## Fourier inversion

- This relation can be inverted to get the intensity distribution, which is what we want
  - $I_\nu(l,m) = \int\int V'_\nu(u,v)exp(2\pi i(ul+vm))dudv$
- This is the fundamental equation of synthesis imaging
- Interferometrists love to talk about the $(u,v)$ plane. Remember that $u,v$ (and $w$) are measured in wavelengths.
- The vector $b=(u,v,w)=(r_1-r_2)/\lambda$ is the baseline

## Special case: PB correction

- If the response of the antenna is direction-dependent,
  - then we are measuring
    - $I_\nu(l,m)D_{1\nu}(l,m)D^\star_{1\nu}(l,m)$ **instead of** $I_\nu(l,m)$
      - (ignore polarisation for now)
    - An easier case is when the antennas all have the same response
      - $A_\nu(l,m) = |D_\nu(l,m)|^2$
      - $V'_\nu(u,v) = \int\int A_\nu(l,m) I_\nu(l,m) exp(-2\pi i(ul+vm)) dl dm$
      - (we just make the standard Fourier inversion and) the divide by the primary beam $A_\nu(l\,.\,m)$)
  - $I_\nu(l,m) = \frac{\int\int V'_\nu(u,v) exp(2\pi i(ul+vm)) du dv}{A_\nu(l,m)}$

  - See imaging lectures

## Minimum detectable temperature

- $\Delta T_{\min} = \dfrac{K_s \; T_{\rm sys}}{\sqrt{(\Delta\nu \;\; t_{\rm int} \;\; n)}} = \Delta T_{\rm rms}$
  - $T_{\rm rms}$ **RMS noise temperature of the system**
  - $\Delta T_{\min}$ **= sensitivity, or minimum detectable temperature, K**
  - $K_s$ **= sensitivity constant, dimensionless**
  - $T_{\rm sys}$ **is the system noise temperature,**
    - $T_{\rm sys} = T_A + T_{\rm LP}[(1/\epsilon) - 1] + (1/\epsilon)T_{rx}$
  - $T_{\rm LP}$ **is the physical temperature of the transmission line (or coaxial line) between antenna and receiver, K**
  - $\epsilon$ **transmission line efficiency** ($0 < \epsilon < 1$)
  - $T_A$ **is the antenna noise temperature,**
  - $T_{rx}$ **is the receiver noise temperature,**
  - $\Delta\nu$ **is the bandwidth and**
  - $t_{\rm int}$ **is the post detection integration time and**
  - $n$ **is the no. of records averaged**
- **For good sensitivity, you need low** $T_{\rm sys}$ **(low** $T_{rx}$**), long integrations (**$t_{\rm int}$**) and, for continuum, large bandwidth (**$\Delta\nu$**).**

## Minimum detectable temperature and flux density

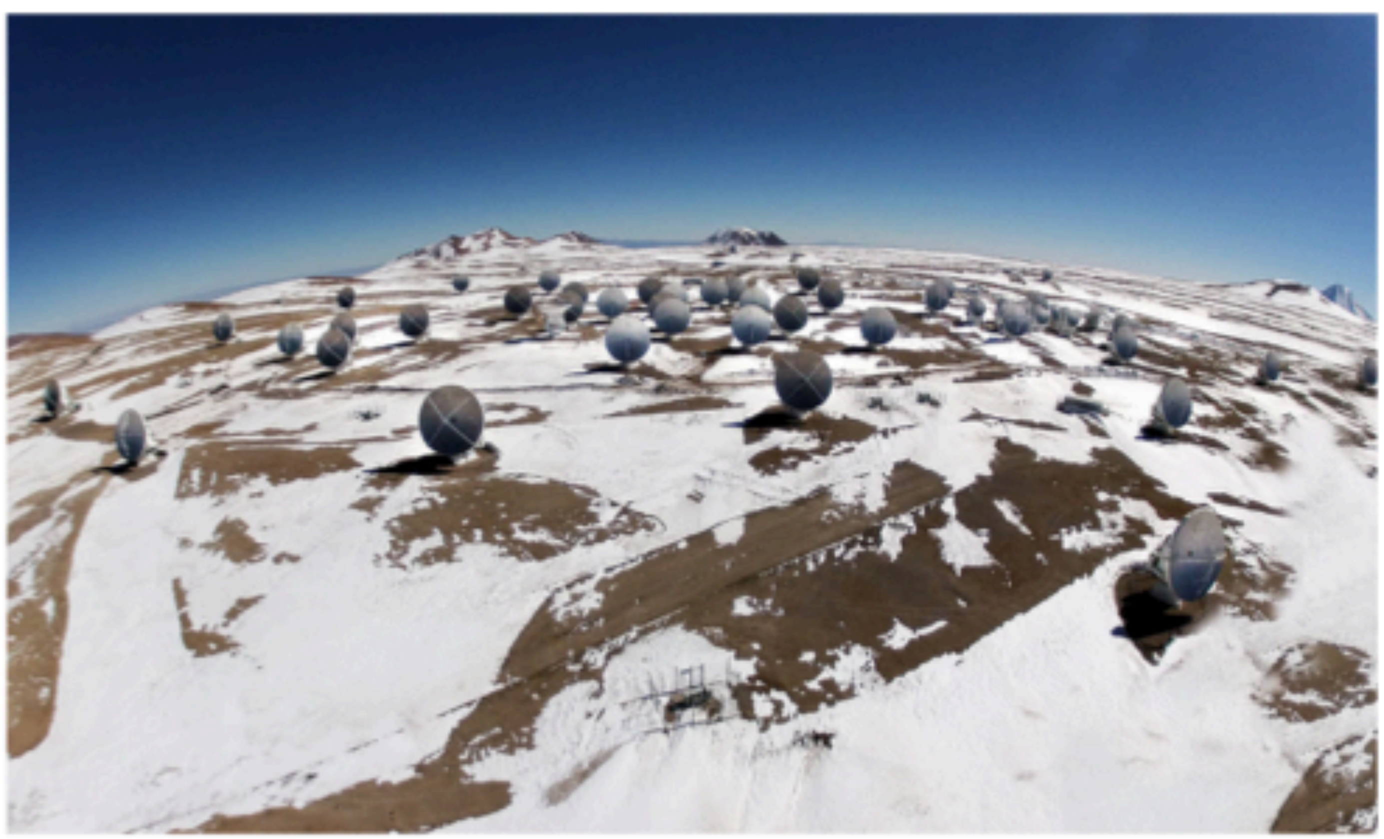
- $\Delta S_{\min} = \frac{2k}{A_{\text{eff}}} \times \frac{K_s\ T_{\text{sys}}}{\sqrt{(\Delta\nu\ t_{\text{int}}\ n)}}$
  - $S_{\min}$ = **minimum detectable flux density**
  - $k$ **is the Boltzmann's constant**
  - $A_{\text{eff}}$ **is the effective area of the antennas**
  - $K_s$ = **sensitivity constant, dimensionless**
  - $T_{\text{sys}}$ **is the system noise temperature,**
    - $T_{\text{sys}} = T_A + T_{\text{LP}}[(1/\epsilon) - 1] + (1/\epsilon)T_{rx}$
  - $T_{\text{LP}}$ **is the physical temperature of the transmission line (or coaxial line) between antenna and receiver, K**
  - $\epsilon$ **transmission line efficiency (**$0 < \epsilon < 1$**)**
  - $T_A$ **is the antenna noise temperature,**
  - $T_{rx}$ **is the receiver noise temperature,**
  - $\Delta\nu$ **is the bandwidth and**
  - $t_{\text{int}}$ **is the post detection integration time and**
  - $n$ **is the no. of records averaged**
- **For good sensitivity, you need low** $T_{\text{sys}}$ **(low** $T_{rx}$**), long integrations (**$t_{\text{int}}$**) and, for continuum, large bandwidth (**$\Delta\nu$**).**

# Noise

- $S_{\rm rms} = \frac{2k}{A_{\rm eff}} \times \frac{T_{\rm sys}}{\sqrt{N_A(N_A - 1)t_{\rm int}\Delta\nu}}$
  - $S_{\rm rms}$ **RMS noise level**
  - $T_{\rm sys}$ **is the system temperature,**
  - $A_{\rm eff}$ **is the effective area of the antennas,**
  - $N_A$ **is the number of antennas,**
  - $\Delta\nu$ **is the bandwidth and**
  - $t_{\rm int}$ **is the integration time and**
  - $k$ **is the Boltzmann's constant**
- **For good sensitivity, you need low** $T_{\rm sys}$ **(receivers), large** $A_{\rm eff}$ **(big, accurate antennas), large** $N_A$ **(many antennas), long integrations (**$t_{\rm int}$**) and, for continuum, large bandwidth (**$\Delta\nu$**).**

## Antennas collect radiation

- ✦ Specification, design and cost are frequency-dependent .
  High-frequency: steerable dishes (5 – 100 m diameter)
  - ✦ Low-frequency: fixed dipoles, yagis, ….
  - ✦ Ruze formula efficiency = $= exp(-(\frac{4\pi\sigma}{\lambda})^2$
  - ✦ Surface rms error $\sigma < \lambda/20$
  - ✦ sub-mm antennas are challenging (surface rms $<$ 25 $\mu$m for 12 m ALMA antennas); offset pointing $<$ 0.6 arcsec rms



## Receivers

- Detect radiation
- Cryogenically cooled for low noise (at high frequencies)
- Normally detect two polarization states
  - separate optically
- Optionally, in various combinations:
  - Amplify RF signal
    - Then
      - either digitize directly (possible up to ~10's GHz)
      - or mix with phase-stable local oscillator signal to make intermediate frequency (IF) → two sidebands (one or both used) → digitize
      - a mixer is a device with a non-linear voltage response that outputs a signal at the difference frequency
- Digitization typically 3 – 8 bit
- Send to correlator

## Time and frequency distribution

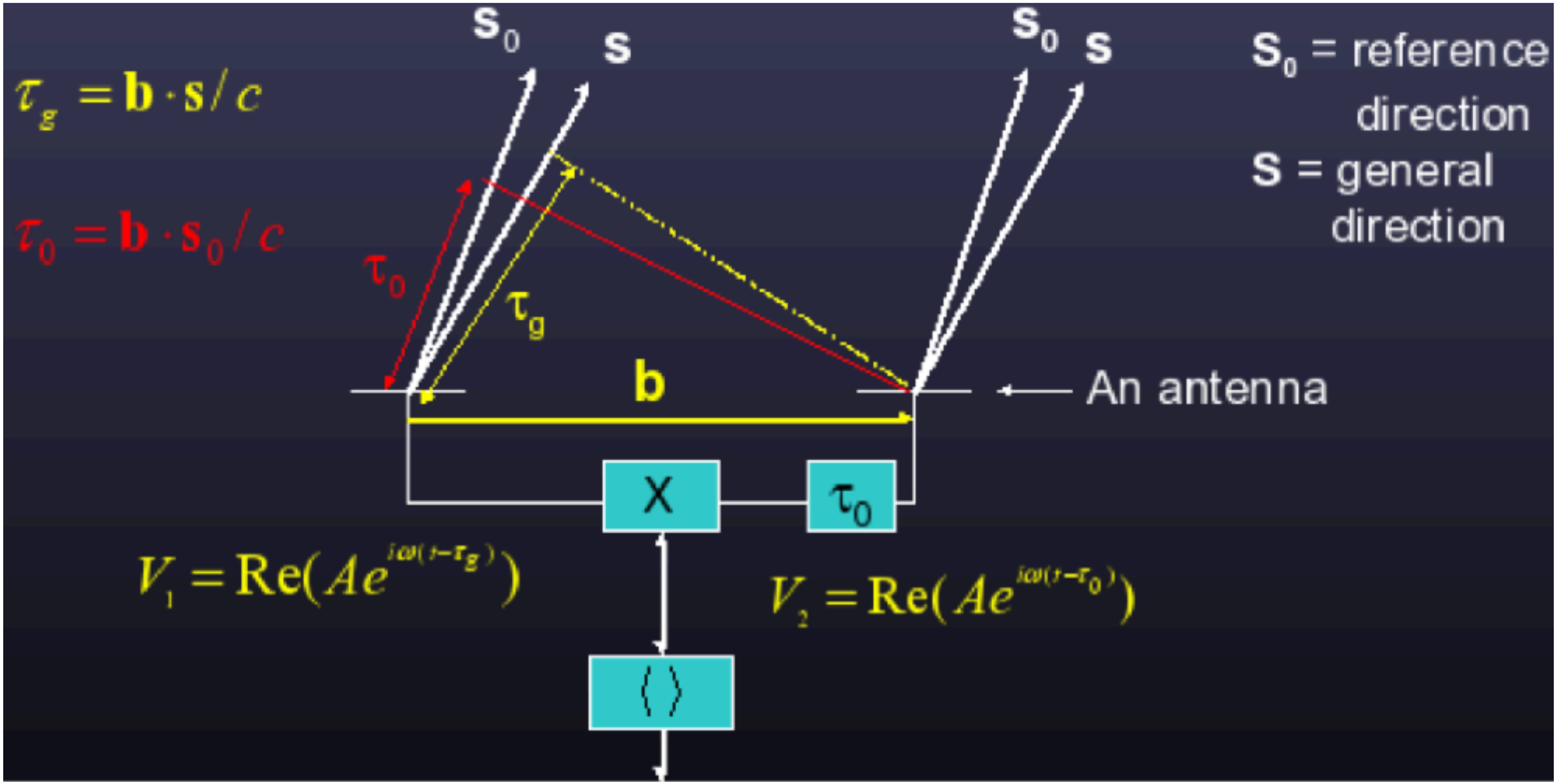
- These days, often done over fibre using optical analogue signal
  - Master frequency standard (e.g. H maser)
  - Must be phase-stable – round trip measurement
    - Slave local oscillators at antennas
    - Multiply input frequency
    - Change frequency within tuning range

# Delay

- An important quantity in interferometry is the time delay in arrival of a wavefront (or signal) at two different locations, or simply the delay, $\tau$.
- This directly affects our ability to calculate the coherence function
  - Examples:
    - Constant ("cable") delay in wave-guide or electronics
    - geometrical delay
    - propagation delay through the atmosphere
  - Aim to calibrate and remove all of these accurately
  - Phase varies linearly with frequency for a constant delay
    - $\Delta\phi = 2\pi\tau\Delta\nu$
    - Characteristic signature

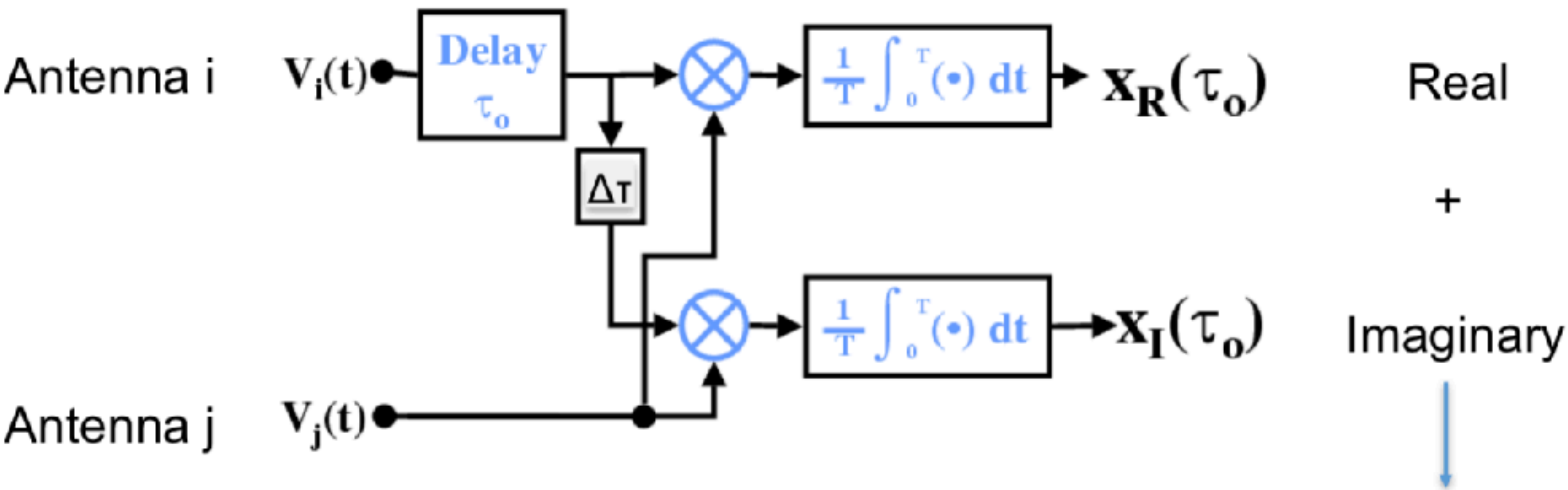
## Delay tracking

- The geometrical delay $\tau_0$ for the delay tracking centre can be calculated accurately from antenna position + Earth rotation model.
- Works exactly only for the delay tracking centre.
  - Maximum averaging time is a function of angle from this direction.



# What does a correlation do?

- Takes digitized signals from individual antennas;
  - calculates complex visibilities for each baseline



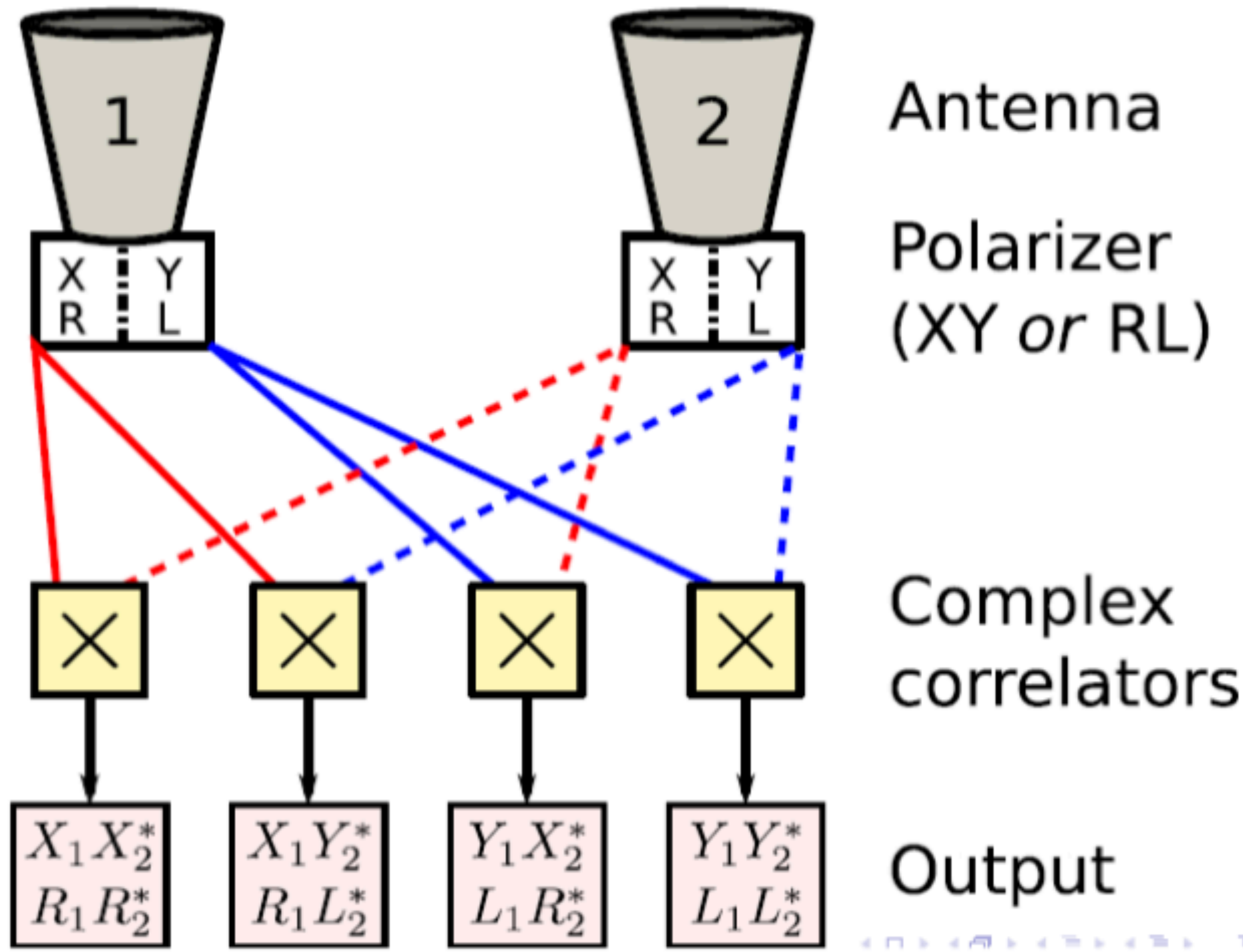

- $x_R = x_{ij}(\tau_0)$
- $x_I = x_{ij}(\tau_0 + \Delta\tau)$, **with** $\Delta\tau = 1/\left(4\nu_0\right)$ $(\Delta\phi = 90°)$.

or amplitude
And phase

# Stokes Parameters and visibilities

- This assumes the perfect case:
  - no rotation on the sky (not true for most arrays, but can be calculated and corrected)
- perfect system

- Circular feeds
  - $V_I = V_{RR} + V_{LL}$
  - $V_Q = V_{RL} + V_{LR}$
  - $V_U = V_{RL} - V_{LR}$
  - $V_V = V_{RR} - V_{LL}$



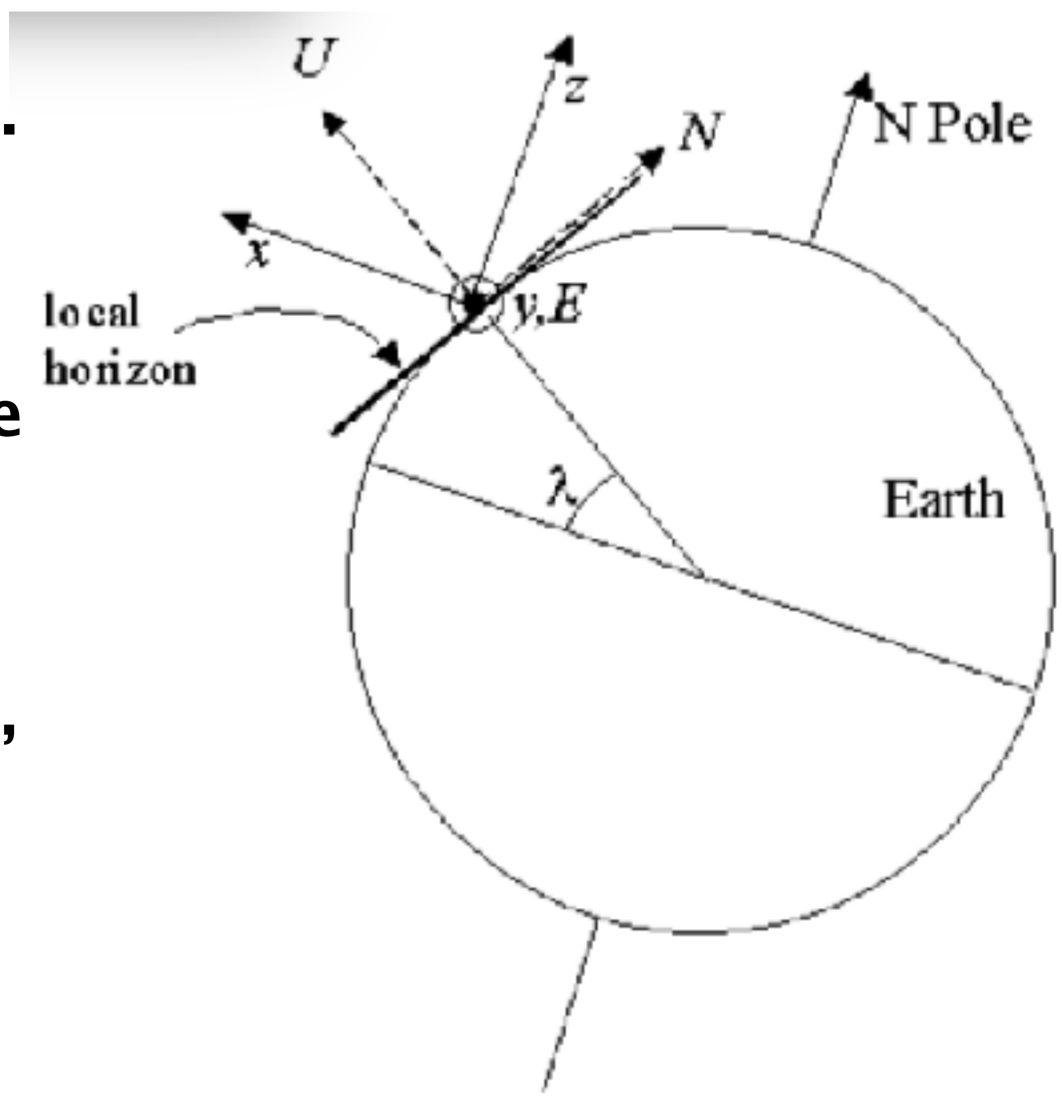
# Spectroscopy

- We make multiple channels by correlating with different values of lag, $\tau_0$. This is a delay introduced into the signal from one antenna with respect to another.
  - For each quasi-monochromatic frequency channel, a lag $\tau$ is equivalent to a phase-shift $2\pi\tau\nu$, i.e.,
    - $V(u,v,\tau) = \int V(u,v,\nu) exp(2\pi i \tau \nu) d\nu$
  - This is another Fourier transform relation with complementary variables $\nu$ and $\tau$, and can be inverted to extract the desired visibility as a function of frequency.
    - $V(u,v,j\Delta\nu) = \Sigma_k V(u,v,k\Delta\tau) exp(-2\pi ijk\Delta\nu\Delta\tau)$
  - In practice, we do this digitally, infinite frequency channels:
  - Each spectral channel can then be imaged (and deconvolved) individually.
    - The final product is a data cube, regularly gridded in two spatial and one spectral coordinate.
- I have described an "XF" correlator.
    - The Fourier Transform step can be done first ("FX").

# Obtaining (u,v) from an antenna array

- A synthesis imaging radio instrument consists of a number of radio elements (radio dishes, dipoles, or other collectors of radio emission), which represent measurement points in $u, v$ space. We now need to describe how to convert an array of dishes on the ground to a set of points in $u, v$ space.

- ENU coordinates to x, y, z
  - The first step is to determine a consistent coordinate system. Antennas are typically measured in units such as meters along the ground. We will use a right-handed coordinate system of East, North, and Up $(E, N, U)$. These coordinates are relative to the local horizon, however, and will change depending on where we are on the spherical Earth. It is convenient in astronomy to use a coordinate system aligned with the Earth's rotational axis, for which we will use coordinates $x, y, z$ as shown in Figure. Conversion from $(E, N, U)$ to $(x, y, z)$ is done via a simple rotation matrix:



# Obtaining (u,v) from an antenna array

- Baseline and spatial frequencies
  - Note that the baselines are differences of coordinates, i.e. for the baseline between two antennas we have a vector
    - $B = (B_x, B_y, B_z) = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
  - This vector difference in positions can point in any direction in space, but the part of the baseline that matters in calculating $u, v$ is the component perpendicular to the direction $\theta_0$ (the phase center direction), which we called $B_{\text{proj}}$.
    - Let us express the phase centre direction as a unit vector $s_0 = (h_0, \delta_0)$, where $h_0$ is the hour angle (relative to the local meridian) and $\delta_0$ is the declination (relative to the celestial equator). Then
      - $B \,.\, s_0 = B cos\theta_0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & -\sin\lambda & \cos\lambda \\ 1 & 0 & 0 \\ 0 & \cos\lambda & \sin\lambda \end{bmatrix} \begin{bmatrix} E \\ N \\ U \end{bmatrix}$$

# Obtaining (u,v) from an antenna array

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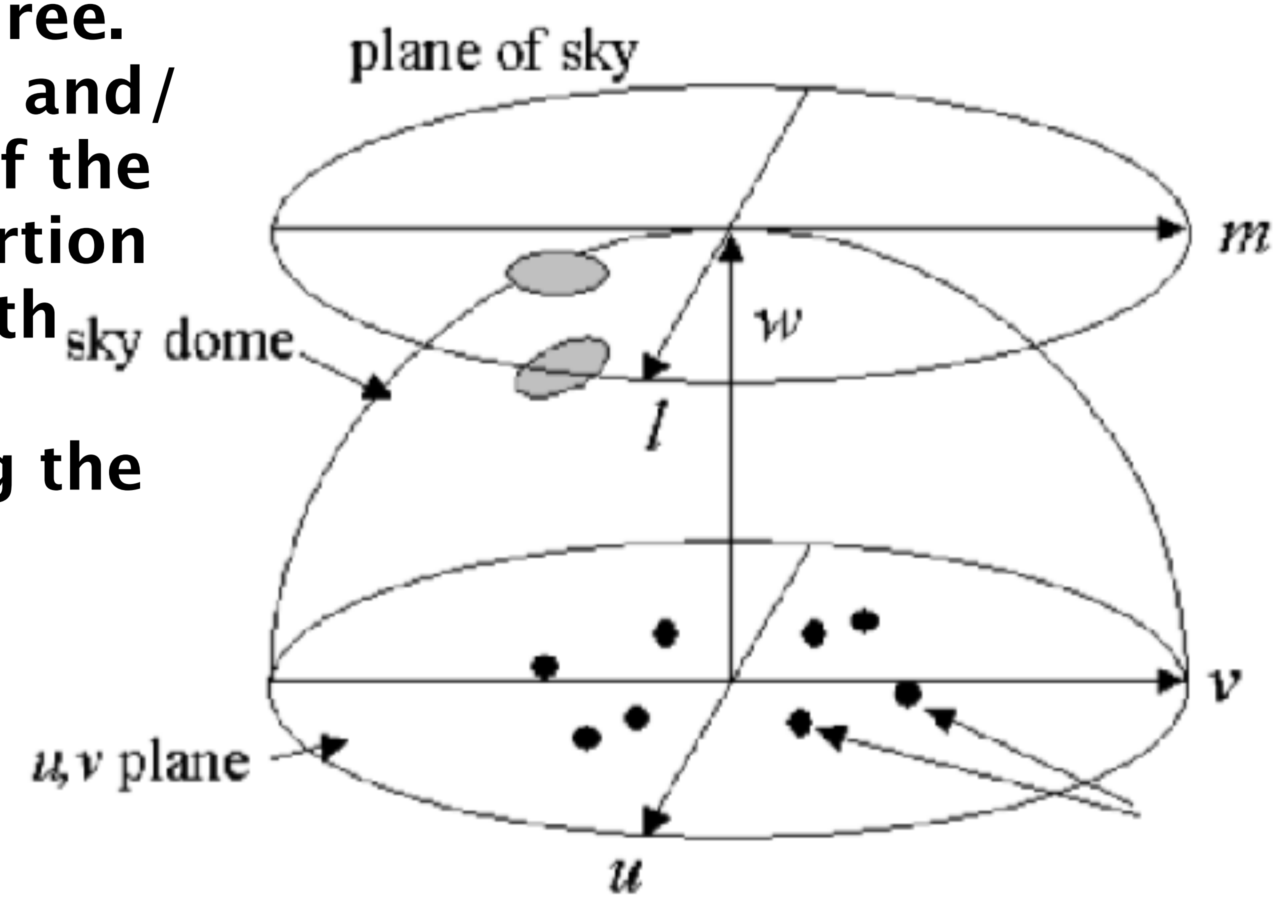
      - $B \,.\, s_0 = B cos\theta_0$
  - Recall that the spatial frequencies $u, v$ are just the distances expressed in wavelength units, so we can get the $u, v$ coordinates from the baseline length expressed in wavelength units from the following coordinate transformation:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \sin h_o & \cos h_o & 0 \\ -\sin \delta_o \cos h_o & \sin \delta_o \sin h_o & \cos \delta_o \\ \cos \delta_o \cos h_o & -\cos \delta_o \sin h_o & \sin \delta_o \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

## Obtaining (u,v) from an antenna array

- Notice that we have introduced a spatial frequency $w$, which we must include to be accurate.
  - However, if we limit our image to a small area of sky near the phase centre (small angular coordinates $l, m$), then we can get away with considering only $u, v$ coordinates.
  - Ignoring causes distortion that is akin to projecting a section of the sky dome only a flat plane. The condition for this to be valid is
    - $1/2(l^2 + m^2)w \ll 1$.
  - For $w = 1000\ \lambda\mathrm{s}$ for example,
    - we could map out to about 1/30 radian or a little over 1 degree.
  - (As we use higher frequencies and/or longer baselines, the part of the sky we can map without distortion gets smaller. We will henceforth ignore the $w$ coordinate and assume that we are measuring the sky in a small region near the phase centre.)



# Obtaining (u,v) from an antenna array

- Note that $u, v$ depend on the Hour Angle, so as the Earth rotates and the source appears to move across the sky, the array samples different $u, v$ at different times.
  - Figure shows an example with antenna locations, corresponding $u, v$ points at a single instant in time, and the $u, v$ points over many hours in time. The $u, v$ points trace out portions of ellipses, called $u, v$ tracks, and sample more of the $u, v$ plane. Making a map over a long period of time is called Earth Rotation Synthesis.