Astronomical Techniques II
Lecture 12 - Self-Calibration and some more

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Calibration errors - impact on the image

1. The origin of calibration errors
2. The impact of calibration errors
    1. Deconvolution equation - no longer valid
3. Sidelobes of the PSF $\sim \dfrac{1}{\sqrt{N(N-1)}}$
4. Show up as increased RMS in the map

Problems with ordinary calibration

1. Rely on frequent observations of radio sources of known structure, position and strength to determine the calibration solutions
2. $\tilde{V}_{i,j}(t) = g_i(t)\ g_j^*(t)\ G_{i,j}(t)\ V_{i,j}(t) + \mathcal{E}_{i,j}(t) + \epsilon_{i,j}(t)$
3. $\tilde{V}_{i,j}(t) = g_i(t)\ g_j^*(t)\ V_{i,j}(t) + \epsilon_{i,j}(t)$
4. $g_i(t)$
   1. Instrumental part (KJy^{-1}, $SEFD$) - slowly varying
   2. Propagation part (troposphere and ionosphere) - faster varying
5. Drawbacks
   1. $g_i(t)$s come from a time and direction different from that of interest!
   2. Residual errors (frequency and baseline length dependent) remain
   3. For stronger sources - dominate the error budget
   4. Strength of the available calibrator
   5. Presence of any resolved structure or other confusing sources

The idea of *self-calibration*

1. Allow the element gains to be free parameters in the imaging process
2. Impact of self-calibration
    1. Constraints = No. of measured visibilities = $N(N-1)/2$
    2. Instrumental DoF = N
    3. Constraints available for the emission in the sky = $N(N-1)/2 - N$ (amplitude) and $N(N-1) - (N-1)$ (phase)
    4. Loss of information of absolute position of the source
    5. Loss of information of absolute strength of the source

Redundant Calibration

1. Consider a 1-D array of N elements with uniform spacing between antennas, *d* (Westerbork, Ooty)
2. Redundant measurements in the *uv* plane for all but the longest baseline
    1. Overdetermined - Of the N(N-1)/2 measurements, only N-1 are independent

Self-calibration

1. Basic premise - even after including the additional DoF of element gains, the job of estimation of an adequate model for $I(l, m)$ is still overdetermined
2. Similar to Clean - use plausible assumptions about $I(l, m)$ to interpret measured visibilities
3. Objective - Deduce $\hat{I}$, the FT of which, $\hat{V}$, after correction for instrumental gains is consistent with the measured visibilities.
4. $S = \sum_k \sum_{i,j \quad i \neq j} w_{i,j}(t_k) |\tilde{V}_{i,j}(t_k) - g_i(t_k) g_j^*(t_k) \hat{V}(i,j)(t_k)|^2$
5. $S = \sum_k \sum_{i,j \quad i \neq j} w_{i,j}(t_k) |\hat{V}_{i,j}(t_k)|^2 |X_{i,j}(t_k) - g_i(t_k) g_j^*(t_k)|^2$, where
$$X_{i,j}(t_k) = \frac{\tilde{V}_{i,j}(t_k)}{\hat{V}_{i,j}(t_k)}$$

Self-calibration - practical implementation

1. Make an initial model of the source, $\hat{I}$
2. Use the previous equation to convert it into a point source model
3. Solve for g_is
4. Compute the corrected visibilities

$$V_{i,j,corr}(t) = \frac{\tilde{V}_{i,j}(t)}{g_i(t)\; g_j^*(t)}$$

5. Build a new model using $Vi, j, corr(t)$
6. Iterate till satisfied

Closure Phase and Amplitude

1. $\tilde{\phi}_{i,j}(t) = \phi_{i,j}(t) + \theta_i(t) - \theta_j(t) + noise$, where

   $\theta_i(t) = arg\ g_i(t)$
2. $\tilde{C}_{i,j,k} = \tilde{\phi}_{i,j}(t) + \tilde{\phi}_{j,k}(t) + \tilde{\phi}_{k,i}(t)$
3. $\tilde{C}_{i,j,k} = \phi_{i,j}(t) + \phi_{j,k}(t) + \phi_{k,i}(t) + noise$
4. $\tilde{C}_{i,j,k} = C_{i,j,k} + noise$
5. $\Gamma_{i,j,k,l} = \dfrac{|\tilde{V}_{i,j}(t)||\tilde{V}_{k,l}(t)|}{|\tilde{V}_{i,k}(t)||\tilde{V}_{j,l}(t)|}$
6. Iterative least-squares techniques to make $\hat{V}_{i,j}(t)$ consistent with $\tilde{V}_{i,j}(t)$
7. It can formally be shown that *self-calibration* is equivalent to using clsoure quantities (Cornwell and Wilkinson, 1981)

Misc.

1. Relationship with Apadtive Optics
2. Why does *self-cal* work?
   1. Most successful for dense *uv* coverages for arrays with largeish N (few tens) and good SNR
   2. Sources are simple and can be represented by a small number of DoF
   3. For a large N interferometer, it still remains a vastly over determined problem
   4. No formal proof of convergence of self-calibration is available

Driving *self-cal*

1. Initial model - usual calibration and subsequent imaging - good enough
2. Model *must not* contain any features due to calibration errors
3. Images at near by frequencies, higher/lower resolutions useful
4. One can even start from a point source model and slowly move towards a detailed model of a source which is many many resolution elements across
5. Prudent to solve only for phases to begin with
6. Use of weighting schemes
7. Choice of averaging time
8. Non-uniqueness of images

Baseline based errors

1. Random time varying pointing errors (jitter)
2. Non-isoplaneticity in the ionosphere
3. Departures of the primary beam from the reference primary beam
4. Correlator problems (bias, incorrectly set sampling levels)
5. Local RFI

A formalism for 3-D imaging

1. $$V(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\mathcal{A}(l,m)\, I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm+w(\sqrt{1-l^2-m^2}-1))}\, dl\, dm$$

2. $$V(u,v,w)\, e^{-2\pi i w} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\mathcal{A}(l,m)\, I(l,m)}{\sqrt{1-l^2-m^2}} \delta(n - \sqrt{1-l^2-m^2})\, e^{-2\pi i(ul+vm+wn)}\, dl\, dm\, dn$$

1. $$I^{D(3)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(u,v,w)\, S(u,v,w) e^{-2\pi i w} e^{2\pi i(ul+vm+wn)} du\, dv\, dw$$

2. $I^{D(3)} = I^{(3)} \bigstar B^{D(3)}$ where,

$$I^{(3)}(l,m,n) = \frac{\mathcal{A}(l,m)\, I(l,m)}{\sqrt{1-l^2-m^2}}\, \delta(n - \sqrt{1-l^2-m^2})$$