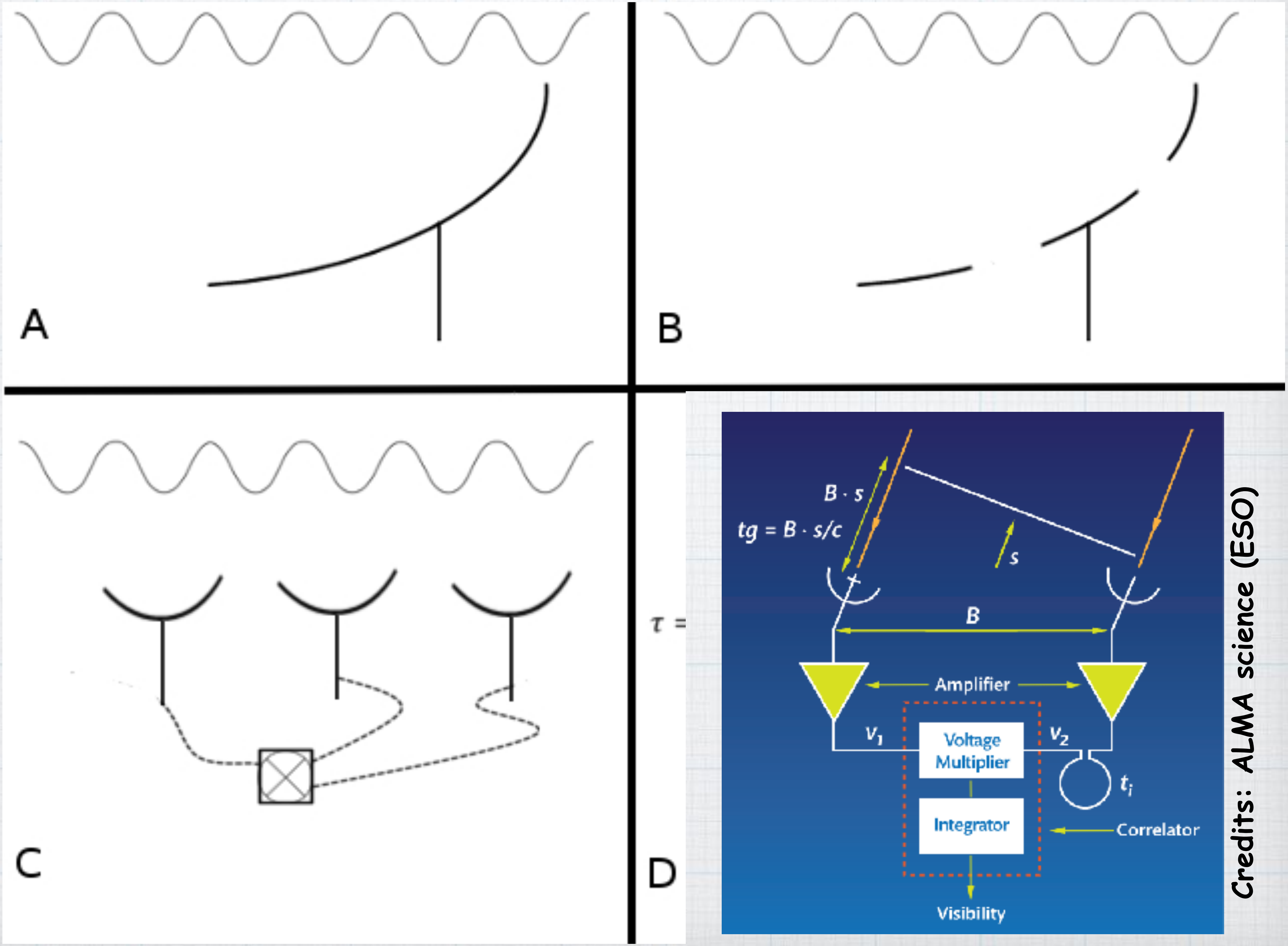
build an interferometer

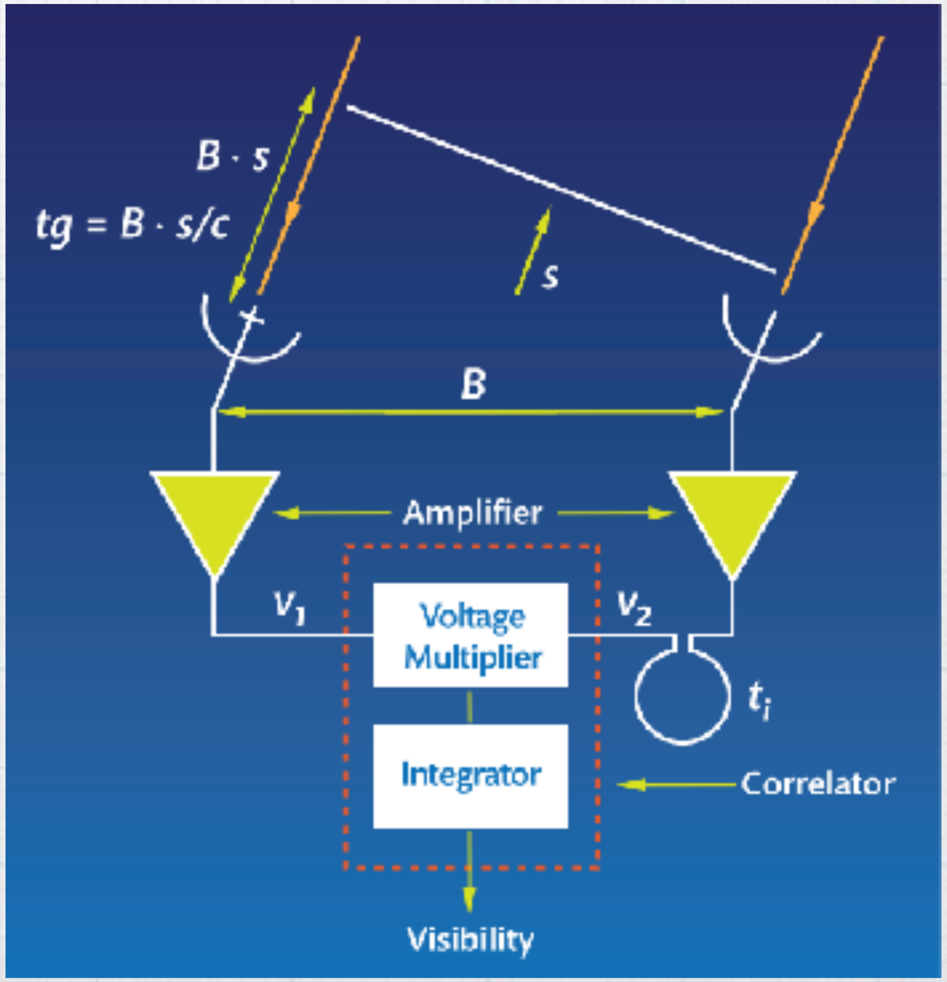


Credits: ALMA science (ESO)

GMRT: A radio interferometer

Van Cittert-Zernicke Theorem

- An interferometer measures the interference pattern produced by pairs of apertures.
  - The interference pattern is directly related to the source brightness: (for small fields-of-view) the complex visibility is the 2D Fourier transform of the brightness on the sky.

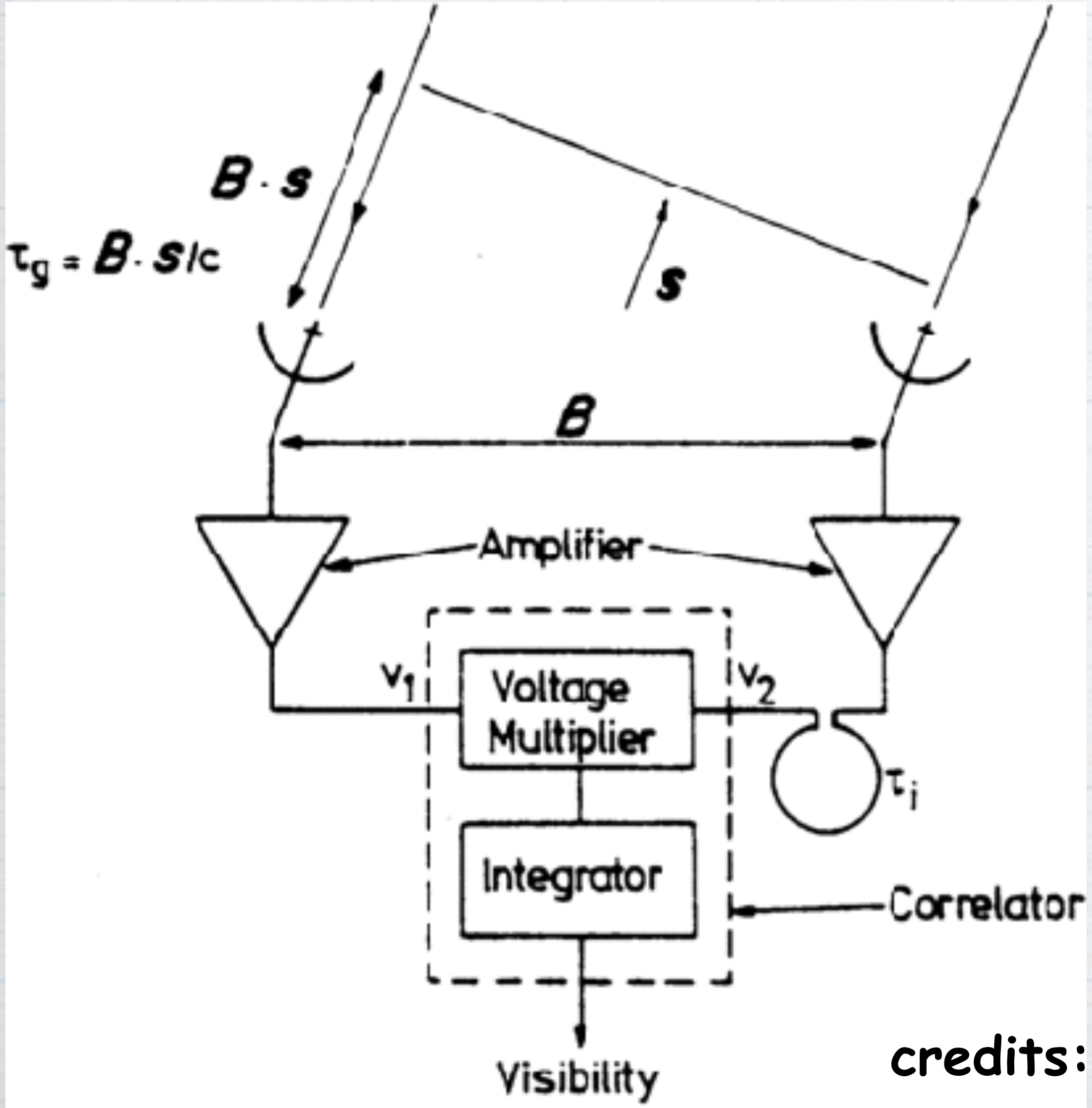


Credits: ALMA science (ESO)

Two antennas (imaging)

- **interferometry**
  - **Correlate the signal**
  - **Correlation = multiplication + integration**

- An interferometer measures the interference pattern produced by pairs of apertures.



- The interference pattern is directly related to the source brightness: (for small fields-of-view) the complex visibility is the 2D Fourier transform of the brightness on the sky.

$$V(u, v) = \int\int I(l, m)\ e^{-2\pi i(ul+vm)}\ dl\ dm$$

credits: 1996 NRAO Synthesis Imaging Summer School

Imaging arrays (include tricks)

- Celestial sources do not vary on human timescales
  - i.e., their statistical parameters do not vary!

- Synthesize a large aperture using repeated observations with a few antennas whose spacings can be varied
  - e.g., by mounting the antennas on tracks

- -or- by tracking the source as it rises and sets
  - The Earth's rotation changes the projected separation between the antennas
  - Thus, one can get a good coverage of the Fourier (u,v) plane without moving the antennas

Aperture synthesis

- A correlation interferometer measures one component of the image Fourier transform
  - The component corresponding to the spatial frequency d/λ
- Assuming that
  - the fov is small and/or
    - the measurements are all in a single plane;
  - the observation bandwidth is small compared to central-frequency;
  - instrumental and propagation effects have been calibrated.
- Given enough interferometers one can measure enough components of Fourier transform and do a Fourier inversion to get the image
- One can thus synthesise a telescope of size equal to the array size

Sensitivity

$$P_a = k_B\ T_a\ \Delta\nu$$

$$P_N = k_B\ T_{sys}\ \Delta\nu$$

$$P_a = (1/2)\ \eta_a\ A\ S_\nu\ \Delta\nu$$

$$(\eta_a\ A)/(2\ k_B) = T_a/S_\nu$$

$$\langle P_i \rangle = a_i \langle (s_i + n_i)^2 \rangle = a_i \left[\langle S_i^2 \rangle + \langle n_i^2 \rangle \right]$$

$$= G_i\ k_B (T_{ai} + T_{sysi}) \Delta\nu$$

sq. of the rms fluctuation of a Gaussian random noise

= expectation value of the variable squared - square of the mean

$$\sigma^2 \langle P_{ij} \rangle = (a_i a_j)/\eta_s{}^2 \langle [(s_i + n_i)(s_j + n_j)]^2 \rangle - (G_{ij}^2/\eta_s^2) k_B^2 S_c^2 \Delta\nu^2$$

$$\Delta S_{ij} = \frac{1}{\eta_s} \times \sqrt{\frac{T_{sysi}\ T_{sysj}}{2\ \Delta\nu\ \tau_{acc}}}$$

$$\Delta I_{rms} = \frac{1}{G} \frac{T_{sys}}{\sqrt{(N(N-1)/2) \times N_{pol} \times 2 \times \Delta\nu \times \Delta t}}$$

Imaging arrays

- Standard calibration and imaging
    - (DI instrumental effects)
    - and DD instrumental + propagation effects
        - correction for w-term and for PB
            - image plane correction
            - Fourier plane correction
            - pointing self-calibration
    - Mosaicing
    - w/ advanced image parameterisation
        - multi-scale (CLEAN) deconvolution
        - multi-frequency synthesis (imaging)
        - full polarisation (Stokes) calibration and imaging

Telescope sensitivity

- Noise limit for imaging with interferometric radio telescopes

$$\sigma = \frac{T_{\mathrm{sys}}}{A_{\mathrm{eff}} \times \sqrt{(\Delta\nu \times \Delta t)}}$$

- Sensitivity improvements achieved by
  - wide band receivers,
  - long integration times
  - more antennas
  - long baselines
    - B_{max} ~100 km @200 MHz, the confusion noise is ~1 μJy beam^{-1}.

Measurement equation formalism

$$\overrightarrow{V}_{ij}^{Obs}(\nu,t) = G_{ij}\ W_{ij}(\nu,t) \int P_{ij}\ M_{ij}(s,\nu,t)\ \vec{I}(s,\nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

$$V(u,v,w) = \int\int I(l,m)\ e^{-2\pi i\left[ul + vm + w(\sqrt{(1-l^2-m^2)} - 1)\right]}\ \frac{dl\ dm}{\sqrt{(1-l^2-m^2)}}$$

Recall (a special case): $V(u,v) = \int\int I(l,m)\ e^{-2\pi i(ul+vm)}\ dl\ dm$

(a.k.a. **van Cittert Zernicke** theorem)

Measurement equation

$$\vec{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \vec{b}_{ij}.\vec{s}}\ d\vec{s}$$

DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
(A-projection)
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,...

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

Imaging challenges at low-ν

- Wide-field imaging
  - account for direction dependent (DD) effects
  - PB: time, frequency and polarisation dependence
  - w-term
- Wide-band imaging
  - … plus frequency dependence of the sky brightness
- Data volume $\propto N_{\mathrm{ant}}^2 \times N_{\mathrm{channel}} \times t$
- Sky brightness $\Longrightarrow$ multi-scale deconvolution
- Ionospheric effects $\Longrightarrow$ need for DD solvers