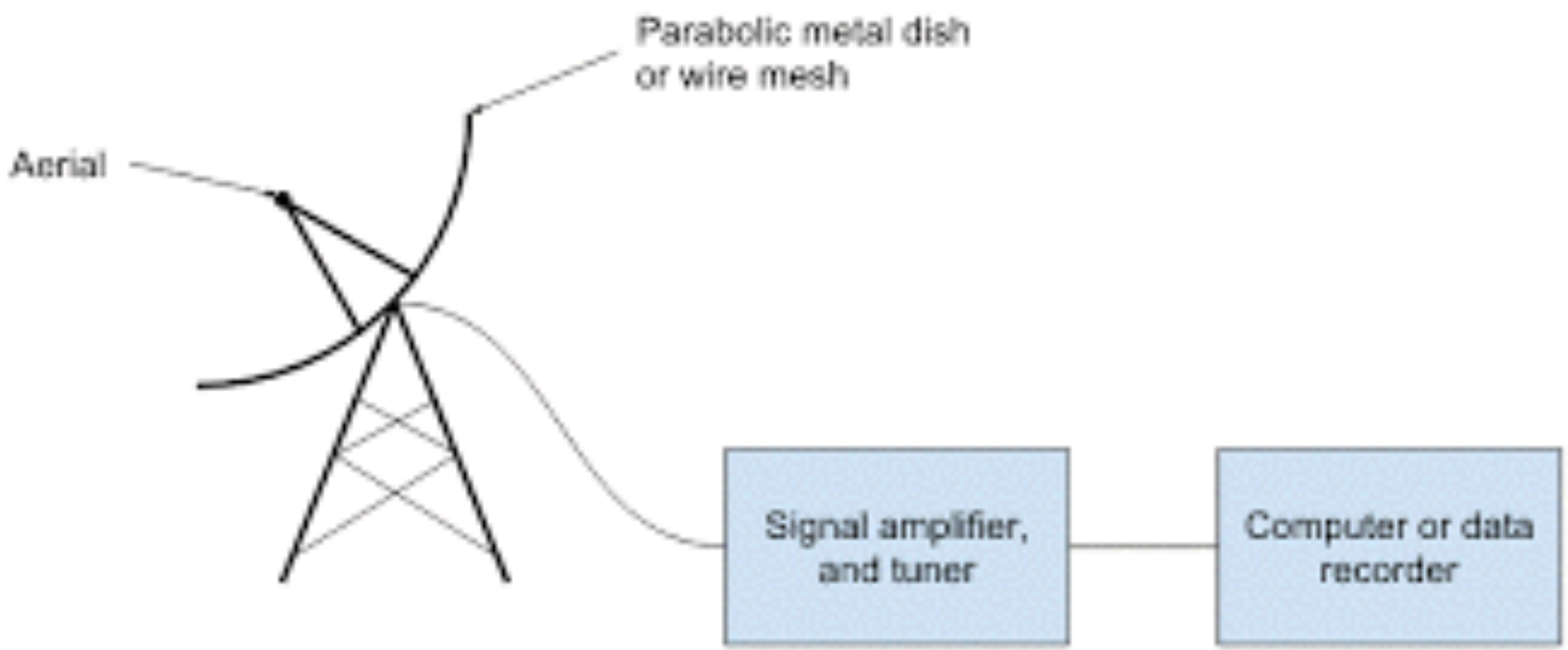
A basic radio telescope

- Single dish
  - Feed
  - Receiver (FE)
  - Mount
  - Transmission lines
  - Receiver (BE)
  - ABC
  - Mechanical
    - Azimuth drive
    - Elevation d



Radio telescope

- The region of transition between a free space and a guided wave or vice-versa.
- For a radio telescope, the antenna acts as a collector of radio waves.
- The response of an antenna as a function of direction is given by the "antenna pattern".
  - By reciprocity this pattern is the same for both receiving and transmitting.

Working single dish radio telescope

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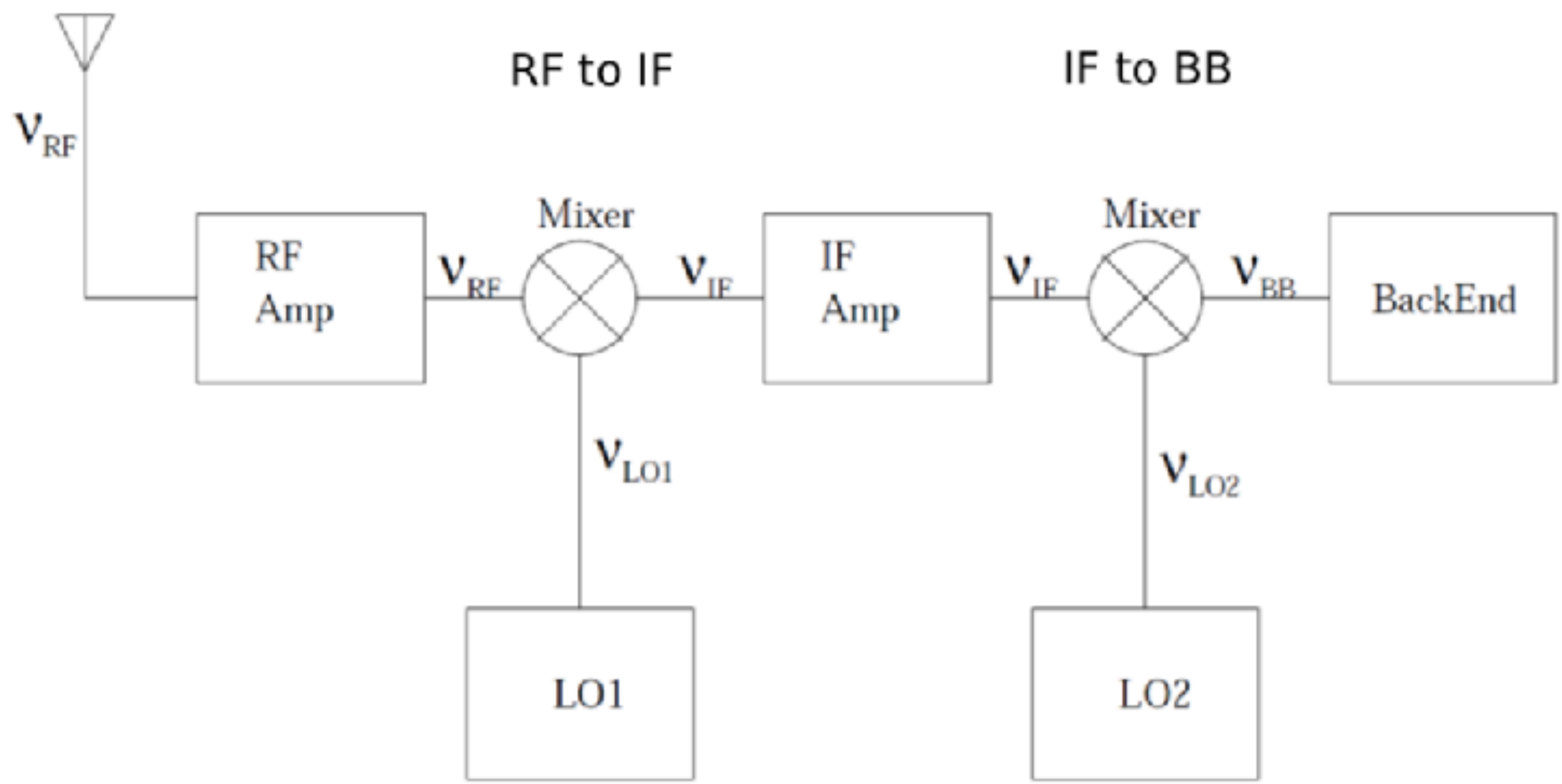
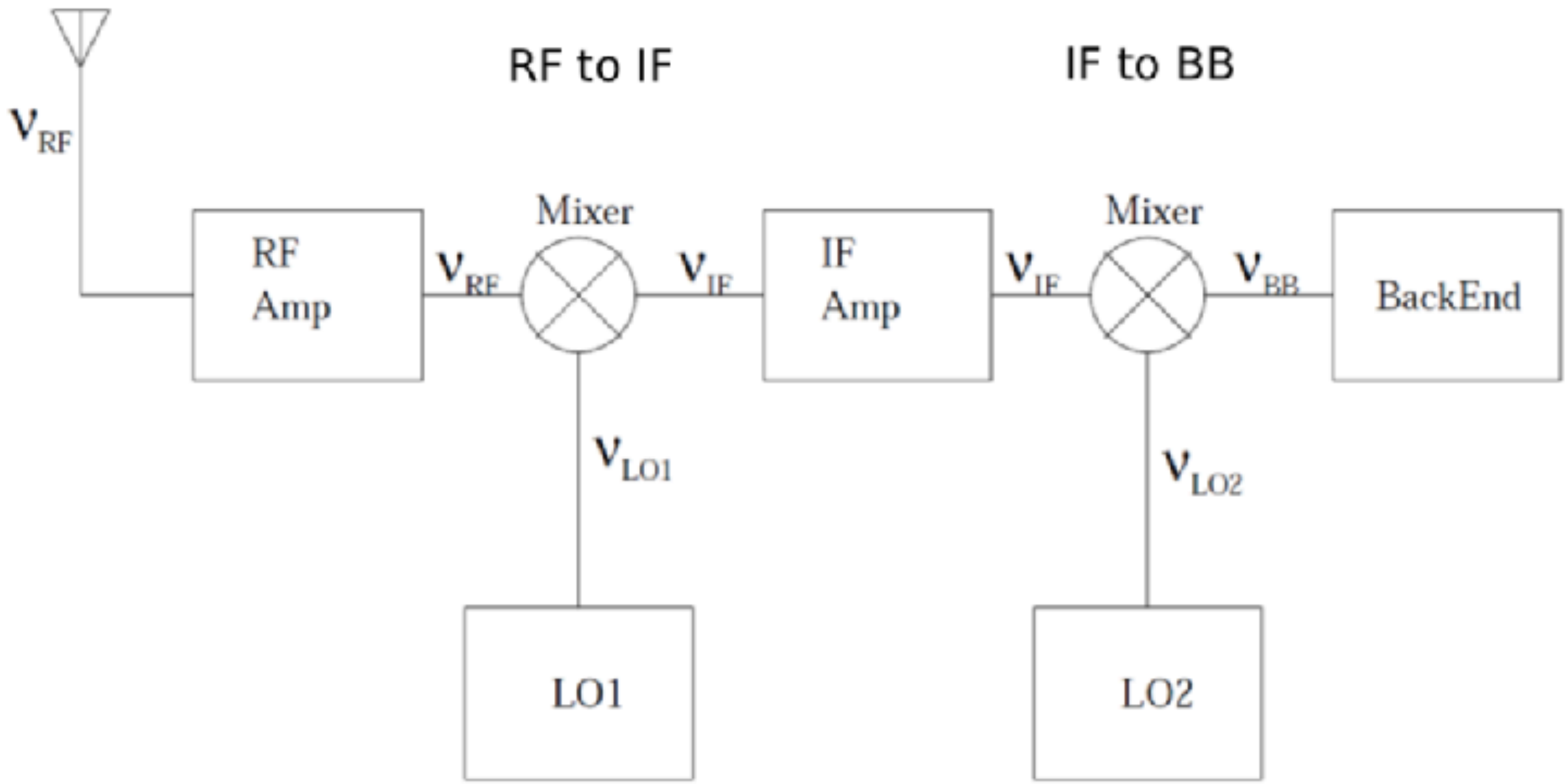


Working single dish radio telescope

- EM waves impinge on the antenna and create a fluctuating voltage – frequency is the same as of the incoming wave called Radio frequency.
  - Needs amplification: Low Noise Amplifier at the receiver Front-End amplifies the signal.
- Mixer: changes the frequency of the incoming signal; pure sine wave by tunable signal generator – Local Oscillator.
  - Mixing – also called heterodyning.



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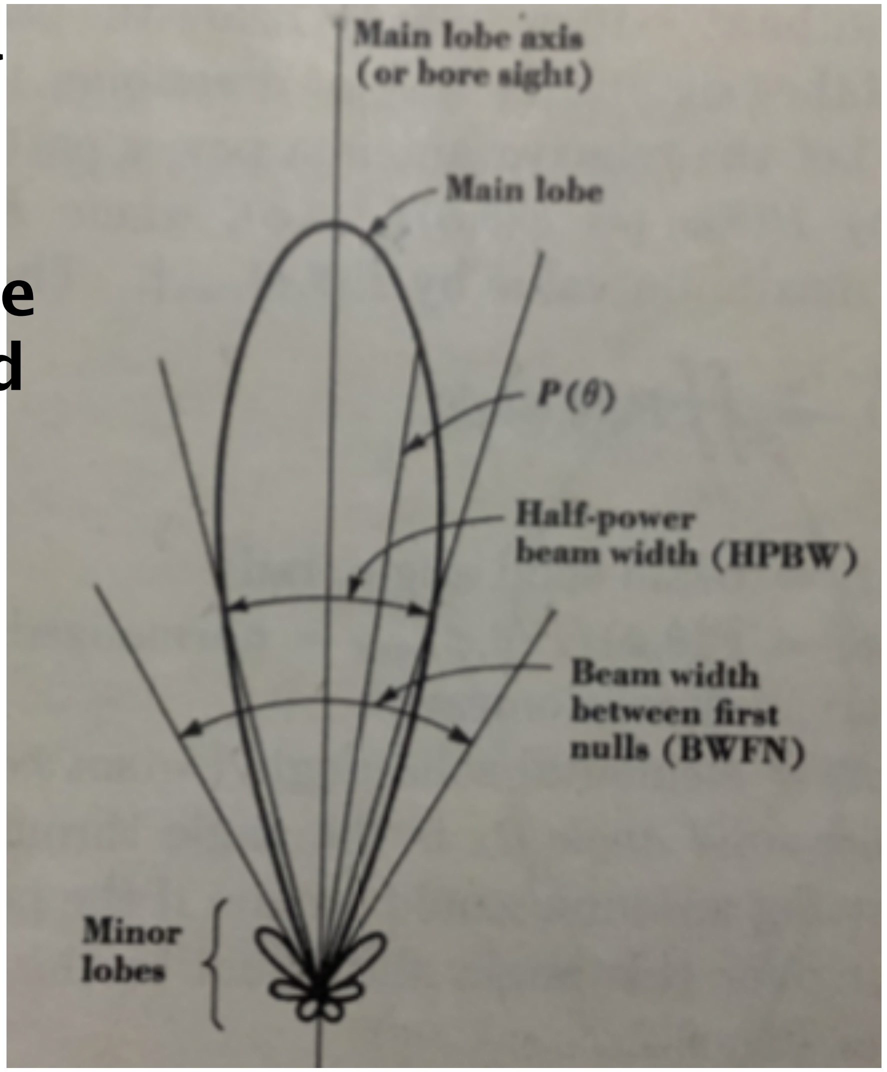
  - Mixing – also called heterodyning.
- Brings the signal to an intermediate frequency: allows the use of same backend for a number of different frequencies observed.
  - Sometimes more mixers: IFs that minimise transmission losses and those that are optimal for best amplification – super-heterodyne systems.
    - Another stage of amplification followed by a mixer to convert the signal to Baseband.
    - Passed to a backend: square-law detector / correlation / pulsar backend

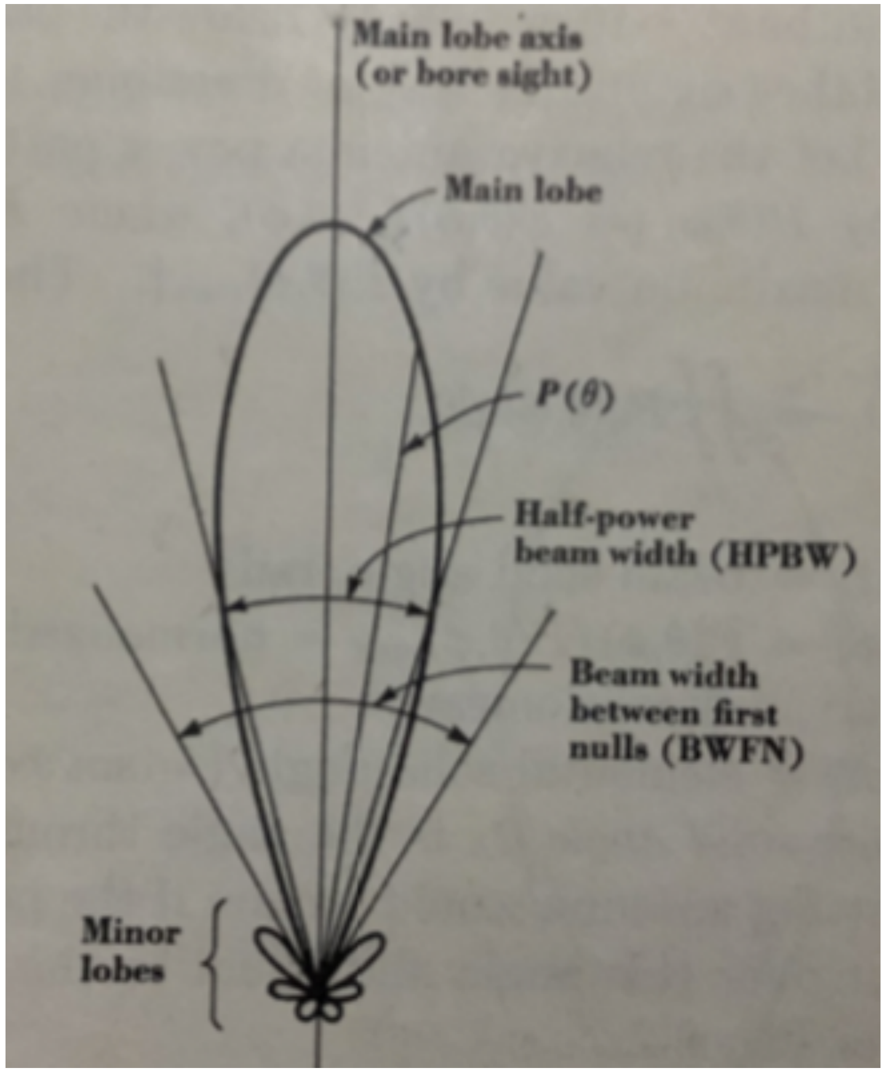
Effective aperture

- Antenna's ability to absorb the waves that are incident on it is measured by the quantity "effective aperture", A_e.
  - $A_e = \dfrac{\text{power density available at the antenna terminals}}{\text{flux densiy of the wave incident on the antenna}}$
  - $= \dfrac{W\ Hz^{-1}}{W\ m^{-2}\ Hz^{-1}}.$
- Also called effective area of the antenna. It is a function of direction, thus:
  - $A_e = A_e(\theta, \phi)$
- The power pattern of the antenna describes the directional response of an antenna (normalized to unity at the maximum):
  - $P(\theta, \phi) = \dfrac{A_e(\theta, \phi)}{A_e^{\max}}$ and
  - $\Theta_{\text{HPBW}} \simeq \dfrac{\lambda}{D}$



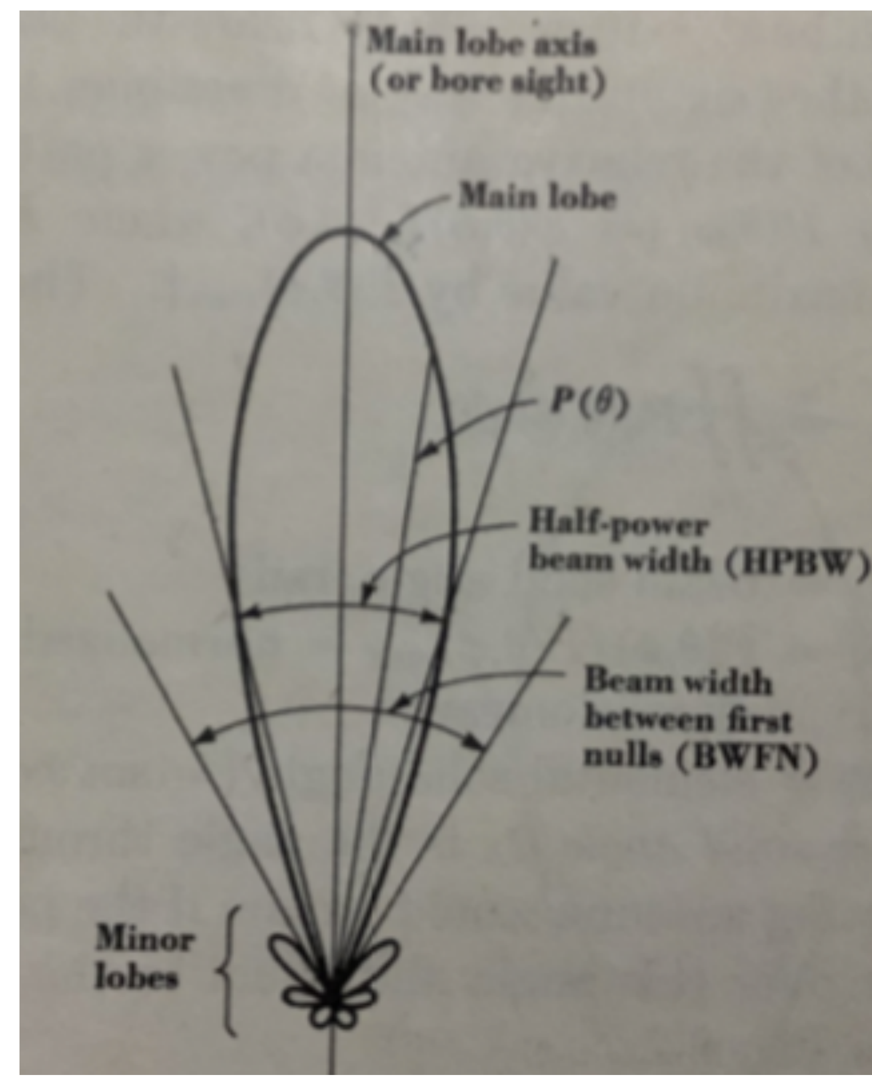
Directivity, gain and aperture efficiency

- Another measure of the response of the antenna as a function of direction is described by "directivity":
  - $$D(\theta, \phi) = \frac{\text{power emitted into } (\theta, \phi)}{(\text{total power emitted})/4\pi}$$
  - $$= \frac{4\pi P(\theta, \phi)}{\int P(\theta, \phi) d\Omega}$$
- Aperture efficiency is the ratio of the maximum effective aperture and the geometric cross sectional area of the reflector:
  - $$\eta = \frac{A_e^{\max}}{A_g}$$



Gain and Directivity

- $$G(\theta,\phi) = \frac{\text{power emitted into } (\theta,\phi)}{\text{total poer input}/4\pi} \times \eta$$
- **Gain is often given in decibels (dB) which is:**
  - $G(\text{dB}) = 10\log(\text{G})$
    - **The convenience is that when there are amplifiers in succession the total gain is simply the addition.**



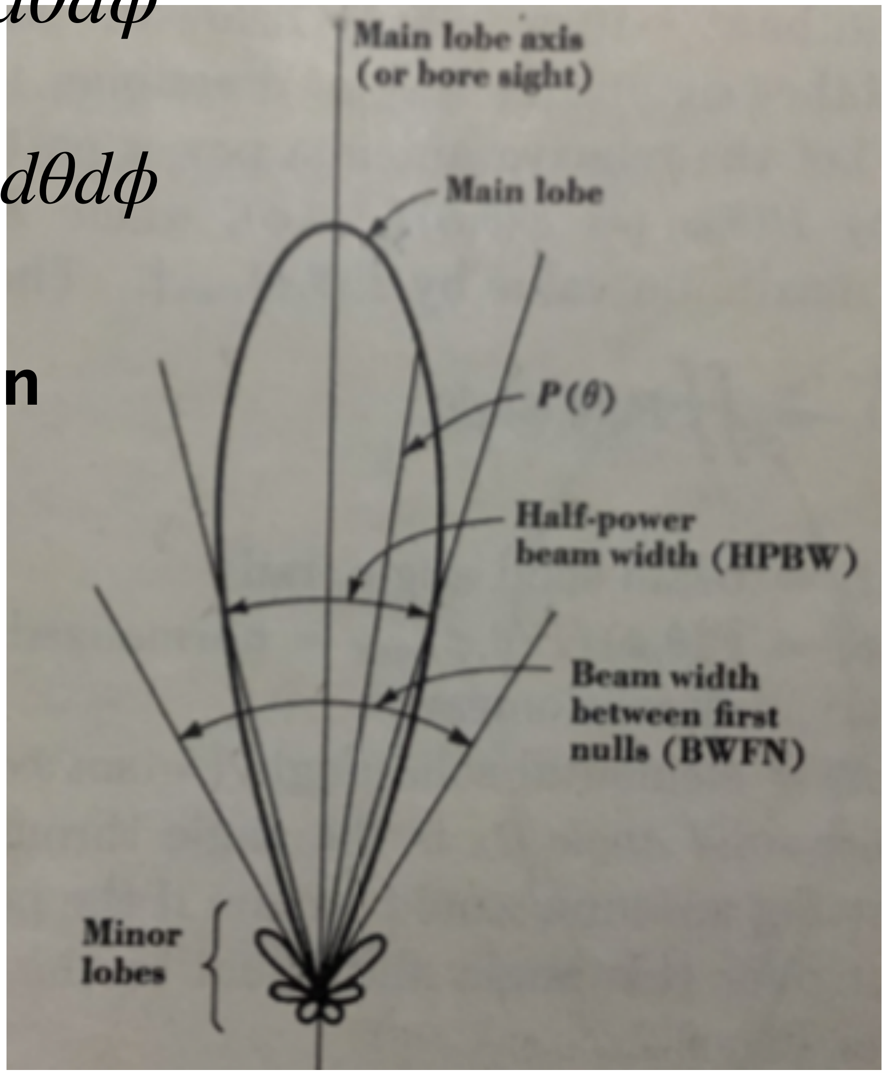
Effect of pattern in observed sky

- Consider observing a sky brightness distribution $B(\theta)$ with a telescope having a power pattern as shown. Then the power available at the antenna terminals is
- $$W(\theta') = \frac{1}{2}\int B(\theta)A_e(\theta - \theta')d\theta$$

$$W(\theta', \phi') = \frac{1}{2}\int B(\theta, \phi)A_e(\theta - \theta', \phi - \phi')sin(\theta)d\theta d\phi$$

$$T_A(\theta', \phi') = \frac{A_e^{\max}}{\lambda^2}\int T_B(\theta, \phi)P(\theta - \theta', \phi - \phi')sin(\theta)d\theta d\phi$$

- **Antenna temperature** is the weighted average of the sky temperature – the weighting function is the power pattern of the antenna.



Relation between directivity gain and effective aperture

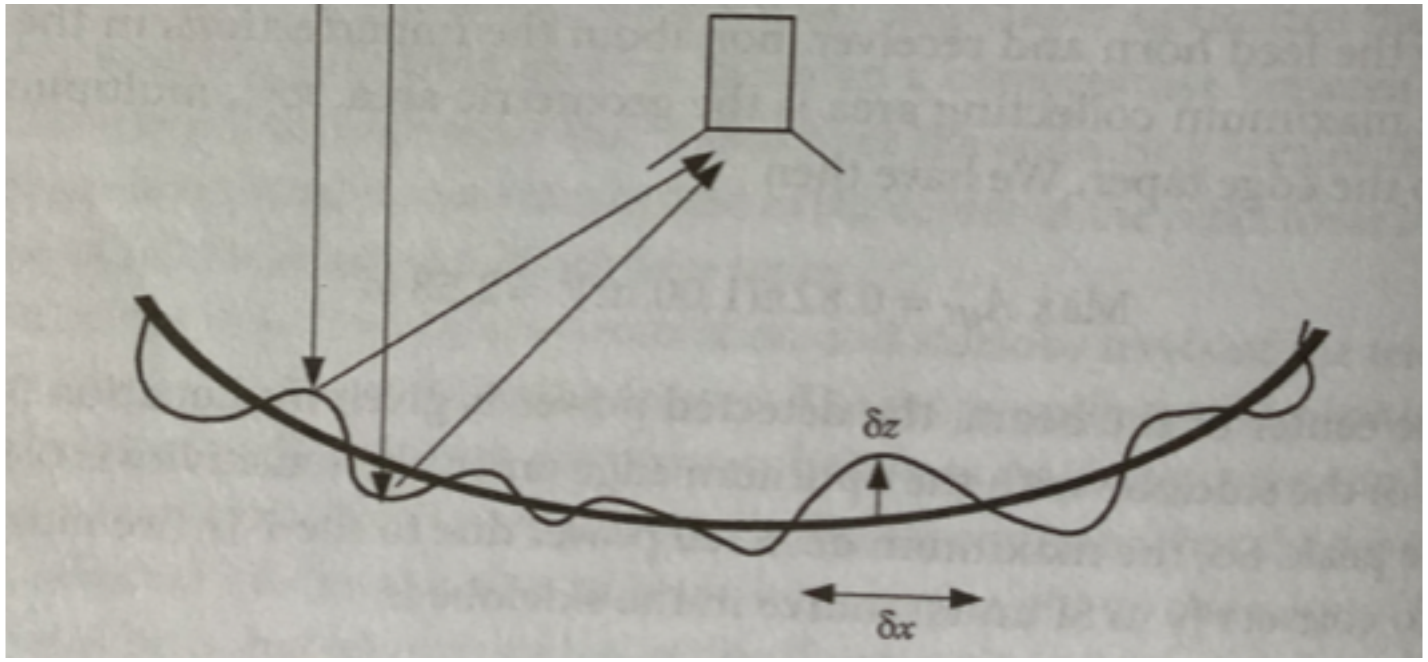
- Consider an antenna terminated in a resistor and the entire setup placed in a blackbox at temperature T.
  - At thermal equilibrium, the power flowing from resistor to antenna is
    - $P_{R\to A} = kT$
  - And that flowing from the antenna to the resistor is
  - $$P_{R\to A} = \frac{A_e^{\max} kT}{\lambda^2} \int P(\theta,\phi)\, d\Omega$$
  - Since the net power is zero, we can equate the two and get
    - $$A_e^{\max} = \frac{\lambda^2}{\int P(\theta,\phi)\, d\Omega}$$
    - Maximum effective aperture is determined by the shape of the power pattern alone.

Relation between directivity gain and effective aperture

- For a reflecting telescope
  - $P(\theta, \phi) d\Omega \sim \Omega^2_{\mathrm{HPBW}} \sim (\frac{\lambda}{D})^2$
  - And thus
  - $A_e^{\max} \sim D^2 \simeq \frac{\lambda^2}{\int P(\theta, \phi) d\Omega}$
  - The maximum effective aperture scales like the geometric area of the reflector, hence,
  - $A_e = A_e^{\max} \times P(\theta, \phi) = \frac{\lambda^2 \times P(\theta, \phi)}{\int P(\theta, \phi) d\Omega}$
  - And hence
  - $D(\theta, \phi) = \frac{4\pi}{\lambda^2} A_e(\theta, \phi)$
  - (recall $D(\theta, \phi) = \frac{4\pi \times P(\theta, \phi)}{\int P(\theta, \phi) d\Omega}$)

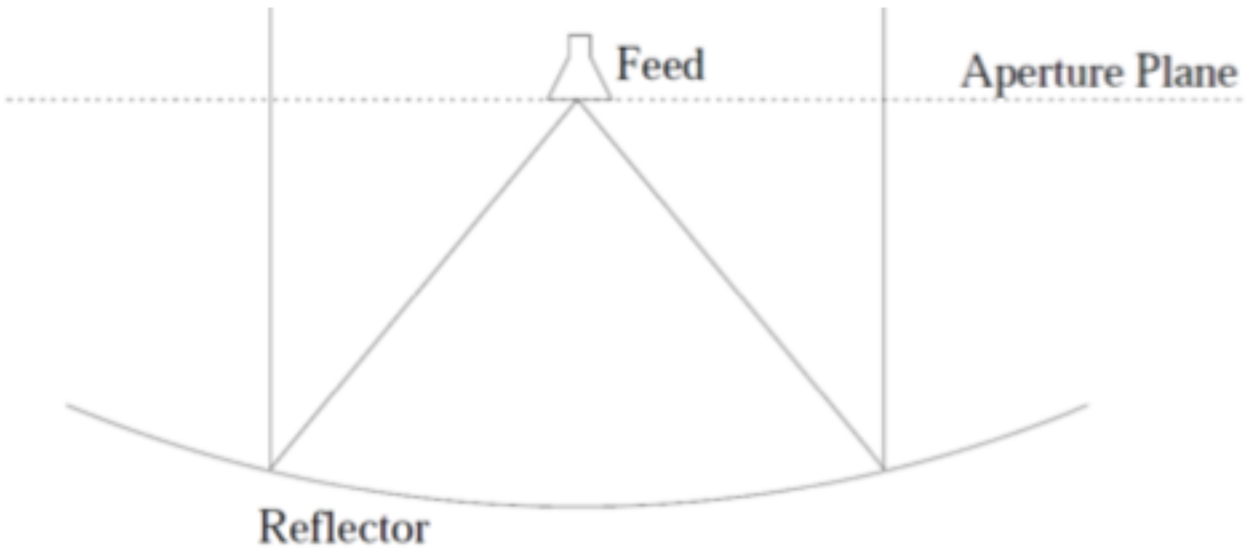
Surface errors

- Imperfections in the reflecting surface
- Cause path length differences
- Reduce on-axis sensitivity of the telescope – loss in effective collecting area.
  - Ruze equation
    - $$\eta_s = \exp[-(\frac{4\pi\sigma}{\lambda})^2] = \frac{\text{ratio of collecting area with error}}{\text{ratio of collecting area without error}}$$
    - (H.W.)



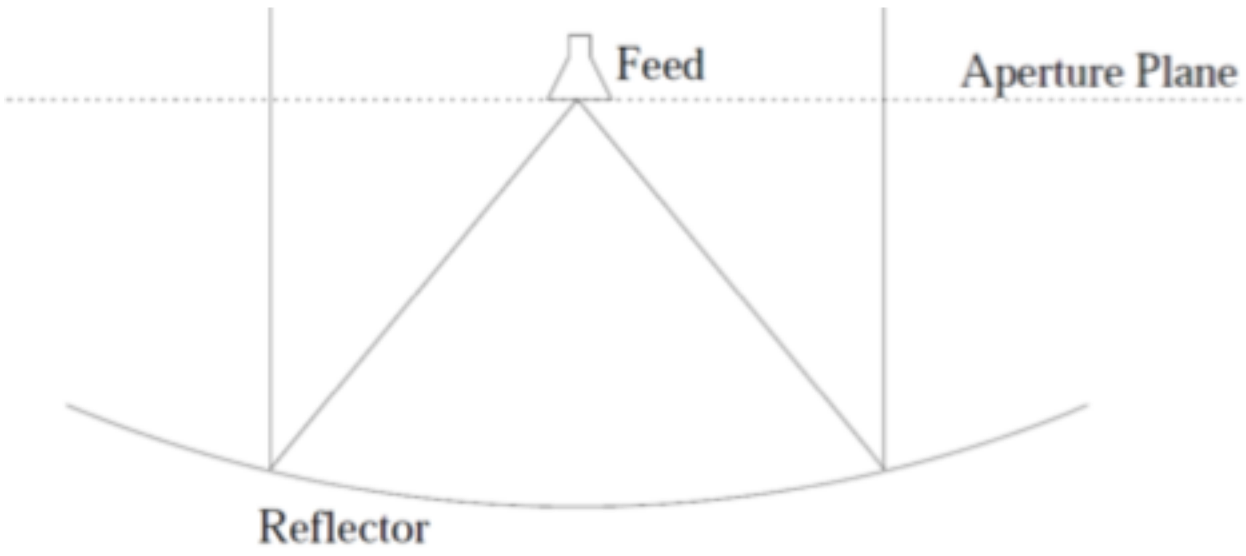
Aperture illumination

- The beam pattern of the feed determines the illumination of the primary reflector.
  - Illumination affects the angular resolution, sensitivity level in the side-lobes and effective collecting area.
- Ideally would like to have uniform sensitivity from centre to the edge of the dish - but we do not want unwanted radiation from the ground to be picked up.
- A quantity that describes how the feed's beam is distributed on the primary reflector is called edge taper: ratio of sensitivity at the centre to that at the edge.
  - A more tapered illumination will have a broader main beam or equivalently smaller effective aperture but also lower side-lobes than uniform illumination. If the illumination is high towards the edges there will be a lot of spillover.



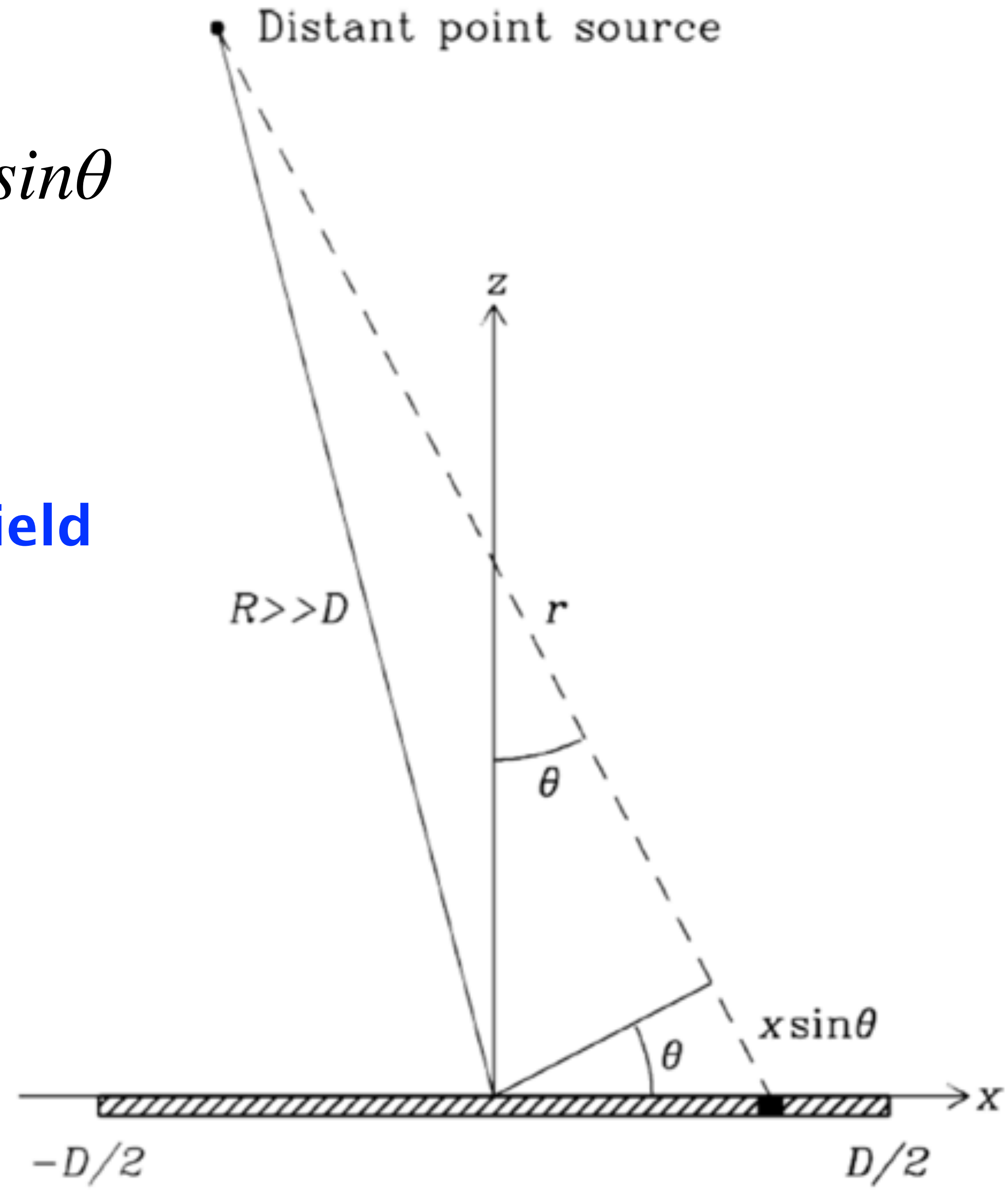
Aperture illumination

- Aperture of a reflector is the plane through which all the rays pass. Beam pattern of an antenna is its power gain as a function of direction.
- Huygen's principle:
  - Aperture can be treated as a collection of small elements acting individually.
  - Each point in a wave front can be regarded as an imaginary source – and the wave at at other point is the addition of the contribution of each of these point sources.
  - Finite size of the dish results in diffraction.



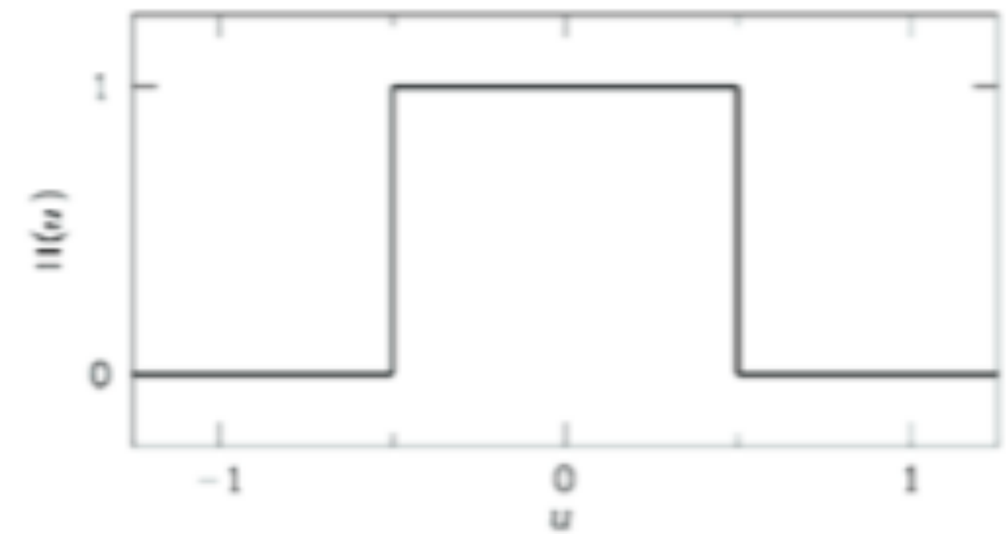
Aperture illumination: 1D aperture of width D

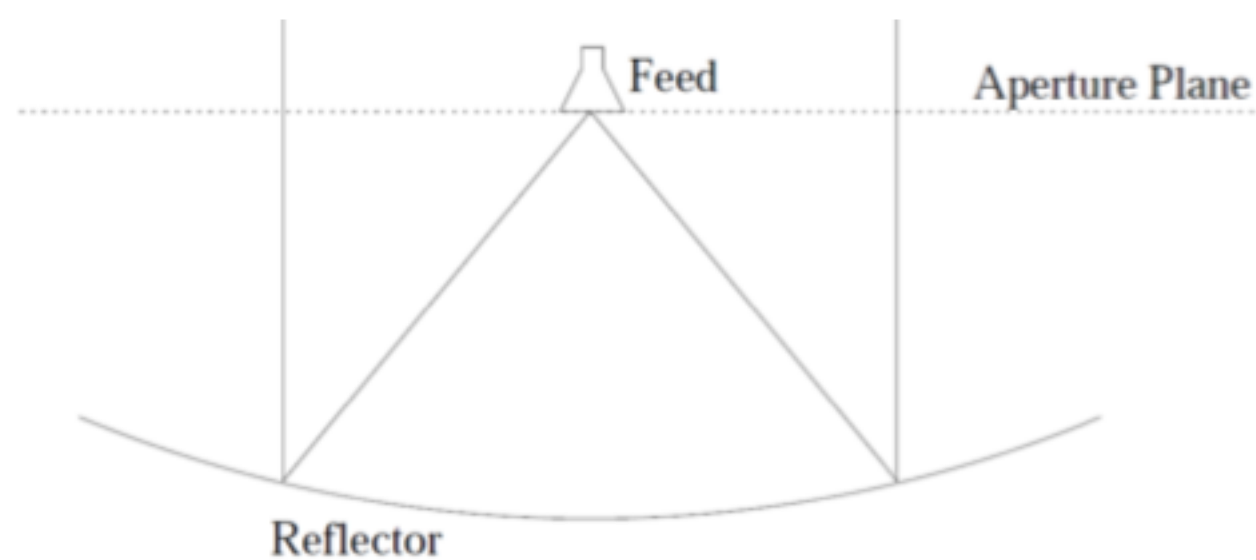
- The electric field in the far field regime due to the element x to $x + dx$ is
- $$df \propto g(x)\frac{exp(-i2\pi r(x)/\lambda)}{r(x)}dx$$
- $r \approx R + xsin\theta = R + xl$ where $l \simeq sin\theta$
- $df \propto g(x)exp(-i2\pi xl/\lambda)dx$
- $$f(l) = \int_{\text{aperture}} g(u)e^{-12\pi lu}du$$
  - Fourier transform of the aperture field pattern.



1D uniformly illuminated aperture

- $g(u) = \text{constant}$
- $-\frac{D}{2\lambda} < u < +\frac{D}{2\lambda}$
- $f(l) = \int_{-1/2}^{+1/2} exp(-i2\pi lu)du$
  - Where $l = sin\theta$
- $= \frac{e^{-i\pi lu}}{-i\pi lu}$ **(plus limits)**
- $\frac{e^{-i\pi l} - e^{i\pi l}}{-i2\pi l}$
- $= \frac{sin(\pi l)}{\pi l}$





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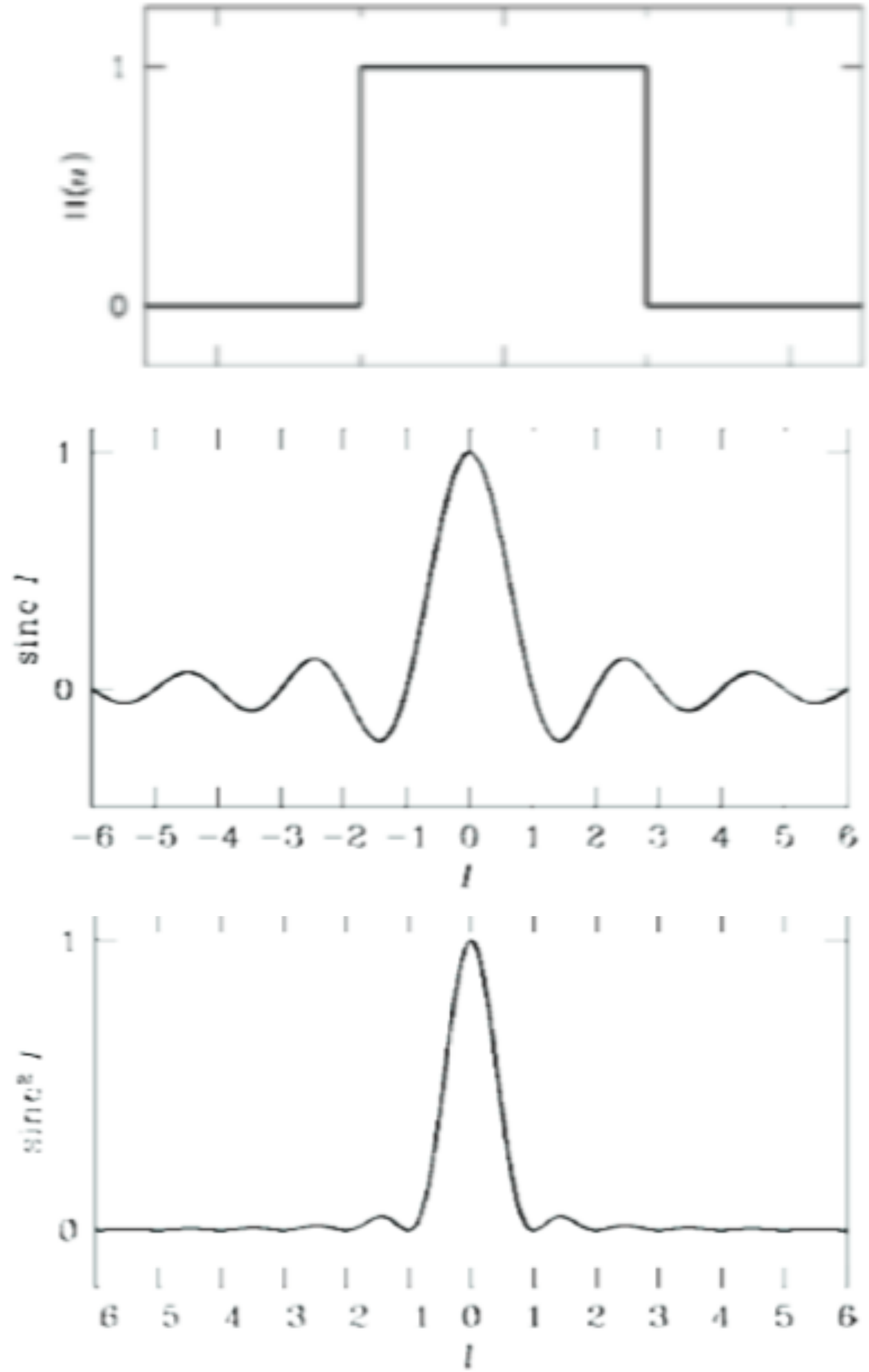
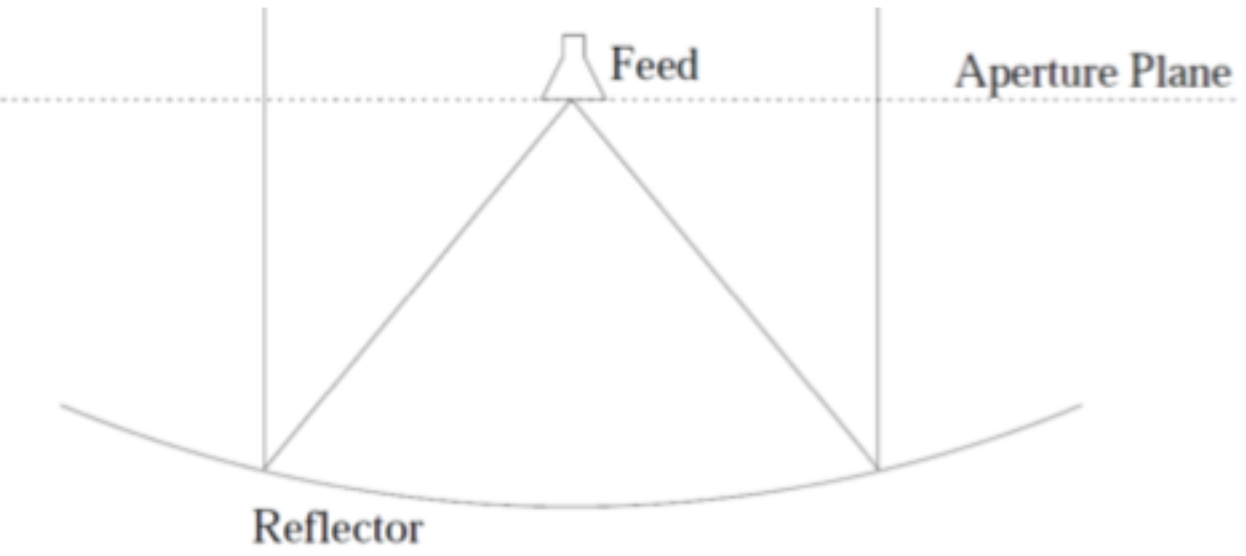
- $\dfrac{e^{-i\pi l} - e^{i\pi l}}{-i2\pi l}$
- $= \dfrac{sin(\pi l)}{\pi l}$
- $f(\theta) = \dfrac{D}{\lambda} sinc(\dfrac{\theta D}{\lambda})$
- $P(\theta) = f^2(\theta) = (\dfrac{D}{\lambda})^2 sinc^2(\dfrac{\theta D}{\lambda})$



Aperture illumination / blockage

- The feed is located above the reflector and thus blocks the aperture.
  - What is the effect of this on the antenna pattern ?
- Now that we know the aperture plane and far field are related by a Fourier transform, we can find the estimate the effect of the aperture blockage using the properties of FT.
  - A uniform aperture with a width d – what is its FT ?
  - A uniform aperture with with l – what is its FT ?



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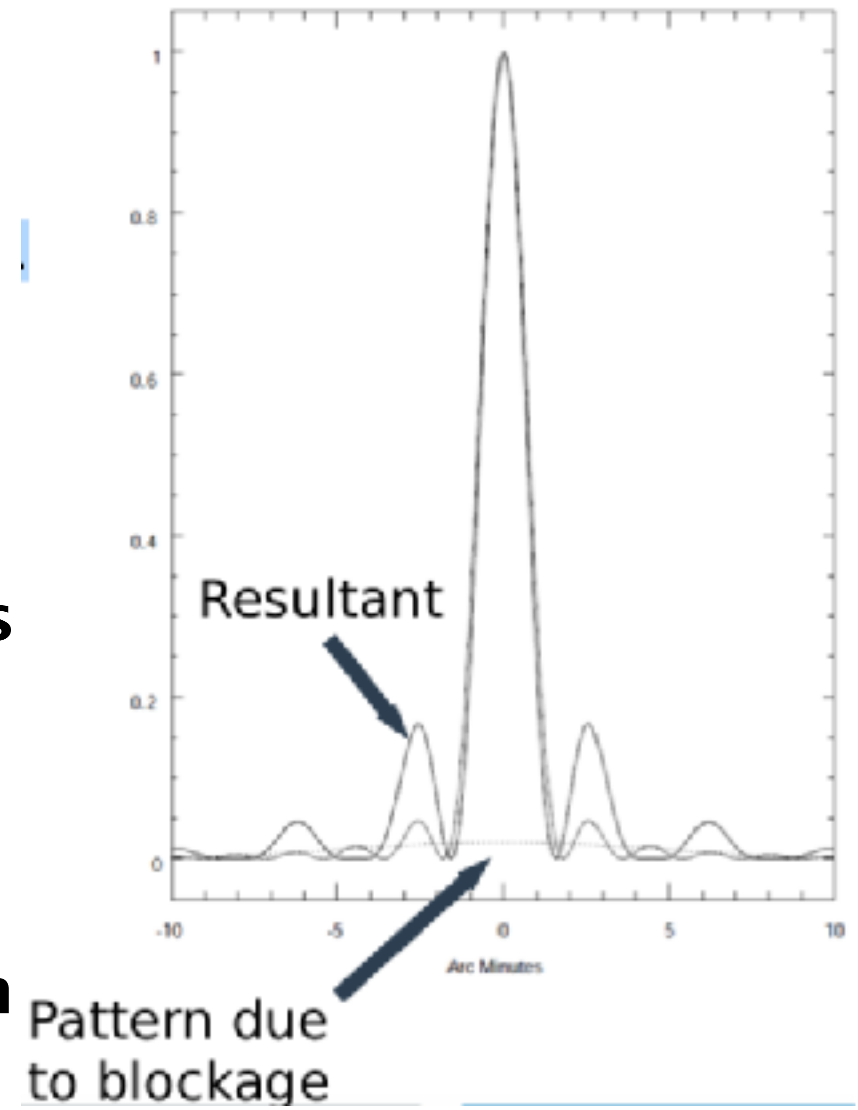
  - $\mu = \lambda / l$
  - $E(\mu) \propto \dfrac{sin(\pi l u/\lambda)}{\pi\mu} - \dfrac{sin(\pi d\mu/\lambda)}{\pi\mu}$
  - Should be minimised for a good beam, offset feeds to eliminate blockage!



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Application: power pattern at one from a signal from another

- For a reflecting telescope
  - Consider sending information from antenna 1 with gain $G_1(\theta, \phi)$ and input power P_1 to antenna 2 with directivity $D_2(\theta, \phi)$ at a distance R away.
  - The flux density at antenna 2 is:
  - $S = \frac{P_1}{4\pi R^2} G_1(\theta, \phi)$
    - (factor G states that the power is not distributed isotropically)
    - ower available at antenna 2 is:
      - $W = A_{2e} S = \frac{P_1}{4\pi R^2} G_1(\theta, \phi) A_{2e}$
        - (recall $D(\theta, \phi) = \frac{4\pi}{\lambda^2} A_e(\theta, \phi)$)
  - After substituting for the effective aperture,
    - $W = (\frac{\lambda}{4\pi R})^2 P_1 G_1(\theta, \phi) D_2(\theta', \phi')$
      - called as Friis transmission equation