Discuss several topics:

- Standard calibration and imaging
  - (DI instrumental effects)
  - w/ DD instrumental and propagation effects
  - advanced image parameterisations

CALIBRATION AND IMAGING

- Standard calibration and imaging
  - (DI instrumental effects)
- w/ DD instrumental + propagation effects
  - correction for *w*-term and for PB
    - image plane correction
    - Fourier plane correction
    - pointing self-calibration
  - Mosaicing
- w/ advanced image parameterisation
  - multi-scale CLEAN (deconvolution)
  - multi-frequency synthesis (imaging)
  - full polarisation (Stokes) calibration and imaging

TELESCOPE SENSITIVITY

- Noise limit for imaging with interferometric radio telescopes

$$\sigma = \frac{T_{\mathrm{sys}}}{A_{\mathrm{eff}} \times \sqrt{(}\Delta\nu \times \Delta t)}$$

TELESCOPE SENSITIVITY

- Noise limit for imaging with interferometric radio telescopes

$$\sigma = \frac{T_{\rm sys}}{A_{\rm eff} \times \sqrt{(\Delta\nu \times \Delta t)}}$$

- Sensitivity improvements achieved by
  - wide band receivers,
  - long integration times
  - more antennas
  - long baselines

$$\sigma_{\rm confusion} \propto (\nu^{-2.7}/B_{\rm max}^2)$$

    - B_{max} ~100 km @200 MHz, the confusion noise is ~1 μJy beam^{-1}.

IMAGING CHALLENGES AT LOW FREQ.

- Wide-field imaging
  - account for direction dependent (DD) effects
  - PB: time, frequency and polarisation dependence
  - *w*-term
- Wide-band imaging
  - … plus frequency dependence of the sky brightness
- Data volume $\propto N_{\mathrm{ant}}^2 \times N_{\mathrm{channel}} \times t$
- Sky brightness $\Longrightarrow$ multi-scale deconvolution
- Ionospheric effects $\Longrightarrow$ need for DD solvers

IMPLICATIONS FOR IMAGING

- Long baselines $B_{\max} > 2 \text{ km} \implies \text{DR} > 10^4$
- Wide-field effects:
  - *w*-term, PB effects and ionosphere effects
- Larger data volume

- Wide-field, wide-band, high resolution, high dynamic range imaging using large data sizes
  - a natural consequence of low frequency and high sensitivity imaging.

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MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

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$$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\ dl\ dm$$

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DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,…

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

MEASUREMENT EQUATION

$$\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$$

mutual coherence function

complex amplitude of the radiation emanating from the source in the direction $\vec{s}$

$\vec{s} = \vec{s}_0 + \overrightarrow{\sigma}$ point near the phase centre

time difference between the incoming radiation collected at two antennas separated by $\overrightarrow{b}$

$$d\Omega = \frac{d\vec{s}}{R^2}$$

MEASUREMENT EQUATION

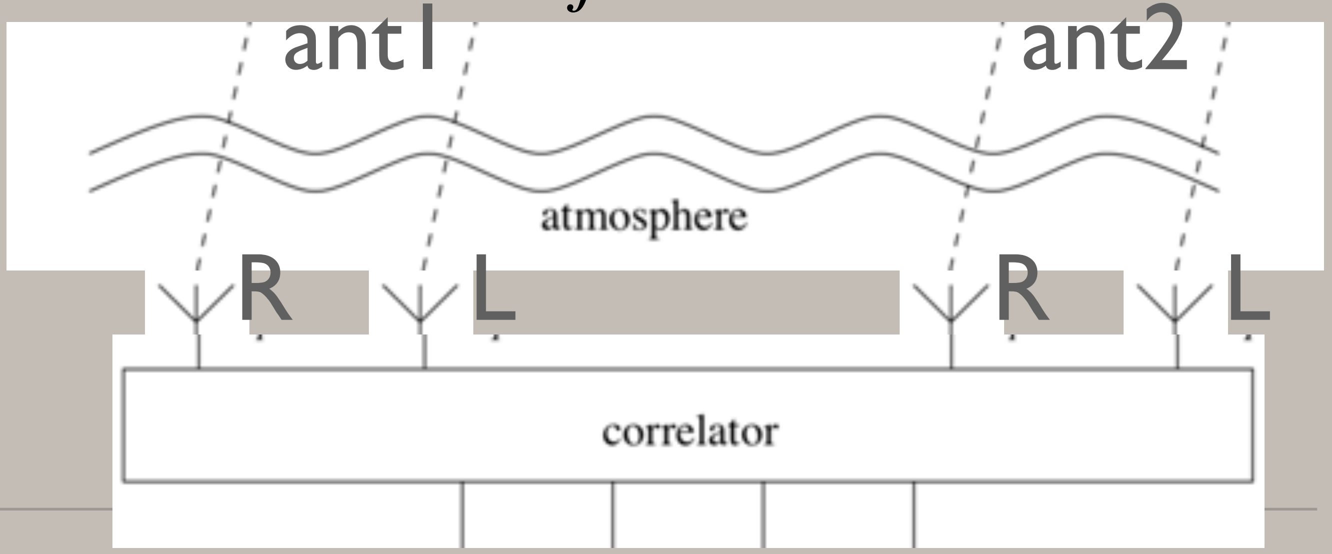
- $\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$
- $V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i (ul + vm + w(n-1))} \, dl \, dm$
  - (for $w \simeq 0, n \simeq 1$)
    - $V(u, v) = \int I(l, m) \; e^{-2\pi i (ul + vm)} \; dl \; dm$
    - (this is van-Cittert Zernike theorem)

MEASUREMENT EQUATION

- $\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$
- $V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i (ul + vm + w(n-1))} \, dl \, dm$
- Polarised radiation:
  - $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
    - (two nominal orthogonal components of incident electric field are measured at each antenna i)

MEASUREMENT EQUATION

- $$\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$$
- $$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i (ul + vm + w(n-1))} \, dl \; dm$$
- Polarised radiation:
  - $\overrightarrow{E_i} = [E^r \; E^l]_i^T$
  - (four cross-correlation products, $\langle \overrightarrow{E_i} \otimes \overrightarrow{E_j^*} \rangle$ per baseline)
  - $\overrightarrow{V_{ij}} = [V^{rr} \; V^{rl} \; V^{lr} \; V^{ll}]_{ij}^T$
  - $\vec{I} = [I^{rr} \; I^{rl} \; I^{lr} \; I^{ll}]^T$



MEASUREMENT EQUATION

- $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
- (Jones matrices describe this modulation for the electric field incident at each orthogonal pair of feeds $\overrightarrow{E_i} = [E^r \ E^l]_i^T$)
  - DI: $J_i^{vis} = [GDC]$
    - (a 2×2 matrix product)
      - complex gains, G,
      - polar'n leakage, D and
      - feed config'n, C.
  - DD: $J_i^{sky} = [EPF]$
    - (a 2×2 matrix product)
      - AIPs, E,
      - PA effects, P and
      - tropospheric / ionospheric effects, and Faraday R'n, F.

MEASUREMENT EQUATION

- $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
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  - DI: $J_i^{vis} = [GDC]$
  - DD: $J_i^{sky} = [EPF]$
    - $K_{ij}^{\{vis,\ sky\}} = [J_i \otimes J_j^{\dagger}]^{\{vis,\ sky\}}$
    - (effect on each baseline ij is described by the outer-product of these antenna-based Jones matrices, a 4×4 matrix!)
- $\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b} \cdot \vec{\sigma}/\lambda} d\Omega$

MEASUREMENT EQUATION

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

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Antenna PB

MT-MFS

non-coplanar
baselines
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- $$\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma} / \lambda} d\Omega$$

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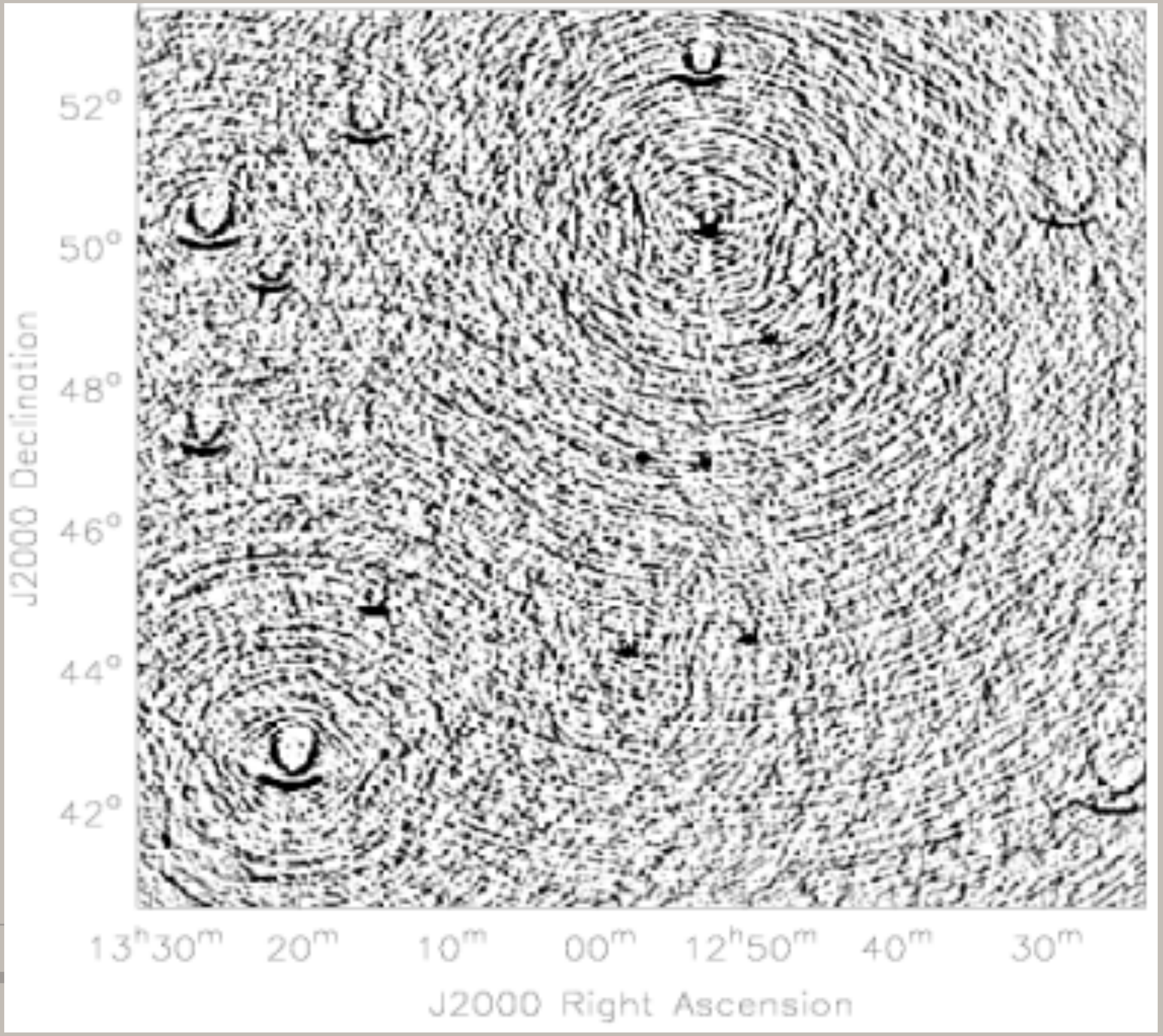
w-TERM

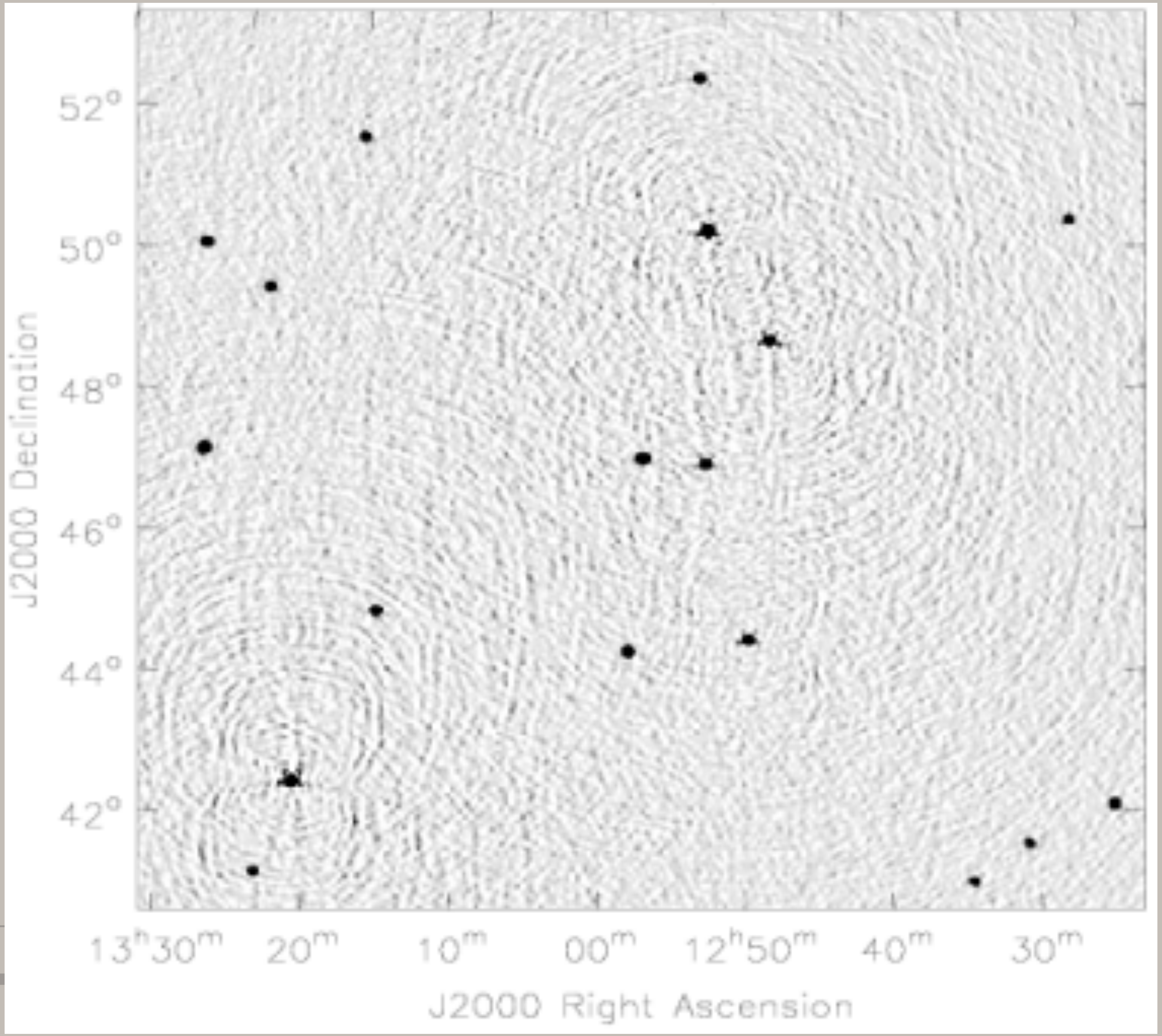
- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\; dm$$
- $$e^{\iota w\sqrt{1-l^2-m^2}}$$



Credits: S. Bhatnagar, synthesis imaging NRAO workshop

w-TERM

- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\ dl\ dm$$
- $e^{\iota w\sqrt{1-l^2-m^2}}$
- divide the FoV into a no. of FACETS



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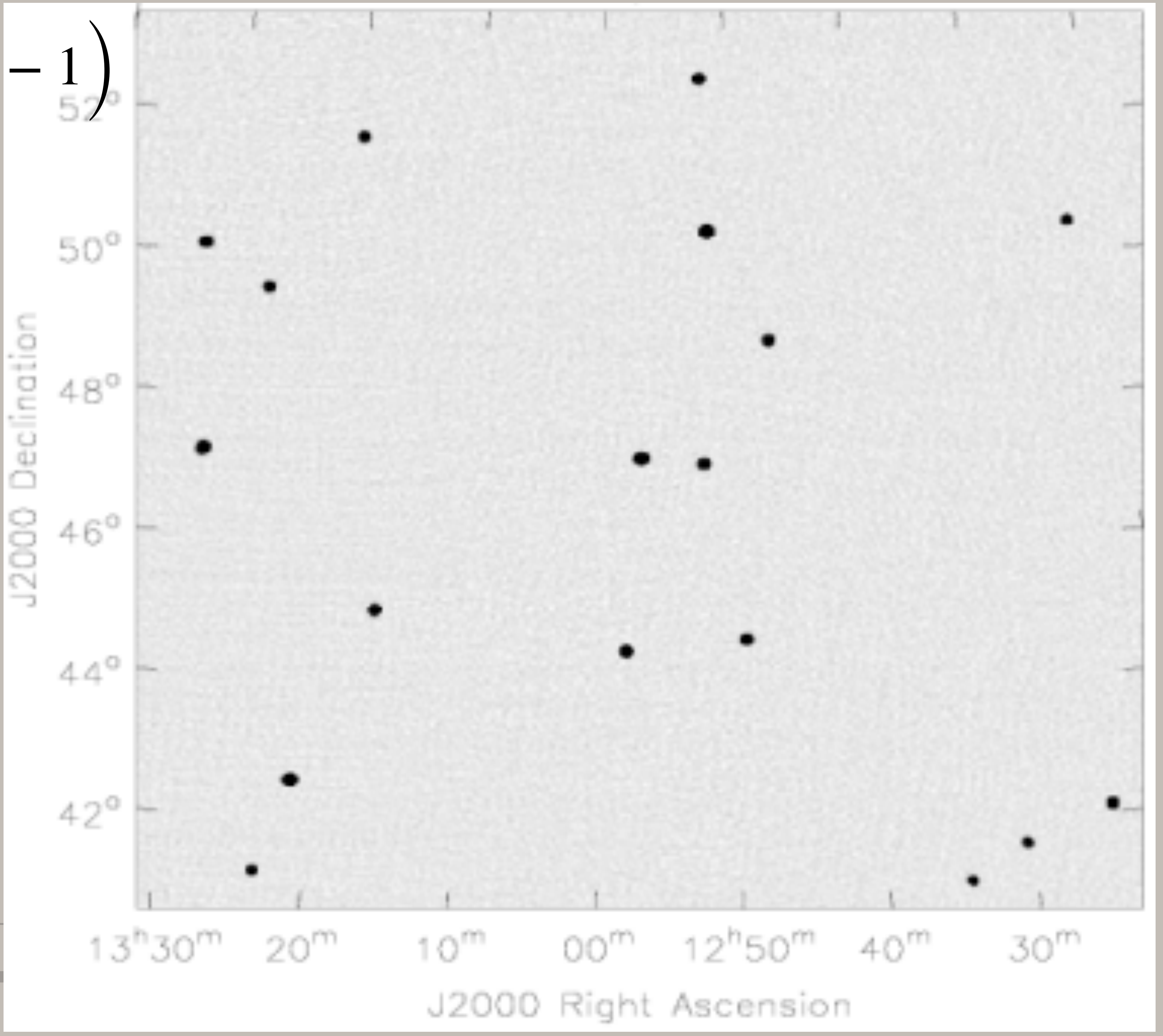
W-TERM

- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\ dl\ dm$$
- $$K_{ij}^{Sky} = e^{w_{ij}\left(\sqrt{1-l^2-m^2}-1\right)}$$
- An order-of magnitude faster than FACETing, and
- for the same amount of computing time provides higher DR images.



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CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma} / \lambda} d\Omega$$

is different for each baseline

- Assumption:
  - sky is (not) variable, and
  - Antenna power pattern is (not) changing!

CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}^{obs}_{ij} = [K^{vis}_{ij}] \int [K^{sky}_{ij}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma} / \lambda} d\Omega$$
  - $$\overrightarrow{V}^{obs}_{cn\times 1} = [K^{vis}_{cn\times cn}][S_{cn\times cm}][F_{cm\times cm}][K^{sky}_{cm\times cm}]\vec{I}^{sky}_{cm\times 1}$$

vector of n visibilities

projection operator describing the uv-coverage

Fourier transfer operator

Pixelated image of sky

CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}^{obs}_{ij} = [K^{vis}_{ij}] \int [K^{sky}_{ij}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma}/\lambda} d\Omega$$
  - $$\overrightarrow{V}^{obs}_{cn\times 1} = [K^{vis}_{cn\times cn}][S_{cn\times cm}][F_{cm\times cm}][K^{sky}_{cm\times cm}]\vec{I}^{sky}_{cm\times 1}$$
    - (this is in block matrix form; and
      - in spatial frequency domain)
  - $$\overrightarrow{V}^{obs}_{cn\times 1} = [K^{vis}_{cn\times cn}][S_{cn\times cm}][G_{cm\times cm}]\overrightarrow{V}^{sky}_{cm\times 1}$$

$[F_{cm\times cm}][K^{sky}_{cm\times cm}][F^{\dagger}_{cm\times cm}]$;
(convolution operator)

CORRECTION FOR **PB**

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  - $\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma}/\lambda} d\Omega$
  - $\overrightarrow{V}_{ij}^{obs}(\nu) = \int \; J_{ij}^{S}(s, \nu, t) \; I(s, \nu) \; e^{i(ul+vm+w(\sqrt{(1-l^2-m^2)}-1))} \; ds$

DATA PB SKY GEOMETRY

  - Visibility depends on time and frequency!

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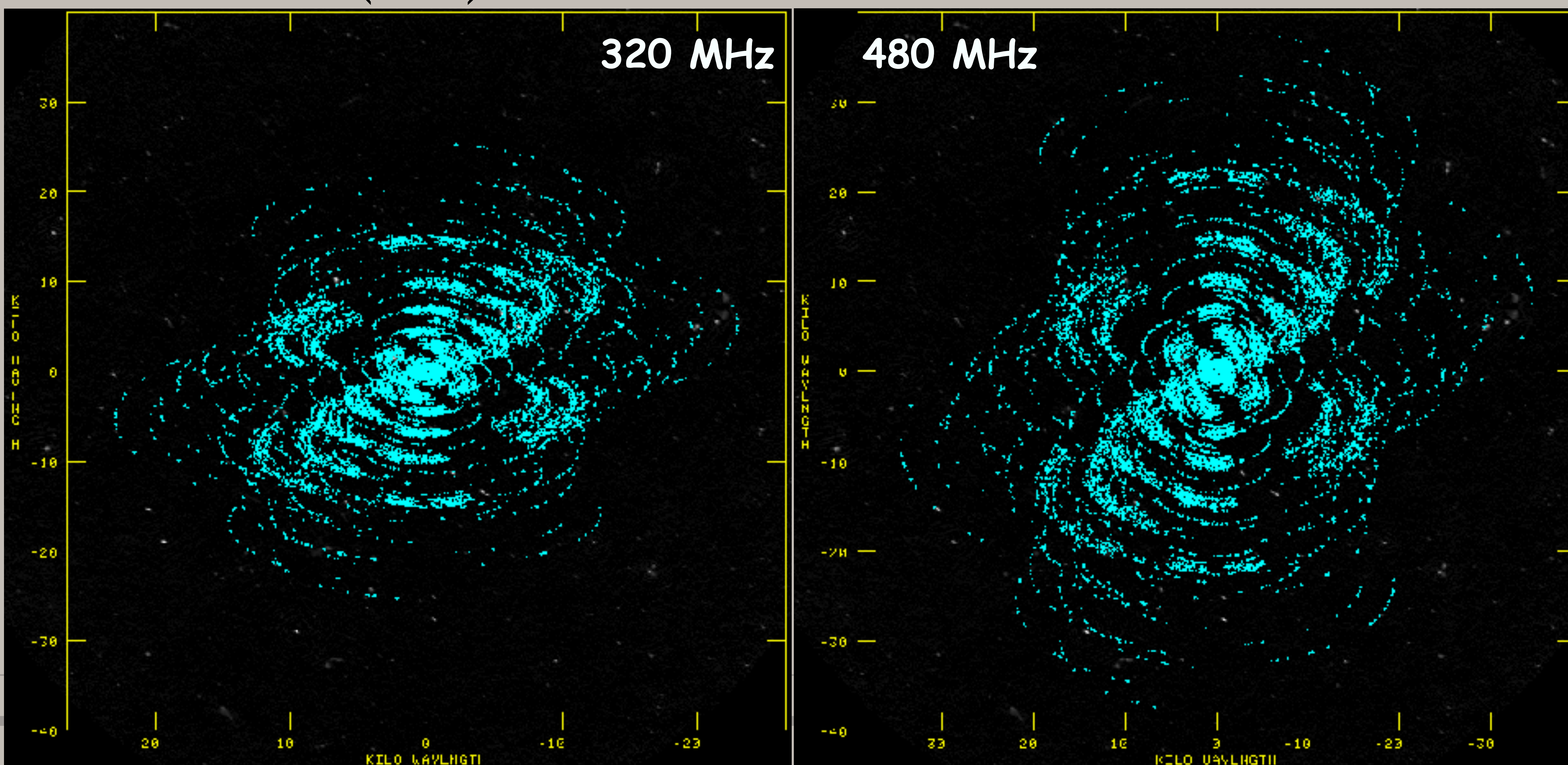
CORRECTION FOR **PB**

- **multi-frequency** synthesis
  - $$I_{\nu}^{sky} = I_{\nu 0}^{sky} \left(\frac{\nu}{\nu 0}\right)^{I_{\alpha}^{sky} + I_{\beta}^{sky} log(\frac{\nu}{\nu 0})}$$



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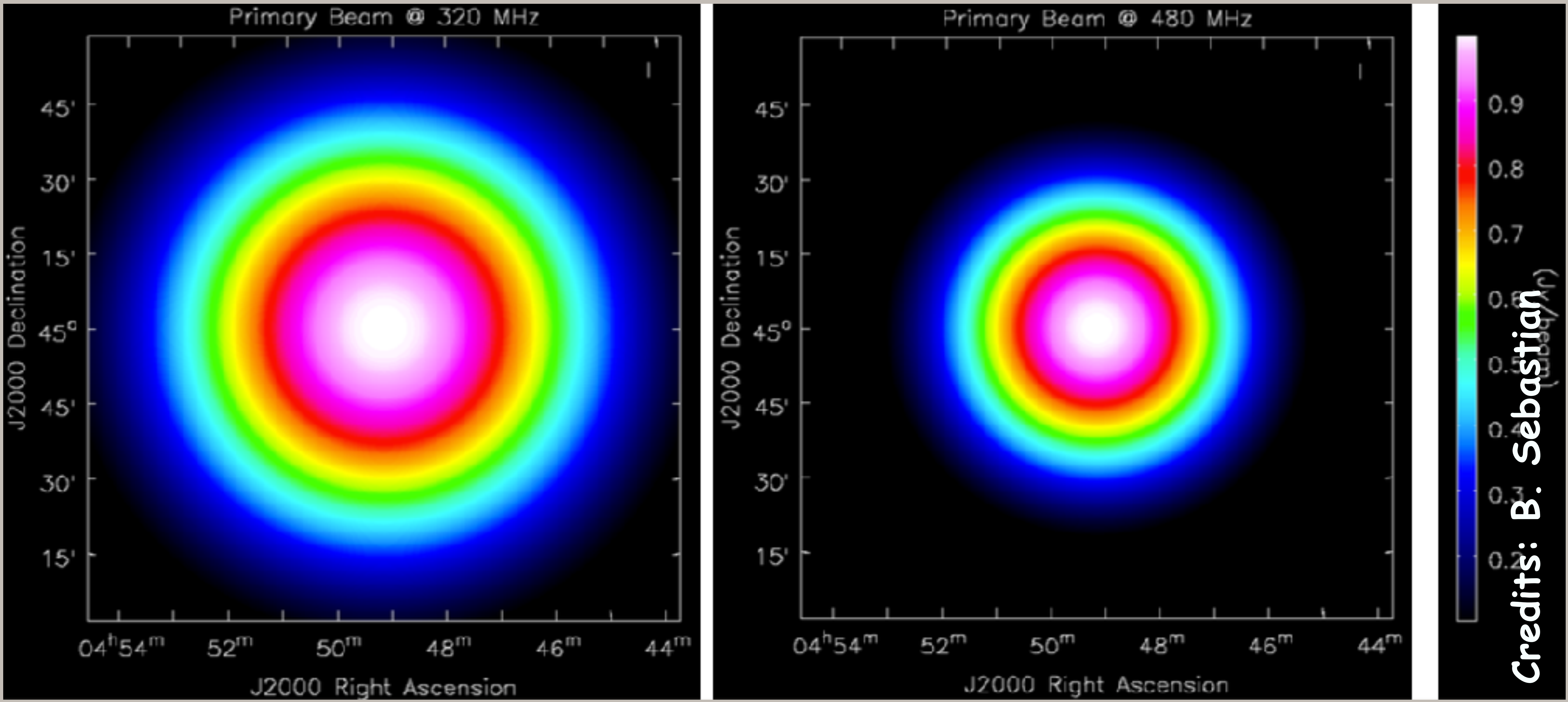
- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{i(ul+vm+w(\sqrt{(}1-l^2-m^2)-1))}\ ds$$



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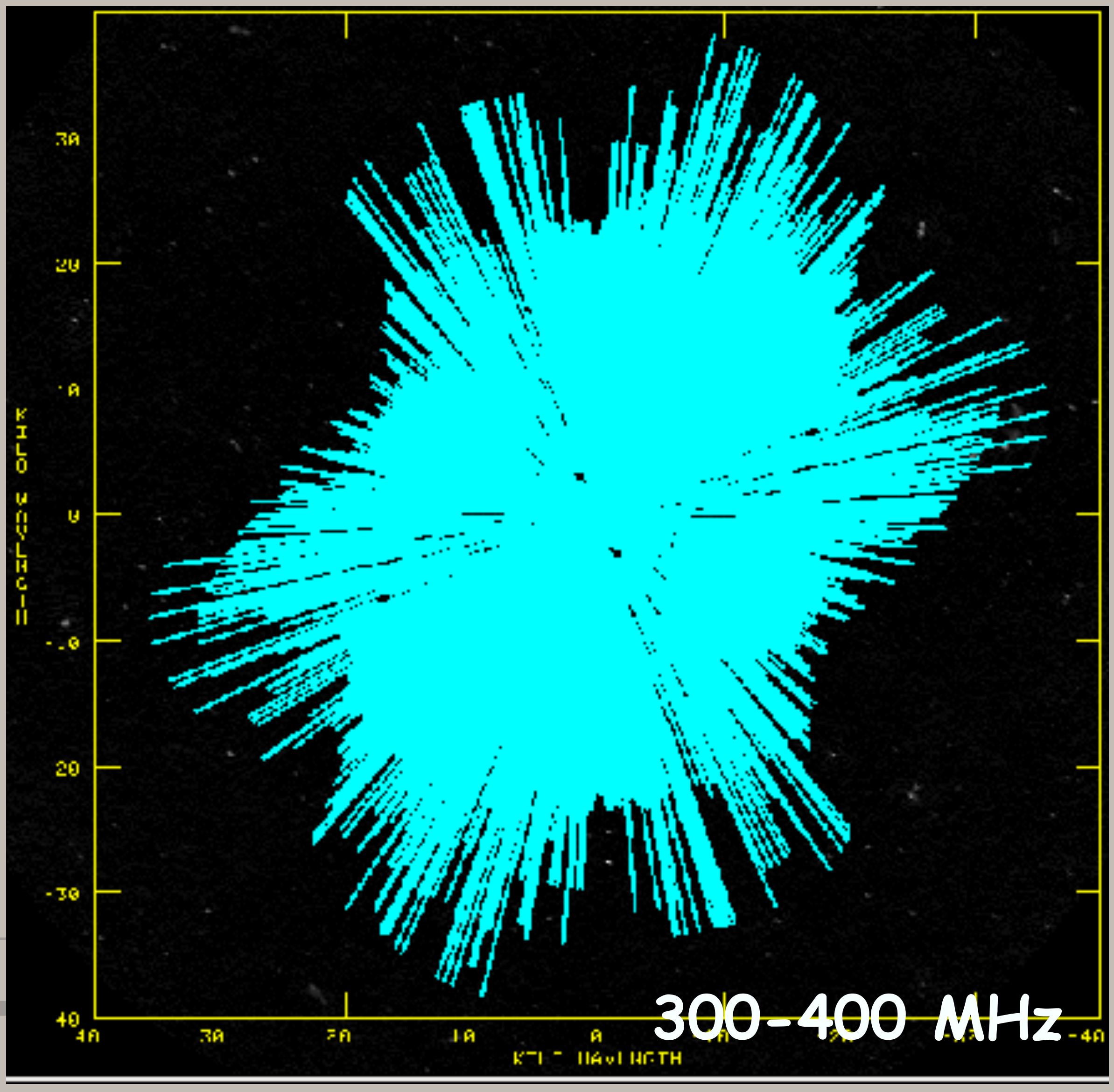
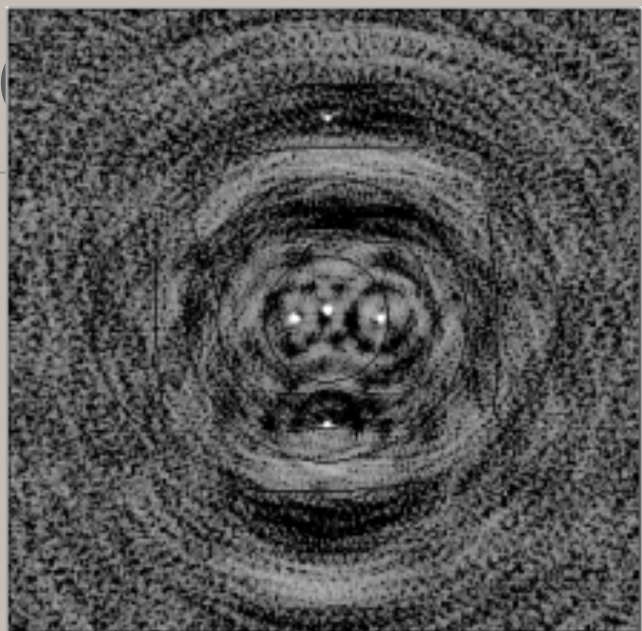
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{i(ul+vm+w(\sqrt{(}1-l^2-m^2)-1))}\ ds$$
- **multi-frequency** synthesis
  - $$I_\nu^{sky} = I_{\nu 0}^{sky}\left(\frac{\nu}{\nu 0}\right)^{I_\alpha^{sky} + I_\beta^{sky} log(\frac{\nu}{\nu 0})}$$
  - $I_0 = I_{\nu_0}$
  - $I_1 = I_\alpha \times I_{\nu_0}$
  - $I_2 = (I_\alpha(I_\alpha - 1)/2 + I_\beta) \times I_{\nu_0}$

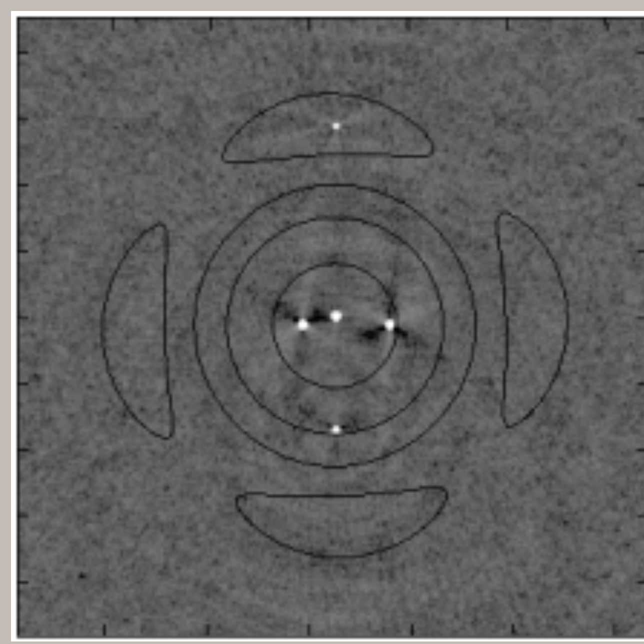


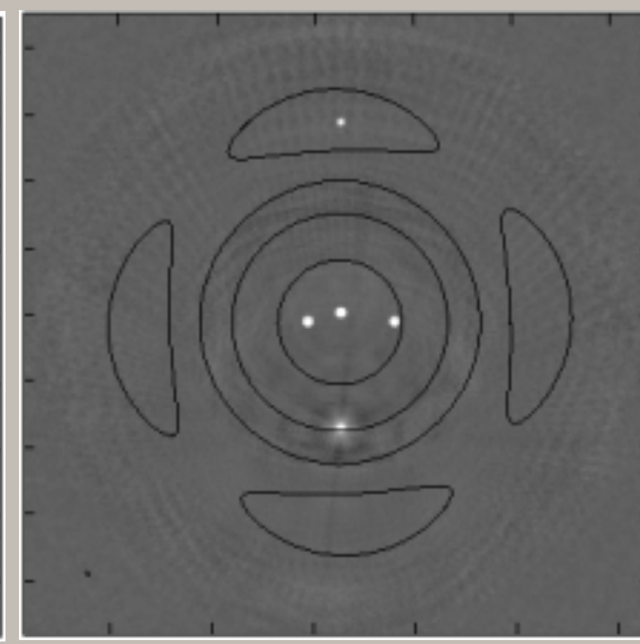
ON FOR **PB**



FT

(standard imaging)





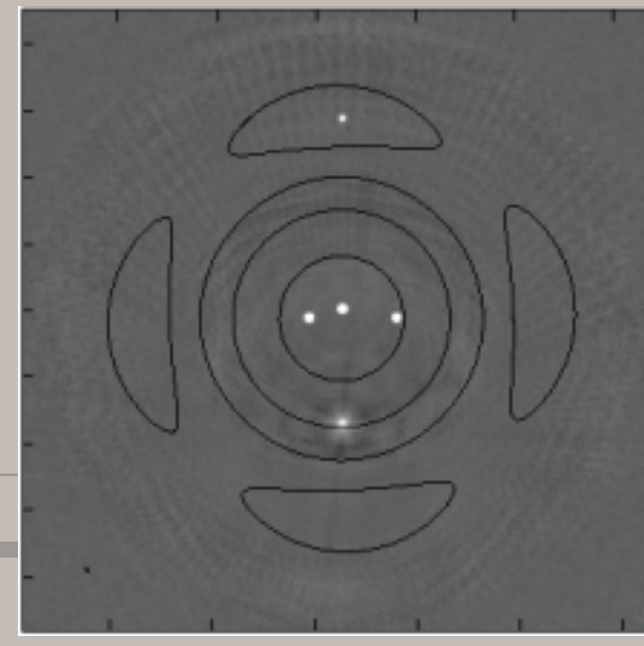
FT

+ MT-MFS

+ A-projection

FT

+ MT-MFS



FT

+ MT-MFS

+ WB A-projection

Credits: S. Bhatnagar (NRAO, USA)

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PEELING: DD CALIBRATION

- antenna based gains are determined in the direction of each compact source.
  - subtract these gains (contribution of compact sources from the observed data using a DFT) and the residual visibilities are imaged again.
- drawbacks of peeling…

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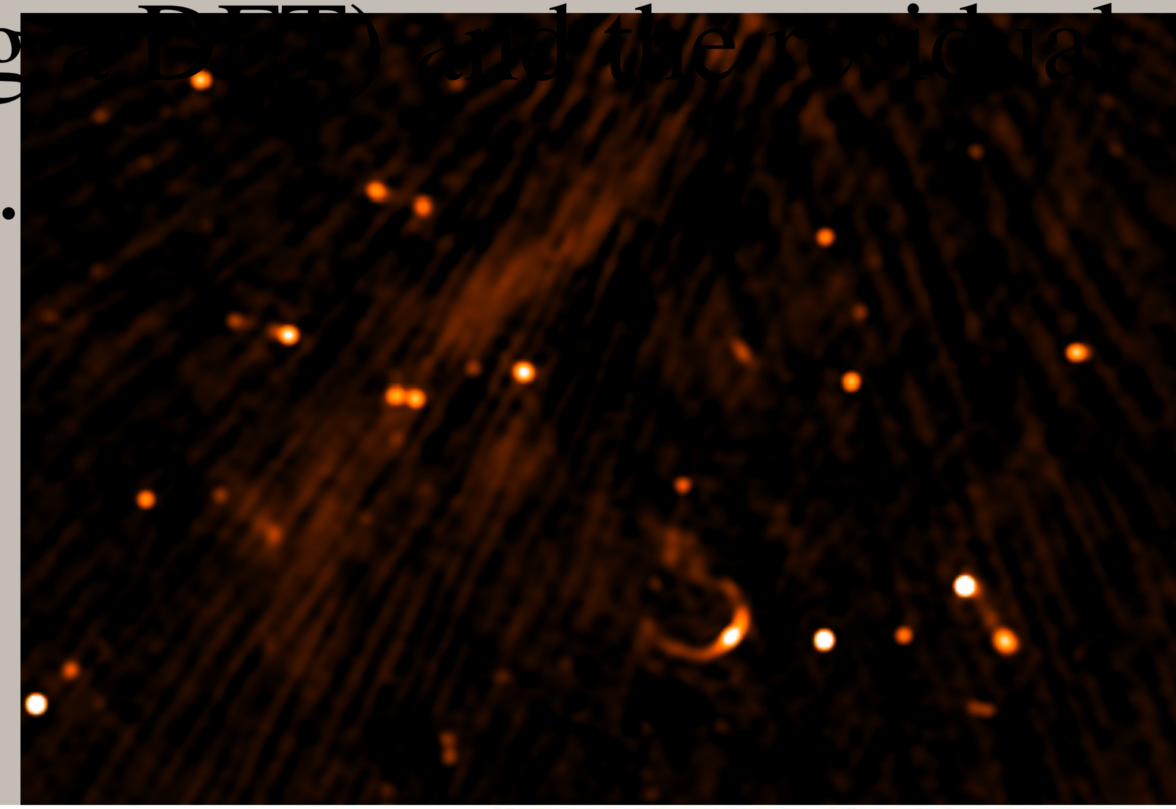
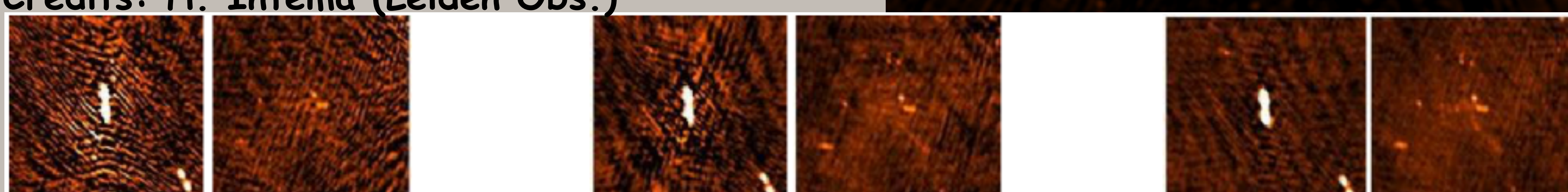


Credits: H. Intema (Leiden Obs.)



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STOKES PARAMETERS

Sir George Gabriel Stokes

- Defined by George in 1852
  - Adopted for astronomy by Chandrasehkar in 1947.
  - Not a vector quantity! Deals with power instead of electric field amplitudes.
    - Can be used for partially polarised radiation.
    - The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)

Sir George Gabriel Stokes

HOW DO WE MEASURE STOKES ….?

- Stokes parameters are the auto-correlation & cross-correlation products returned from the correlator, but input to the correlator can come from different feed types.
- Feeds normally designed to approximate pure linear or circular.
  - Circular feeds – frequency dependent response
  - adds 90^{o} phase to R for L, so:
    - I from $RR + LL$
    - V from $RR - LL$

STOKES PARAMETERS

Sir George Gabriel Stokes

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
  - $RR = \mathscr{A}(RR)e^{i\psi RR} = I + V$
  - $LL = \mathscr{A}(LL)e^{i\psi LL} = I - V$
  - $RL = \mathscr{A}(RL)e^{i\psi RL} = Q + iU$
  - $LR = \mathscr{A}(LR)e^{i\psi LR} = Q - iU$

STOKES PARAMETERS

Sir George Gabriel Stokes

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
- Stokes parameters (as percentages of I)
  - $$I = \frac{(RR + LL)}{2}$$
  - $$\frac{Q}{I} = \frac{\mathrm{Re}(RL + LR)}{RR + LL}$$
  - $$\frac{V}{I} = \frac{RR - LL}{RR + LL}$$
  - $$\frac{U}{I} = \frac{\mathrm{Im}(RL - LR)}{RR + LL}$$

STOKES PARAMETERS

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- *I* – total intensity and sum of any two orthogonal polarisations
- *Q* & *U* – completely specify linear polarisation
- *V* – completely specifies circular polarisation
- Stokes parameters (as percentages of *I)*
- Is it really that simple?

STOKES PARAMETERS

Sir George Gabriel Stokes

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
- Stokes parameters (as percentages of I)
- Is it really that simple?
  - No, there are leakages…
    - The total intensity can leak into the polarised components (I into $\{Q,U,V\}$).

Hans Mueller

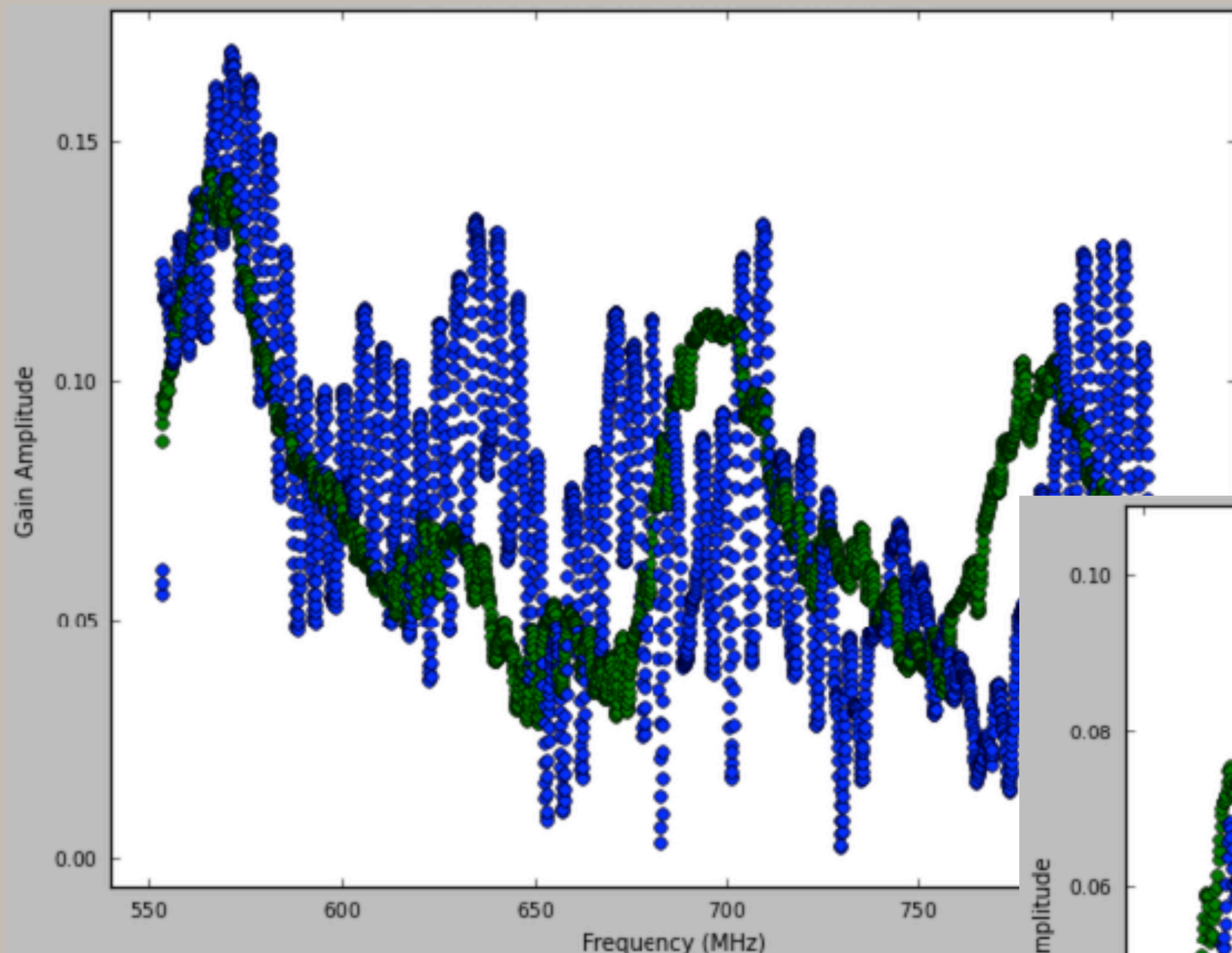
MUELLER MATRIX

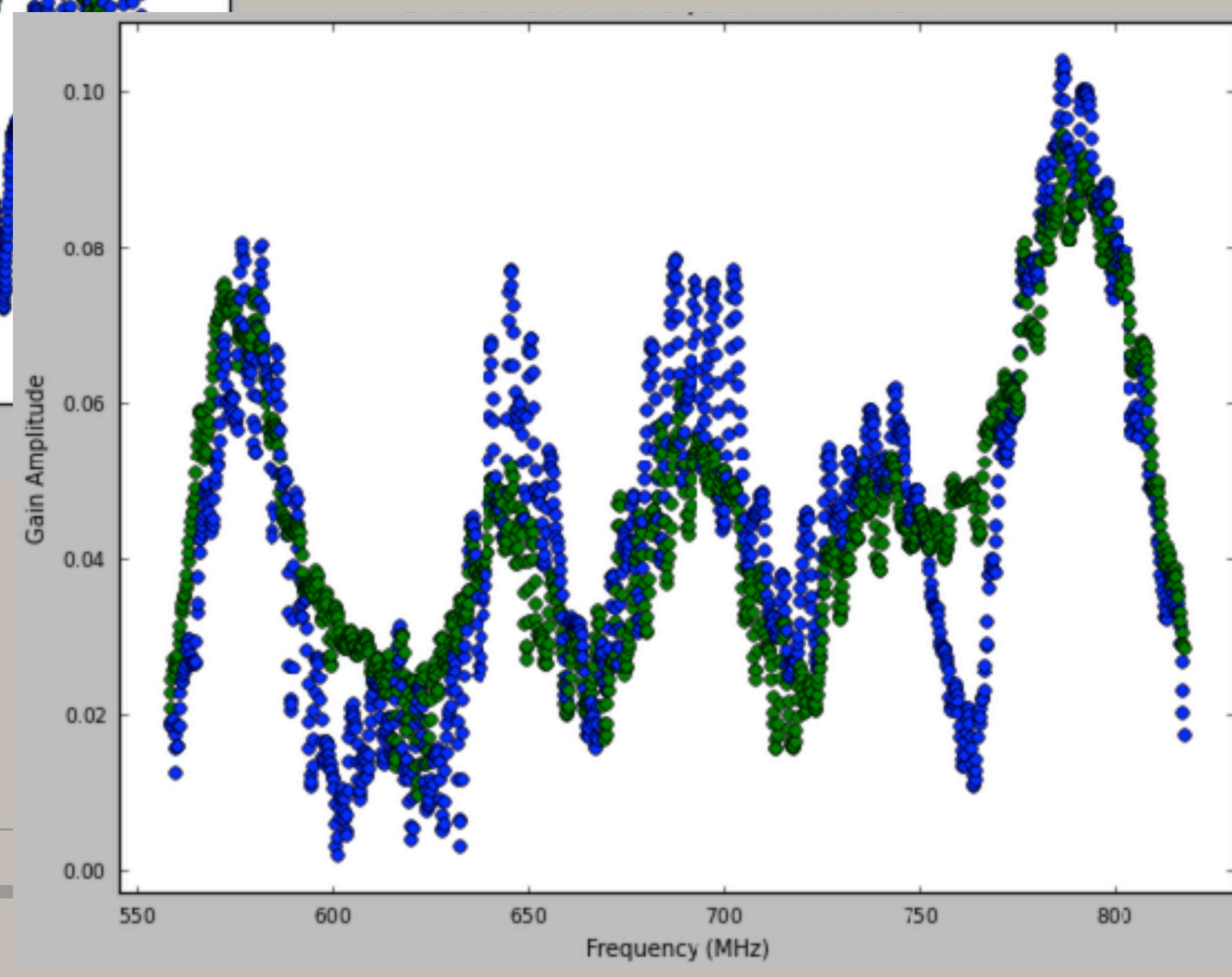
- The leakage of each polarisation into the other can be measured and quantified in a 4×4 matrix (Mueller 1943).

- $$M = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix}$$

- $$\begin{bmatrix} RR + LL \\ RL + LR \\ RL - LR \\ RR - LL \end{bmatrix} = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix} \cdot \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

AMOUNT OF LEAKAGES





POLARISATION CALIBRATION

- Flux density scale
- $I \Leftrightarrow Q$ leakage
- $I \Leftrightarrow U$ leakage
- $I \Leftrightarrow V$ leakage
- Alignment => PA calibration
- Ellipticity, $Q \Leftrightarrow V$
- RL phase, $U \Leftrightarrow V$

Constrained using calibrator with known Stokes parameters

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Constrained using calibrator with known Stokes parameters

Need calibrator with known PA

POLARISATION CALIBRATION

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- Alignment => PA calibration
- Ellipticity, $Q \Leftrightarrow V$
- RL phase, $U \Leftrightarrow V$

Constrained using calibrator with known Stokes parameters

Need calibrator with known PA

Stokes V ~ 0 for most calibrators so no need to worry too much unless you require very high precision

PUTTING THIS ALL TOGETHER

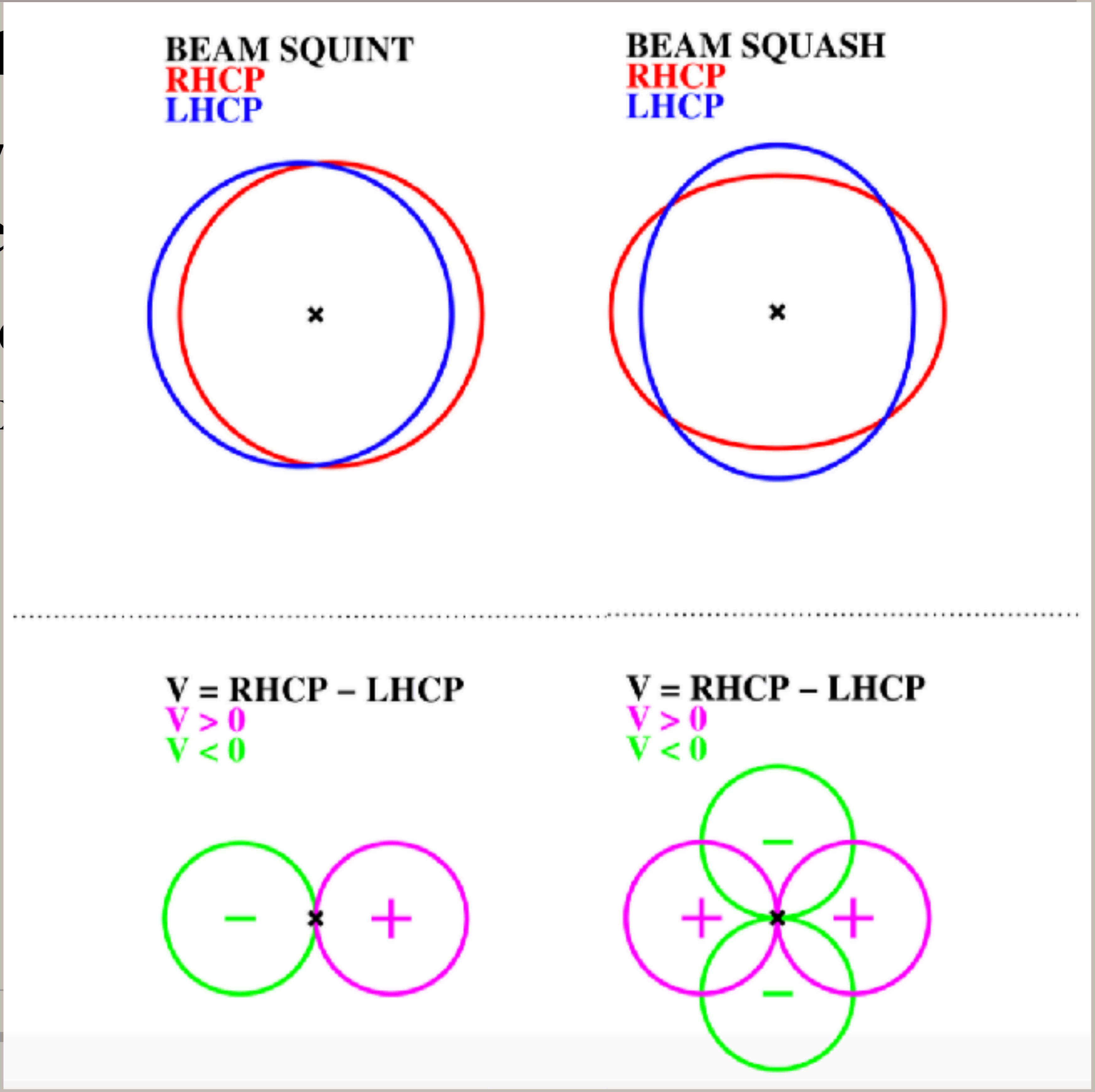
- In the end what we are trying to do is relate products from our correlator to the intrinsic polarised radiation from the source.
- So we need to correct the raw correlator outputs for
  - imperfections in the receiver (leakages).
    - The orientation of the receiver with respect to the telescope structure.
      - a.k.a. the changing parallactic angle.
  - Any measured propagation related polarisation effects (e.g. Faraday rotation).

BEAM EFFECTS

- For point sources, all of the previous is fine.
- What if the source you are looking at is extended compared to the telescope beam?
  - There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are…
    - Squint
    - Squash

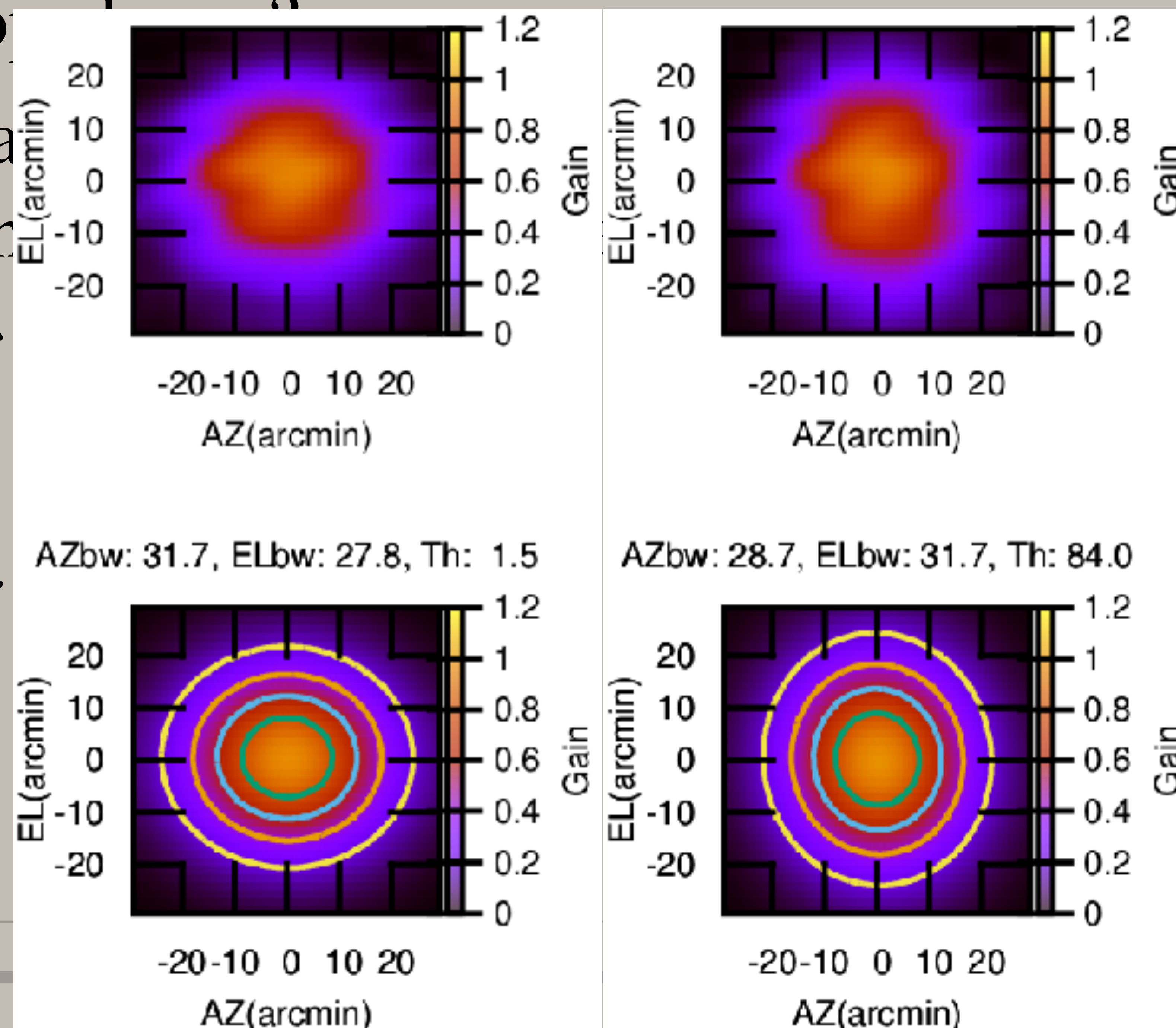
BEAM EFFECTS

- For point sources, al
- What if the source y
compared to the tele
  - There are instrum
measurement of e
    - Squint
    - Squash



BEAM EFFECTS

- For point sources, all of the previous is fine.
- What if the source you are looking at is extended compared to the telesco
  - There are instrumenta measurement of exten
    - Squint
    - Squash



Credits: S.N. Katore (NCRA-TIFR)

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\; W_{ij}(\nu, t) \int P_{ij}\; M_{ij}(s, \nu, t)\; \vec{I}(s, \nu)\; e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\; d\vec{s}$$

DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,…

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \overrightarrow{b_{ij}} . \vec{s}}\ d\vec{s}$$

DATA

Weights

Mueller matrix

full-polarization vector of the sky brightness distribution

BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$
  - $$M_{ij}(\vec{s}, \nu, t) = E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t)$$

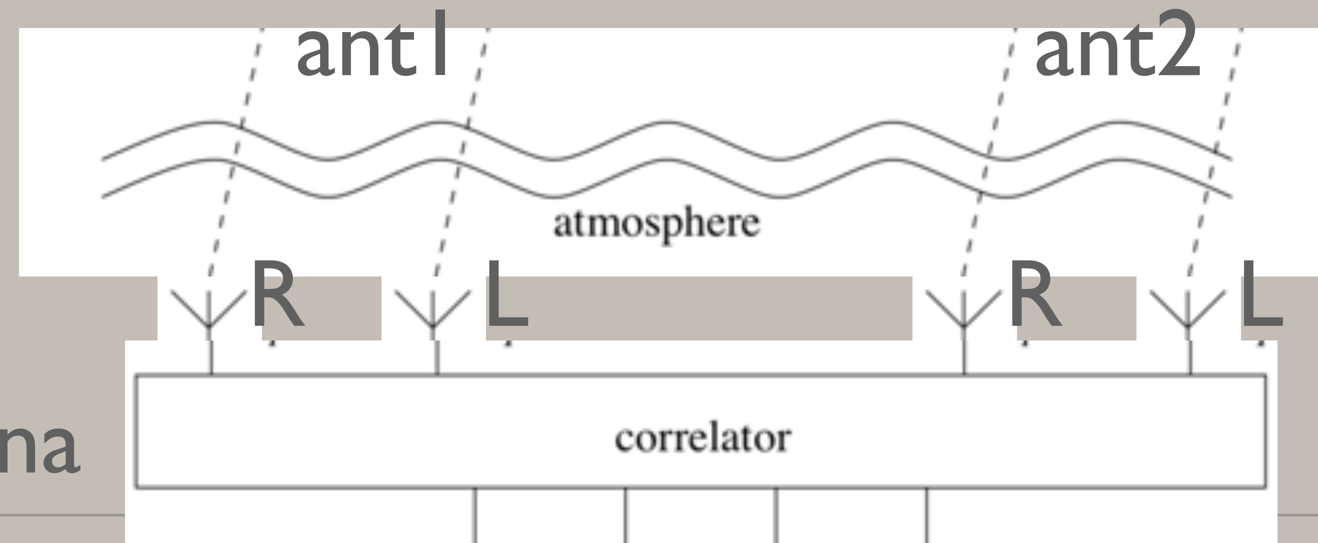
BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu,t) = W_{ij}(\nu,t)\int M_{ij}(s,\nu,t)\ \vec{I}(s,\nu)e^{\imath\overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$
  - $$M_{ij}(\vec{s},\nu,t) = E_i(\vec{s},\nu,t)\otimes E_j^*(\vec{s},\nu,t)$$
  - $$\overrightarrow{V}_{ij}^{Obs}(\nu,t) = W_{ij}(\nu,t)\mathscr{F}\left[\left(E_i(\vec{s},\nu,t)\otimes E_j^*(\vec{s},\nu,t)\right)\cdot\vec{I}(\vec{s},\nu)\right]$$
    - $$= W_{ij}(\nu,t)\left[A_{ij}\star\overrightarrow{V}_{ij}\right]$$

BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \overrightarrow{b_{ij}} . \vec{s}}\ d\vec{s}$$
  - $$M_{ij}(\vec{s}, \nu, t) = E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t)$$
  - $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \mathscr{F} \left[\left(E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t) \right) \cdot \vec{I}(\vec{s}, \nu) \right]$$
    - $$= W_{ij}(\nu, t) \left[A_{ij} \star \overrightarrow{V}_{ij} \right]$$
    - $$\text{where,}\ A_{ij} = A_i \otimes A_j^*$$

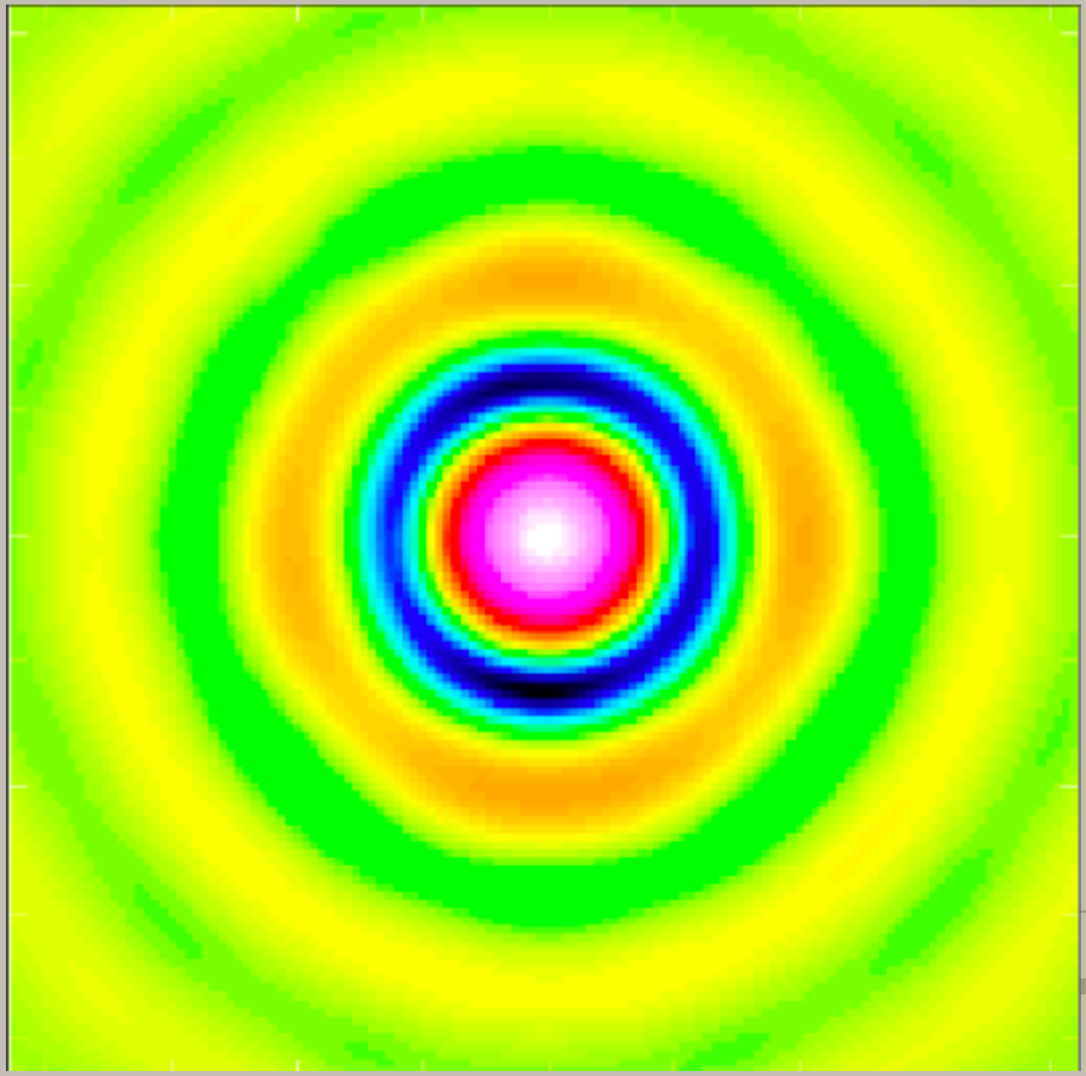
AIPs for two antenna



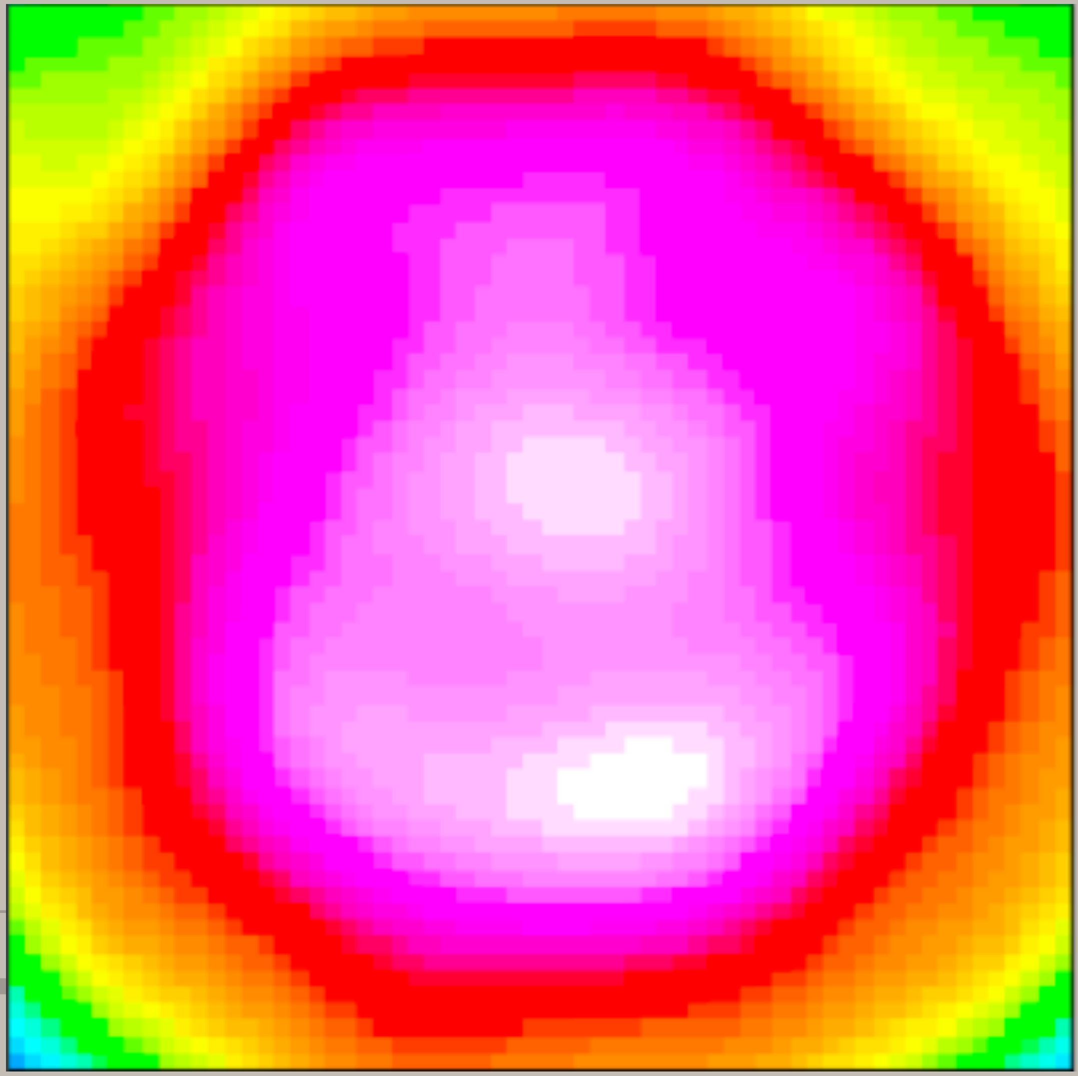
APERTURE ILLUMINATION PATTERN

- Holography data: MeerKAT
  - Obtained by Fourier Transforming the PB Holography measurements

Credits: S. Sekhar (UCT-IDIA)



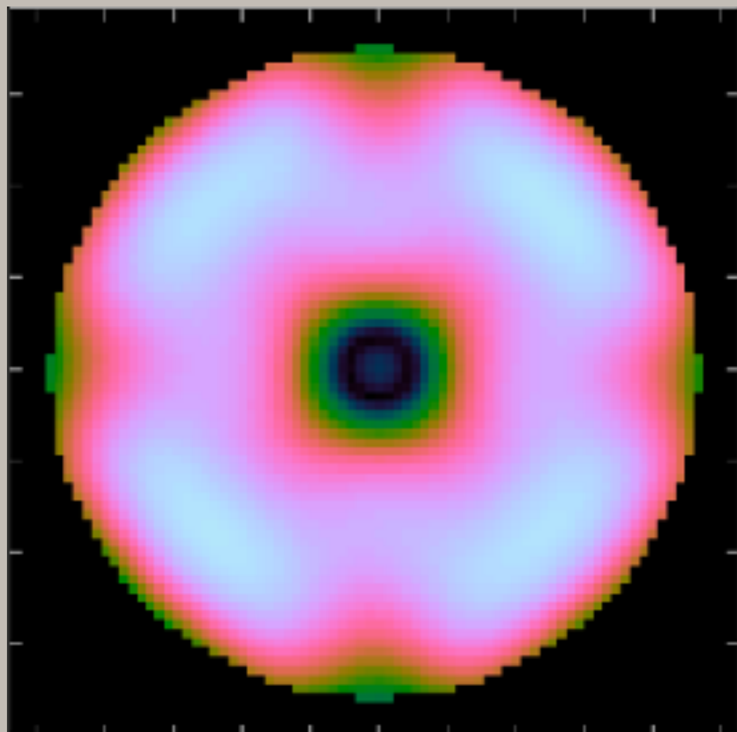
$\Leftrightarrow$

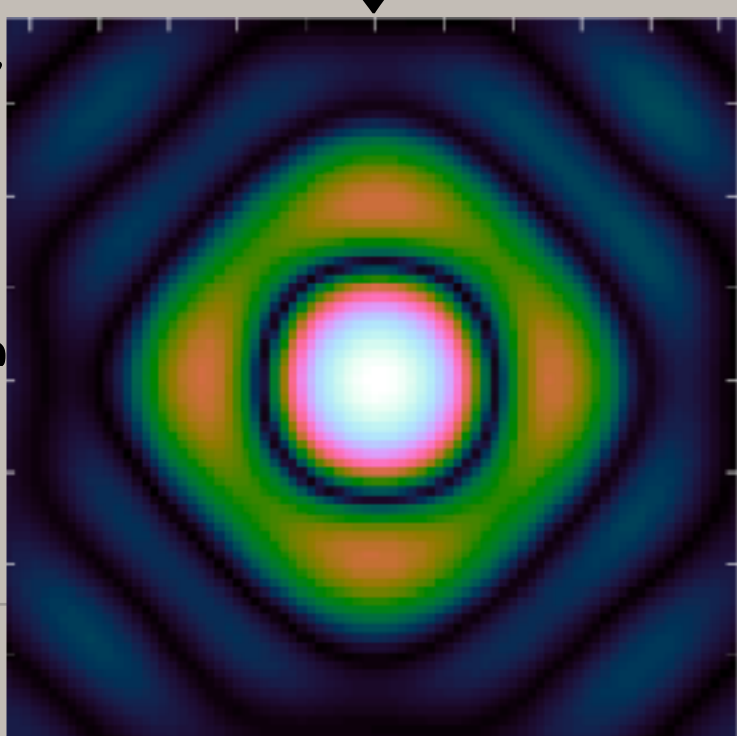


A-to-Z Solver

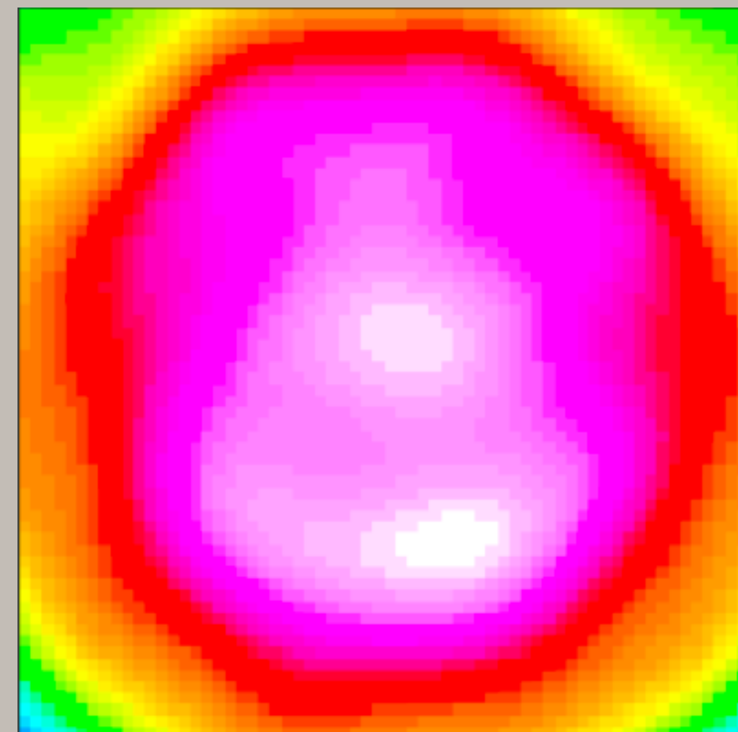
- Use Zernike polynomials to directly model the complex aperture
  - it is a natural domain to model optical aberrations that cause PB weirdness
  - (Telescope agnostic - does not require ray traced model for different antennas/telescopes, only Holography)
  - Aperture size is fixed, independent of number of measured sidelobes.

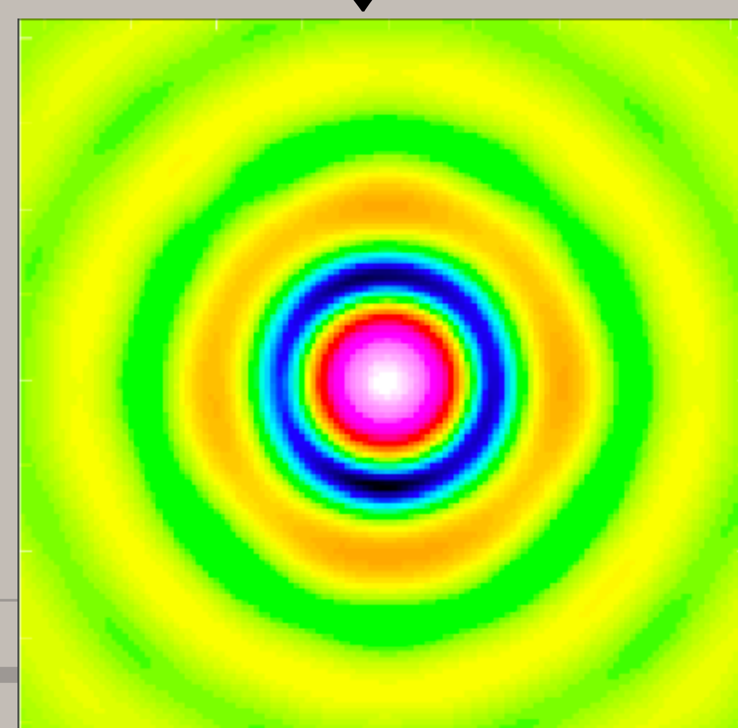
APERTURE ILLUMINATION PATTERN



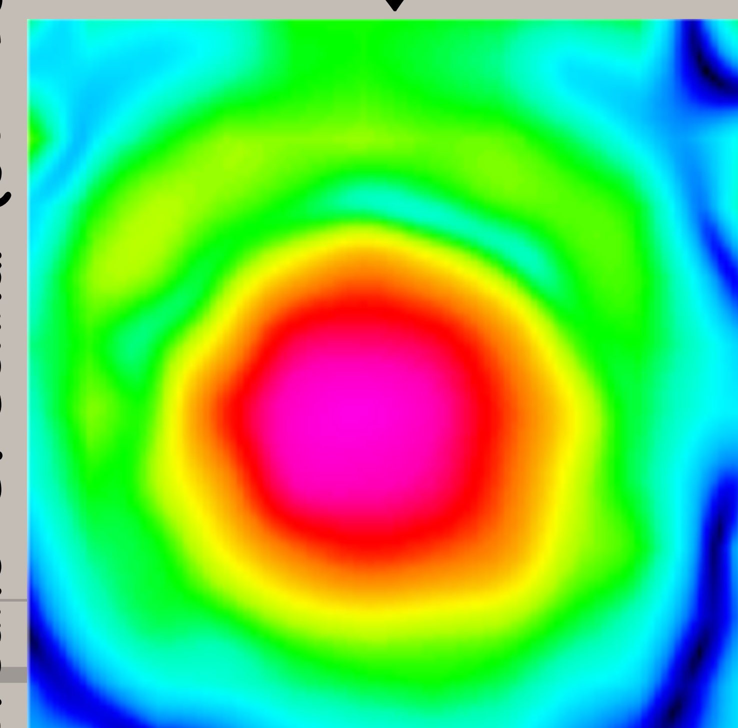


Credits: P. Jagannathan (NRAO)





Credits: S. Sekhar (UCT-IDIA)



Credits: S. Sekhar (UCT-IDIA)

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\iota \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

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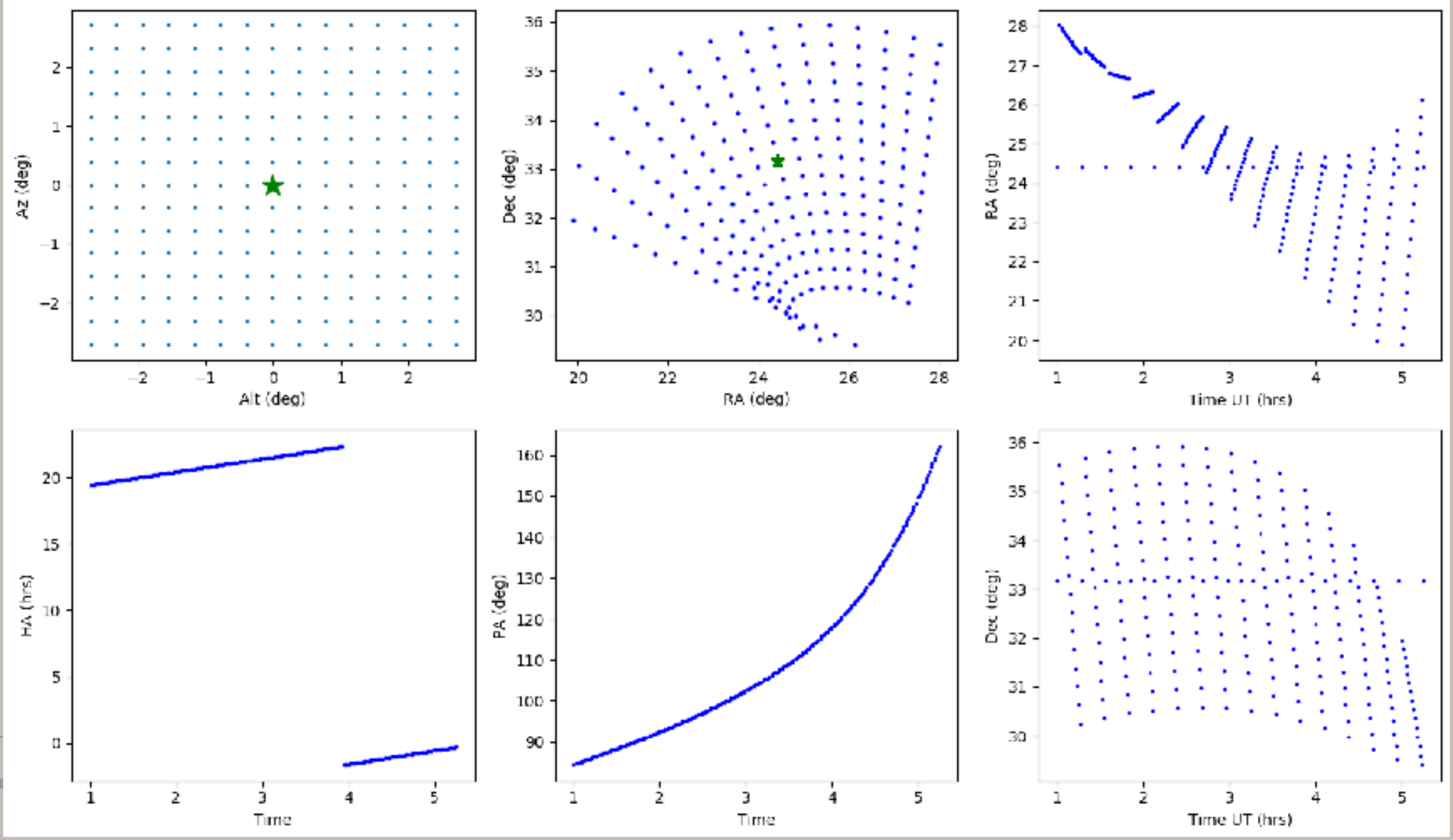
MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

Acknowledgements:
Rau+ 2009, Jagannathan+ 2019,
ASP Conf. Ser. Vol. 180 (1999)

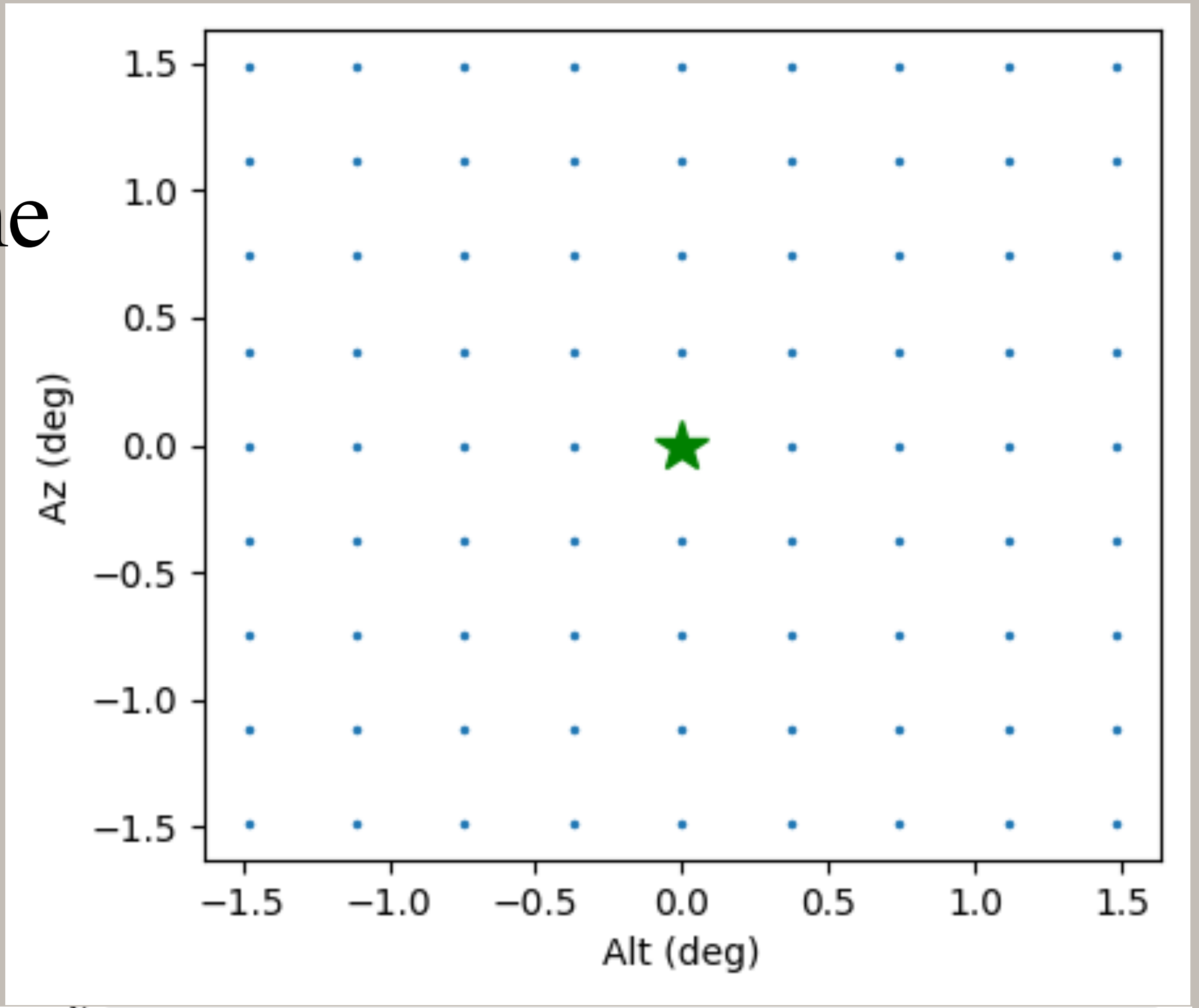
UGMRT DATA (RUN I)

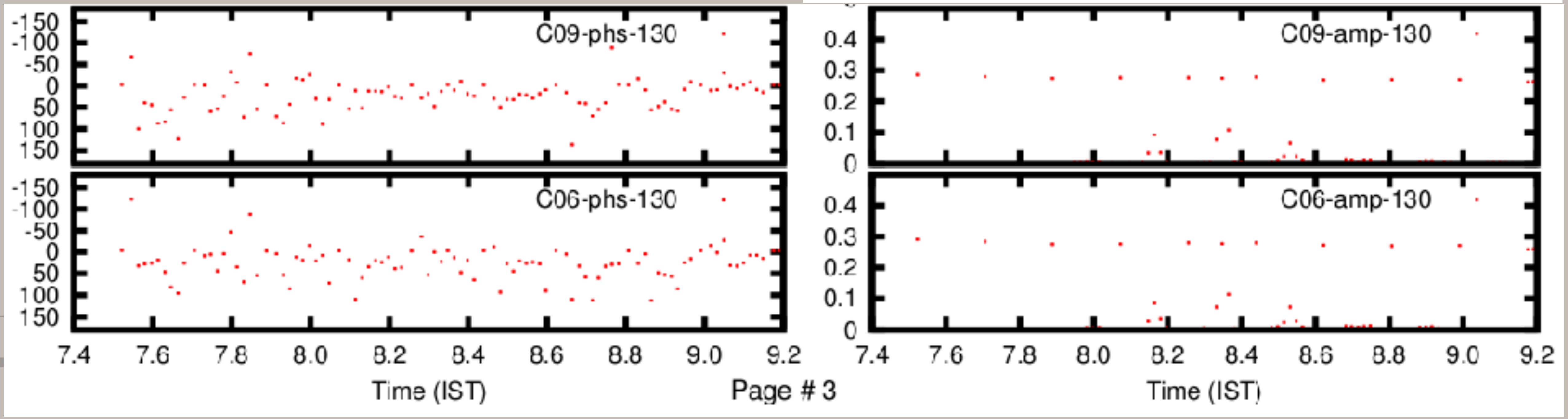
- Holography data



UGMRT DATA

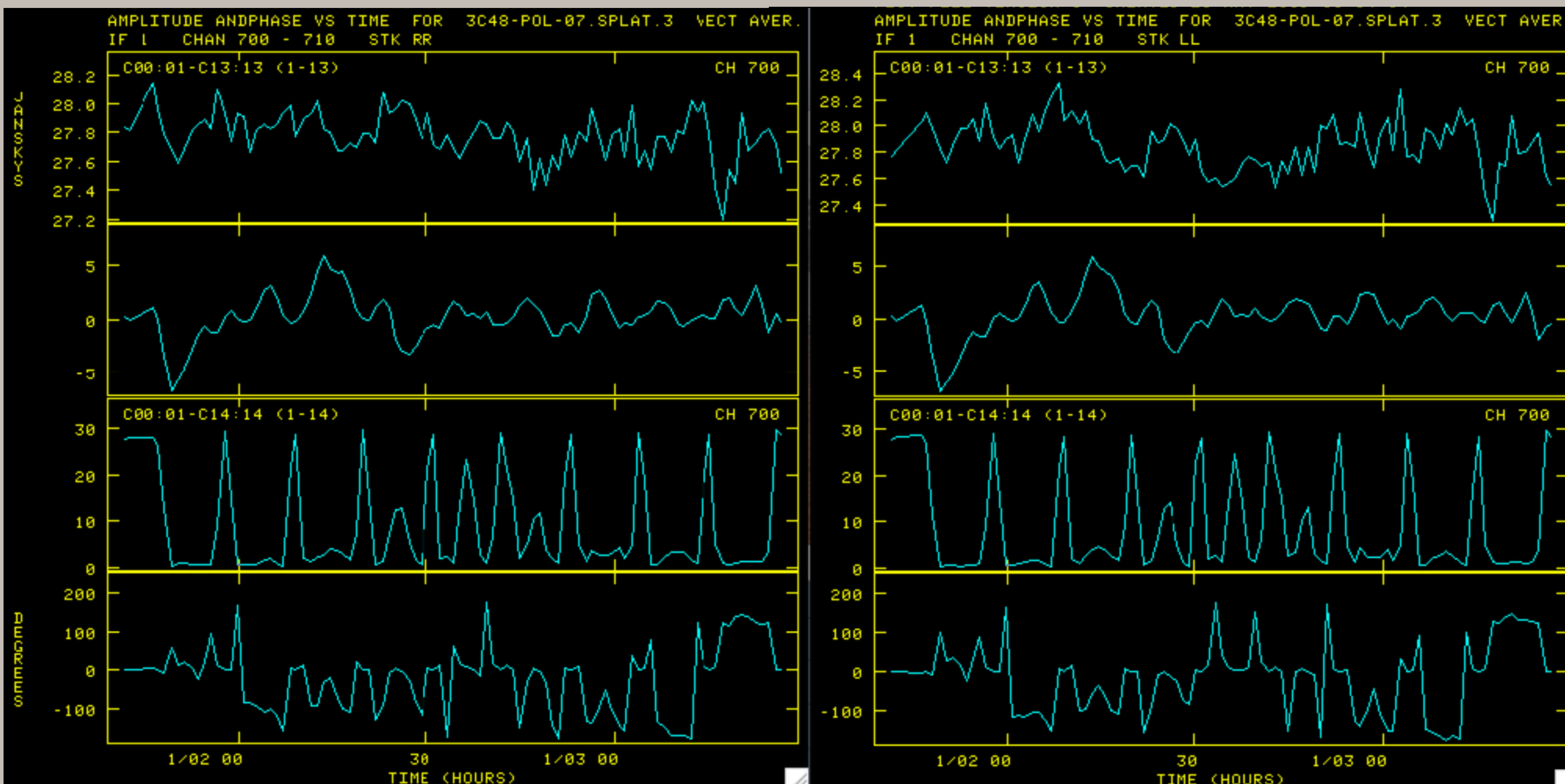
- Holography data
  - scans/data as a function of time





UGMRT DATA

- Holography data
  - scans/data as a function of time



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Chan freq= 0.618310546 GHz Ncor= 4 No. vis=   4875 Sort order= TB
U, V, W are in Kilo wavelengths at the selected frequency

Source PN0001   (0) RA   = 05 42 36.15 DEC = 49 51 7.2

Source= PN0001		Freq= 0.618310546		Sort= TB	700	RR	700	LL	700	RL	700	LR
Time	Ant	U(Kilo)	V(Kilo)	W(Kilo)	Amp	Phas	Amp	Phas	Amp	Phas	Amp	Phas
0/04:13:00.0	1- 2	-0.13	0.53	-0.51	1.700	6	0.200	64	0.801	-167	3.383	56
0/04:13:00.0	1-10	-0.65	0.53	-1.98	0.741	-32	2.011	-65	0.736	-111	1.152	118
0/04:13:00.0	1-12	-0.57	-0.07	-1.60	0.522	-31	0.962	-49	0.493	126	0.108	-133
0/04:13:00.0	1-14	-0.78	0.95	-2.44	1.584	-83	1.423	-114	0.739	-114	1.252	-41
0/04:13:00.0	1-15	1.40	-1.93	4.26	1.253	121	3.503	30	0.863	40	2.568	-163
0/04:13:00.0	1-17	4.39	-6.90	13.72	0.177	143	0.463	122	0.315	81	0.207	85
0/04:13:00.0	1-18	5.45	-10.05	17.56	0.962	-141	0.884	-66	0.641	116	0.957	-66
0/04:13:00.0	1-22	-4.53	-8.48	-10.09	1.206	-93	1.238	-68	0.659	-168	0.333	-27
0/04:13:00.0	1-24	-9.63	-17.25	-21.52	1.759	-71	0.405	5	1.038	-141	1.228	-67
0/04:13:00.0	1-26	-0.22	7.93	-2.81	0.340	-139	1.109	-64	0.379	35	0.885	-95
0/04:13:00.0	1-28	1.05	19.23	-2.49	1.403	-93	0.855	-95	0.616	-131	0.695	-108
0/04:13:00.0	1-30	2.48	31.23	-2.22	1.265	-115	0.579	-93	0.494	-11	0.469	-156
0/04:13:02.7	1- 2	-0.13	0.53	-0.51	2.269	16	1.254	5	0.890	26	1.350	-39
0/04:13:02.7	1-10	-0.65	0.53	-1.98	0.931	5	1.574	-142	0.986	-102	2.230	63
0/04:13:02.7	1-12	-0.57	-0.07	-1.60	0.602	-55	0.982	-167	1.005	144	0.722	-25
0/04:13:02.7	1-14	-0.78	0.95	-2.44	1.524	-101	0.352	147	0.845	120	1.549	-96
0/04:13:02.7	1-15	1.40	-1.93	4.26	1.514	-110	5.170	83	2.118	100	3.091	-114
0/04:13:02.7	1-17	4.39	-6.90	13.72	0.628	31	0.341	16	0.452	-10	0.066	14
0/04:13:02.7	1-18	5.45	-10.05	17.56	0.735	45	0.446	-110	0.336	-53	0.359	-131
0/04:13:02.7	1-22	-4.53	-8.48	-10.09	1.280	-87	1.117	155	0.337	130	0.503	97
0/04:13:02.7	1-24	-9.63	-17.26	-21.52	1.797	-53	1.063	-17	1.000	-110	0.633	-140
0/04:13:02.7	1-26	-0.22	7.93	-2.81	1.228	-96	0.207	158	0.115	-34	0.083	-166
0/04:13:02.7	1-28	1.05	19.23	-2.49	0.582	-179	0.722	-154	0.546	5	0.220	-132
0/04:13:02.7	1-30	2.47	31.23	-2.22	1.570	-105	0.293	83	1.148	-141	1.083	-117
0/04:13:05.4	1- 2	-0.13	0.53	-0.51	1.195	-67	0.936	-16	1.107	25	1.763	-24
0/04:13:05.4	1-10	-0.65	0.53	-1.98	1.170	144	0.846	-136	1.377	-160	0.712	-50
0/04:13:05.4	1-12	-0.57	-0.07	-1.60	0.443	-133	0.688	-120	0.743	152	0.820	-121
0/04:13:05.4	1-14	-0.78	0.95	-2.44	1.988	-169	0.093	-172	0.150	152	0.786	171
0/04:13:05.4	1-15	1.40	-1.93	4.26	1.774	-86	2.564	151	0.833	150	1.401	-66
0/04:13:05.4	1-17	4.39	-6.90	13.71	0.424	53	0.242	-65	0.198	24	0.772	132
0/04:13:05.4	1-18	5.46	-10.05	17.56	0.465	-145	1.164	-159	0.822	-155	0.197	154
0/04:13:05.4	1-22	-4.53	-8.48	-10.09	1.001	-179	0.597	-155	1.050	124	0.958	-96

File = 3C147-10M-C .UVDATA.   1   Vol = 3   Userid = 141   Channel = 700   IF = 1
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Time	Ant	U(Kilo)	V(Kilo)	W(Kilo)	Amp	Phas	Amp	Phas	Amp	Phas	Amp	Phas
0/04:13:00.0	1- 2	-0.13	0.53	-0.51	1.700	6	0.200	64	0.801	-167	3.383	56
0/04:13:00.0	1-10	-0.65	0.53	-1.98	0.741	-32	2.011	-65	0.736	-111	1.152	118
0/04:13:00.0	1-12	-0.57	-0.07	-1.60	0.522	-31	0.962	-49	0.493	126	0.108	-133
0/04:13:00.0	1-14	-0.78	0.95	-2.44	1.584	-83	1.423	-114	0.739	-114	1.252	-41
0/04:13:00.0	1-15	1.40	-1.93	4.26	1.253	121	3.503	30	0.863	40	2.568	-163
0/04:13:00.0	1-17	4.39	-6.90	13.72	0.177	143	0.463	122	0.315	81	0.207	85
0/04:13:00.0	1-18	5.45	-10.05	17.56	0.962	-141	0.884	-66	0.641	116	0.957	-66
0/04:13:00.0	1-22	-4.53	-8.48	-10.09	1.206	-93	1.238	-68	0.659	-168	0.333	-27
0/04:13:00.0	1-24	-9.63	-17.25	-21.52	1.759	-71	0.405	5	1.038	-141	1.228	-67
0/04:13:00.0	1-26	-0.22	7.93	-2.81	0.340	-139	1.109	-64	0.379	35	0.885	-95
0/04:13:00.0	1-28	1.05	19.23	-2.49								
0/04:13:00.0	1-30	2.48	31.23	-2.22								
0/04:13:02.7	1- 2	-0.13	0.53	-0.51								
0/04:13:02.7	1-10	-0.65	0.53	-1.98								
0/04:13:02.7	1-12	-0.57	-0.07	-1.60								
0/04:13:02.7	1-14	-0.78	0.95	-2.44								
0/04:13:02.7	1-15	1.40	-1.93	4.26								
0/04:13:02.7	1-17	4.39	-6.90	13.72								
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0/04:13:02.7	1-24	-9.63	-17.26	-21.52								
0/04:13:02.7	1-26	-0.22	7.93	-2.81								
0/04:13:02.7	1-28	1.05	19.23	-2.49								
0/04:13:02.7	1-30	2.47	31.23	-2.22								
0/04:13:05.4	1- 2	-0.13	0.53	-0.51								
0/04:13:05.4	1-10	-0.65	0.53	-1.98								
0/04:13:05.4	1-12	-0.57	-0.07	-1.60								
0/04:13:05.4	1-14	-0.78	0.95	-2.44								
0/04:13:05.4	1-15	1.40	-1.93	4.26								
0/04:13:05.4	1-17	4.39	-6.90	13.71								
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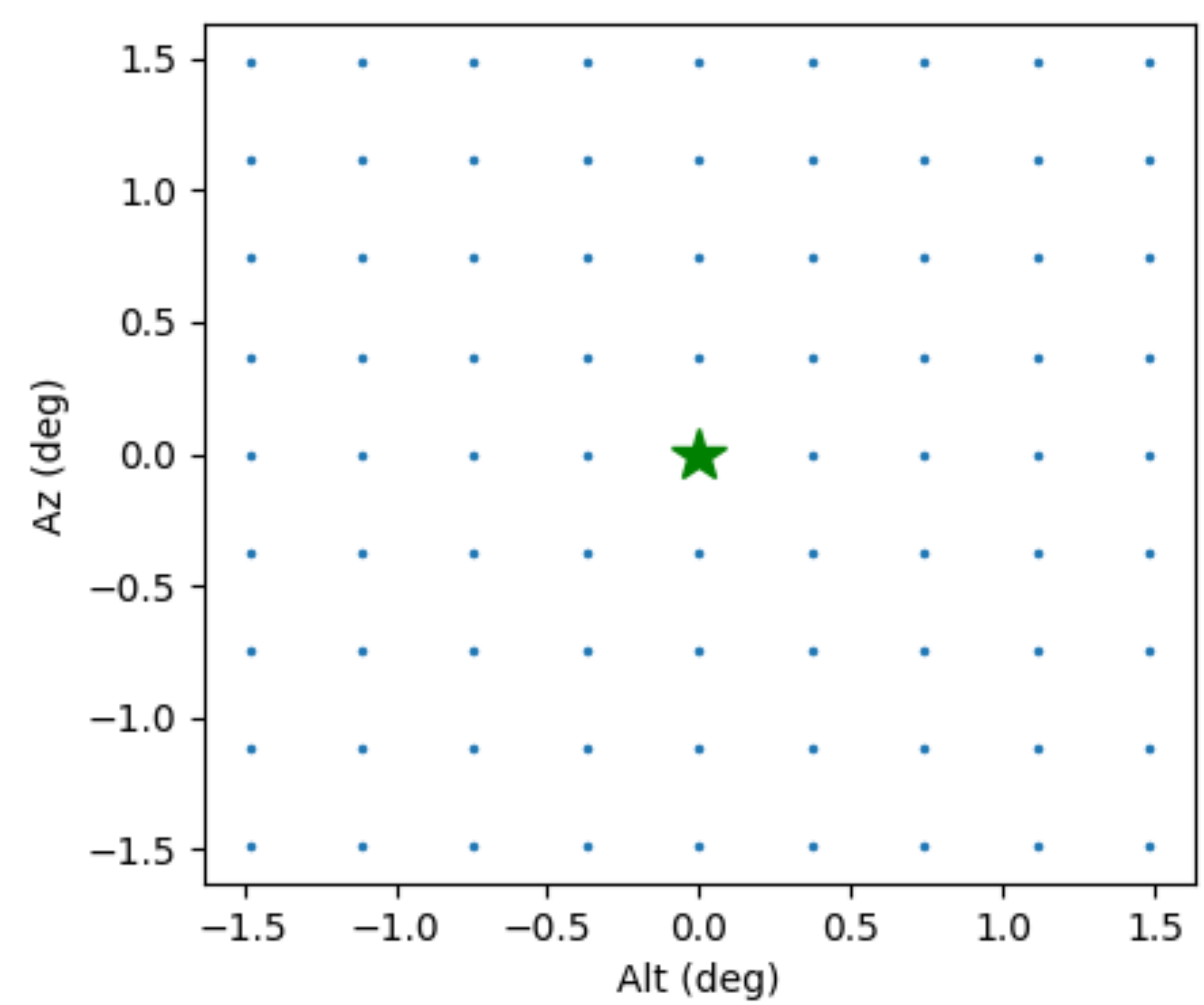
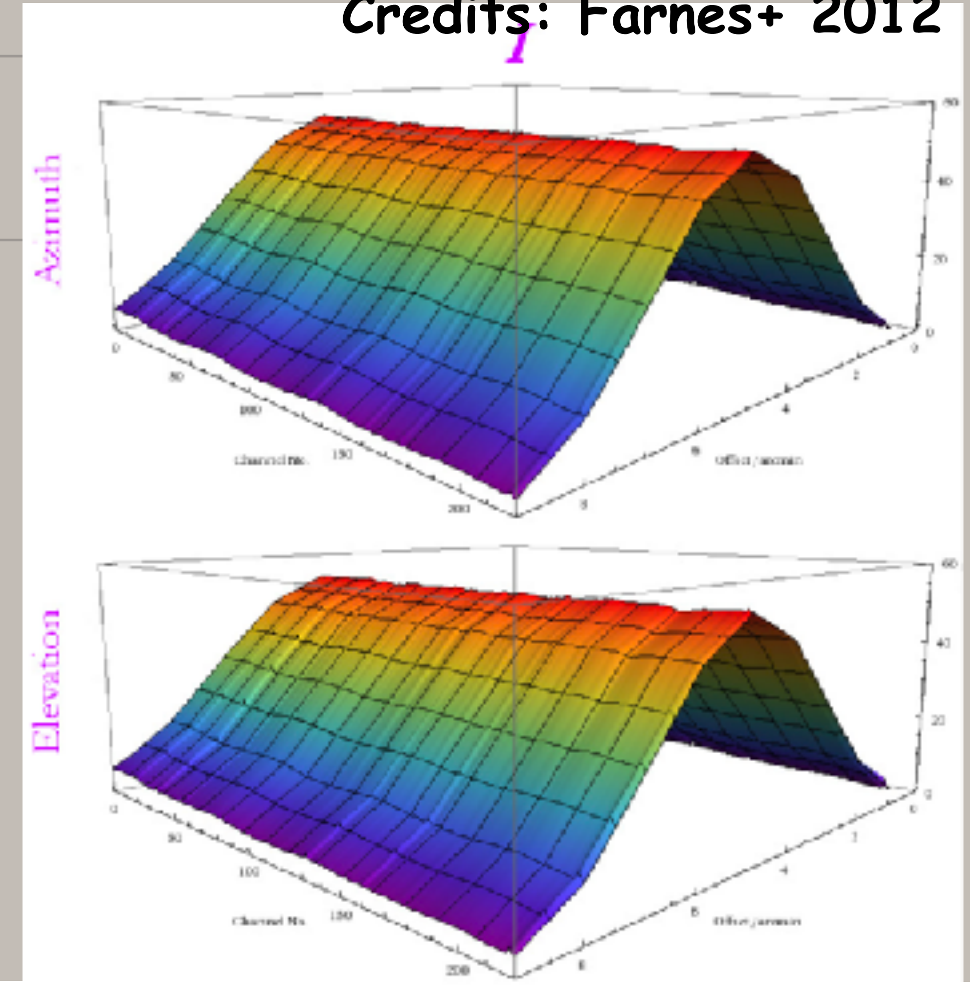


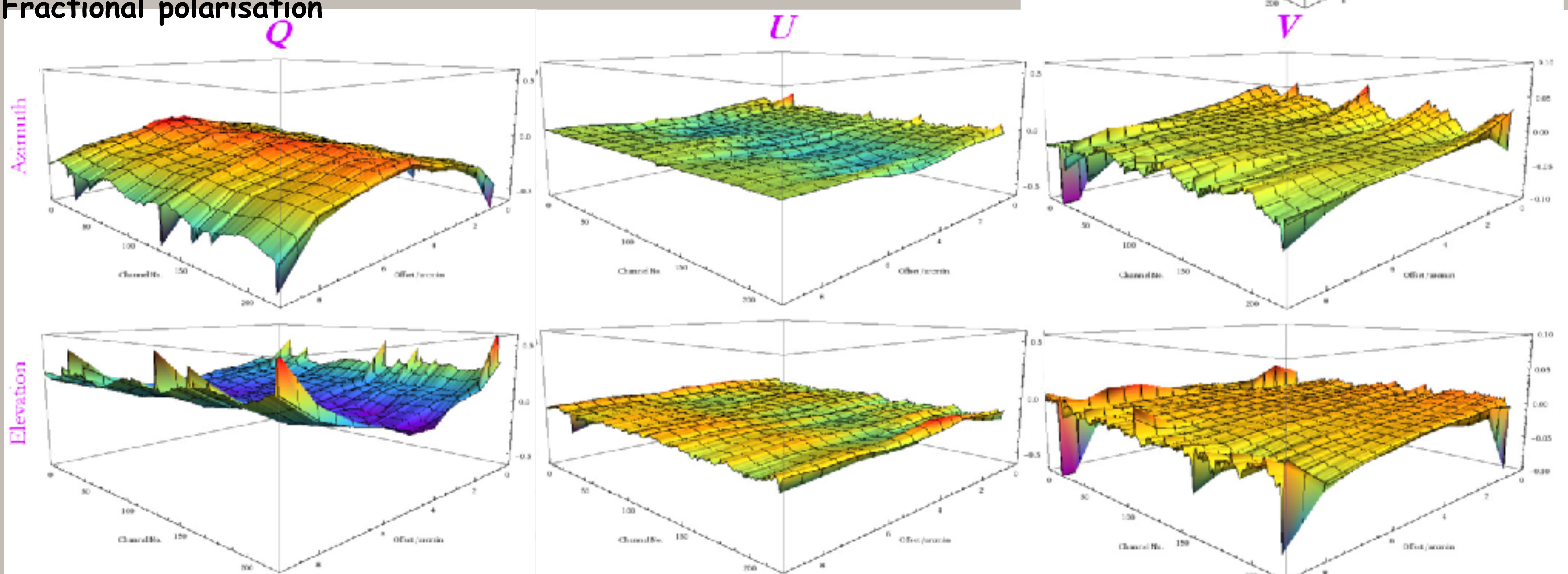
Credits: Farnes+ 2012

NEXT: BEAM PROFILES

- 325 MHz:
  - X-axis offset from phase-centre
  - Y-axis beam response
  - Z-axis channel/frequency

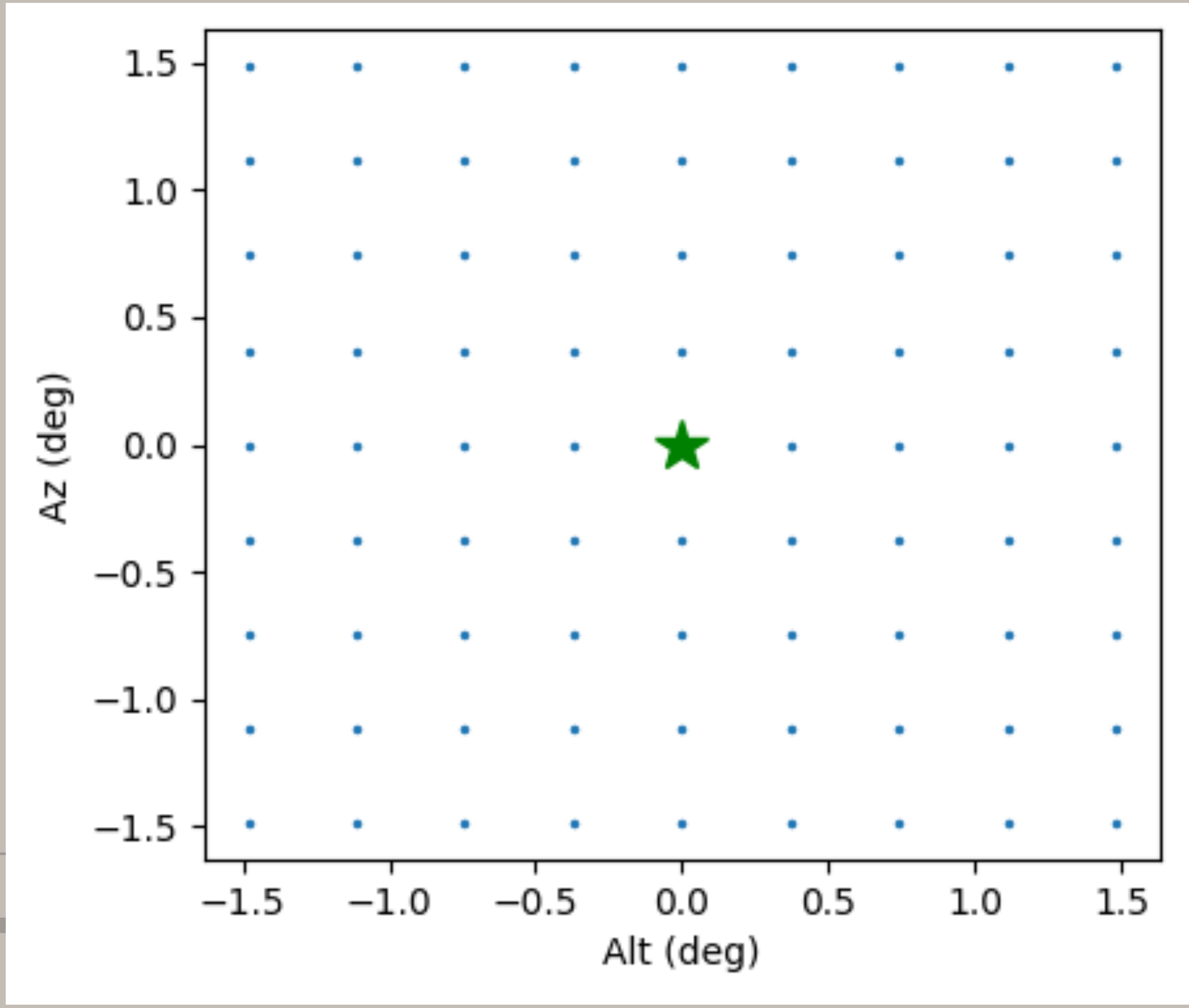


Fractional polarisation



UNDERSTANDING DATA

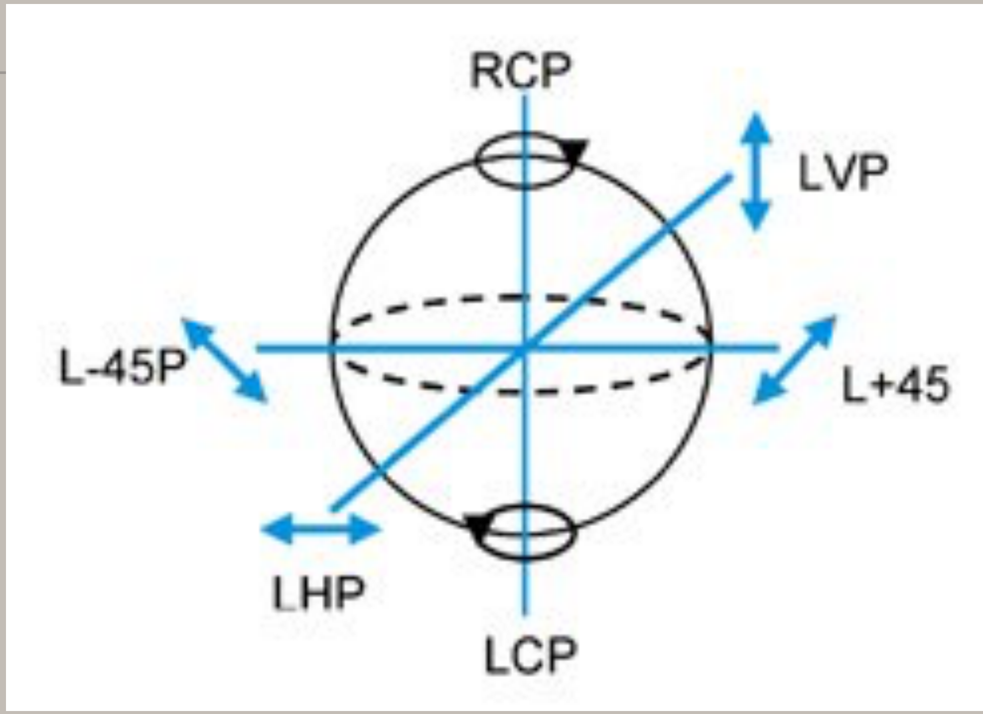
- Holography data
  - scans/data as a function of time
- construct (Stokes) image/beam
- Understanding the polarisation properties of the GMRT dish is fundamental …
- need for
  - "accurate aperture model"
  - CASA implementation

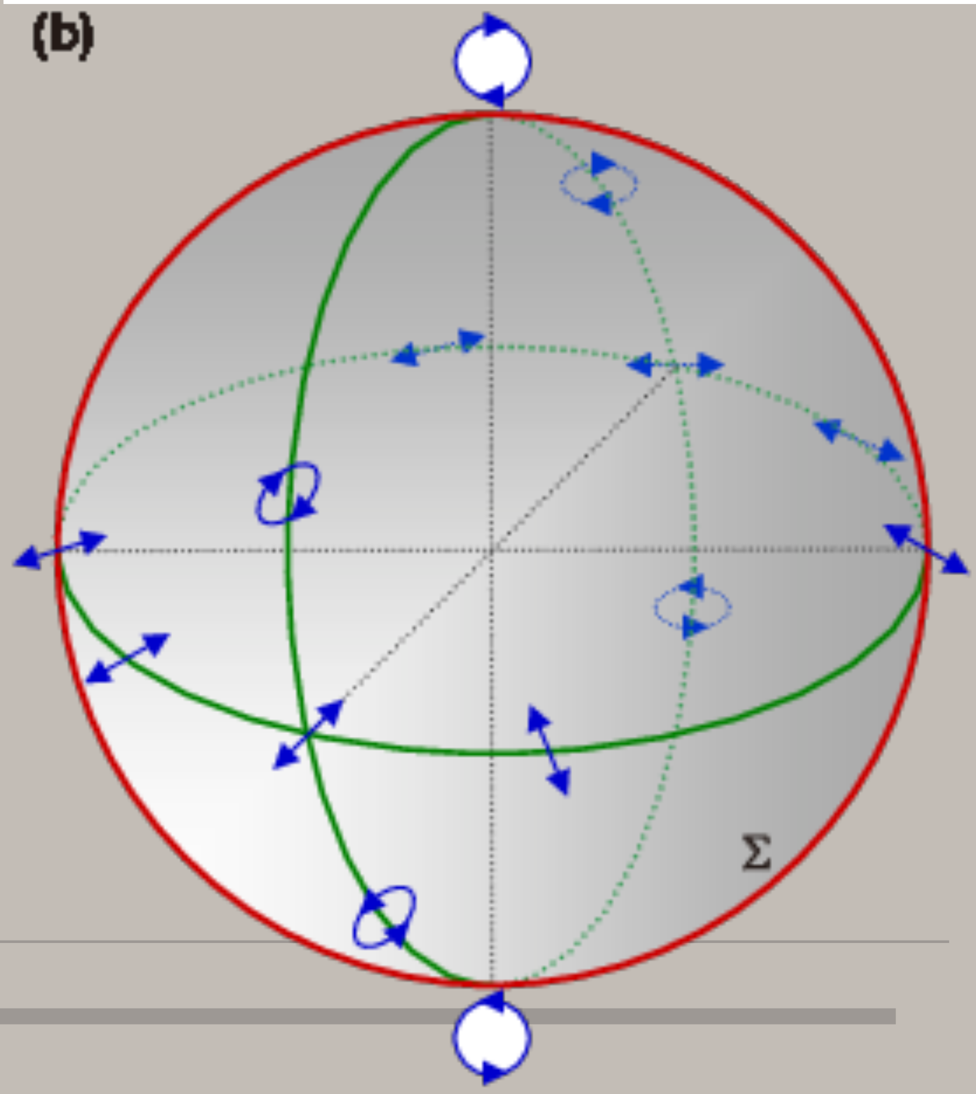


POINCARE SPHERE

Jules Henri Poincaré

- the spherical surface occupied by completely polarised states in the space of the vector
- Poles represent circular polarisations
  - Upper-hemisphere LHCP
  - Lower-hemisphere RHCP
- Equator represents linear polarisations with longitude representing tilt angle
- Latitude represents axial ratio





JONES VECTORS

Robert Clarke Jones

- Jones calculus is a matrix-based means of relating observed to incident fields.
- Vectors describe incident radiation and matrices the response of the instrument.
- The Jones Vector: $\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}$
  - Examples:
  - Linearly (x-direction) polarised wave: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
  - Left-Hand Circularly polarised wave: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

JONES MATRICES

Robert Clarke Jones

- Jones Effect of instrument described by 2x2 matrix:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_0 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

- Simple Examples:
  - Linear polariser: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
  - Left-Hand Circular polariser: $\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

- In practice matrix elements complex.
- Important: Only applicable to *completely* polarised waves