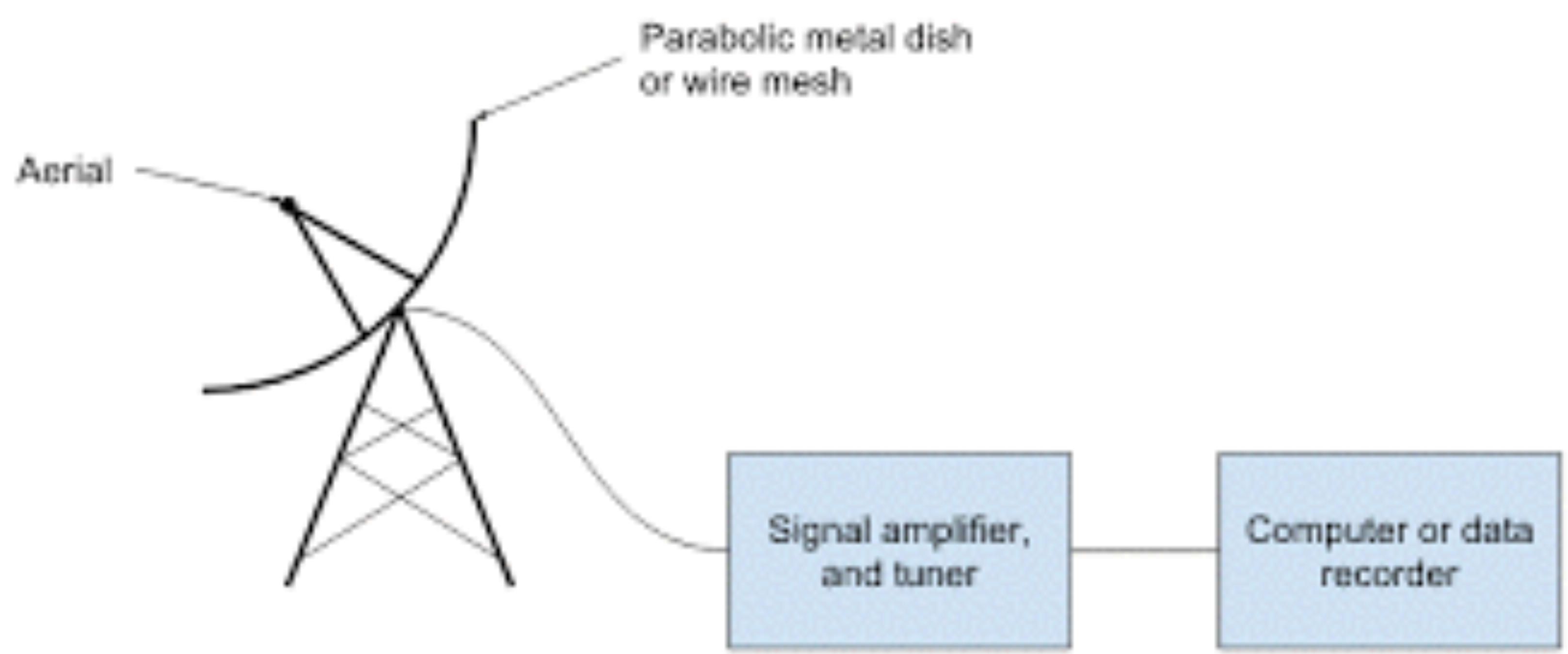
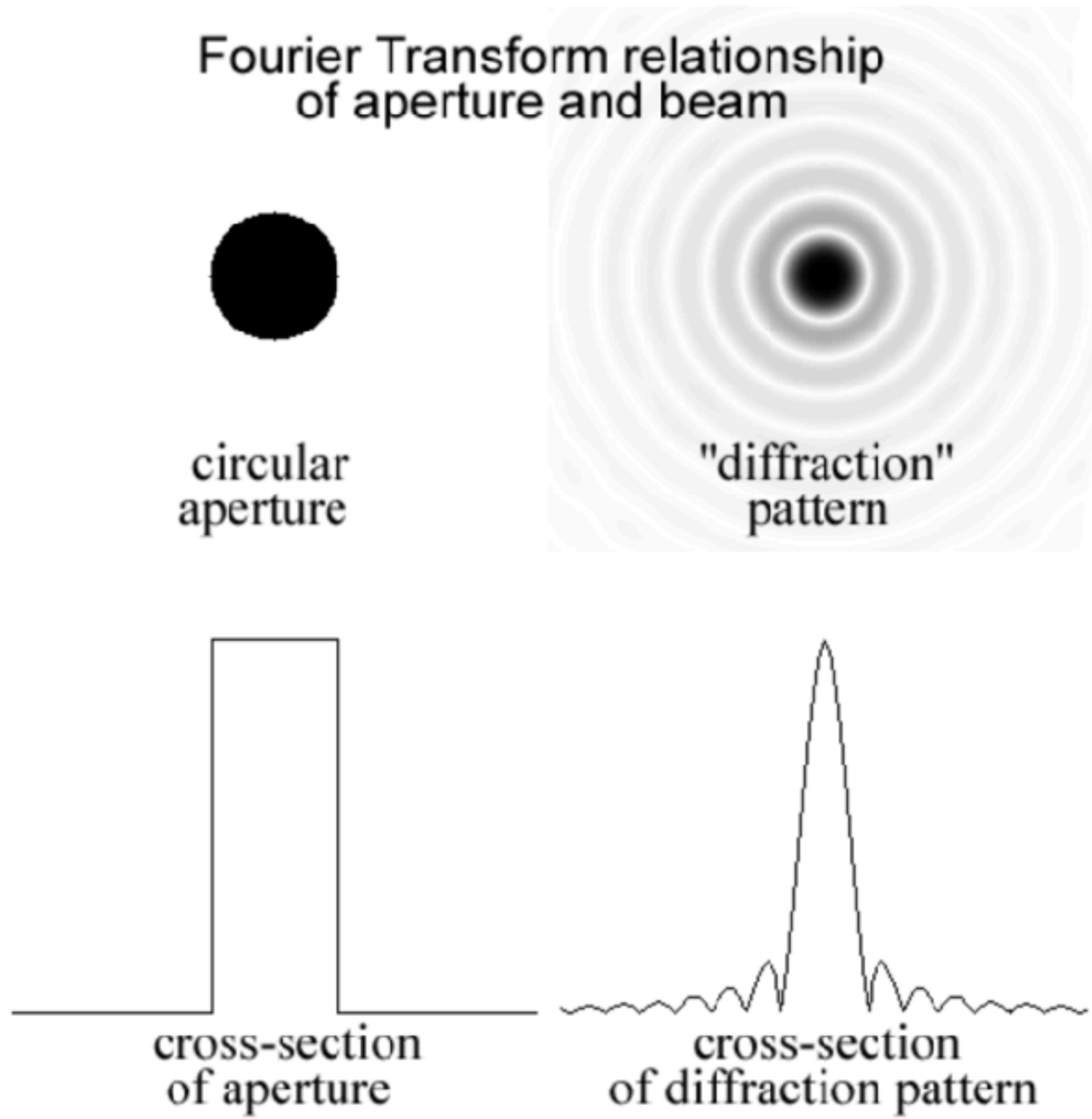
A basic radio telescope

- Single dish
  - Feed
  - Receiver (FE)
  - Mount
  - Transmission lines
  - Receiver (BE)
  - ABC

- Single dish
  - Mechanical
    - Altitude drive
    - Azimuth drive



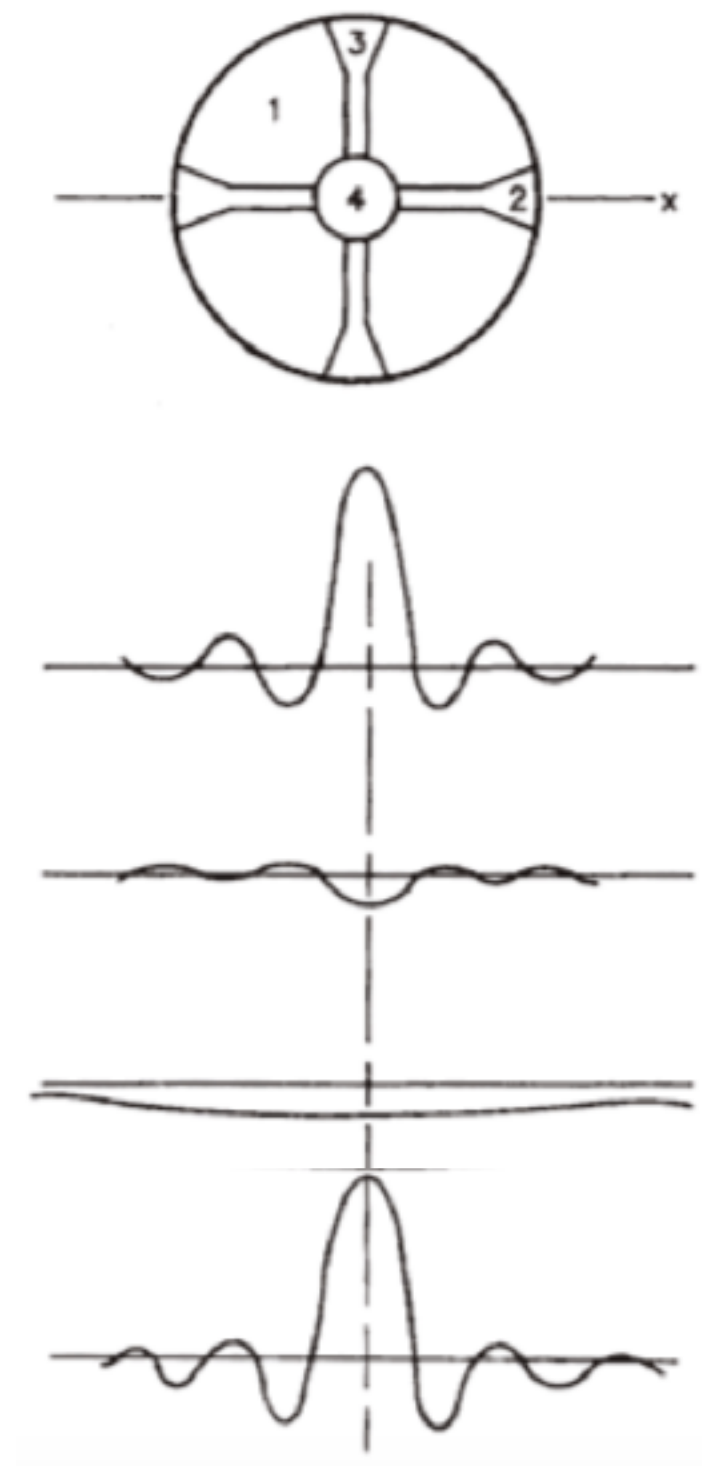
Fourier pairs



Antenna performance

- ✦ FT of unblocked aperture
  - ✦ FT of legs-blockage
  - ✦ FT of feed-blockage
- ✦ Sum of all FTs

✦ $\eta = \eta_{\text{reflector}} \eta_{\text{blockage}} \eta_{\text{feed spillover eff}} \eta_{\text{misc}}$



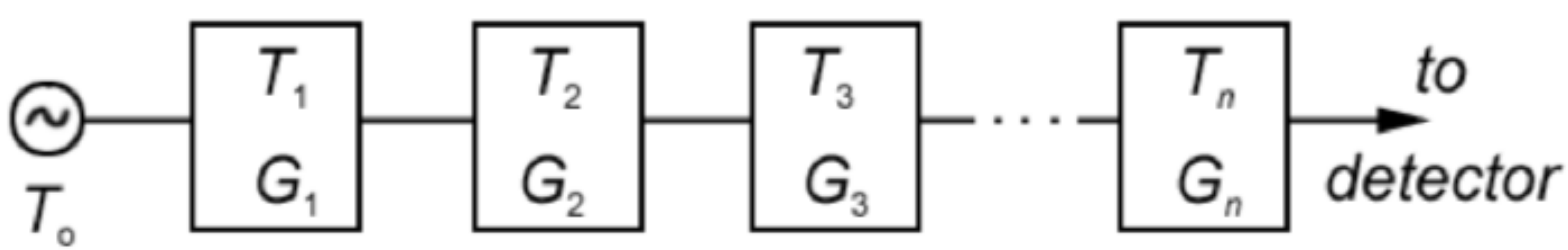
Power vs. temperature

- The power level of the radiation (W) can be traced from its reception by the feed, through the receiving system. The "signal" is generally noise-like (white noise, containing all frequencies in the band). For convenience, we often consider the equivalent noise temperature corresponding to the power level
  - $P = kT\Delta\nu$
    - although we also refer to the power level in decibel milliwatts [dBm], we can consider the power received by the antenna,
  - $P_a = kT_a\Delta\nu$
    - where T_a is the antenna temperature, and the output power of the receiver as
  - $P_{\rm tot} = P_a + P_{\rm sys} \implies T_{\rm tot} = T_a + T_{\rm sys}$
  - $T_{\rm sys}$ is the system temperature, and represents the added noise of the system. It is a figure of merit, and should be kept as low as possible.
  - $T_{\rm tot} = [T_{bg} + T_{\rm sky} + T_{\rm spill}] + [T_{\rm loss} + T_{\rm cal} + T_{\rm rx}]$

    - To see how important the unwanted $T_{\rm sys}$ is, let's compare it with a typical signal. Say we have a point source of flux density 1 Jy [$= 1 = 10^{-26} Wm^{-2}Hz^{-1}$]. If observed with a radio telescope of 10 m diameter, what is T_a?

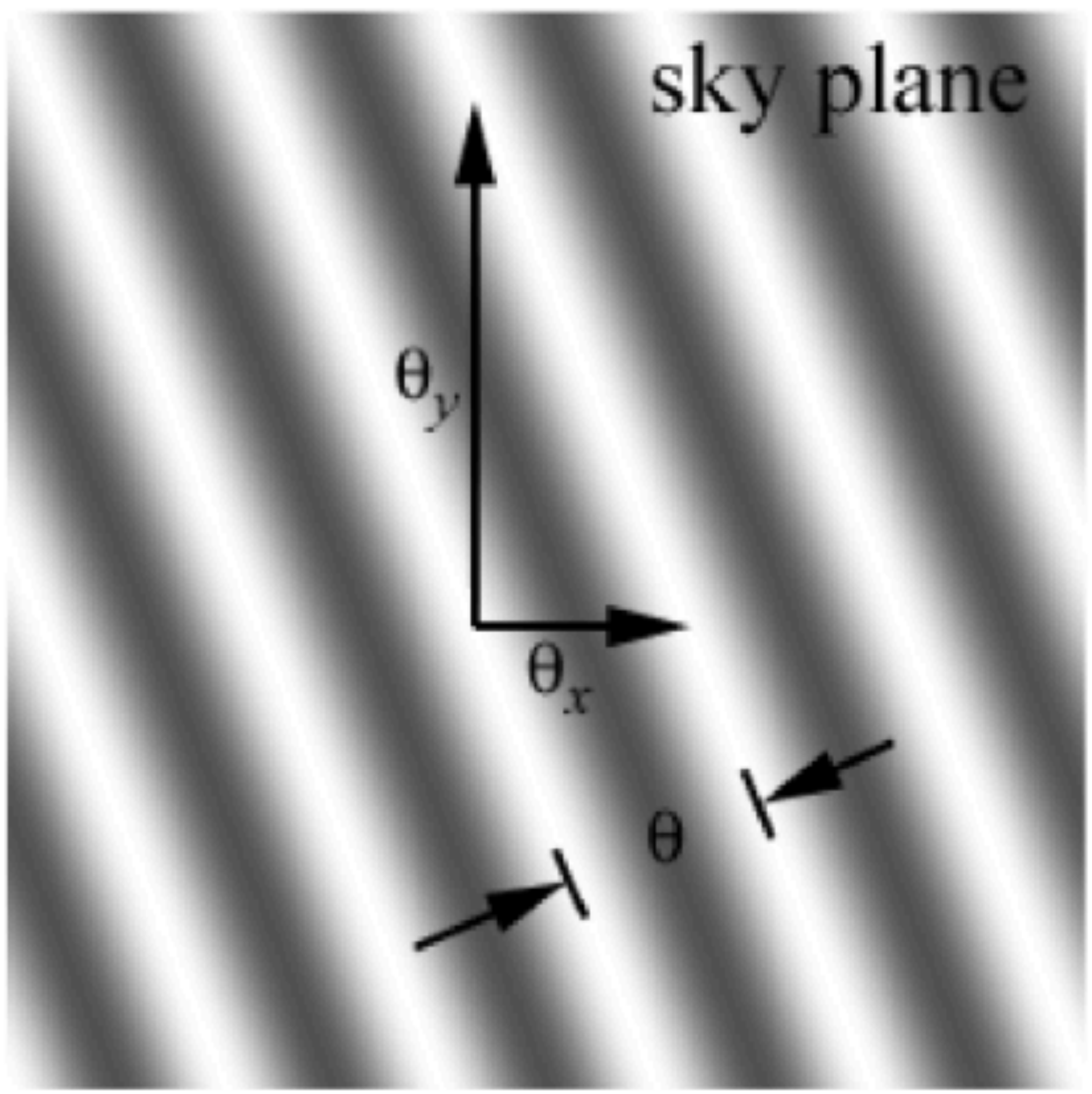
Total system temperature

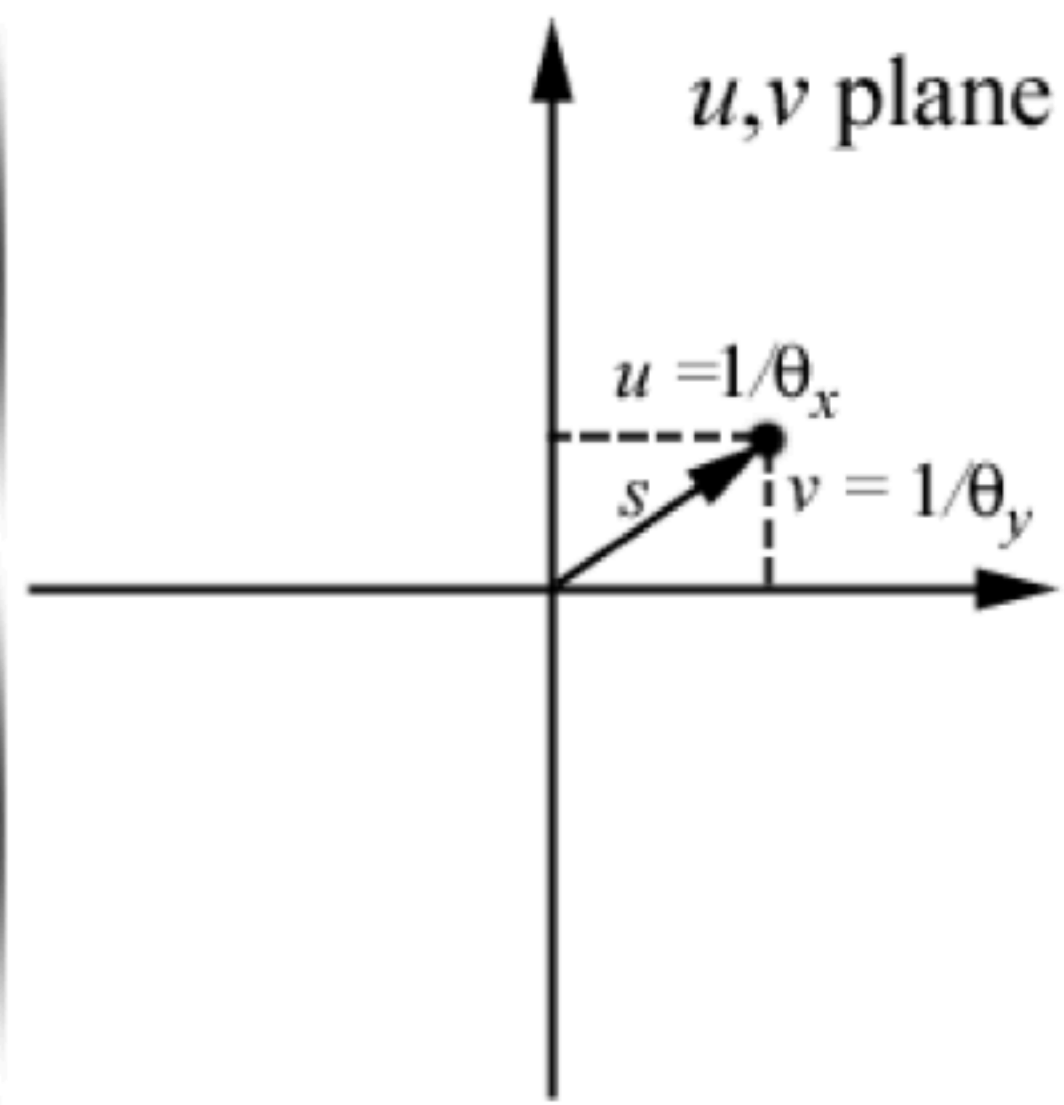
- An actual system is just a linear chain of 2-port devices,
  - where the input (i.e. from the feed) is shown as T_0, and each two-port device is labeled with its noise temperature and gain.
    - Some of the gains may be less than one (i.e. a lossy cable or attenuator). The power output of the whole system will be
  - $P = G_1 G_2 G_3 \ldots G_n k T_0 \Delta\nu + G_1 G_2 G_3 \ldots G_n k T_1 \Delta\nu + G_2 G_3 \ldots G_n k T_2 \Delta\nu + G_3 \ldots G_n k T_n \Delta\nu + \ldots$
  - And the corresponding system temperature is
  - $T_{\rm sys} = T_0 + T_1 + T_2/G_1 + T_3/(G_1 G_2) + \ldots + T_n/(G_1 G_2 \ldots G_{n-1})$
  - You can see that the external temperature (the antenna temperature) just gets added to by all of the noise temperatures of the following devices, but each stage after the first stage gets divided by the total gains of the preceding stages. This makes the first amplifier stage all-important.



FT relationship and inverse

- A point in the u, v plane a distance s from origin has components u and v.
  - In RA, this corresponds to a single baseline, or pair of antennas.
  - The FT of this sampling corresponds to fringes in the sky plane, with angular separation $\theta =$ fringe-spacing.
  - The two corresponding angular coordinates are θ_l and θ_m, which are the fringe separations in the l and m angular directions.

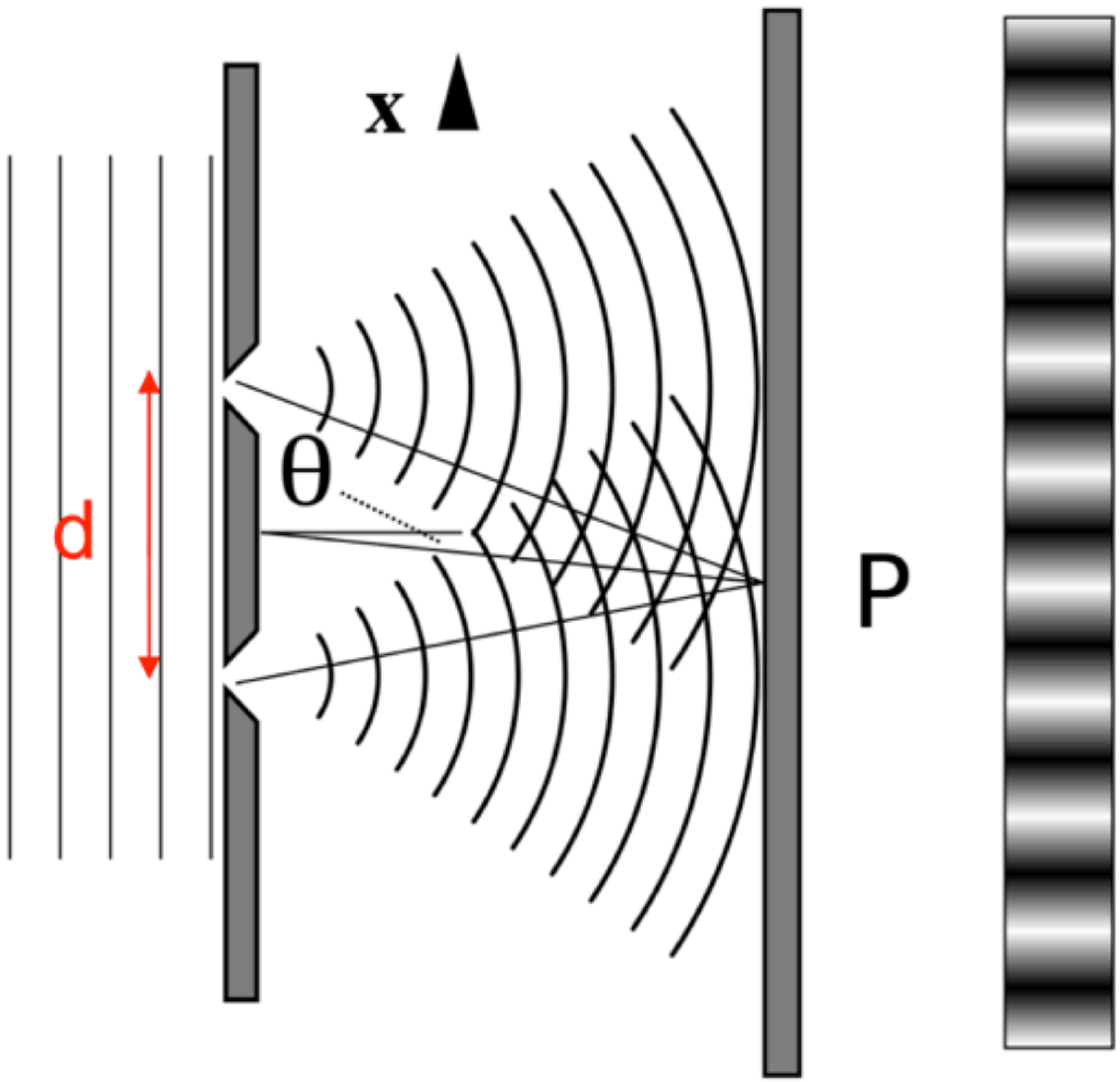




Objective

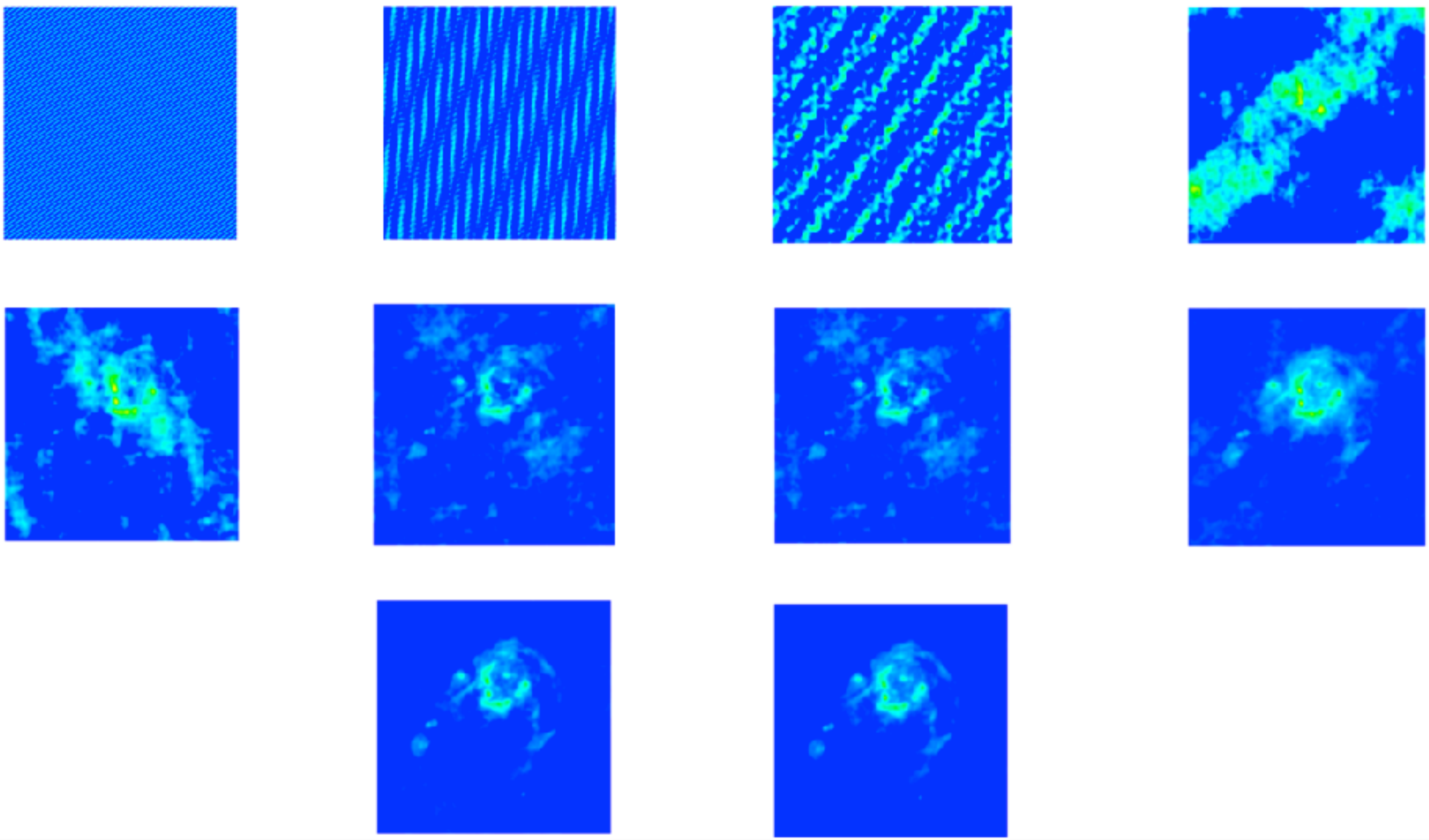
- A more formal approach to radio interferometry using coherence functions
  - A complementary way of looking at the technique
  - Be clear about simplifying assumptions
- Relaxing the assumptions
- How does a radio interferometer work?
  - Follow the signal path
  - Technologies for different frequency ranges

Young's double slit experiment



- Fringes
  - Angular spacing of fringes
    - λ/d
  - Familiar from optics
  - Essentially the way that astronomical interferometers work at optical and IR wavelengths
  - Direct detection

Build up of an interferometer from many slits



But this is not how radio interferometer work in practice

- The two techniques are closely related, and it often helps to think of images as built up of sinusoidal “fringes”
- But radio interferometers collect radiation (“antenna”), turn it into a digital signal (“receiver”) and generate the interference pattern in a special-purpose computer (“correlator”)
- How does this work?
- I find it easiest to start with the concept of the mutual coherence (or correlation) of the radio signal received from the same object at two different places
- No proofs, but I will try to state the simplifying assumptions clearly.

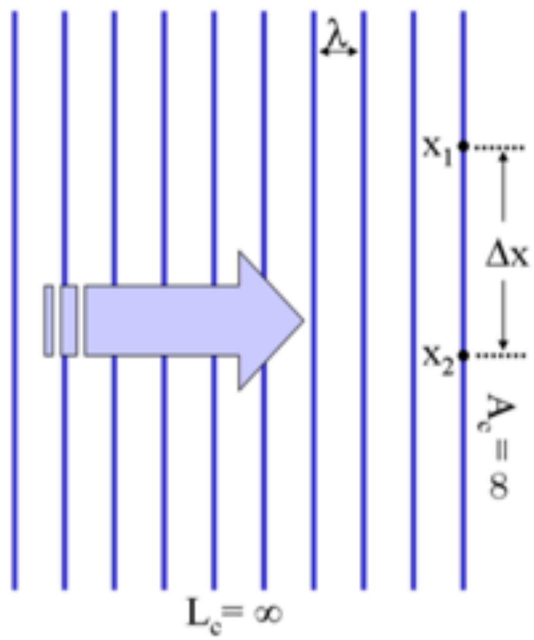
The ideal interferometer

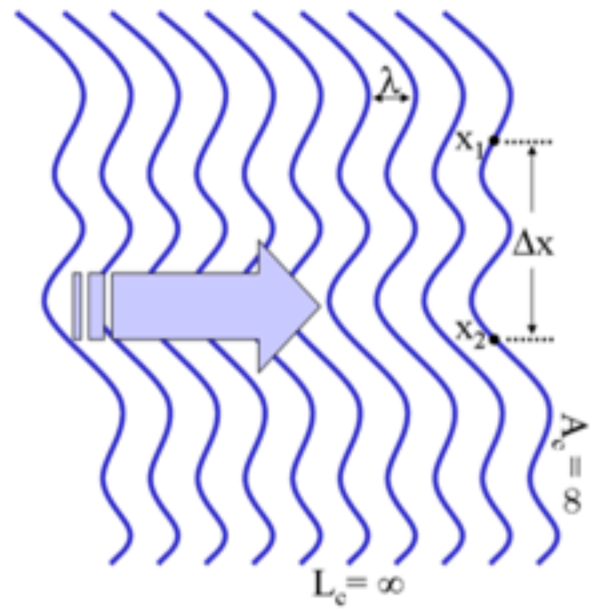
- Astrophysical source, location R, generates a time-varying electric field $E(R,t)$. EM wave propagates to us at point r.
- In frequency components: $E(R,t) = \int E_\nu(R) exp(2\pi i\nu t)d\nu$
  - The coefficients $E_\nu(R)$ are complex vectors (amplitude and phase; two polarisations)
- Simplification 1: radiation is monochromatic
  - $E_\nu(r) = \int\int\int P_\nu(R,r)E_\nu(R)dxdydz$
    - where $P_\nu(R,r)$ is the propagator
- Simplification 2: scalar field (ignore polarisation for now)
- Simplification 3: sources are all very far away
  - This is equivalent to having all sources at a fixed distance – there is no depth information

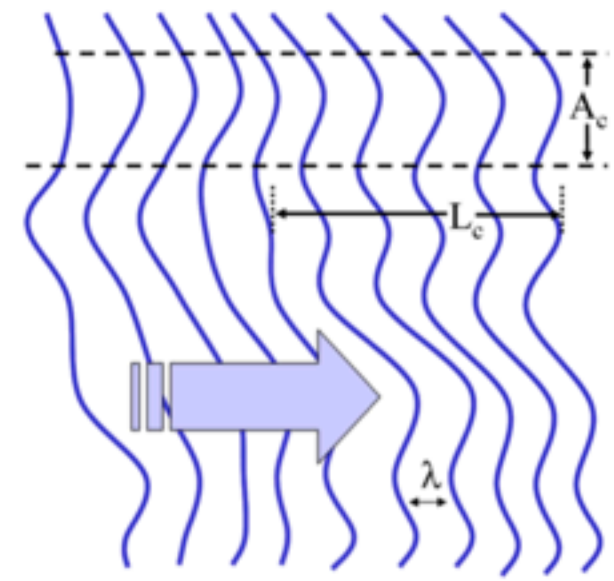
The ideal interferometer

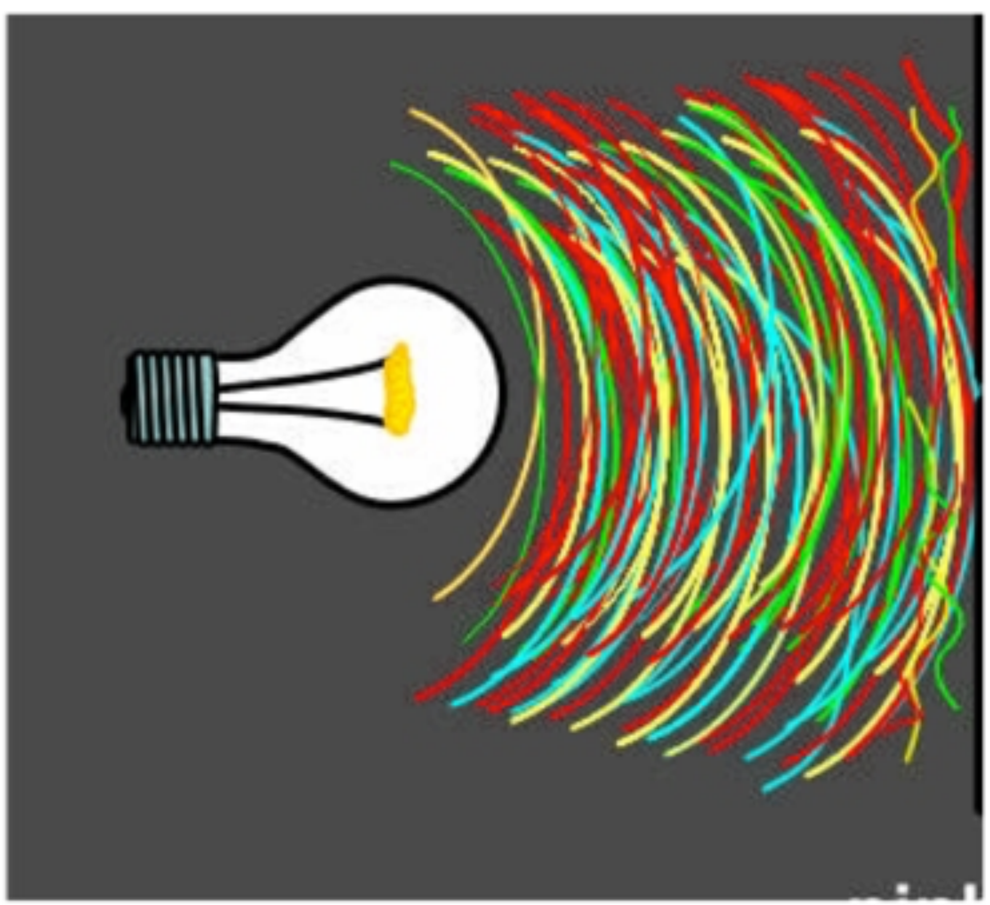
- Simplification 4: space between us and the source is empty
- In this case, the propagator is quite simple (Huygens' Principle), so
  - $E_\nu(r) = \int E_\nu(R) \frac{exp(2\pi i \frac{|R-r|}{c})}{R-r} dA$
    - and dA is the element of area at distance $|R|$
- We can measure is the correlation of the field at two different observing locations. This is
  - $C_\nu(r_1, r_2) = < E_\nu(r_1) E_\nu^\star(r_2) >$
- where $< >$ denotes an expectation value and $\star$ means complex conjugation.
- Simplification 5: radiation from different astronomical objects is not spatially coherent ('random noise')
    - $E_\nu(R_1) E_\nu^\star(R_2) > = 0$ unless $R_1 = R_2$

Spatial and temporal coherence









- ✦ Plane wave (spatially coherent)
- ✦ Varying profile (spatially coherent)
- ✦ Partially coherent

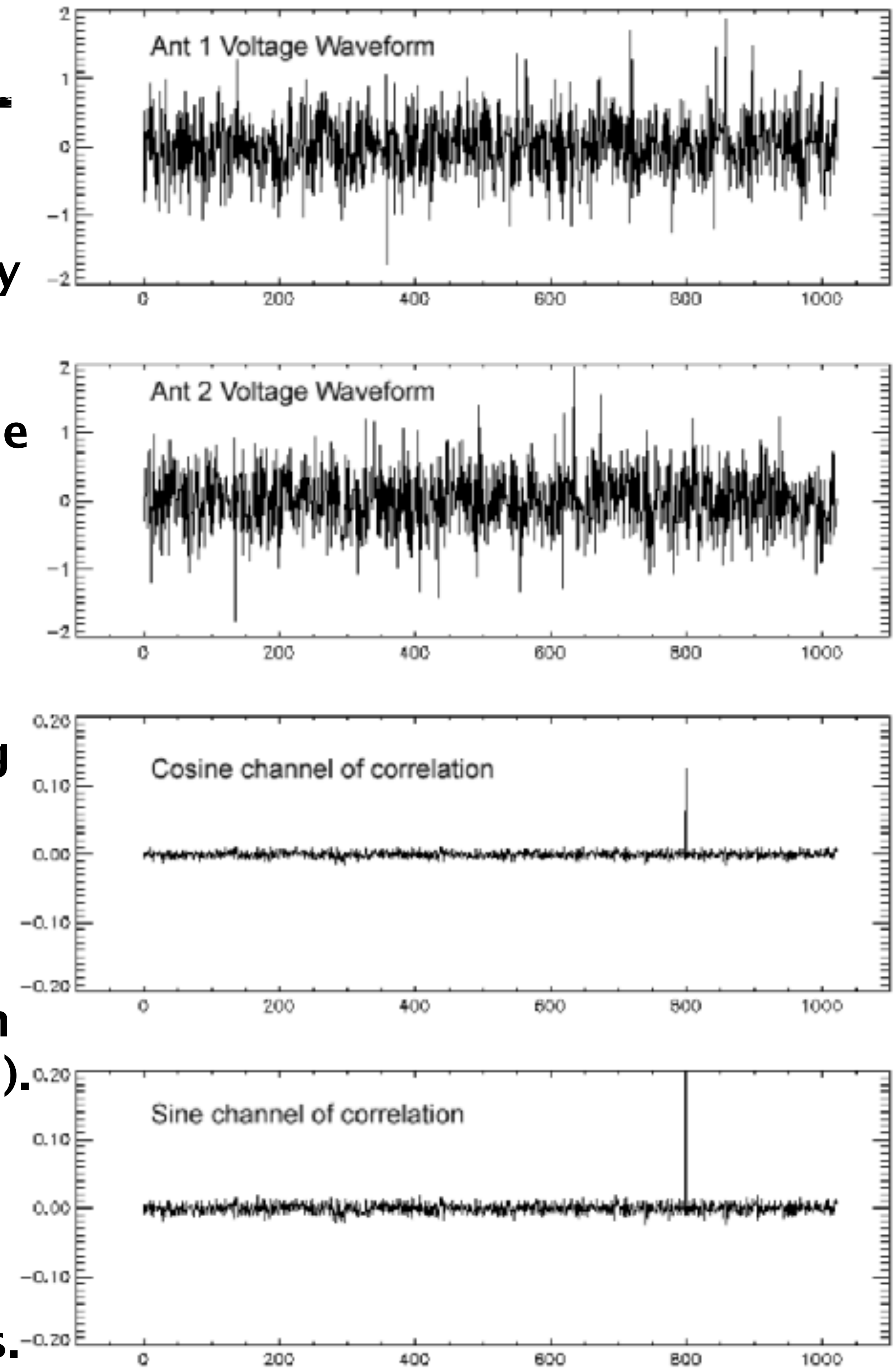
✦ Most radio sources, like incandescent-bulb are broad-band, incoherent emitters

Coherence and correlation

- The IF signal from two antennas/ receivers looks like a noise signal. Part of the waveform is really signal from the source, and part of it (perhaps the largest part) is noise.
  - If they both look the same, how do we tell the difference?
    - The source signal will be correlated between the two antennas, while the noise signal will not.

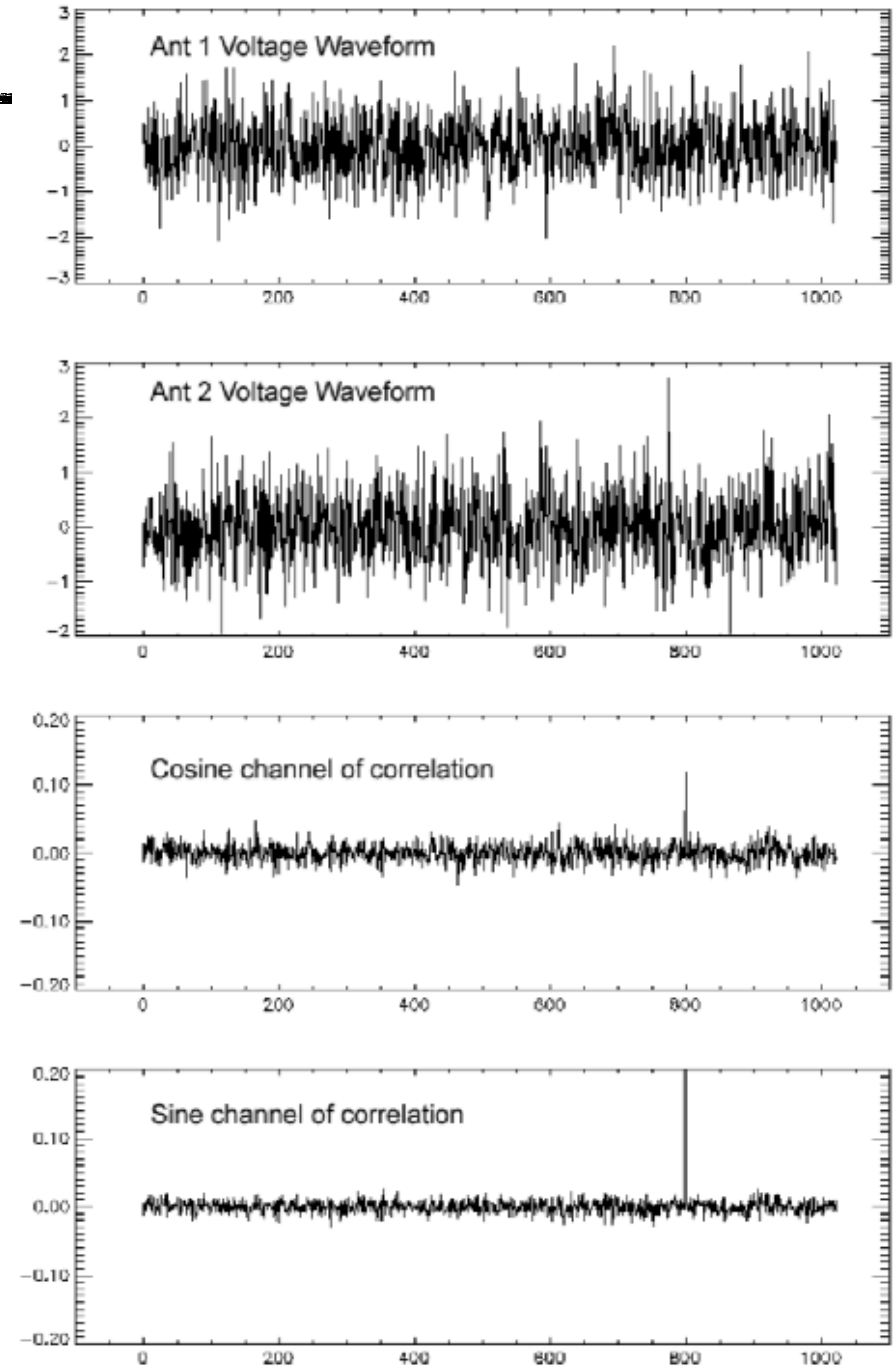
Coherence and correlation

- Two voltage waveforms, with phase 30 degrees, with the waveform for antenna 1 shifted by 800 time samples.
  - The noise level is 1/5 of the signal level in this example. The waveforms appear to have no relation to one another, but when correlated they give the plot in the third panel (cosine channel), which shows a good correlation (spike) at a time lag of 800 samples.
  - Shifting the antenna 1 waveform by 90 degrees and performing the correlation again gives the result shown in the bottom panel (sine channel).
  - The combination of the sine and cosine channels gives an amplitude of 0.268 and phase of 30.2 degrees. The correct values are 0.25 and 30 degrees.



Coherence and correlation

- Two voltage waveforms, with the same characteristics as for earlier Figure, but now the noise level 5 times higher and is now equal to the signal level.
  - Because the noise is uncorrelated, the correlated signal is hardly affected, and
    - gives and amplitude of 0.245 and phase of 30.83 degrees, compared to the correct values of 0.25 and 30 degrees.



The ideal interferometer

- Now write $s = R/|R|$ and $I_\nu(s) = |R|^2 < |E_\nu(s)|^2 >$ (the observed intensity). Using the approximation of large distance to the source again,
  - $C_\nu(r_1, r_2) = \int I_\mu(s) exp(\frac{-2\pi i \nu s \,.\, (r_1 - r_2)}{c}))d\Omega$
  - ($d\Omega$ is an element of solid angle)
- $C_\nu(r_1, r_2)$, the spatial coherence function, depends only on separation, $r_1 - r_2$, so we can keep one point fixed and move the other around.
- It is a complex function, with real and imaginary parts, or an amplitude and phase.
- An interferometer is a device for measuring the spatial coherence function

(u,v,w) coordinates

- **We use a coordinate system (u, v, w), where w is along a reference direction to the phase centre and (u,v) are in the orthogonal plane, with u East–West and v North–South (the (u, v) plane)**
- **In this system:**
  - **– Baseline vector between antennas**
    - $b = (\lambda u, \lambda v, \lambda w)$ **(measure in wavelengths)**
  - **– Unit vector to the phase centre** $s_0 = (0,0,1)$
  - **– Unit vector to some point in the field** $s = (l, m, n)$, **with** $l^2 + m^2 + n^2 = 1$**.**

The Fourier relation

- **Simplification 6**: receiving elements have no direction dependence
- **Simplification 7**: all sources are in a small patch of sky
- **Simplification 8**: we can measure at all values of $r_1 - r_2$ and at all times
- Choose coordinate system so that the phase tracking centre has $s_0 = (0,0,1)$ as in the previous slide
  - $C(r_1, r_2) = exp(-2\pi i w) V'_\nu(u, v)$
  - $V'_\nu(u, v) = \int\int I_\nu(l, m) exp(-2\pi i(ul + vm)) dldm$
- This is a Fourier transform relation between the modified complex visibility V'_ν (the spatial coherence function with separations expressed in wavelengths) and the intensity $I_\nu(l, m)$
- "The Fourier Transform of the spatial coherence function of an incoherent source is equal to its complex visibility":
  - the van Cittert – Zernike Theorem

Fourier inversion

- This relation can be inverted to get the intensity distribution, which is what we want
  - $I_\nu(l, m) = \int\int V'_\nu(u, v) exp(2\pi i(ul + vm)) du dv$
- This is the fundamental equation of synthesis imaging
- Interferometrists love to talk about the (u, v) plane. Remember that u, v (and w) are measured in wavelengths.
- The vector $b = (u, v, w) = (r_1 - r_2)/\lambda$ is the baseline

Simplifications

- 1. Radiation is monochromatic
- 2. Electromagnetic radiation is a scalar field
- 3. Sources are all very far away
- 4. Space between us and the sources is empty
- 5. Radiation is not spatially coherent
- 6. Receiving elements have no direction dependence
- 7. All sources are in a small patch of sky
- 8. We can measure all baselines at all times

False

Sometimes true

Almost always true

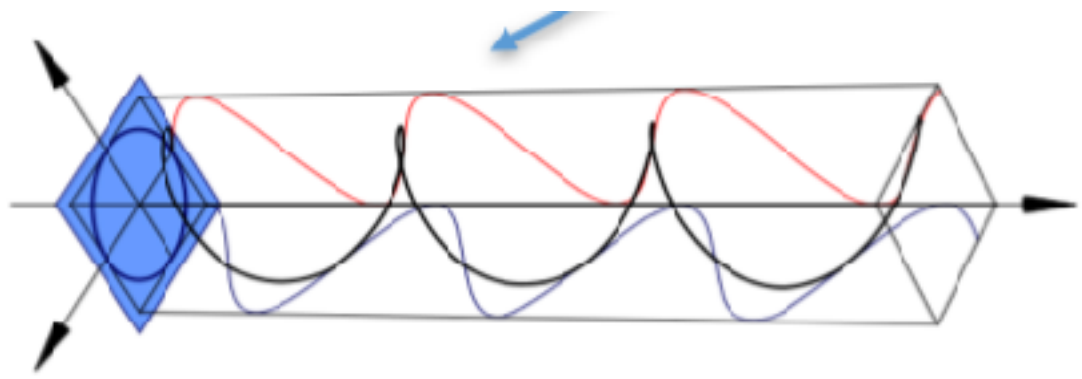
Simplification 1

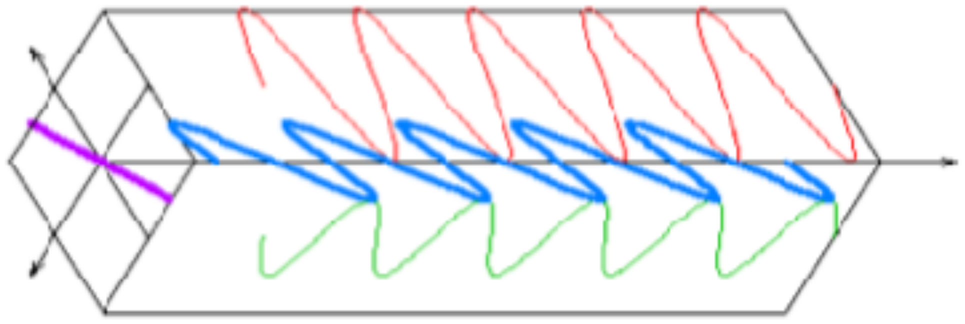
- Radiation is monochromatic
  - We observe wide bands both for spectroscopy (HI, molecular lines) and for sensitive continuum imaging, so we need to get round this restriction.
  - In fact, we can easily divide the band into multiple spectral channels
  - There are imaging restrictions if the individual channels are too wide for the field of view – wait for imaging lecture.
    - Usable field of view $< (\Delta\nu/\nu_0)(l^2 + m^2)^{1/2}$
    - Not usually an issue for modern instruments, which have large numbers of channels

Simplification 2

- ✦ Radiation is field is a scalar quantity
  - ✦ The field is actually a vector and we are interested in both components (i.e. its polarisation).
  - ✦ This makes no difference to the analysis as long as we measure two states of polarisation (e.g. right and left circular, or crossed linear) and account for the coupling between states.
  - ✦ Use the measurement equation formalism for this (calibration and polarisation lectures)

- ✦ Polarisation
  - ✦ Want to image Stokes parameters:
  - ✦ I (total intensity)
  - ✦ Q,U (linear)
  - ✦ V (circular)
  - ✦ Resolve into two (nominally orthogonal) polarization states, either right and left circular or crossed linear.





Simplifications 3 and 4

- Sources are all a long way away
  - Strictly speaking, in the far field of the interferometer,
    - so that the distance is $> D^2/\lambda$ where D is the interferometer baseline
  - True except in the extreme case of very long baseline observations of solar-system objects
- Radiation is not spatially coherent
  - Generally true, even if the radiation mechanism is itself coherent (masers, pulsars)
  - May become detectable in observations with very high spectral and spatial resolution
  - Coherence can be produced by scattering, since signals from the same location in a sources are spatially coherent, but travel by different paths through interstellar or interplanetary medium

Simplifications 5 and 6

- Space between us and the source is empty
- The receiving elements have no direction-dependence
  - Closely related and not true in general.
    - Examples:
      - Interstellar or interplanetary scattering
      - Tropospheric and (especially) ionospheric fluctuations which lead to path/phase and amplitude errors, sometimes seriously direction-dependent
      - Ionospheric Faraday rotation, which changes the plane of polarisation
      - High-frequency antennas are highly directional by design
  - Standard calibration deals with the case that there is no direction-dependence (i.e. each antenna has a single, time-variable complex gain)
  - Direction dependence is becoming more important, especially for low frequencies and wide fields.

Special case: PB correction

- If the response of the antenna is direction-dependent,
  - then we are measuring
    - $I_\nu(l,m)D_{1\nu}(l,m)D^\star_{1\nu}(l,m)$ instead of $I_\nu(l,m)$
      - (ignore polarisation for now)
    - An easier case is when the antennas all have the same response
      - $A_\nu(l,m) = |D_\nu(l,m)|^2$
      - $V'_\nu(u,v) = \int\int A_\nu(l,m)I_\nu(l,m)exp(-2\pi i(ul+vm))dldm$
      - (we just make the standard Fourier inversion and) the divide by the primary beam $A_\nu(l\,.\,m)$)
  - $$I_\nu(l,m) = \frac{\int\int V'_\nu(u,v)exp(2\pi i(ul+vm))dudv}{A_\nu(l,m)}$$
  - See imaging lectures

Simplification 7

- **The field of view is small**
  - **(or antennas are in a single plane).**
    - **Not always true,**
      - **if not:**
      - **(basic imaging equation becomes)**

$$V'_\nu(u,v,w) = \int\int I_\nu(l,m)exp([-2\pi i(ul + vm + (1 - l^2 - m^2)^{1/2}w]/(1 - l^2 - m^2)^{1/2}))dldm$$

  - **No longer a 2D Fourier transform, so analysis becomes more complicated (the "w term")**
  - **map individual small fields ("facets") and combine later, or**
  - **w−projection**
  - **See imaging lectures**

Simplification 8

- We can measure the coherence function for any spacing and time
  - (very wrong!).
- We have a number of antennas at fixed locations on the Earth (or in orbit around it)
  - The Earth rotates
  - We make many (usually) short integrations over extended periods, sometimes in separate observations
    - So effectively we sample at discrete u, v (and w) positions.
    - Implicitly assume that the source does not vary
  - Often a problem when combining observations take over a long time period; some sources vary much faster (e.g. the Sun)
  - Also assume that each integration (time average to get the coherence function) is of infinitesimal small duration.
  - In 2D, this measurement process can be described by a sampling function $S(u, v)$ which is a delta function where we have taken data and zero elsewhere.
  - $I^D_\nu(l, m) = \int\int V'_\nu(u, v)S(u, v)exp(2\pi i(ul + vm))dudv$ is the dirty image
  - The process of getting from $I^D_\nu(l, m)$ to $I_\nu(l, m)$ is deconvolution (examples in other lectures).
  - However, perhaps better to pose the problem in a different way:
    - what model brightness distribution $I_\nu(l, m)$ gives the best fit to the measured visibilities and how well is this model constrained?