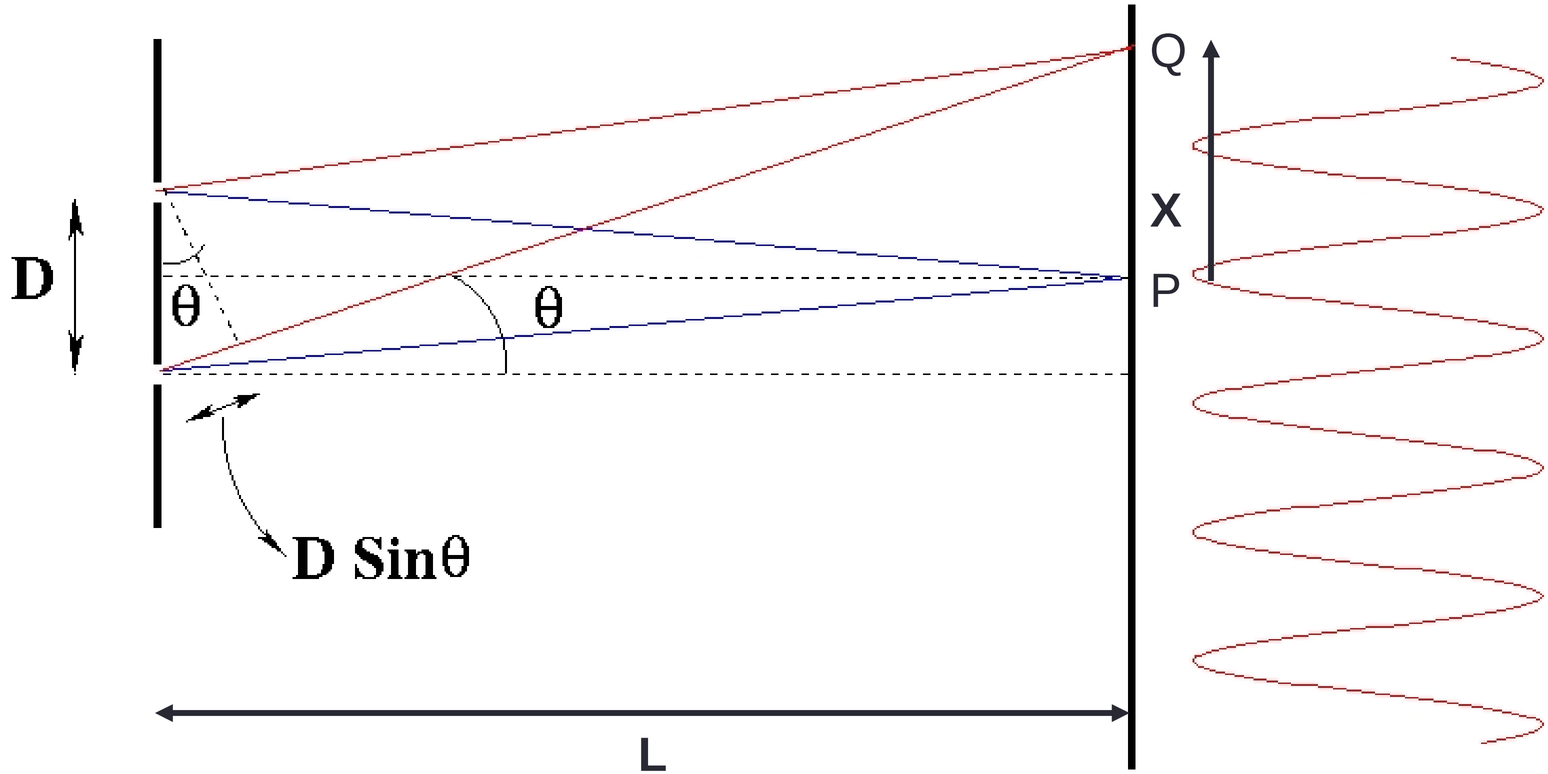
Young's double slit experiment



D↑- F↓

λ↓- F↓

Young's double-slit experiment



Path difference = $n\lambda \Rightarrow$ maxima $X = n\lambda\, L/D$

= $(n+1/2)\lambda \Rightarrow$ minima $X = (n+1/2)\,\lambda\, L/D$

$\lambda \ll D \ll L$

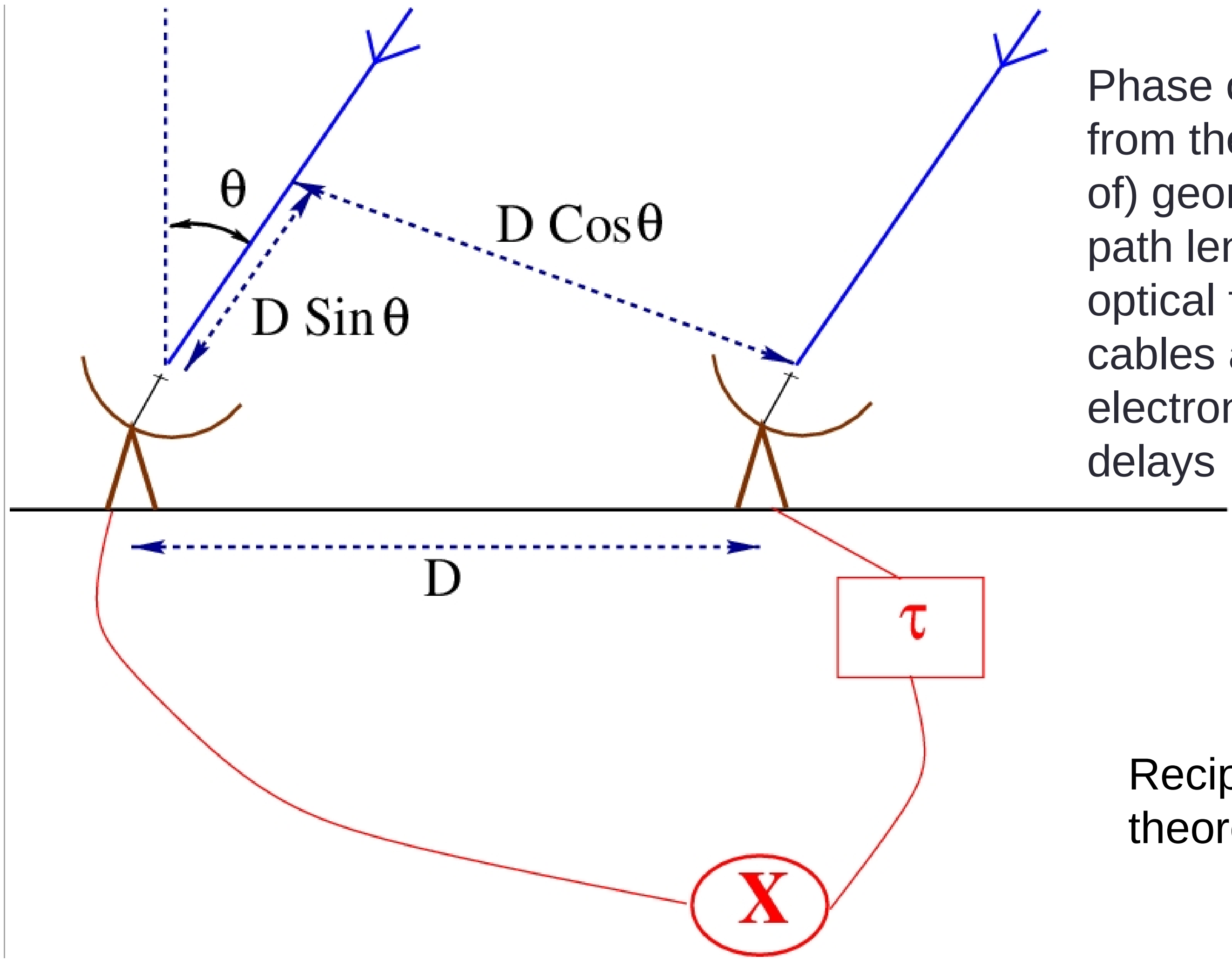
2 element interferometer

- An *antenna* is a device for converting electrical currents in conductors into electromagnetic radiation in space or vice-versa
- Radiating characteristics are identical to receiving characteristics.

- Imagine a radio double slit experiment, except that the slits are now ‘receiving’ rather than ‘emitting’ radio waves – a 2 element interferometer

A two element interferometer

Slits → Telescopes

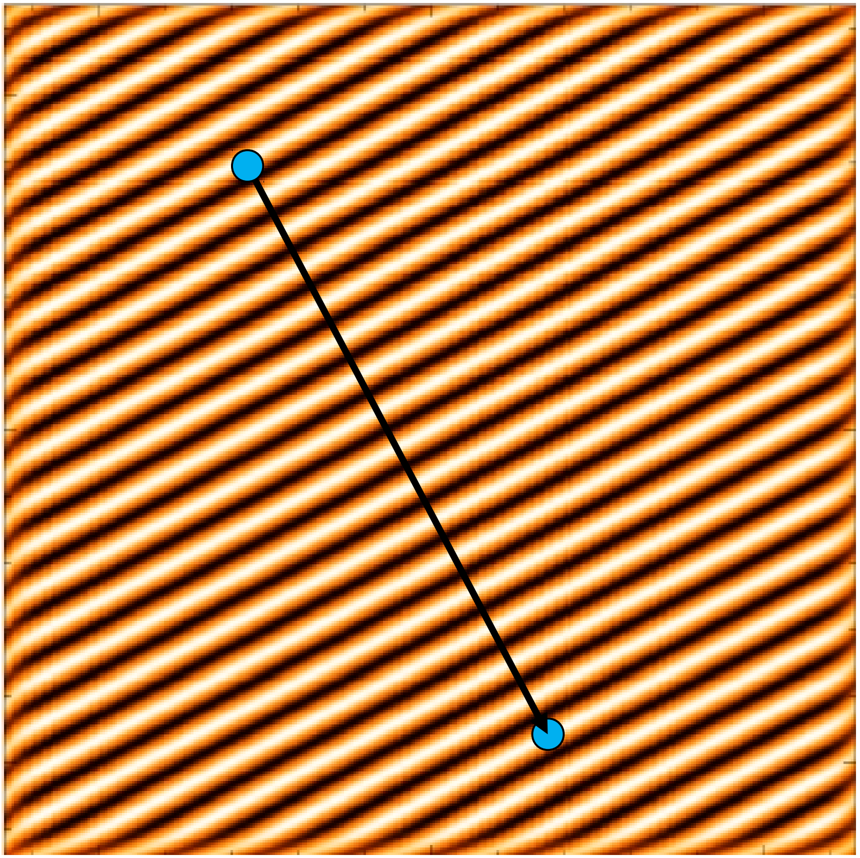


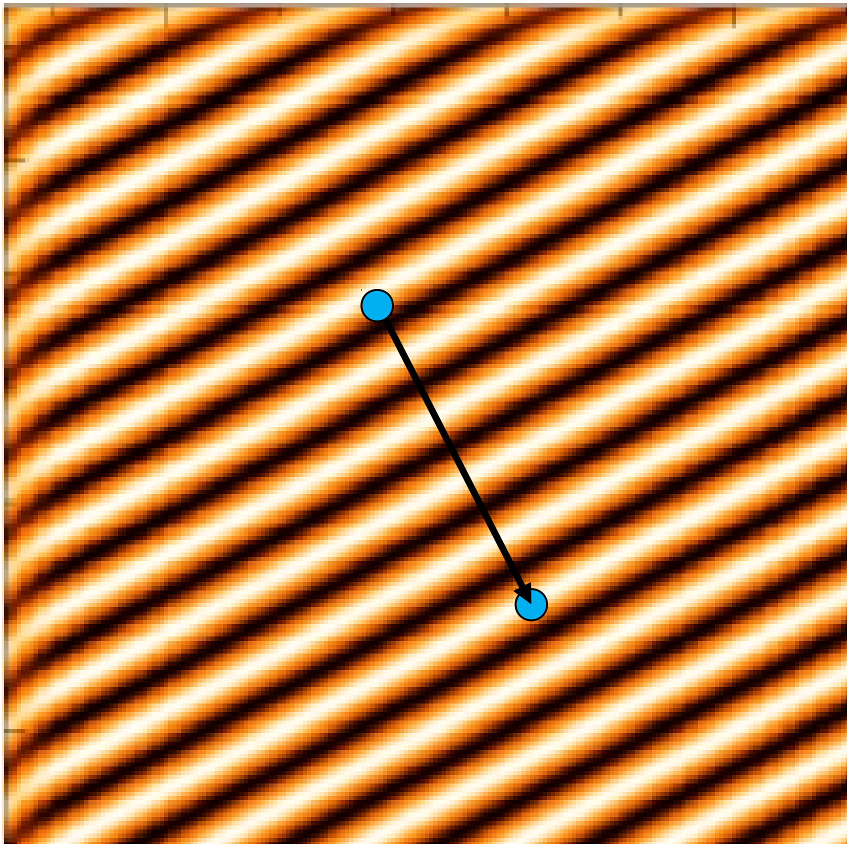
Phase comes from the (sum of) geometric path length, optical fiber, cables and electronic delays

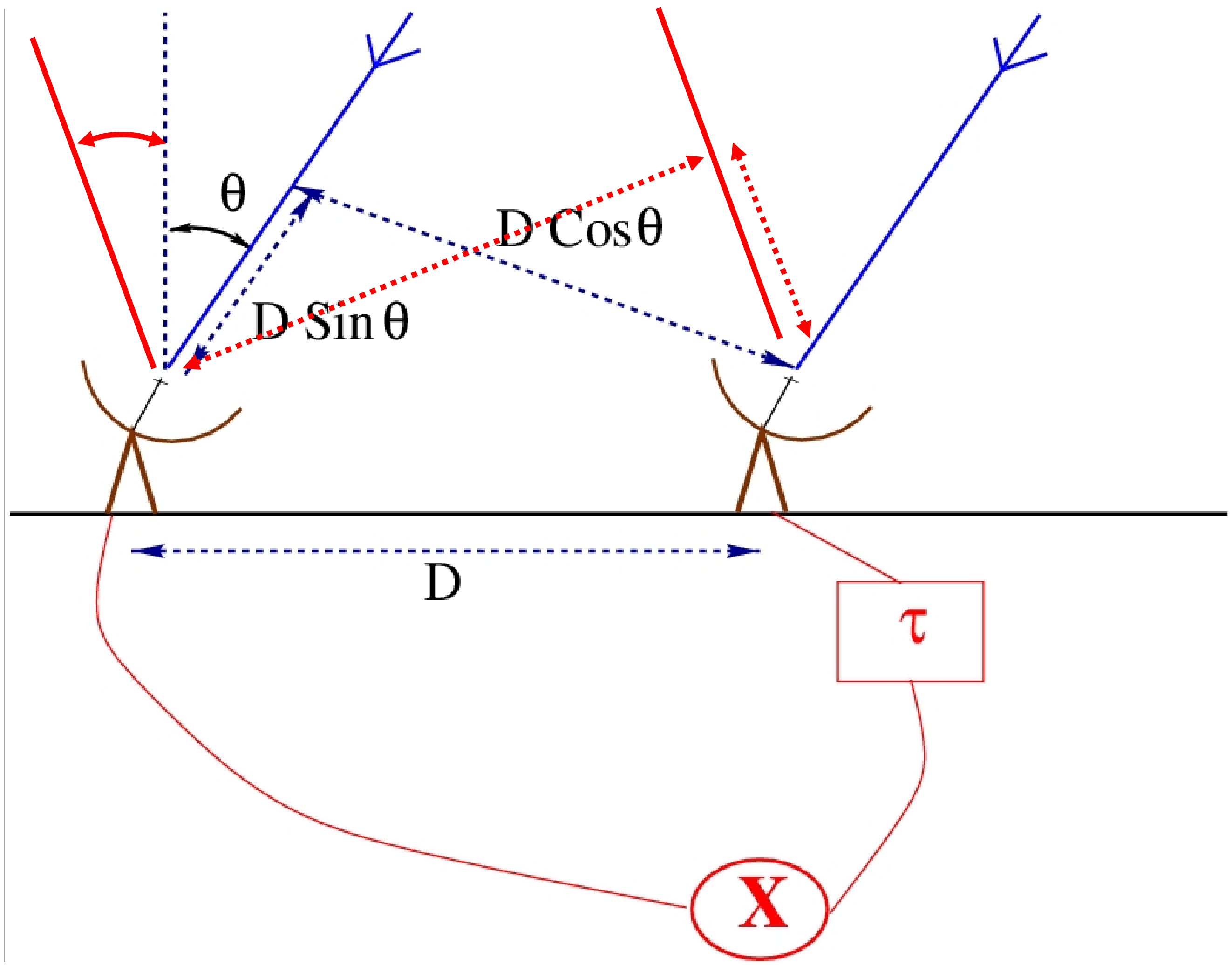
Reciprocity theorem

Screen/Detector → Correlator

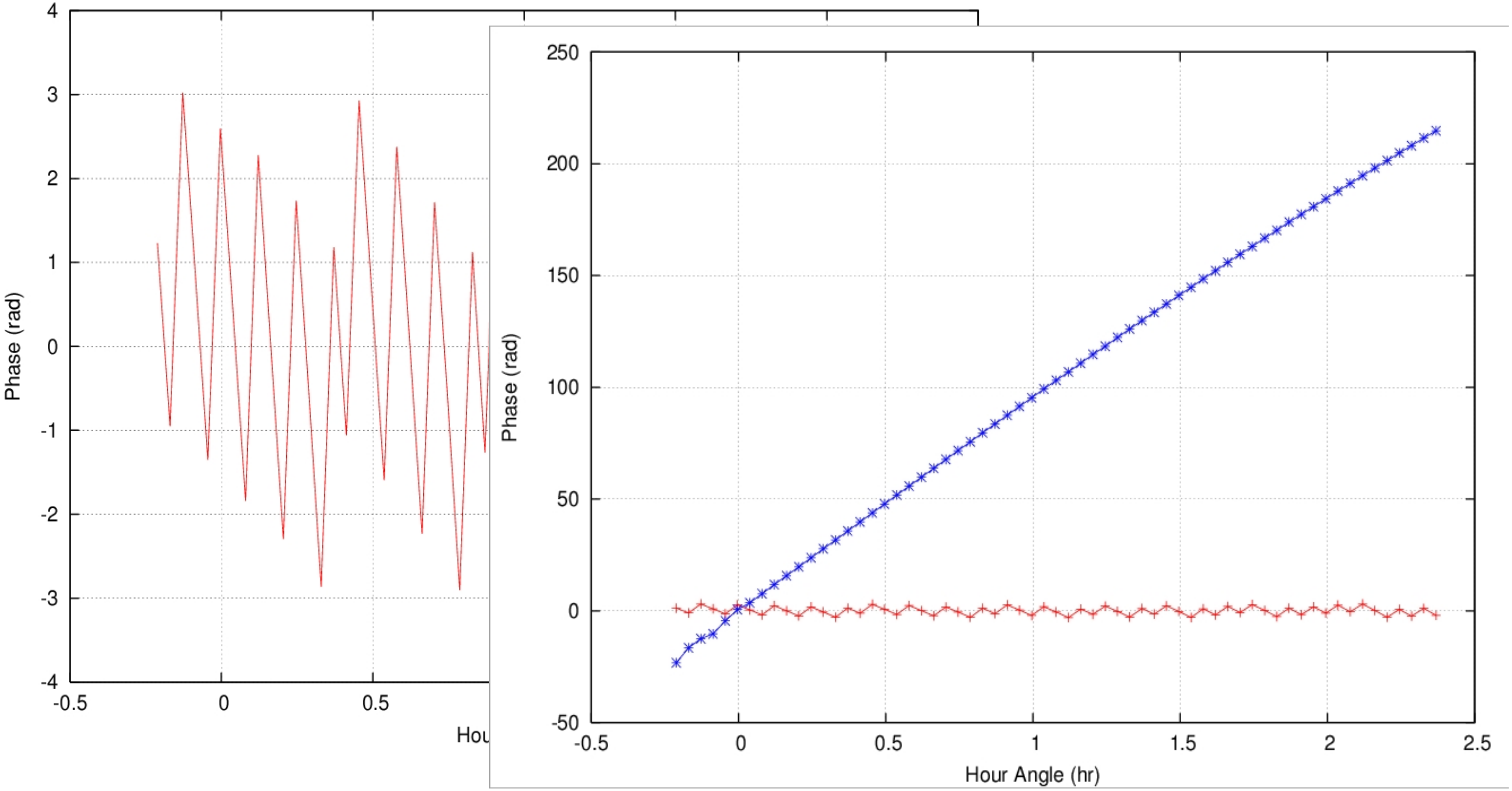
Sky response of an individual baseline






θ
D Cos θ
D Sin θ
D
τ
X

Real life fringes



Sun @ 125 MHz, 26 Apr, 2005, Mileura, Western Australia

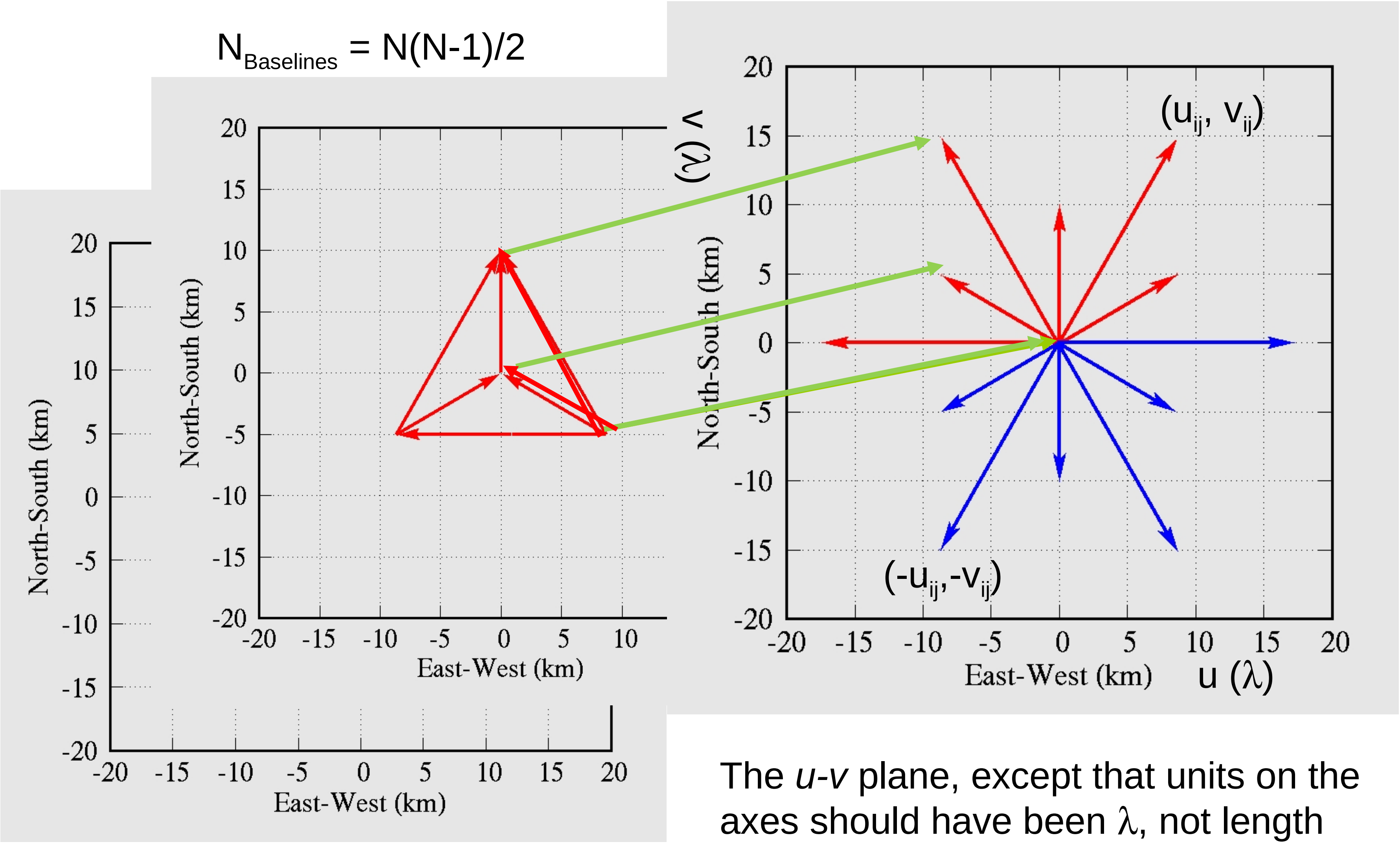
Murchison Widefield Array – Early Deployment effort, phase 2

What are these fringes?

- Young's double slit
  - Fringes are a function of position
  - Constant in time
- Astronomical fringes
  - Arise because the relative motion between the astronomical source and the interferometer changes the effective baseline (D Cosθ)
  - For a given baseline, function of time
  - Assumption: source does not change during the course of the observation
  - *Fringestop – Usually this geometric phase is corrected for in the data, and you do not get to see it.*

Baselines and *u-v plane*

$N_{Baselines} = N(N-1)/2$



The *u-v* plane, except that units on the axes should have been λ, not length

Visibility V(u,v)

- The fundamental Radio Astronomy measurable

  $V_{ij}(u,v,t,\Delta t,\nu_0,\Delta\nu) = \langle V_i(\ldots) \times V_j^*(\ldots,t+\tau,\ldots)\rangle$

- van Cittert Zernike Theorem

  V(u,v) is 2D Fourier Transform of the sky Brightness distribution $B(\theta,\varphi)$

(T(x,y) in the following slides)

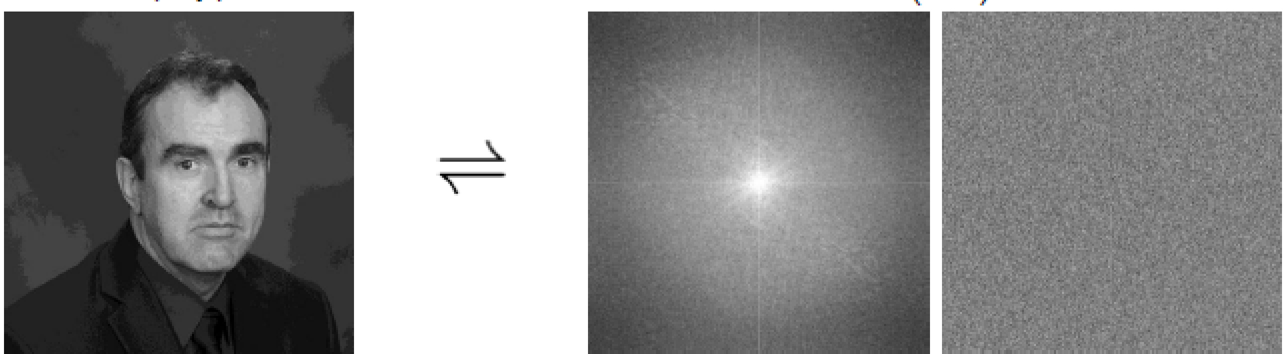
- Incoherent source,
- Small field of view
- Far-field

Visibilities

- each V(u,v) contains information on T(x,y) *everywhere*, not just at a given (x,y) coordinate or within a given subregion

- V(u,v) is a complex quantity
  - visibility expressed as (real, imaginary) or (amplitude, phase)

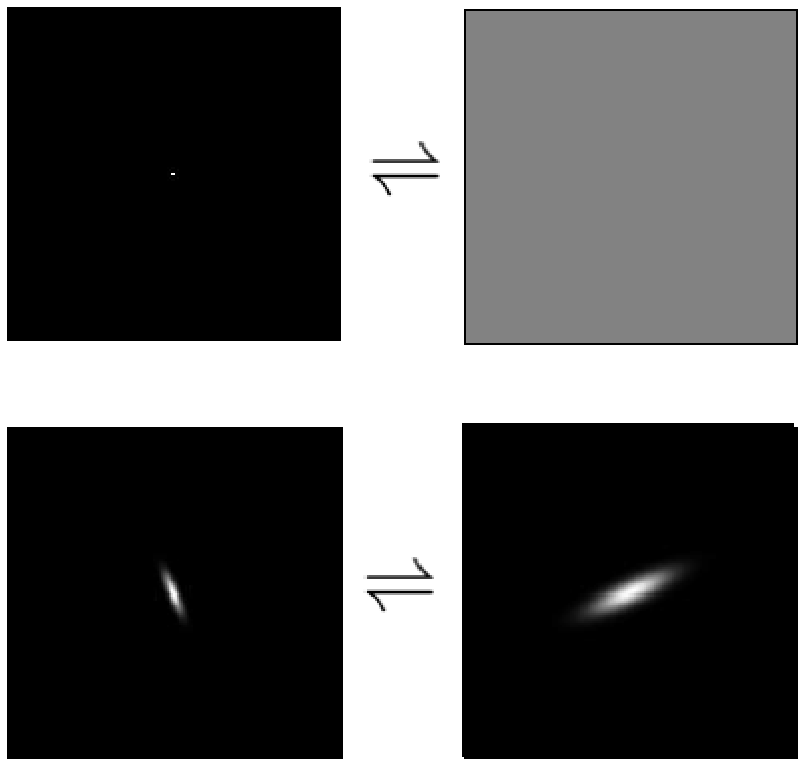
T(x,y) V(u,v)



amplitude phase

Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Example 2D Fourier Transform Pairs



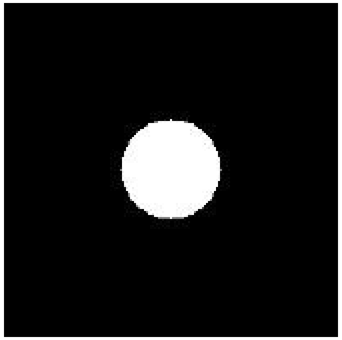
narrow features transform into wide features (and vice-versa)

Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

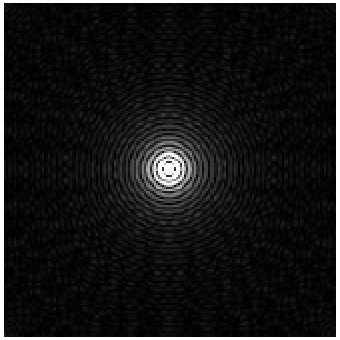
Example 2D Fourier Transform Pairs

T(x,y) amp{V(u,v)}

disk

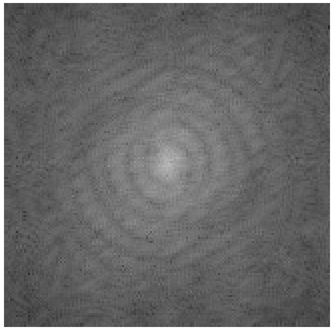


⇌



Bessel

⇌

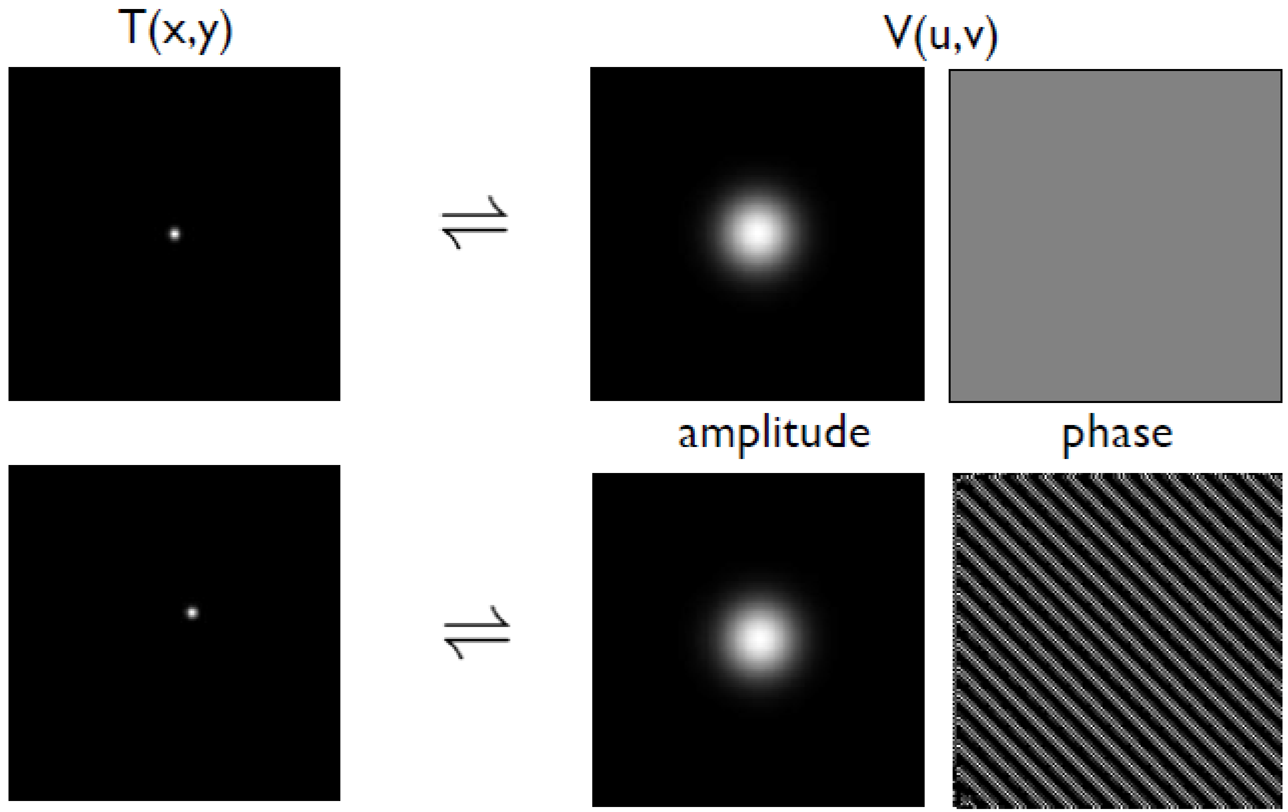


sharp edges result in many high spatial frequencies

Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Amplitude and Phase

- amplitude tells "how much" of a certain spatial frequency
- phase tells "where" this component is located



Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

The Visibility Concept

$$V(u,v) = \int \int T(x,y) e^{2\pi i(ux+vy)} dxdy$$

- visibility as a function of baseline coordinates (u,v) is the Fourier transform of the sky brightness distribution as a function of the sky coordinates (x,y)

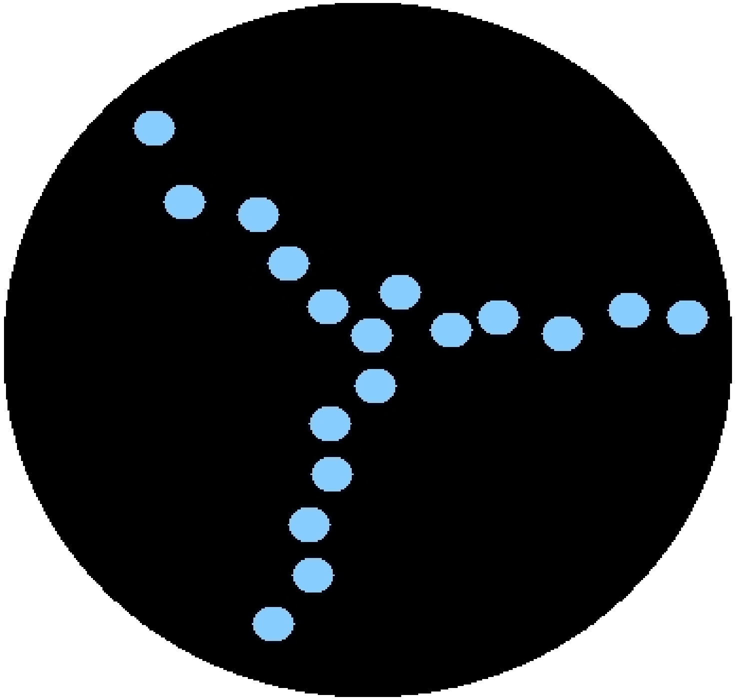
- V(u=0,v=0) is the integral of T(x,y)dxdy = total flux

- since T(x,y) is real, V(u,v) is Hermitian: V(-u,-v) = V*(u,v)
  - get two visibilities for one measurement

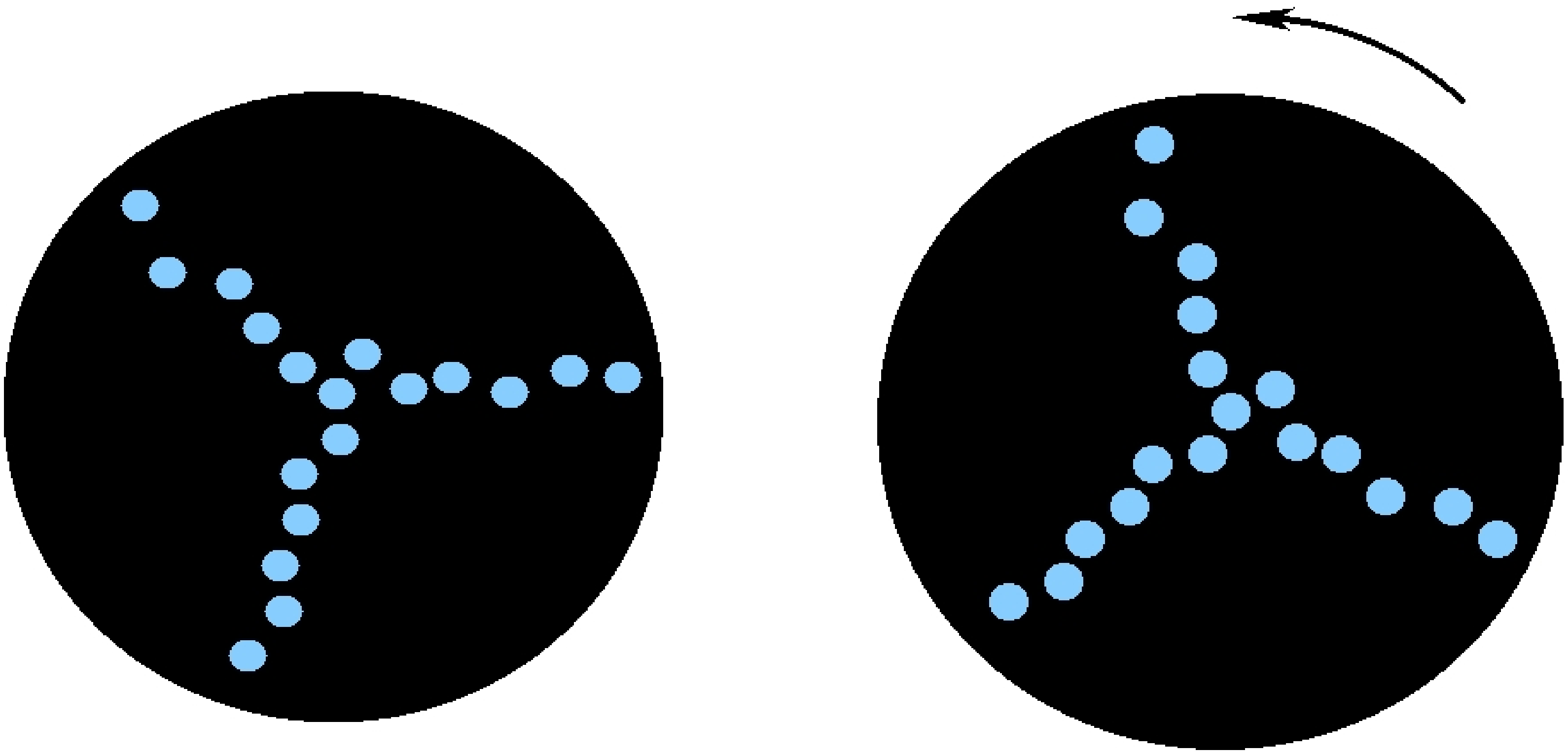
Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

An N element interferometer

- ‘Baselines’ from N elements – N(N-1)/2
- Each of these will lead to a ‘Cosine’ with different orientation and spacing
- The final response of the interferometer will be the superposition of Cosines from all the baselines



Synthesis imaging



VLA - 27 antennas ⇒ 351 baselines

GMRT - 30 antennas ⇒ 435 baselines

MWA – 128 elements ⇒ 8,128 baselines

The mathematical basis

- Brightness distribution in the sky is Fourier transform of the Visibilities

  $B(\theta,\varphi) \leftrightarrow V(u,v)$

  V(u,v) – The quantity measured by a baseline (amplitude, phase / real, imaginary)

- In the uv-plane, we measure visibilities only at a few places i.e. we have a sampling function

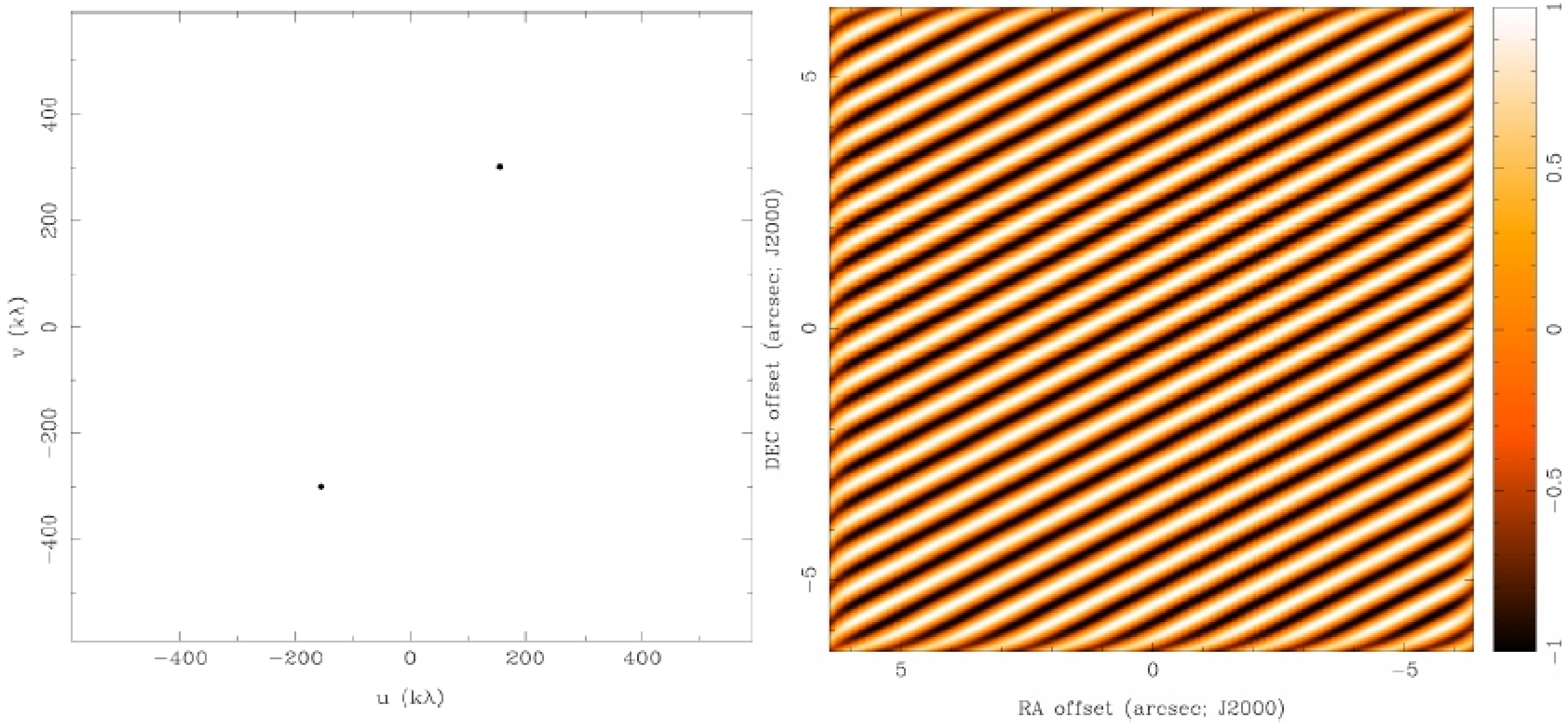
  $S(u,v) = \Sigma_k (u_k, v_k)$

- Point source response of an interferometer (PSF) is Fourier transform of S(u,v)

  $P(\theta,\varphi) \leftrightarrow S(u,v)$

Dirty Beam Shape and N Antennas

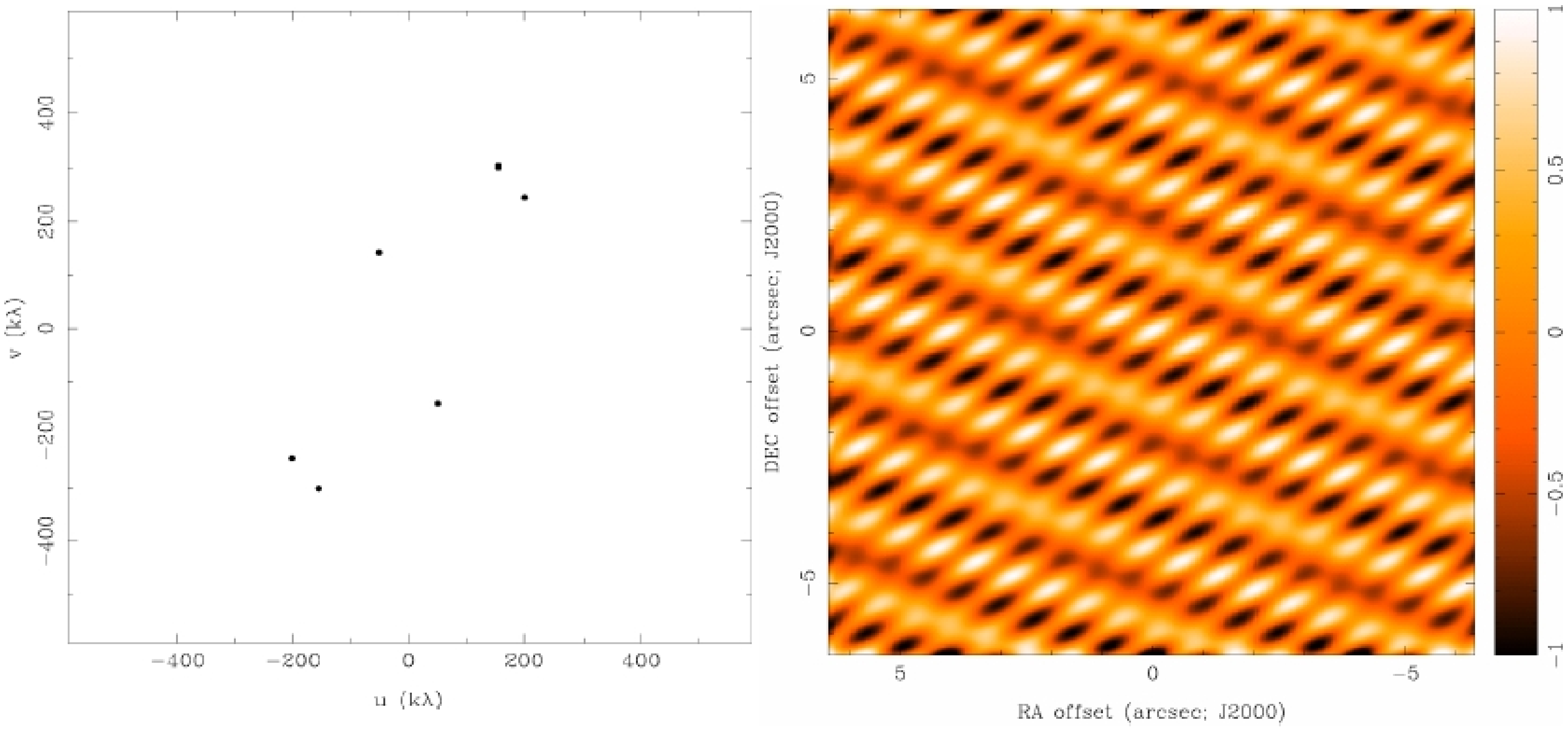
2 Antennas



Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

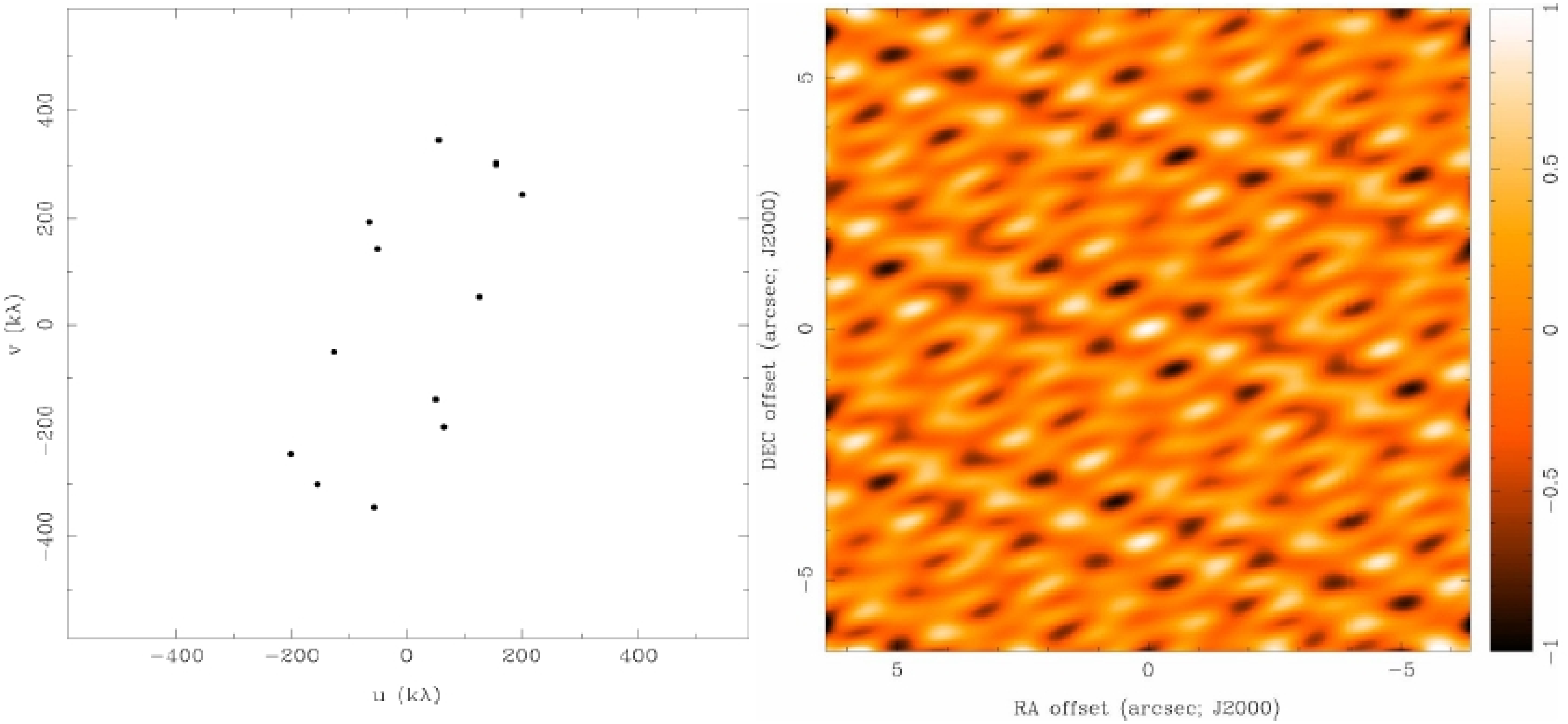
3 Antennas



Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

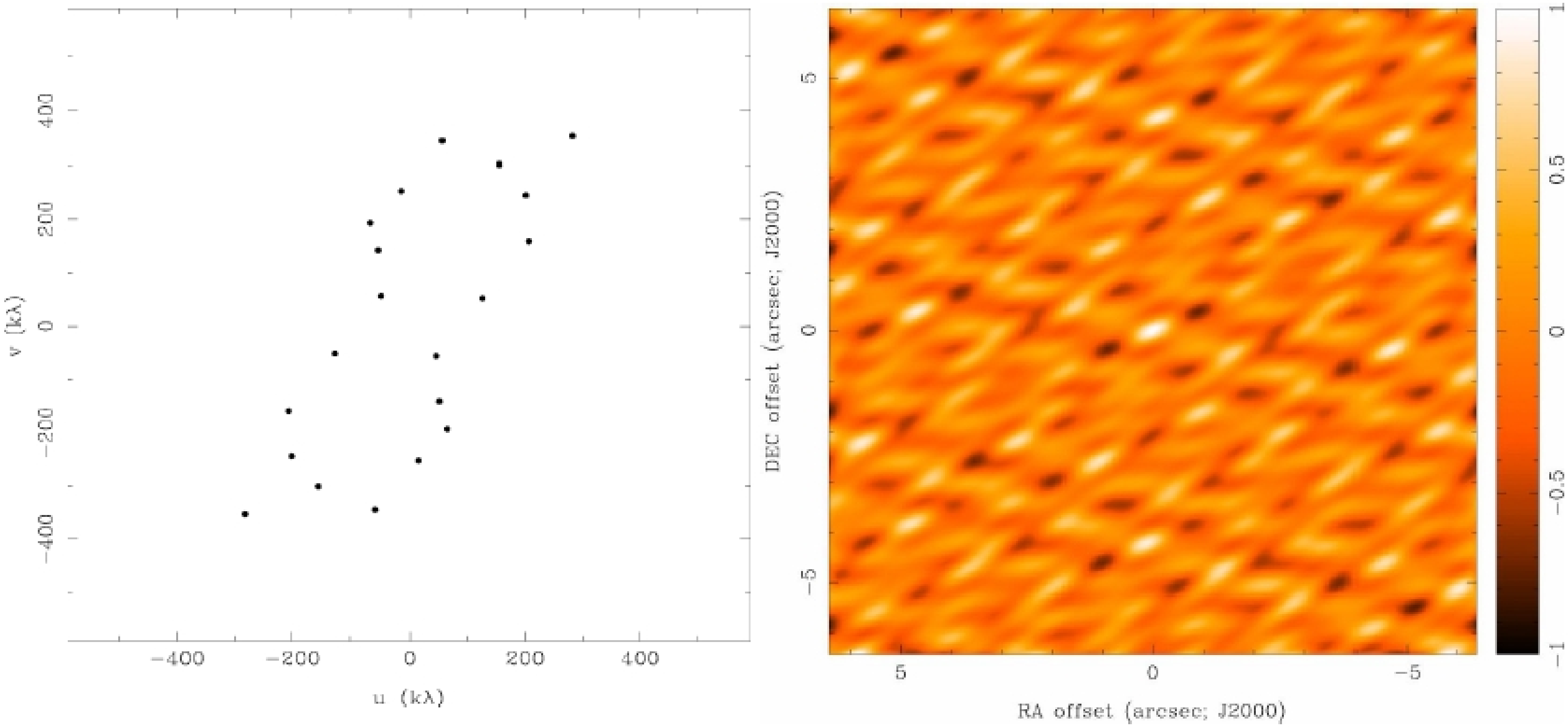
4 Antennas



Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

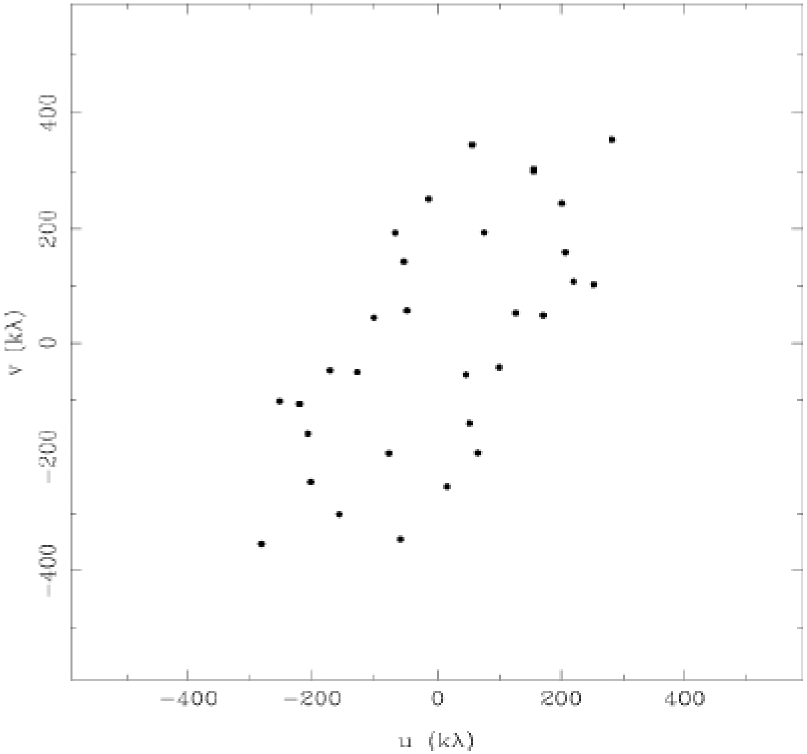
5 Antennas

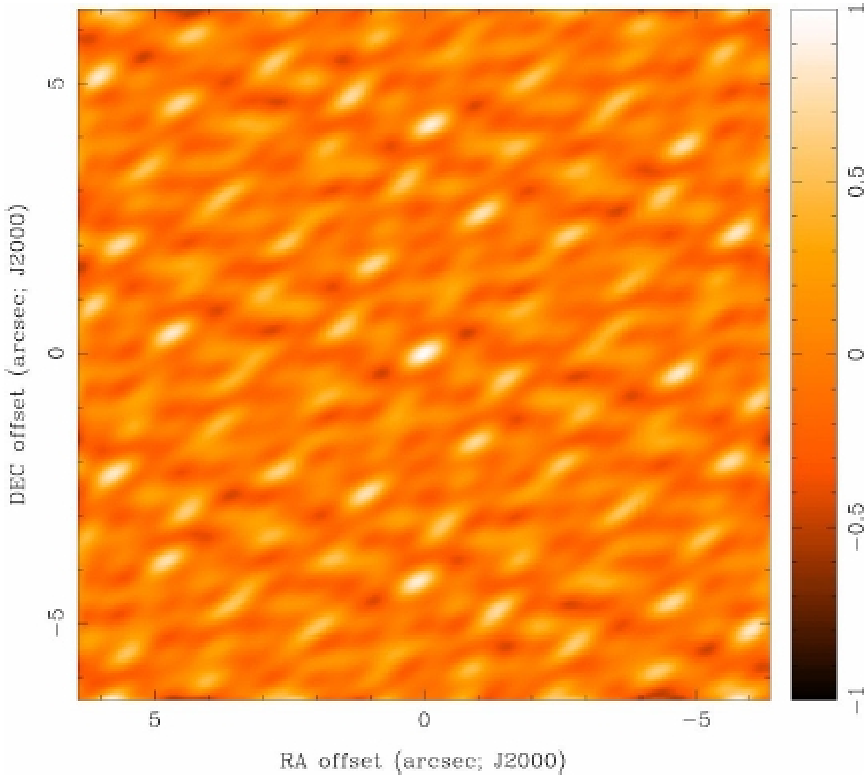


Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

6 Antennas

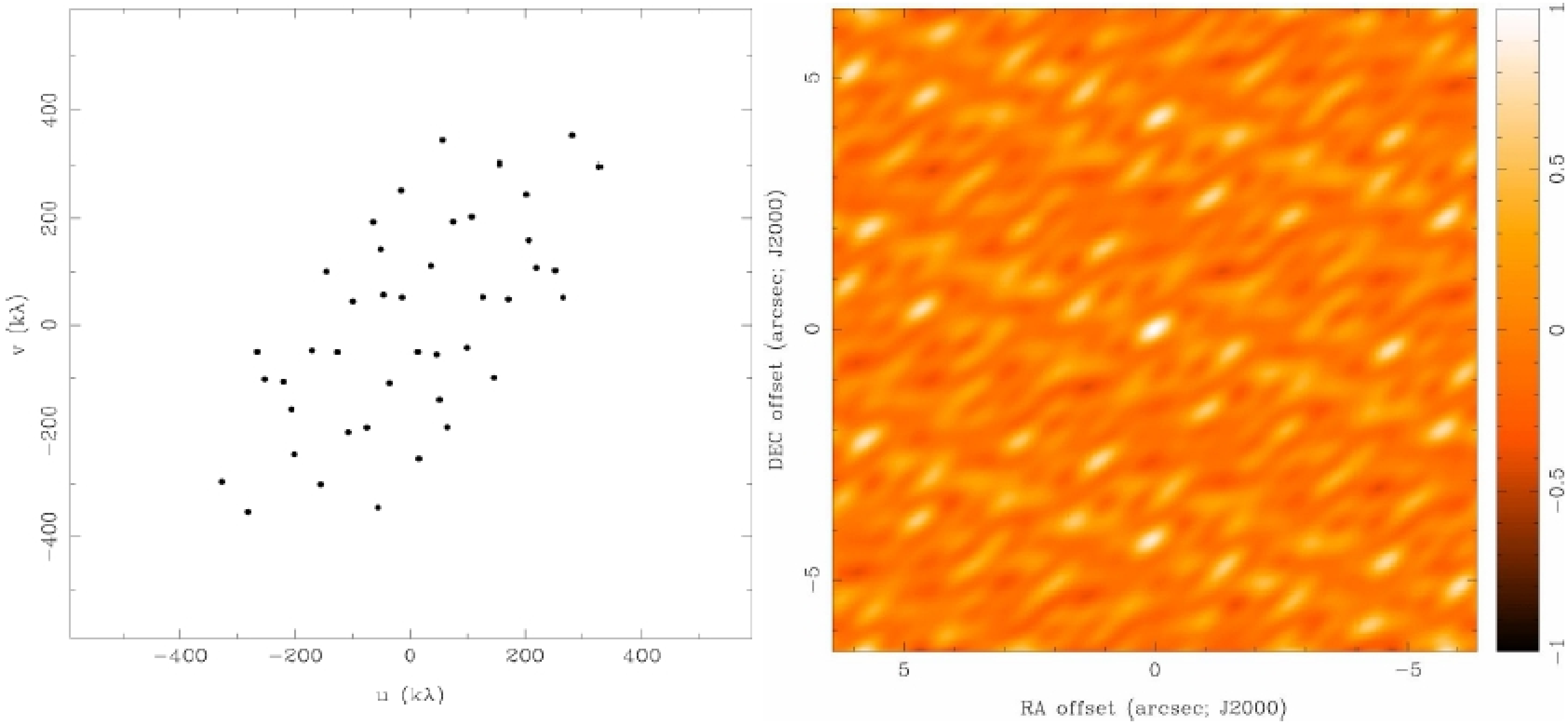




Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

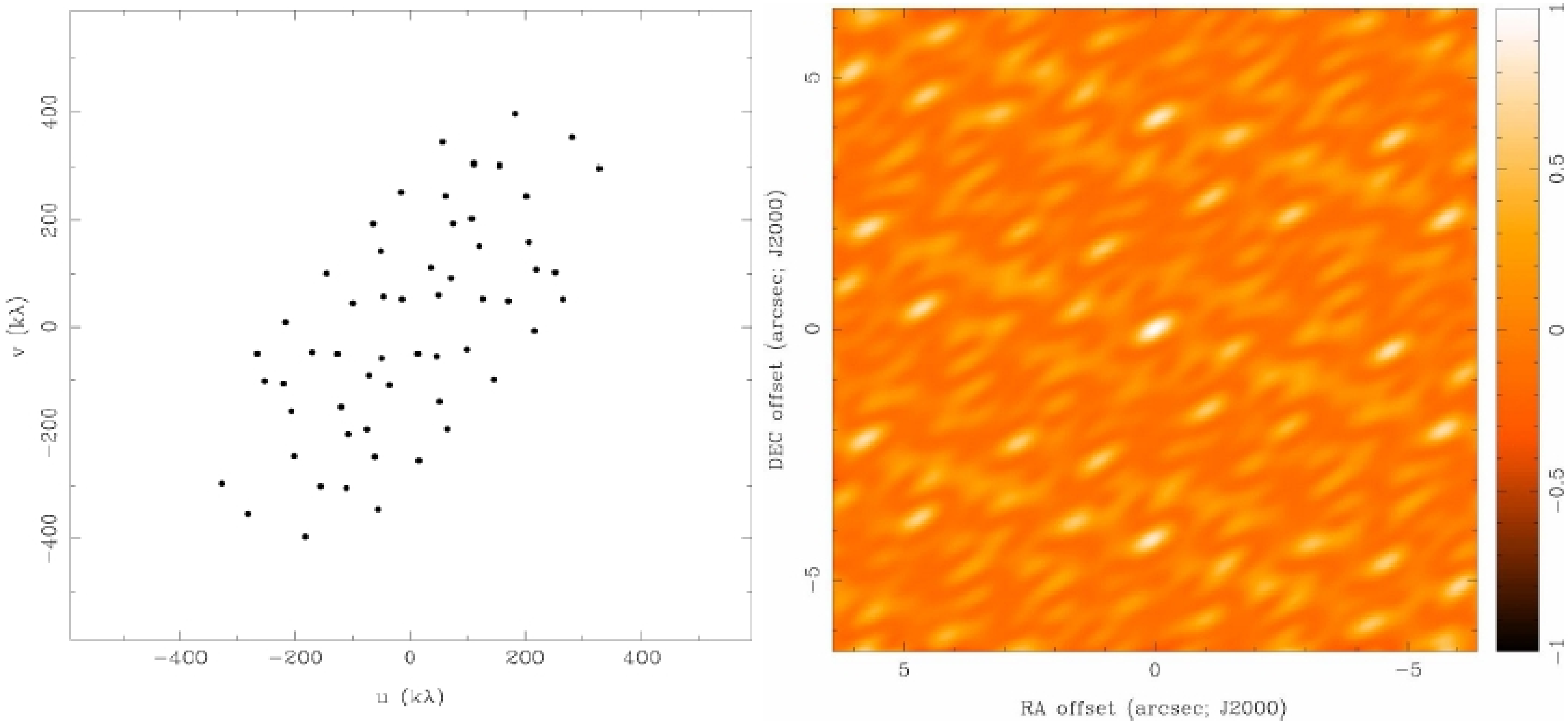
7 Antennas



Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

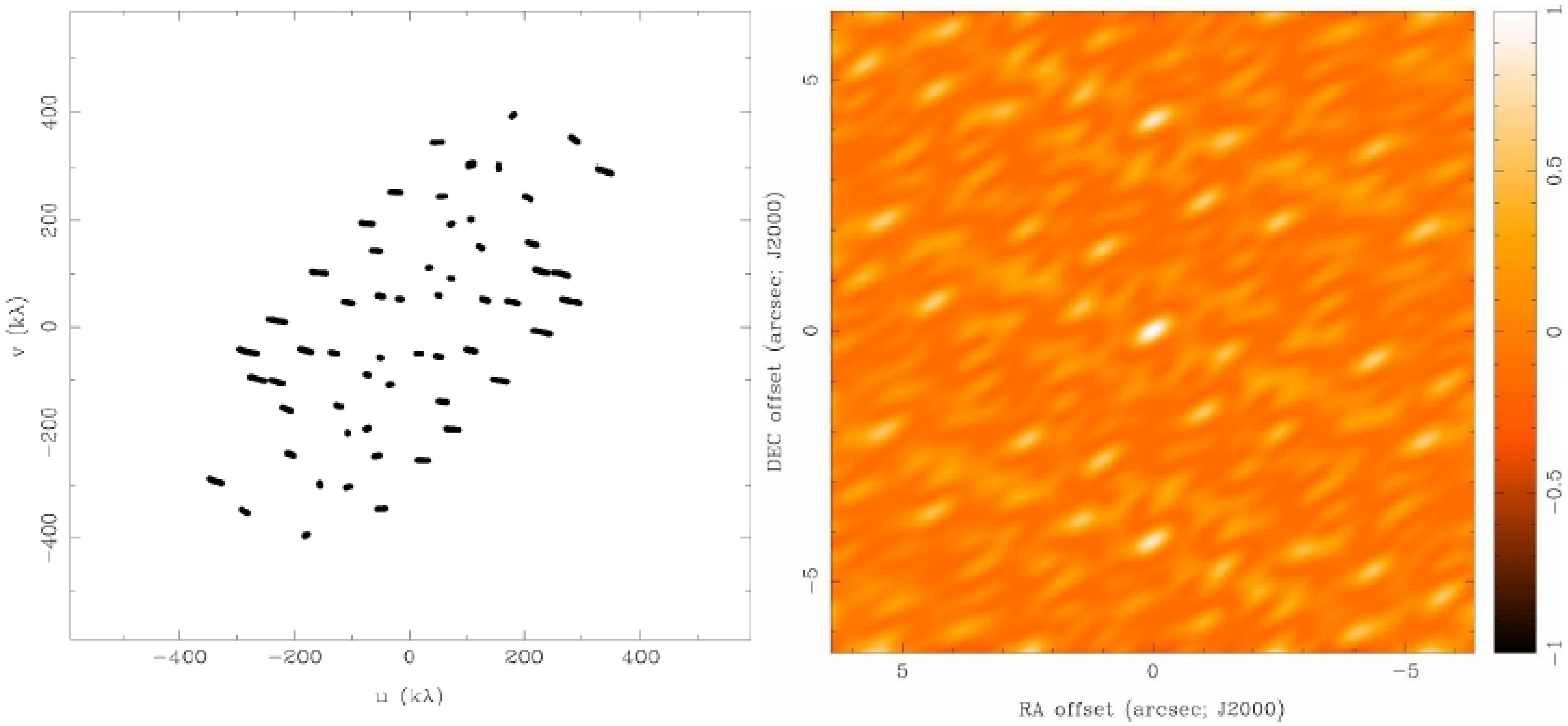
8 Antennas



Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

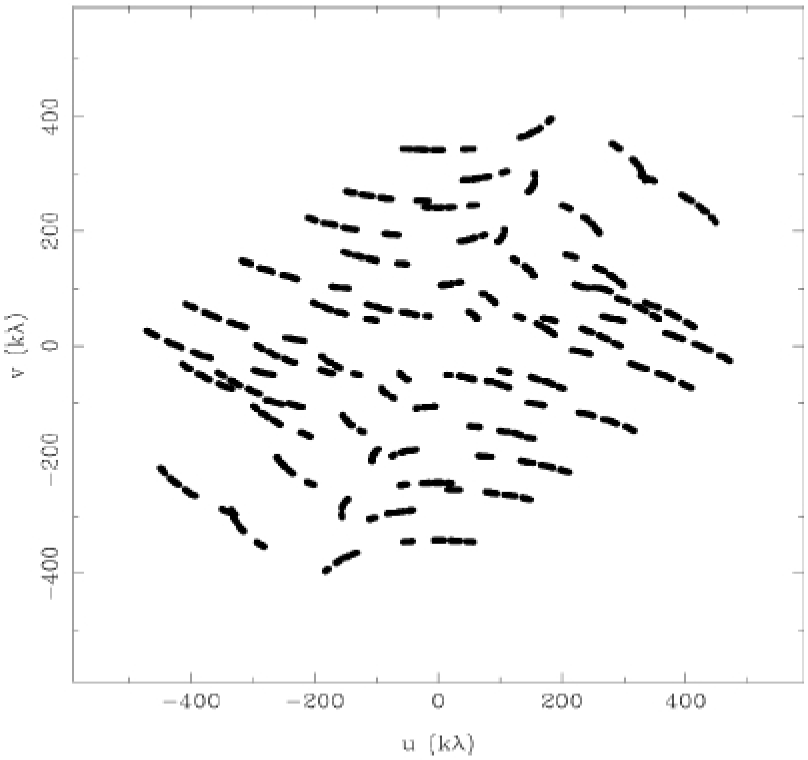
8 Antennas x 30 samples

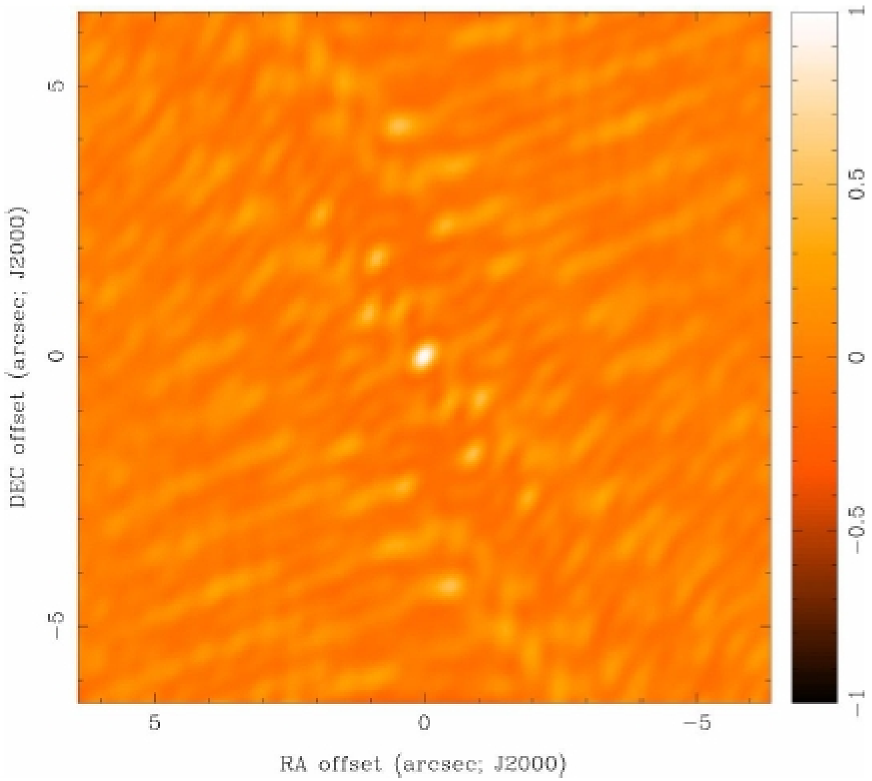


Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

8 Antennas x 120 samples

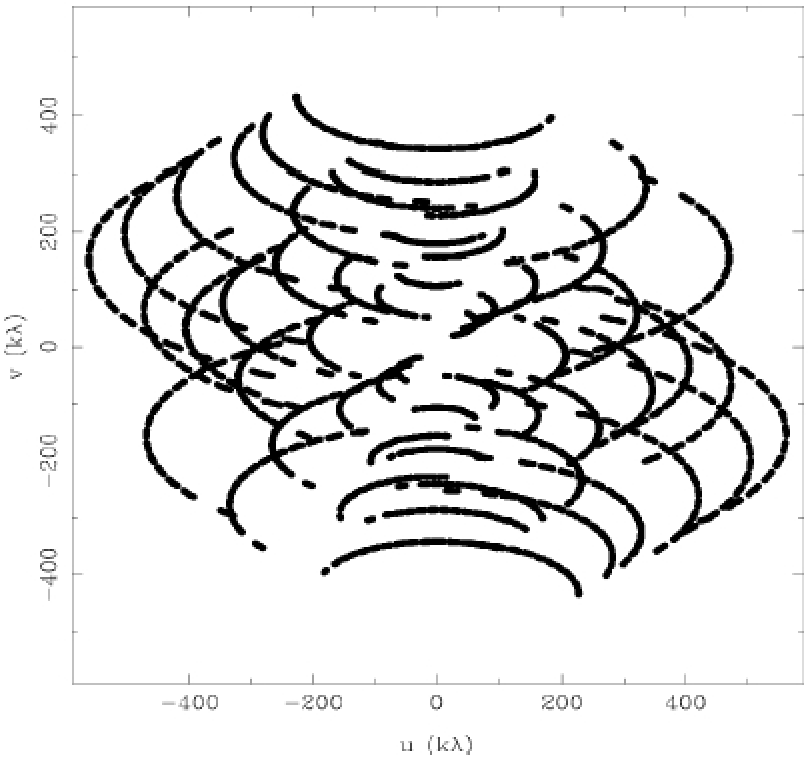


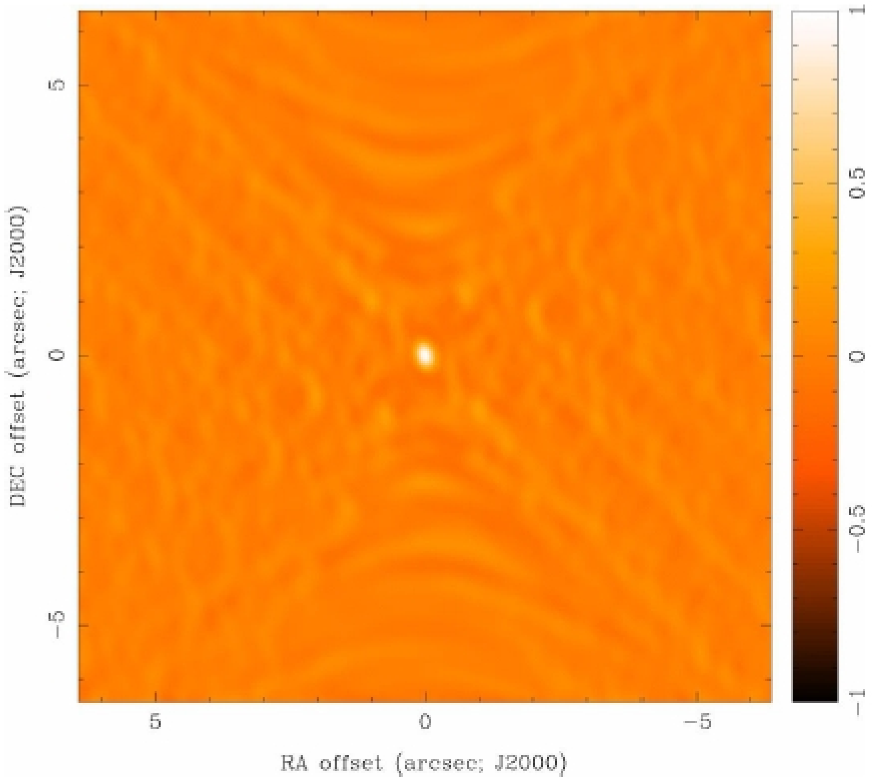


Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

Dirty Beam Shape and N Antennas

8 Antennas x 480 samples





Courtesy David J. Vilner, Harvard-Smithsonian Center for Astrophysics, USA

So what do we finally have?

- $B^S(\theta,\varphi) = FT\ (S(u,v) \times V(u,v))$
- From convolution theorem

  $B^S(\theta,\varphi) = P(\theta,\varphi) \otimes B(\theta,\varphi)$

  $\otimes$ - convolution

  $P(\theta,\varphi) = FT\ S(u,v)$; $B(\theta,\varphi) = FT\ V(u,v)$

The FT of sampled visibilities gives the True sky Brightness distribution convolved with the Point Spread Function.

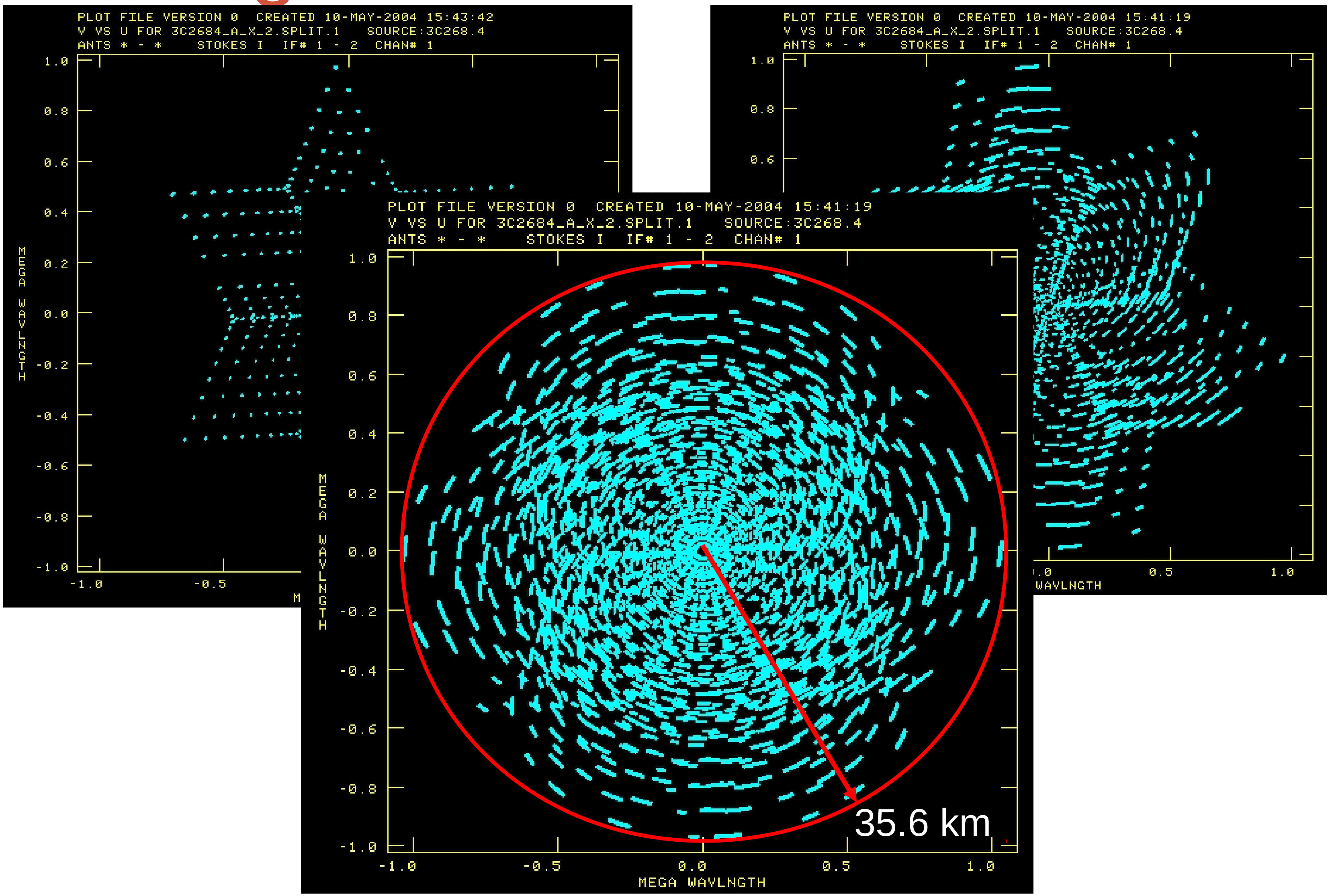
‘***Dirty image***’ is True image convolved with the ‘***Dirty beam***’.

A real life example

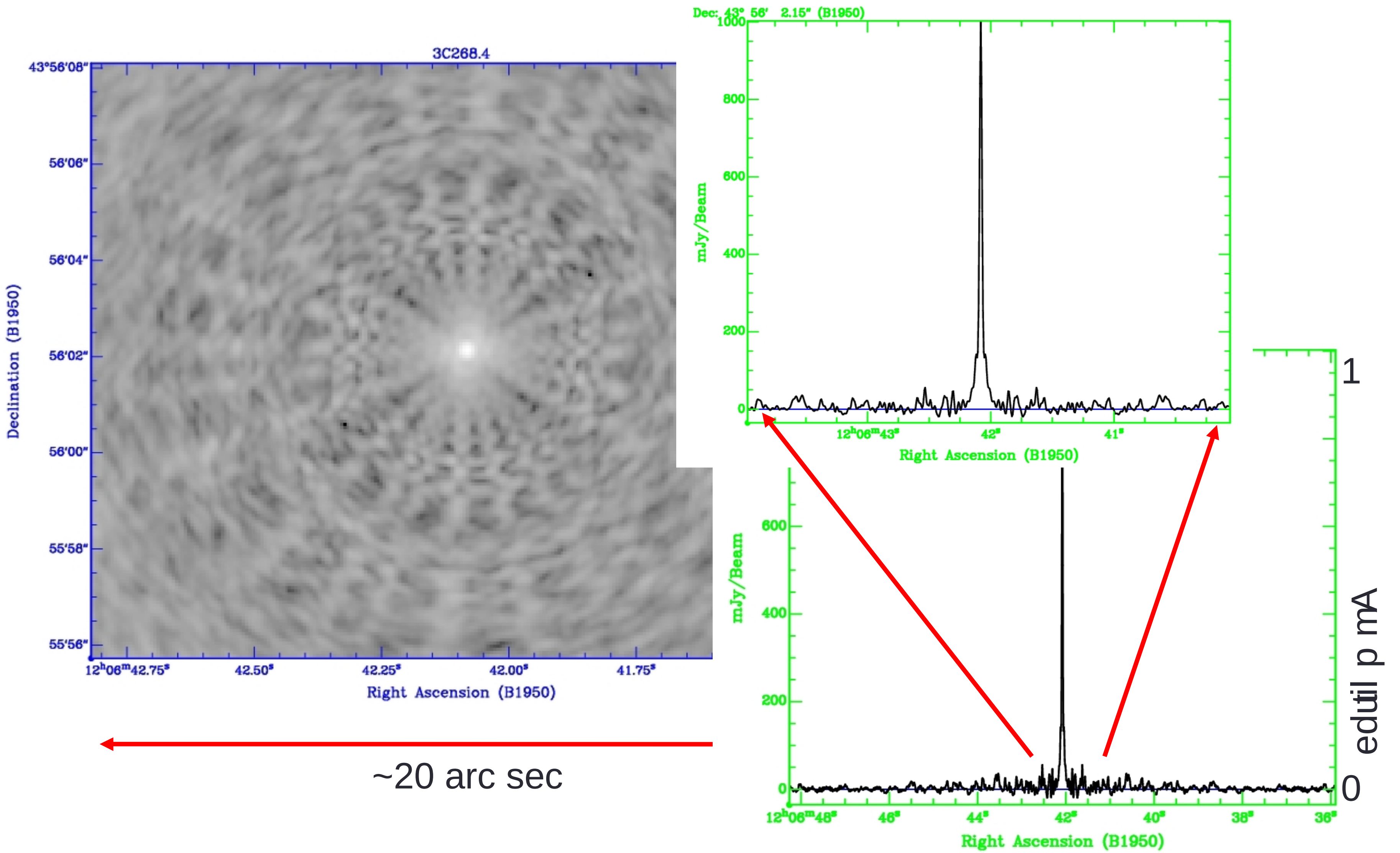
- The Very Large Array (VLA), NM
- 8.43 GHz (λ = 3.56cm)
- 3C268.4

- Data courtesy Colin Lonsdale, MIT Haystack Observatory

Array configuration and u-v coverage



The interferometer response function (Point Spread Function)



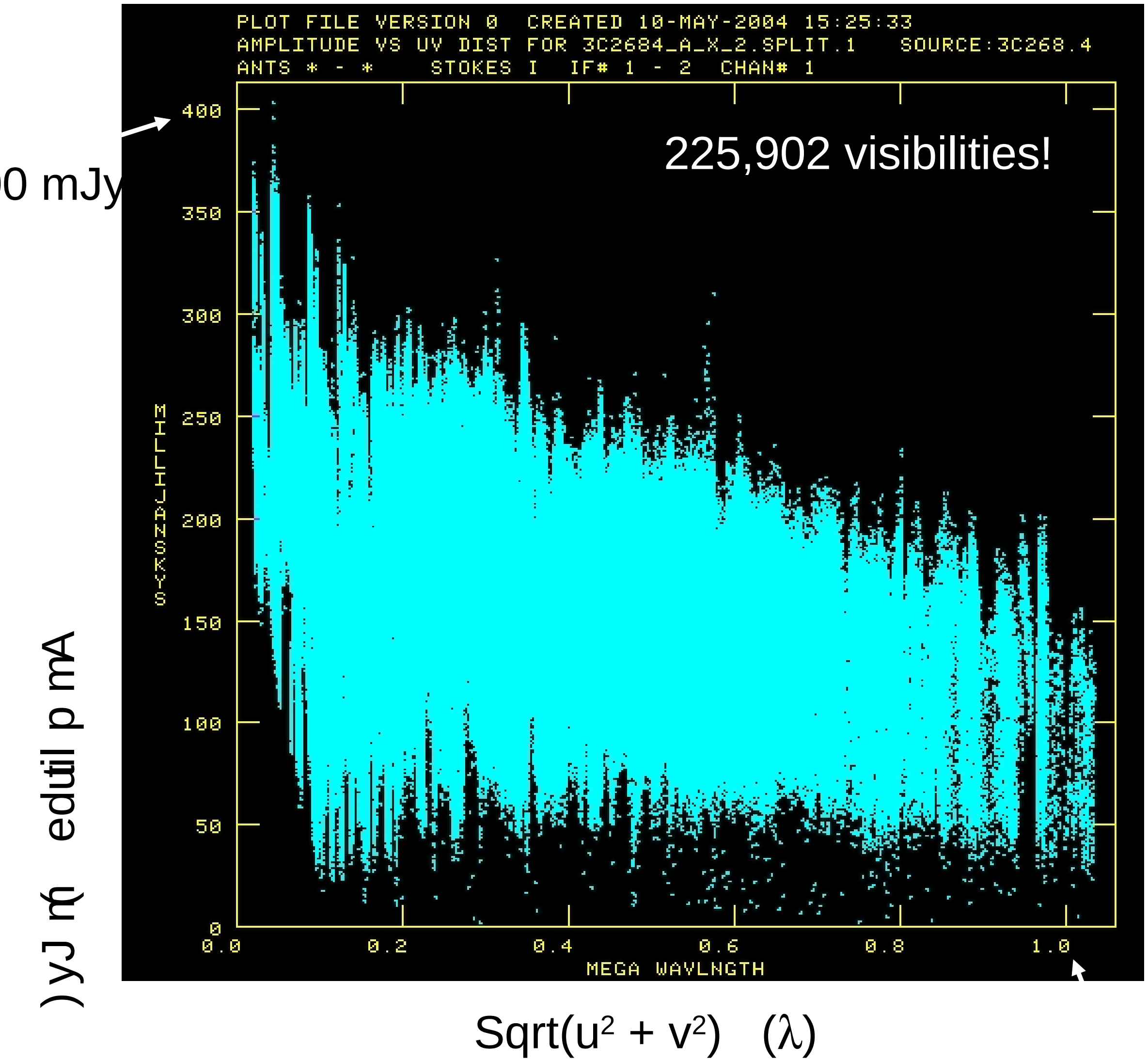
The measured cross-correlations

A typical FM radio station ~0.1 W Hz^{-1} placed at the distance of the Sun ($1.5x10^8$ km)
⇒~35 Jy at Earth

VLA sensitivity at 8 GHz ~$45x10^{-6}$ Jy (10 min, 86 MHz)

In 10 min VLA can detect a source as strong as a typical FM station ~88 AU away!

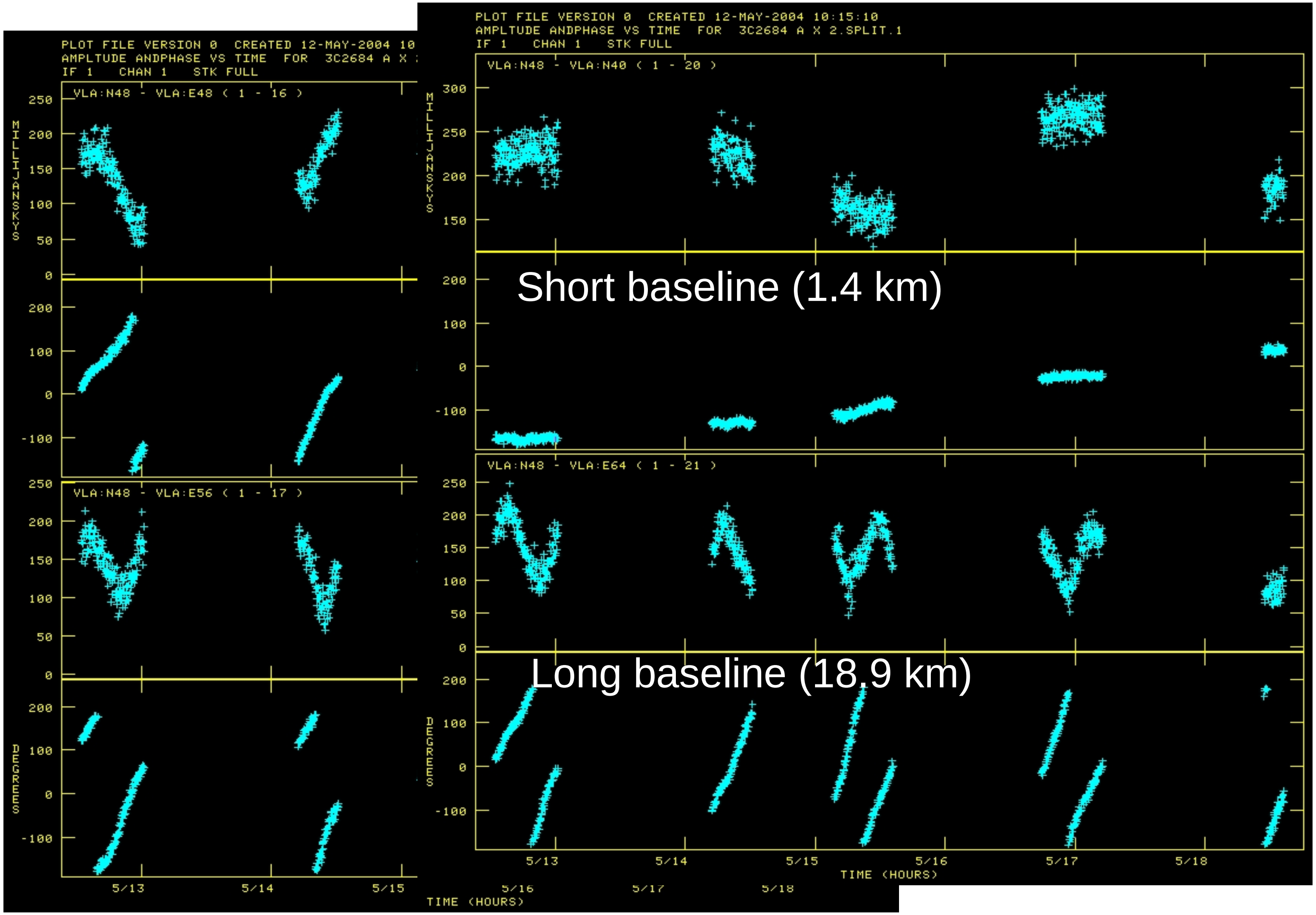
1 Jy = 10^{-26} W m^{-2} Hz^{-1}



The cross-correlations…

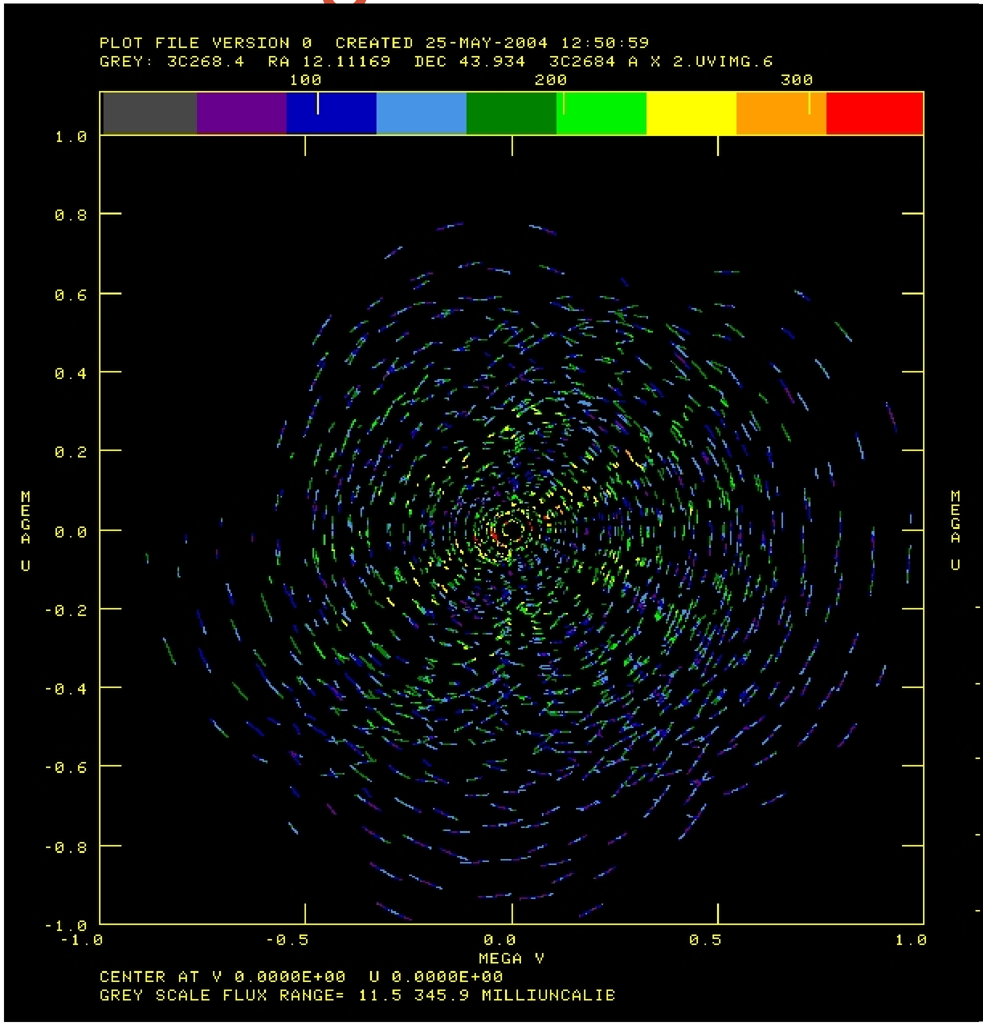
250 mJy

360 deg

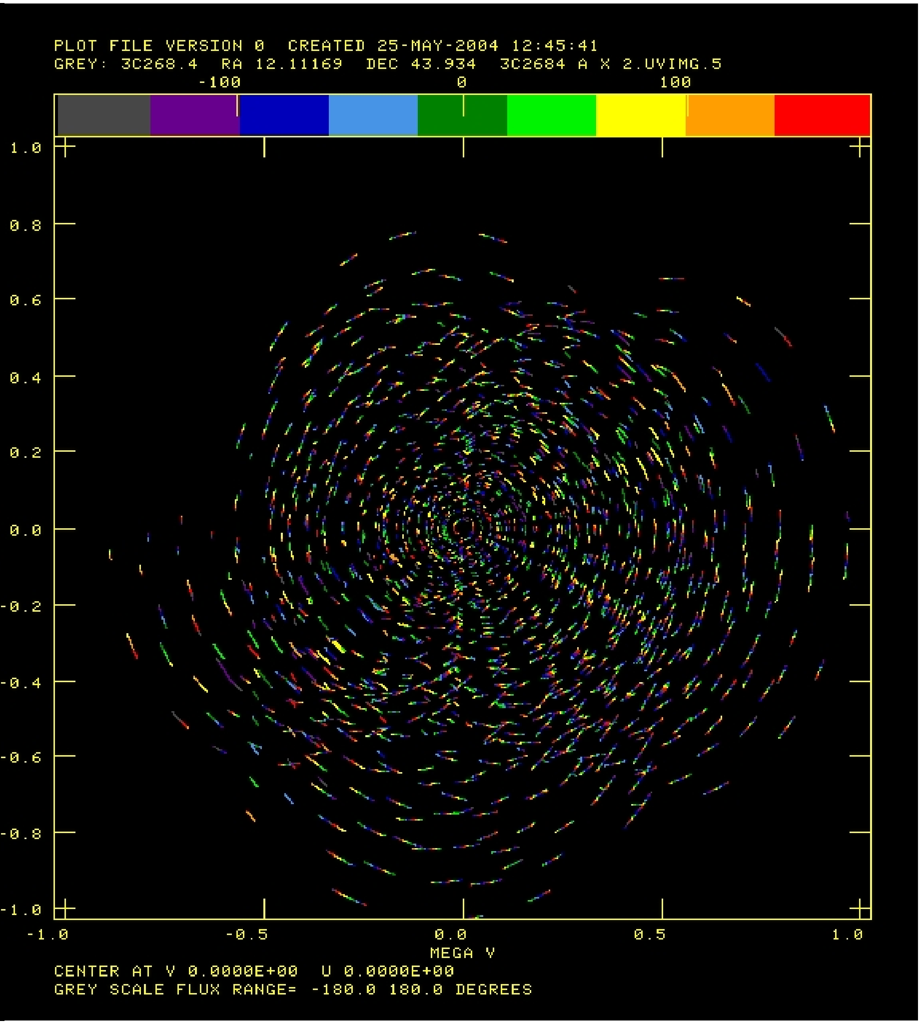


Time (hours) ~7 hrs

The gridded visibilities



Amplitude

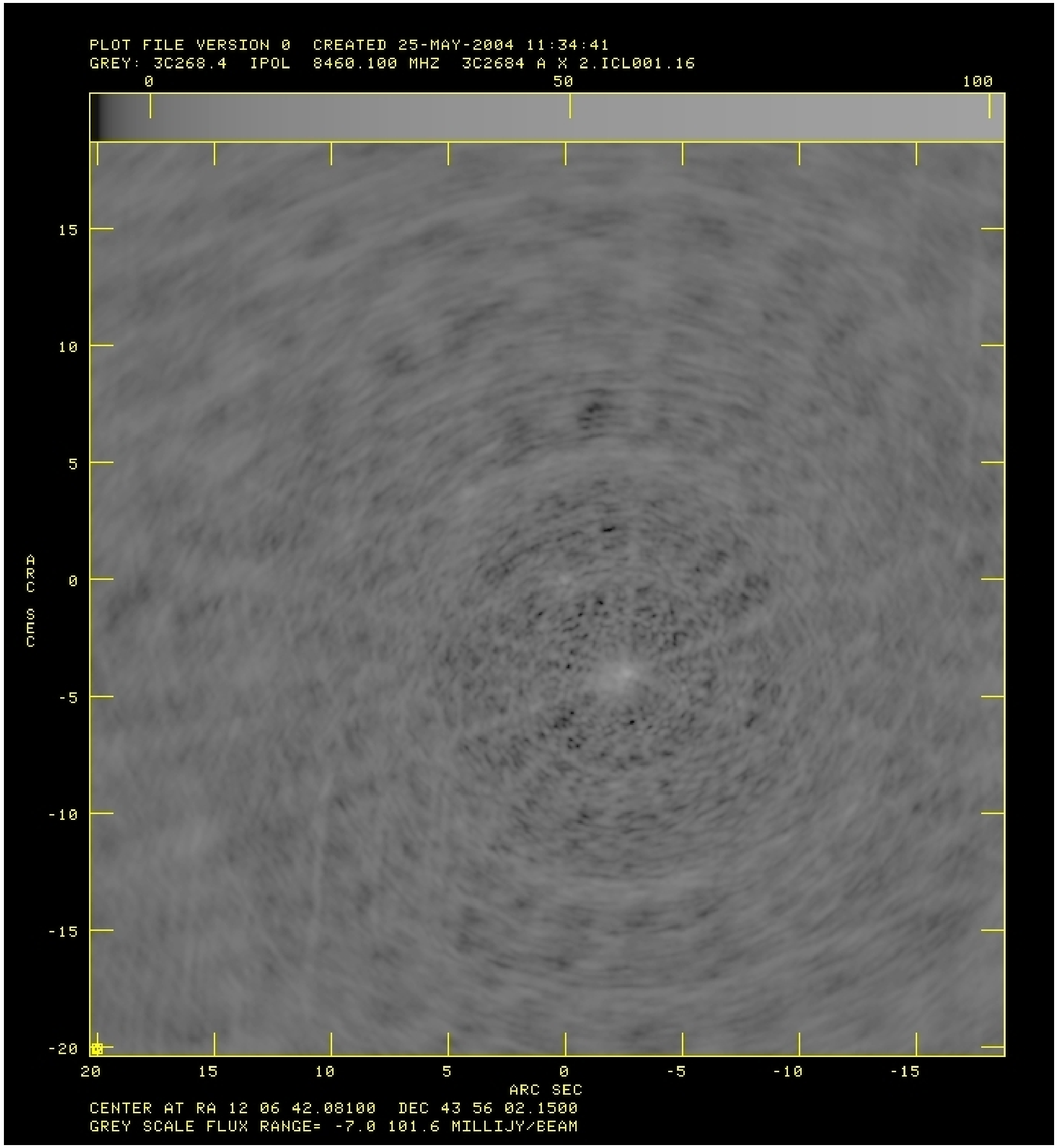


Phase

FT of gridded visibilities

The *dirty* map

Convolution of the PSF with the Brightness distribution



Log scale

The problem of deconvolution

- The measurements from any instrument are really the *convolution* of the *transfer function* of the instrument and the input signal.
- In order to figure out the true input signal, it is necessary to *deconvolve* the *transfer function* from the measurements
- Radio Astronomy solutions
  - CLEAN algorithm(s)
  - Maximum Entropy Method(s)

The CLEAN approach

- Assumption – Astronomical sources can be represented as a sum of discreet point sources
  - Locate the brightest point in the map
  - Subtract a PSF of amplitude ($0<x<I_0$) centered at the brightest pixel and note down the strength and the location of the PSF subtracted
  - Loop over subtracting sources till the strength of the brightest pixel drops to the noise level
  - The final map is the collection of all the point sources which had been subtracted with the residual noise from the dirty map added to it

The *CLEANed* map

Actually, CLEANed and *Self-calibrated* map

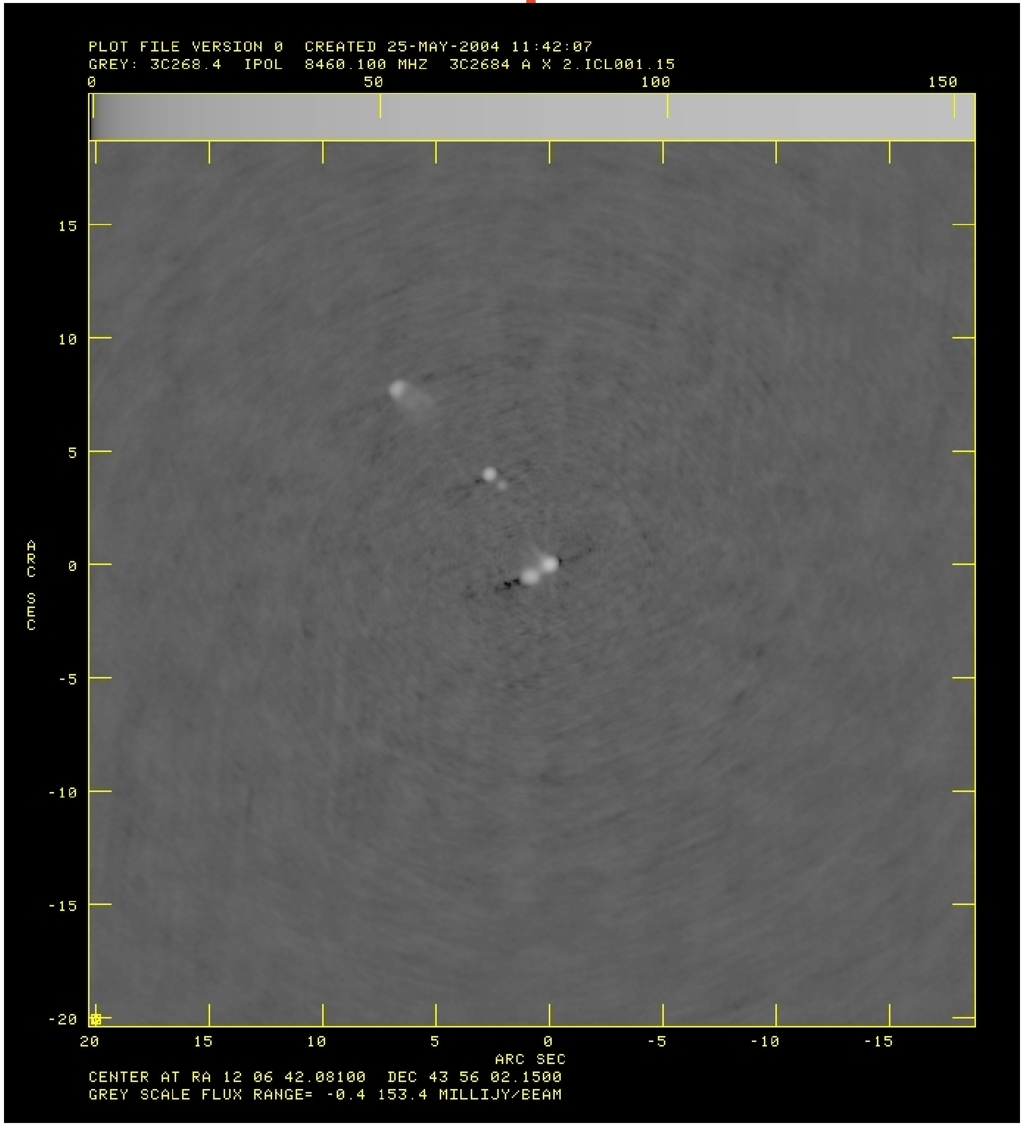
~50,000 Clean iterations

~4000 Clean components

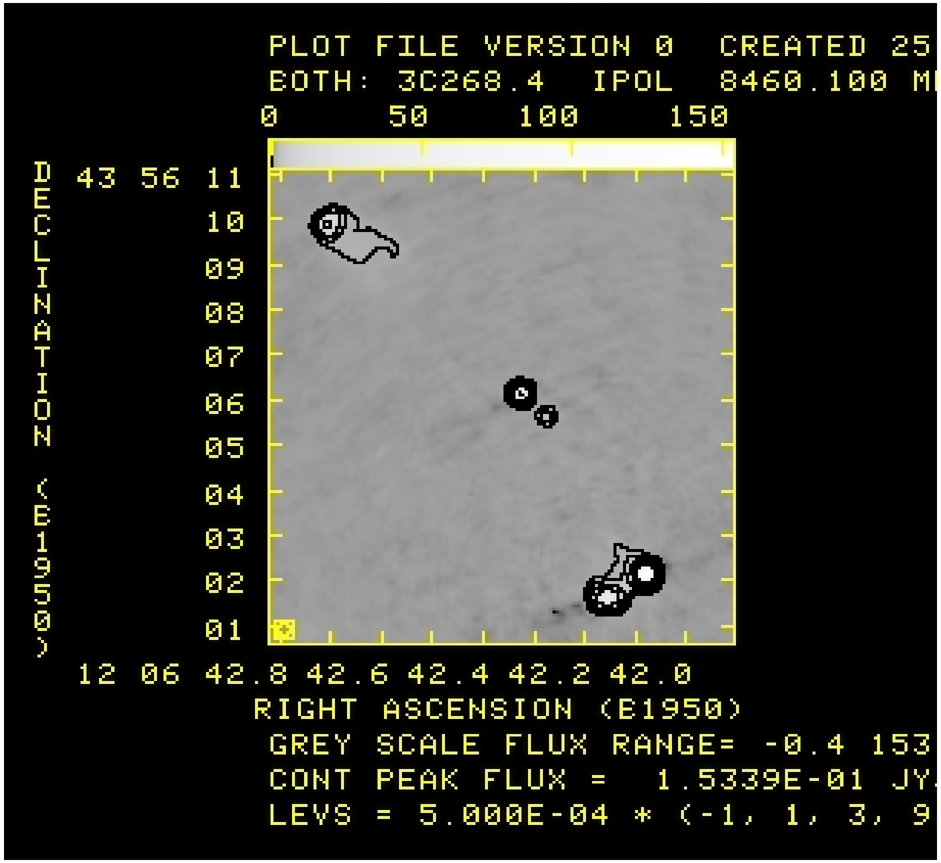
Dynamic range ~5000

Noise ~30 μJy/beam

Log scale



A comparison with other results



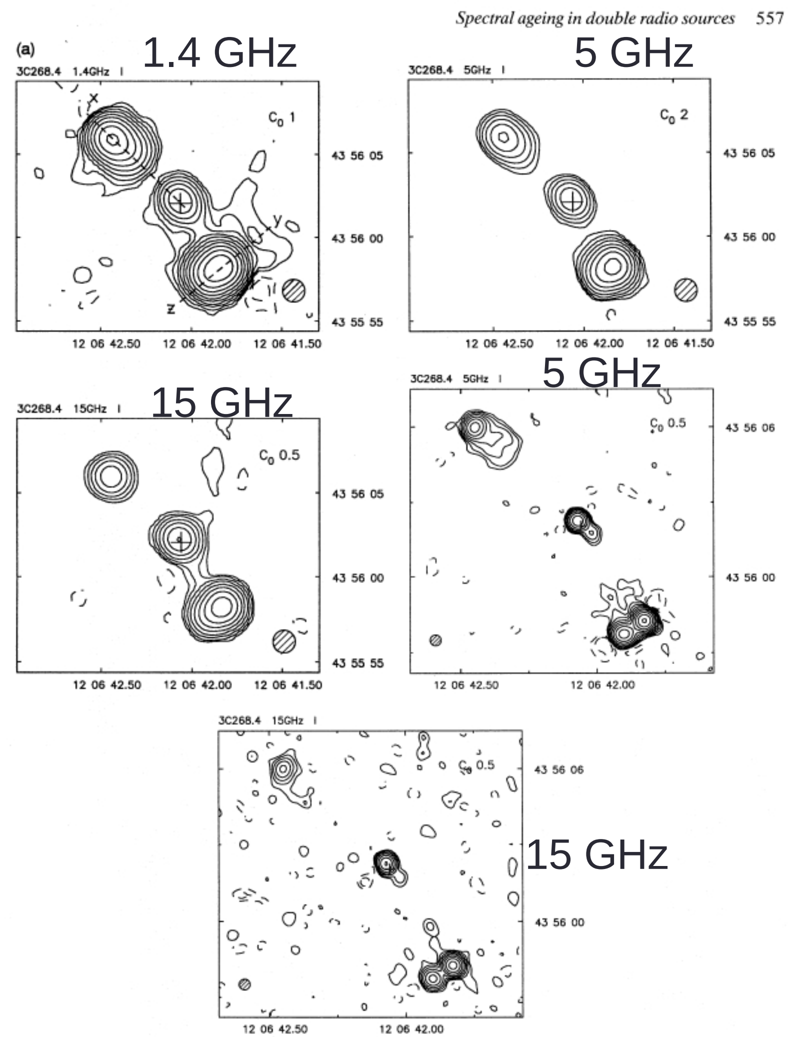
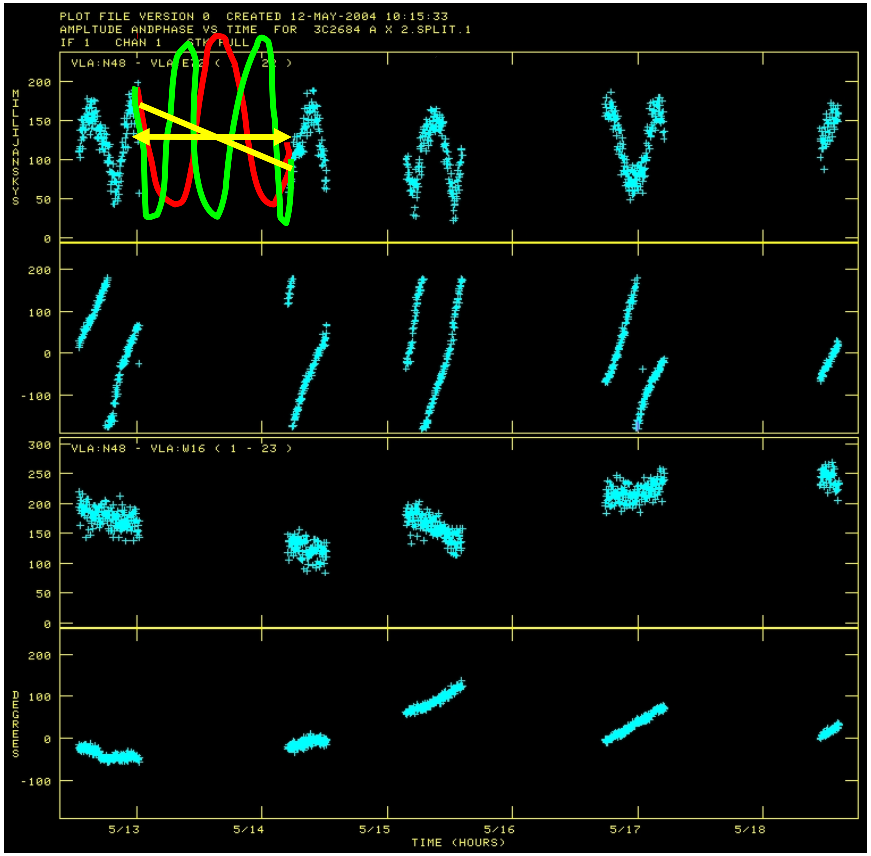


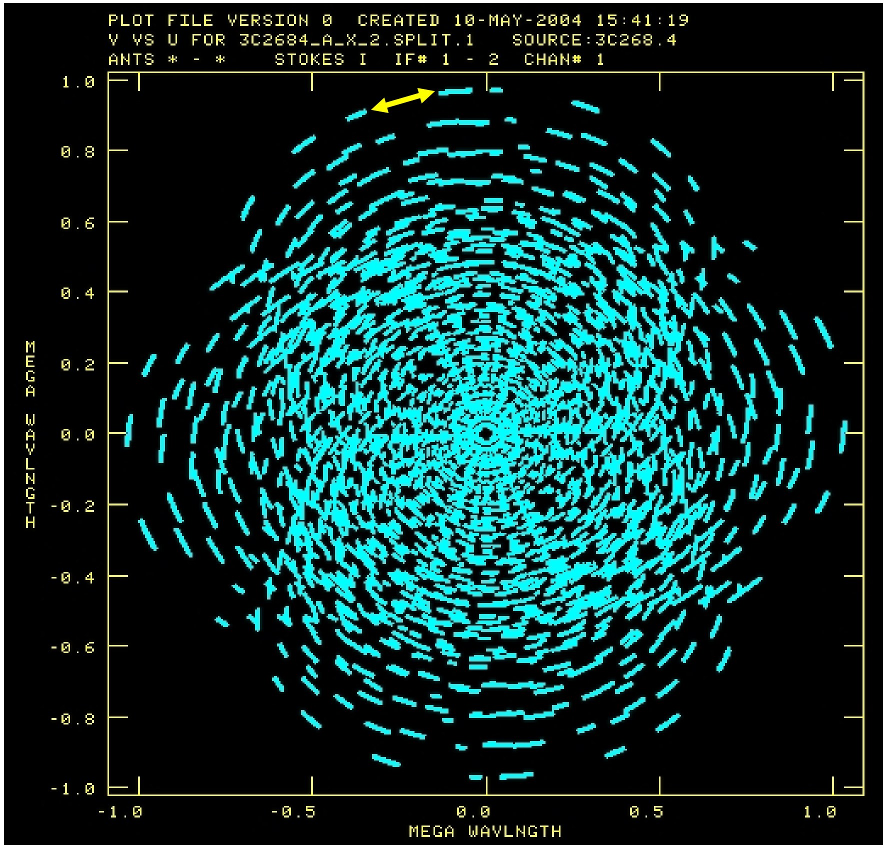
Figure 10. (a) Total intensity maps and (b) strip profiles of total intensity at 1.4 GHz, spectral index and age along the lobe axes indicated by the letters in (a), for 3C268.4. See the caption to Fig. 4 for further details.

Some caveats about radio imaging

- Like optical images, the size of the synthesised aperture (lens, mirror) limits the resolution
- In addition, images are made using an *incompletely filled lens* $\Rightarrow$ some of the information is missing
- The imaging process interpolates or extrapolates to fill in this missing information
- Amounts to fabricating data in absence of measurements!
- Implications
  - Images are consistent with data but not necessarily unique
  - Imaging process also might lead to some artifacts in the image (recognisable)

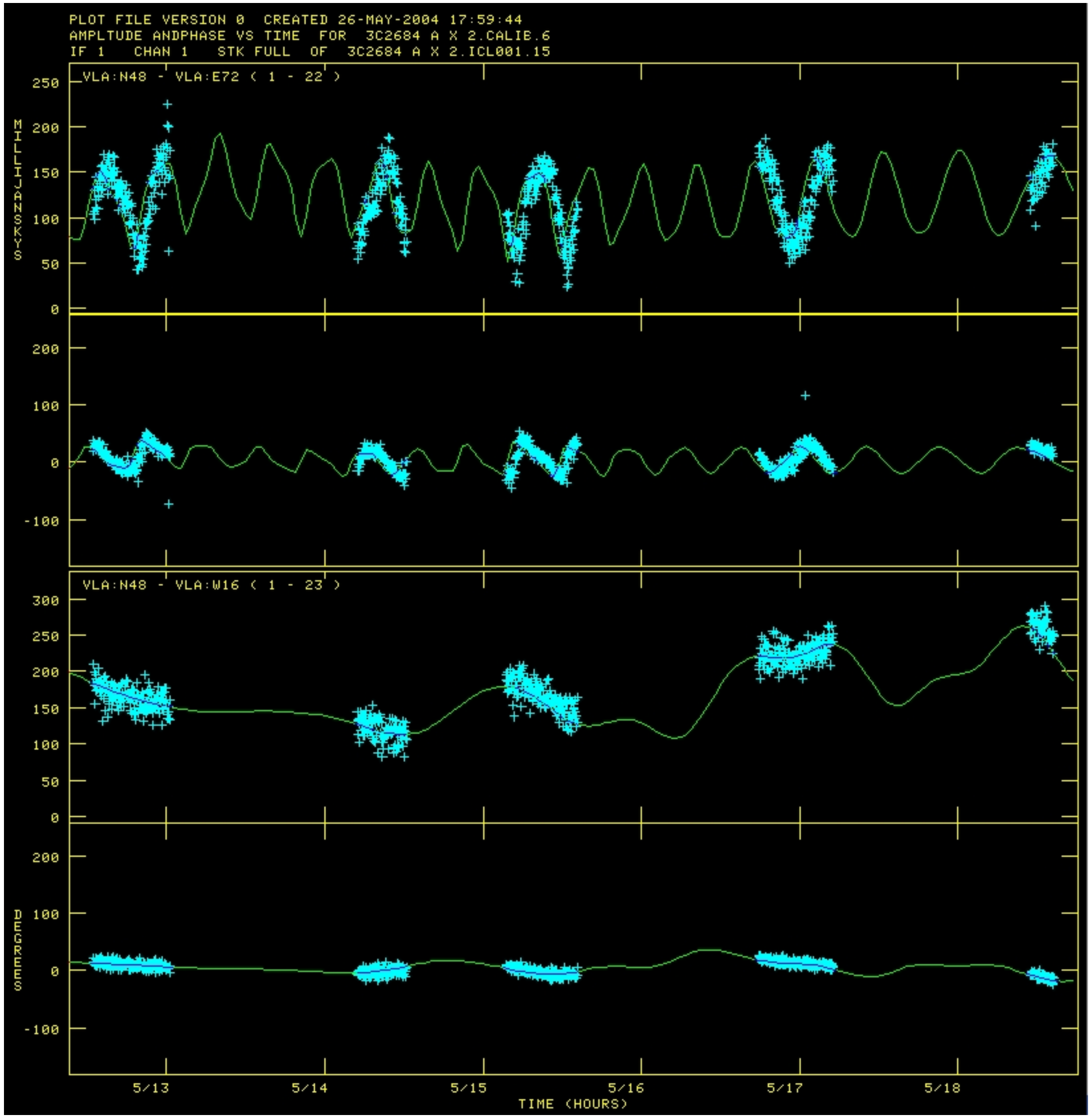
Caveats contd.



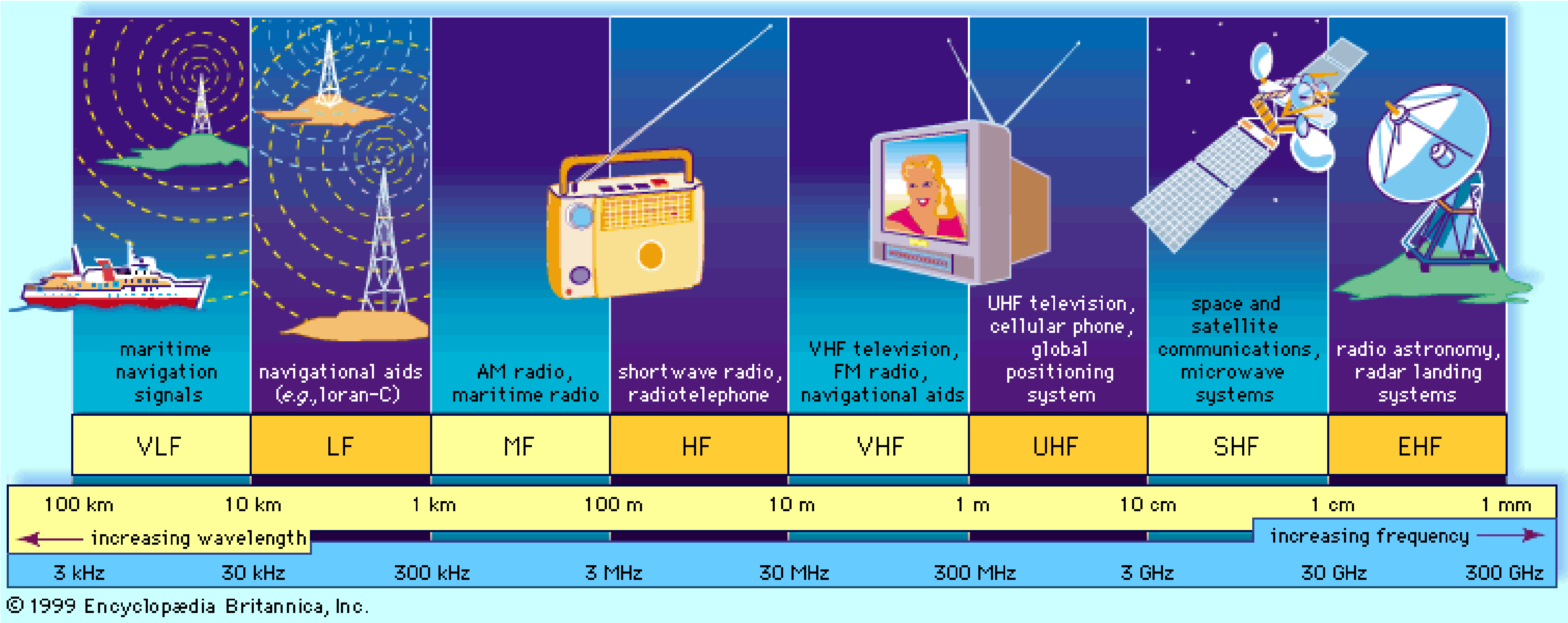


The CLEAN model

Actually, clean + self calibration model



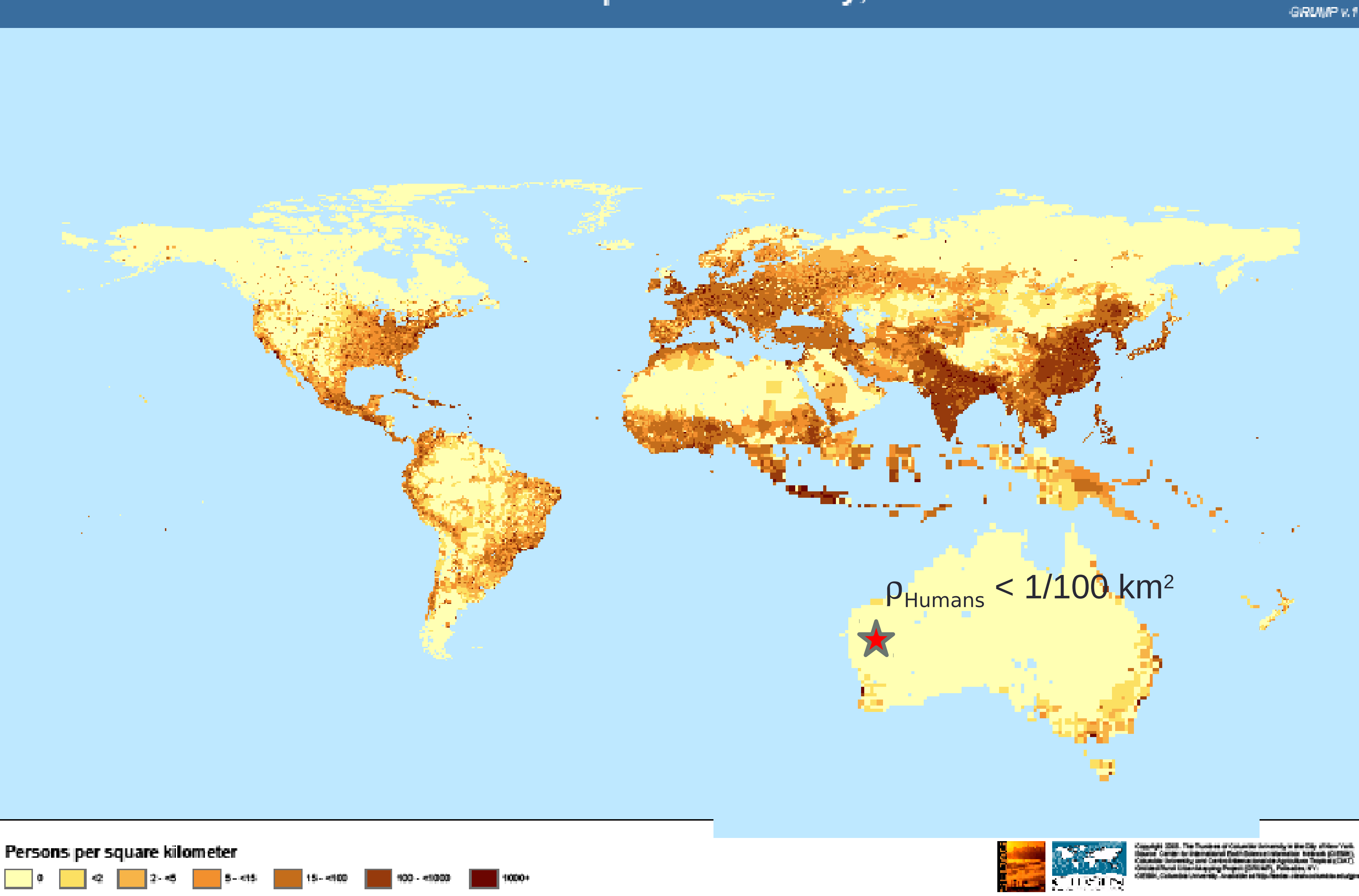
Radio analog of dark-sky problem



Human presence = radio pollution

Cell phones, chord less phones, garage door openers, keyless entry systems, computers, florescent lights, petrol vehicles, mircowave ovens, bluetooth devices,

The World: Population Density, 2000



$\rho_{Humans} < 1/100\ km^2$

Looking ahead …

- The new/next gen. of radio telescopes (incomplete list)
  - Square Kilometre Array (Australia, South Africa)
  - Low Frequency Array (The Netherlands, Europe)
  - Murchison Widefield Array (Australia)
  - Australian SKA Pathfinder (Australia)
  - MEERKAT (South Africa)
  - Jansky Very Large Array (US)
  - Atacma Large Millimetre/Submillimetre Array (Chile)
  - Upgraded GMRT (India)
- 1-2 orders of magnitude improvements in sensitivity and imaging fidelity $\Rightarrow$ active research in calibration and imaging algorithms
- Systematic explorations of the low frequency part of the spectrum (< few 100 MHz) – unprecedented – new phenomenon, new objects, discovery potential
- Exciting diverse new science – Cosmology, early universe, transients, studies, solar and heliospheric science, …

References

- Radio Astronomy, J. D. Kraus, 1986, Cygnus-Quasar Books, Powell, Ohio
- Tools of Radio Astronomy, 2nd edition, 1996, K. Rohlfs and T. L. Wilson, Springer-Verlag, Berlin
- Interferometry and Synthesis in Radio Astronomy – A. R. Thompson, J. M. Moran and G. W. Swenson, 2001, Wiley Interscience, New York
- Synthesis Imaging in Radio Astronomy II, Ed. G. B. Taylor, C. L. Carilli and R. A. Perley, 1999, Astronomical Society of the Pacific Conference Series, Vol 180, San Francisco
- Low Frequency Radio Astronomy, 3rd Ed., Eds.