Astronomical Techniques II
Lecture 10 - Imaging

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Fourier Imaging

1. $\mathcal{A}(l, m)\ I(l, m) =$
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} V(u, v)\ e^{-2\pi i(ul+vm)} d\nu\ du\ dv$$
2. Assumptions
   1. $|\frac{\Delta\nu}{c}\vec{b}.(\vec{s} - \vec{s_0})| << 1$
   2. $|w(l^2 + m^2)| << 1$
3. $V(u, v)$ - $W\ m^{-2}\ Hz^{-1}$
4. $I(l, m)$ - $Jy\ beam^{-1}$
5. In reality we only have $V(u_k, v_k)$

Fourier Imaging

1. $I^D(l,m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)\ V'(u,v)\ e^{-2\pi i(ul+vm)} du\ dv$

   where

   $S(u,v)$ - Sampling Function

2. DFT
   1. $I^D(l,m) = \frac{1}{M}\sum_{k=1}^{M} V'(u_k,v_k)e^{2\pi i(u_k l + v_k m)}$
   2. Computational cost for a $N \times N$ image - $O(N^4)$

FFT Imaging

1. Requires regular gridding
2. Computational cost - $O(N) \times O(N\ logN) = N^2\ logN$

The sampling function

1. $S(u, v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)$
2. $V^S(u, v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)\ V'(u_k, v_k)$
3. $V^S = S\ V'$
4. $I^D = \mathcal{F} V^S$
5. $I^D = \mathcal{F} S \bigstar \mathcal{F} V'$

Weighting Functions - controlling the beam shape

1. $W(u,v) = \sum_{k=1}^{M} R_k \ T_k \ D_k \ \delta(u-u_k, v-v_k)$
2. $V^W(u,v) = \sum_{k=1}^{M} R_k \ T_k \ D_k \ \delta(u-u_k, v-v_k) \ V'(u_k, v_k)$
   1. R_k - Reliability $\frac{T_{sys}}{\sqrt{\Delta t \ \Delta\nu}}$
   2. T_k - Tapering function
   3. D_k - Density weighting function

The Tapering Function

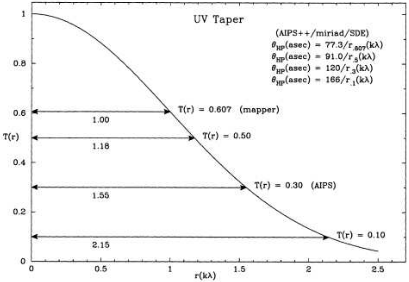


Figure 7–1. A Gaussian (u, v) taper with dispersion $\sigma = 1$ km.

1. Usually $T_k = T(u_k, v_k) = T(u_k)T(v_k) = T(r)$
2. Most useful when the relevant part of the (u, v) plane is densely sampled and is not truncated by the edge of the (u, v) plane
3. Inner (u, v) limit

Impact of tapering on the PSF

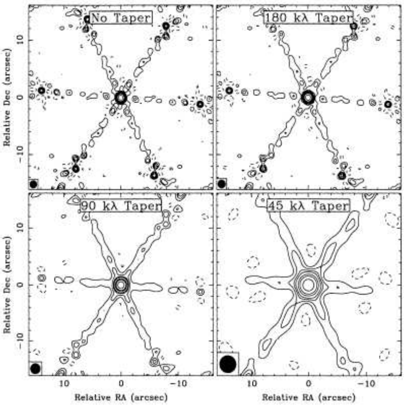


Figure 7–2. The effect of a Gaussian taper on the point source response of a VLA snapshot in the **A** configuration at 20-cm wavelength. As a narrower Gaussian taper (i.e., a heavier tapering) is applied, the half-power width of the point spread function increases and the inner sidelobes are reduced.

Density weighting function

1. Natural: $D_k = 1$
2. Uniform: $D_k = \frac{1}{N_s(k)}$
3. Robust:

PSF Weighting Example

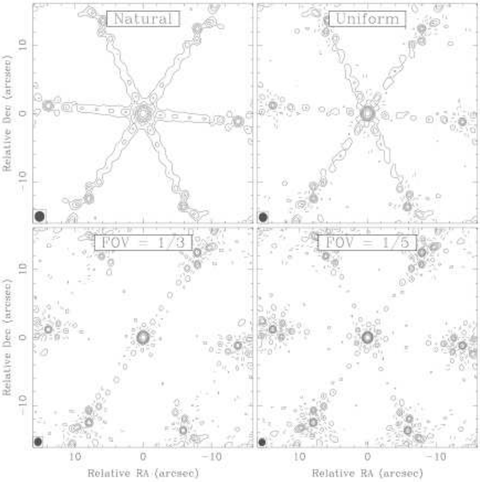


Figure 7–3. The effect of different weighting functions on a VLA 'snapshot' image of a point source.

Trade-offs involved

1. High resolution
2. Sensitivity
3. Good sidelobe performance (low sidelobe levels)

Gridding

1. Interpolation
2. Convolution
   1. Predictable impact on the images
   2. Convolve V^W with some C and then sample this convolution at centre of each cell of the *grid*
   3. $C = 0$, outside some small bounded region, A_C, support size.
   4. $\sum_{k=1}^{M} C(u_c - u_k, v_c - v_k)\ V^W(u_k, v_k)$
   5. $V^R = R\left(C \bigstar V^W\right) = R(C \bigstar (W\ V'))$, where

      $$R = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - \frac{u}{\Delta u},\ k - \frac{v}{\Delta v})$$

Dirty Image

1. $\tilde{I}^D = \mathcal{F} V^R$
2. $\tilde{I}^D = \mathcal{F} R \bigstar \left[(\mathcal{F} C) \left(\mathcal{F} V^W \right) \right]$
3. $\tilde{I}^D = \mathcal{F} R \bigstar [(\mathcal{F} C) (\mathcal{F} W \bigstar \mathcal{F} V')]$
4. $(\mathcal{F} R)(l, m) = \Delta u \, \Delta v \, \coprod(l \Delta u \, , m \Delta v) =$

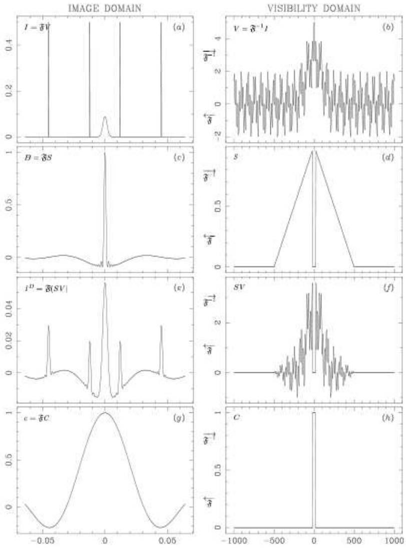
   $\Delta u \, \Delta v \, \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - l \Delta u, \; k - m \Delta v)$

   1. $\tilde{I}^D$ is periodic in l and m, with a period of $1/\Delta u$ and $1/\Delta v$, respectively
   2. Aliasing, due to convolution with the resampling function, $\mathcal{F} R$

Dirty Image

1. FFT generates one period of $\tilde{I}^D$
2. To image $N_l\ \Delta\theta_l$ rad, grid spacing should satisfy $N_l \Delta u = \dfrac{1}{\Delta\theta_l}$
3. $N_l \times N_m$ FFT yields a discreetly sampled version of $\tilde{I}^D$.
4. Primary field of view - $|l| < N_l\ \Delta\theta_l/2$; $|m| < N_m\ \Delta\theta_m/2$
5. $c = \mathcal{F}C$
6. $\tilde{I}_c^D(l, m) = \dfrac{\tilde{I}^D(l, m)}{c(l, m)}$ - Corrected Dirty Image
7. $\tilde{B}_c^D(l, m) = \dfrac{\tilde{B}^D(l, m)}{c(l, m)}$ - Corrected Dirty Beam (PSF)

Imaging Process



Imaging Process