Electrodynamics and Radiative Processes I

Lecture 8 – Bremsstrahlung Radiation II

Bhaswati Bhattacharyya

bhaswati@ncra.tifr.res.in

IUCAA-NCRA Graduate School

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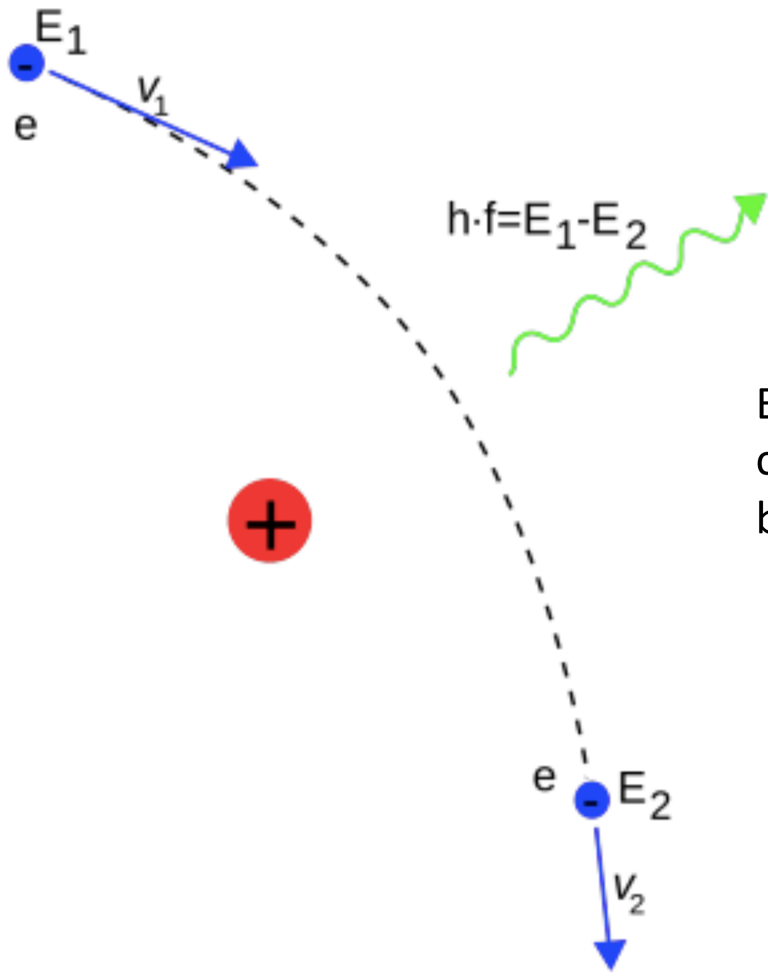
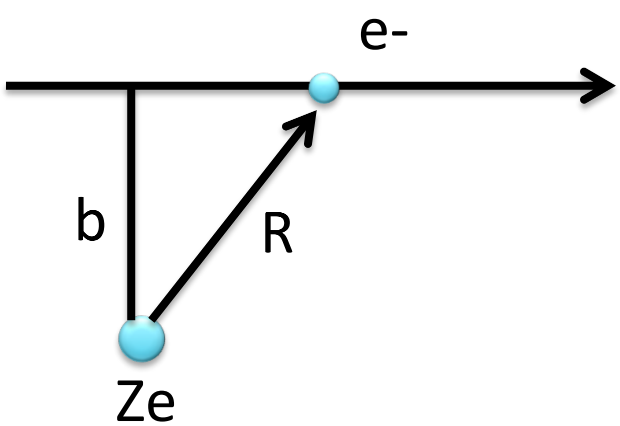
Recap : Bremsstrahlung in Astrophysics



Bremsstrahlung radiation is emitted when a charged particle is deflected (decelerated) by another charge.

https://apatruno.files.wordpress.com/2016/09/lecture52.pdf

Recap: Bremsstrahlung
Emission from a single speed electron



Assume: electron moves rapidly and its path is straight line

Consider an electron of charge −e moving past an ion of charge Ze with impact parameter b

Dipole moment **d**= -e **R**

2nd derivative of dipole moment

$$\ddot{\mathbf{d}} = -e\dot{\mathbf{v}}$$

Fourier transform

$$-\omega^2\hat{\mathbf{d}}(\omega) = -\frac{e}{2\pi}\int_{-\infty}^{\infty}\dot{\mathbf{v}}e^{i\omega t}dt.$$

Recap: Bremsstrahlung
Emission from a single speed electron

Collision time : time interval over which electron and ion are close enough to interact

$$\tau = \frac{b}{v}$$

$$-\omega^2 \hat{\mathbf{d}}(\omega) = -\frac{e}{2\pi}\int_{-\infty}^{\infty} \dot{\mathbf{v}} e^{i\omega t} dt.$$

Case-1 ωτ>>1 the exponential of the integral oscillates rapidly and integral is small
Case-2 ωτ <<1 exponential is essentially unity

$$\hat{\mathbf{d}}(\omega) \sim \begin{cases} \dfrac{e}{2\pi\omega^2}\Delta\mathbf{v}, & \omega\tau \ll 1 \\ 0, & \omega\tau \gg 1, \end{cases}$$

Δv change of velocity during collision

Recap: Bremsstrahlung
Emission from a single speed electron

Recall Spectrum of dipole radiation

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2$$

$$\hat{\mathbf{d}}(\omega) \sim \begin{cases} \dfrac{e}{2\pi\omega^2}\Delta\mathbf{v}, & \omega\tau \ll 1 \\ 0, & \omega\tau \gg 1, \end{cases}$$

So Spectrum of Bremsstrahlung radiation

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Emission from a single speed electron

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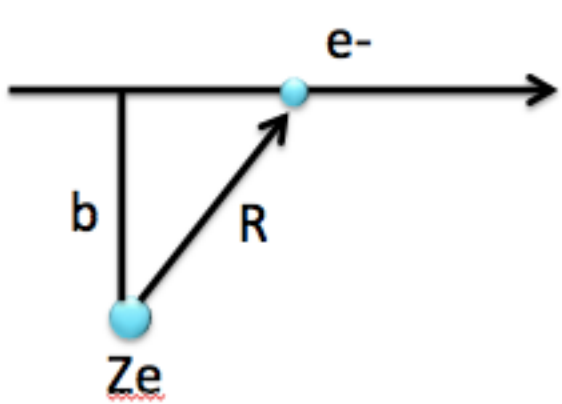
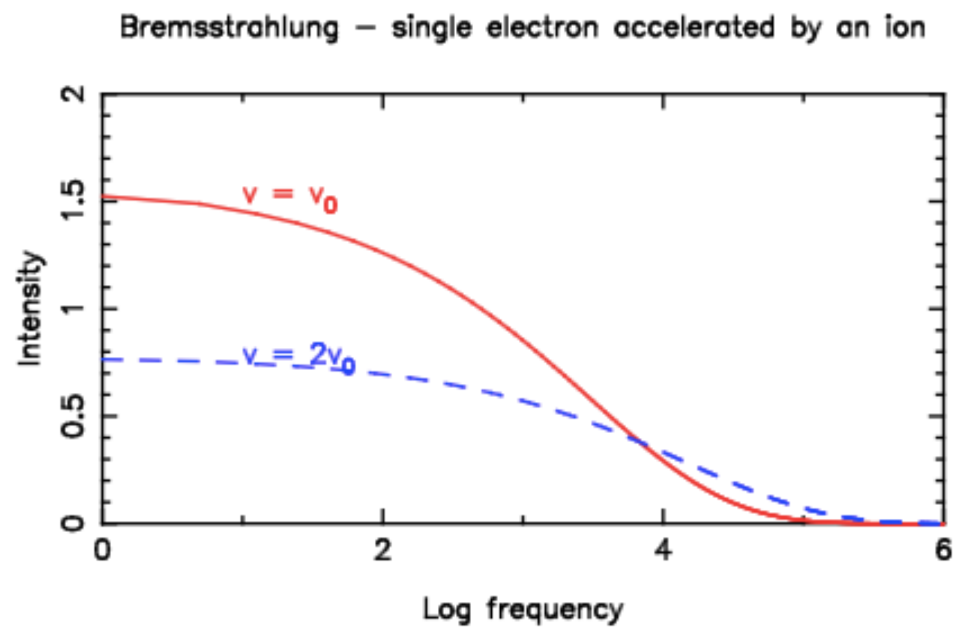
Recap: Bremsstrahlung
Emission from a single speed electron

Emission from a single speed electron

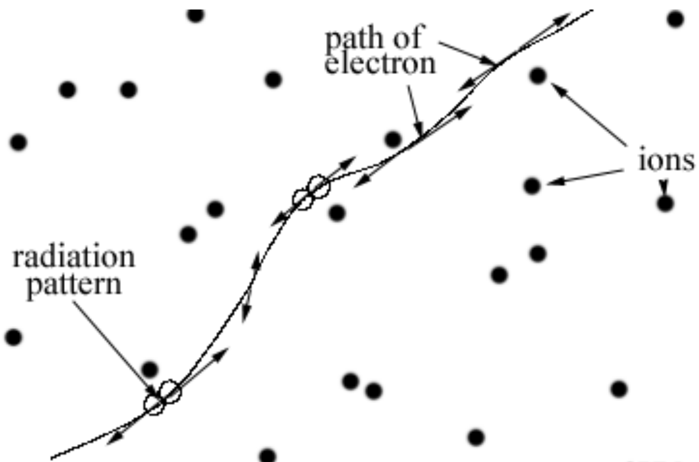
$$\frac{dW(b)}{d\omega} = \begin{cases} \dfrac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2}, & b \ll v/\omega \\ 0, & b \gg v/\omega. \end{cases}$$





Recap: Bremsstrahlung
Emission from multiple single speed electron

Bunch of electrons, all with the same speed, v, which interact with a bunch of ions.



$$\frac{dW}{d\omega\, dV\, dt} = n_e n_i 2\pi v \int_{b_{\min}}^{\infty} \frac{dW(b)}{d\omega}\, b\, db$$

$$b_{\max} \equiv \frac{v}{\omega} \qquad b_{\min}^{(1)} = \frac{4Ze^2}{\pi m v^2} \qquad b_{\min}^{(2)} = \frac{h}{mv}$$

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16\pi e^6}{3\sqrt{3}\, c^3 m^2 v} n_e n_i Z^2 g_{ff}(v,\omega)$$

$$g_{ff}(v,\omega) = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

Thermal Bremsstrahlung Recap

Free electrons are accelerated (decelerated) in the coulomb field of ionised nuclei and radiate energy

When a charged particle accelerates it emits radiation.

Consider a charged particle at a specific impact parameter(b) and velocity(v).

Acceleration is a function of b, v and Z.

Intensity spectrum via Fourier Transform.

Integrate (exact details tricky – gives rise to the Gaunt Factor $\bar{g}_{ff}$, which is a function of ν,T,Z).

Include term for collision rate (depends on number densities n_e and n_i). Integrate over v .

Assume plasma in thermal equilibrium →Maxwellian distribution of v .

Bremsstrahlung
Layout

(1) Emission from single speed electron
    pick rest frame of ion
    calculate dipole radiation
    correct for quantum effects (Gaunt factor)

(2) Emission from collection of electron
    Thermal bremsstrahlung(with numbers)
    Free-Free Absorption
    Non-thermal bremsstrahlung

(3) Relativistic bremsstrahlung (Virtual Quanta)

Thermal Bremsstrahlung Emission

$$\frac{dW}{dV\,dt\,d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT} \bar{g}_{ff}$$

particle energy

high energy cutoff

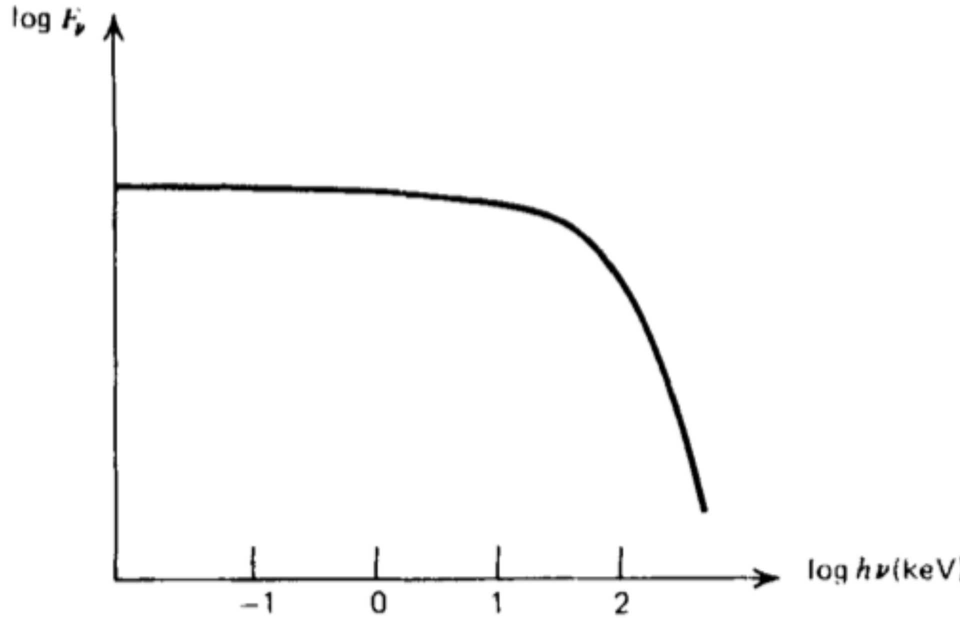
Lower limit of electron velocity from the condition $h\nu \leqslant \frac{1}{2}mv^2$

$$v_{\min} \equiv (2h\nu/m)^{1/2}$$

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8\times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff} = 4\pi j_\nu^{ff}.$$

$$\varepsilon^{ff} \equiv \frac{dW}{dt\,dV} = 1.4\times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B$$

Thermal Bremsstrahlung spectra



Thermal Bremsstrahlung Emission

Photon frequency vs temperature and cut off frequency

Consider an electron moving at a speed of v =1000 km s^{-1}.

Thermal Bremsstrahlung Emission
Photon frequency vs temperature and cut off frequency

Consider an electron moving at a speed of $v = 1000$ km s^{-1}.

In case it would radiate all its kinetic energy in a single interaction
$\Delta E = h\nu = 1/2\ mv^2$

So frequency(maximum) of the emitted radiation is $\Delta E/h \sim 7 \times 10^{14}$ Hz

Now assuming that the electron is part of population of particles for which v is the typical velocity, temperature $T = \Delta E/k \sim 3 \times 10^4$ K

Thermal Bremsstrahlung Emission

Photon frequency vs temperature and cut off frequency

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

Cut off frequency depends only on T and is set when the exponential is equal to 1

$h\nu = kT$ $\qquad$ $\nu_c = \frac{k}{h} T \quad \sim 2\times10^{10}$ Hz

So one can determine cut-off frequency for warm plasma (e.g. HII region, T~10^4 K), and hot plasma (in clusters of galaxies, T~10^8 K), and cut off for hot plasma will be at a higher frequency.

Thermal Bremsstrahlung Emission

Photon frequency vs temperature and cut off frequency

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Cut off frequency depends only and is set when the exponential is equal to 1

$h\nu = kT$ $\nu_c = \frac{k}{h} T$ ~2×10^{10} Hz

So one can determine cut-off frequency for warm plasma (e.g. HII region, T~10^4 K), and hot plasma (in clusters of galaxies, T~10^8 K), and cut off for hot plasma will be at a higher frequency.

Observationally one measures the the cutoff frequency and calculate the plasma temperature and hence velocity.

Total emissivity does not depend on frequency (except the cut – off)

→ Spectrum is Flat

Bremsstrahlung Emissivity

ε_ν^{ff} → (energy/frequency/volume/time).

j_ν → (energy/frequency/volume/time/*solid angle*).

ε_ν^{ff} is exactly the same as P_ν in $j_\nu = \frac{1}{4\pi} P_\nu$

Thermal Bremsstrahlung Emission (Cooling time)

$3/2(n_e+n_p)\,kT$

Cooling Time $T_{cool} = \dfrac{\text{Energy content of a gas}}{\text{Rate at which energy is being radiated}}$

$$\varepsilon^{ff} \equiv \frac{dW}{dt\,dV} = 1.4\times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B$$

$$\tau_{cool} = 6\times 10^{3} T^{1/2} n_e^{-1} \bar{g}_{ff}\, yr$$

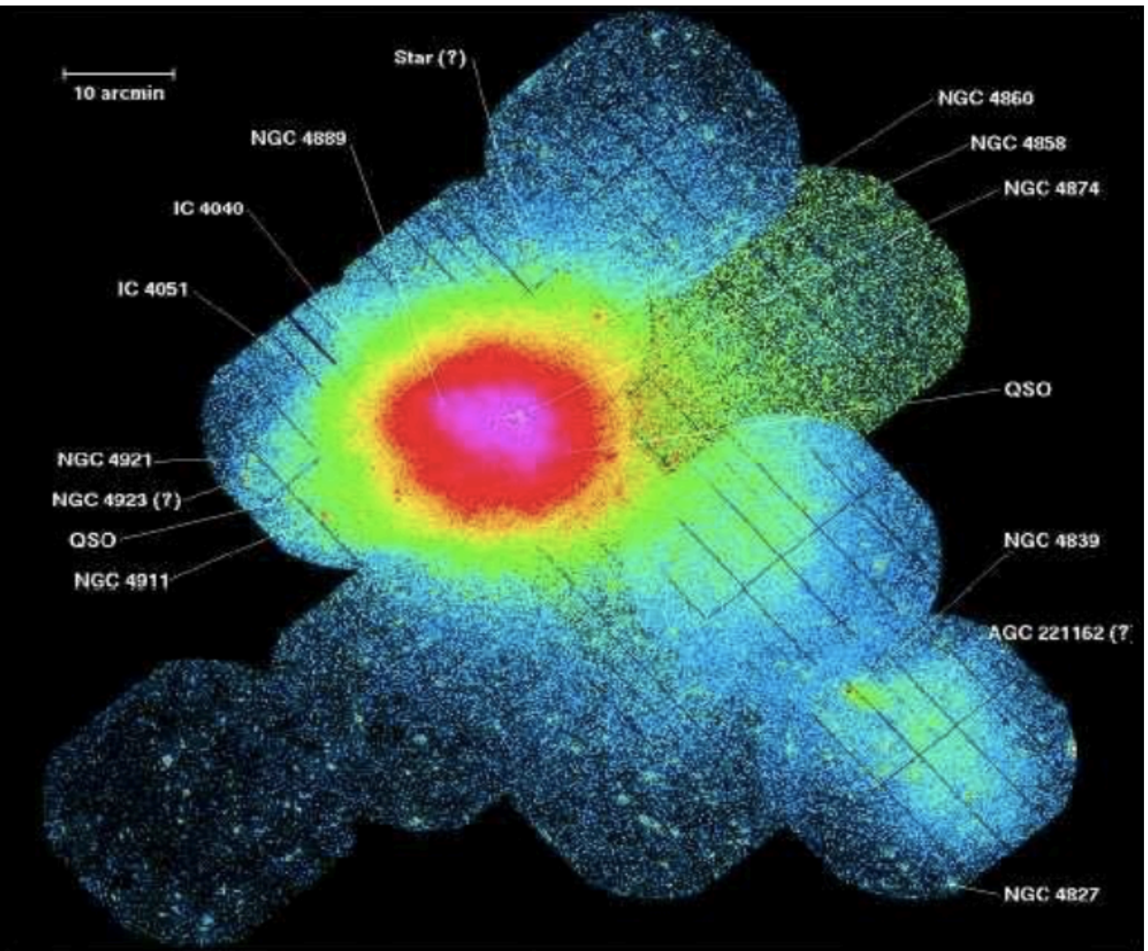
Bremsstrahlung is the main cooling process at temperatures above T~ 10^7 K

All galaxy clusters have bremsstrahlung emission

Thermal Bremsstrahlung Example
Cluster of galaxies

A galaxy cluster, or cluster of galaxies, is a structure consisting hundreds to thousands of galaxies bound together by gravity.

Coma cluster (ROSAT image)



hot gas →thermal bremsstrahlung

Radiative cooling time > 10 Gyr

Radio + X- ray emitters

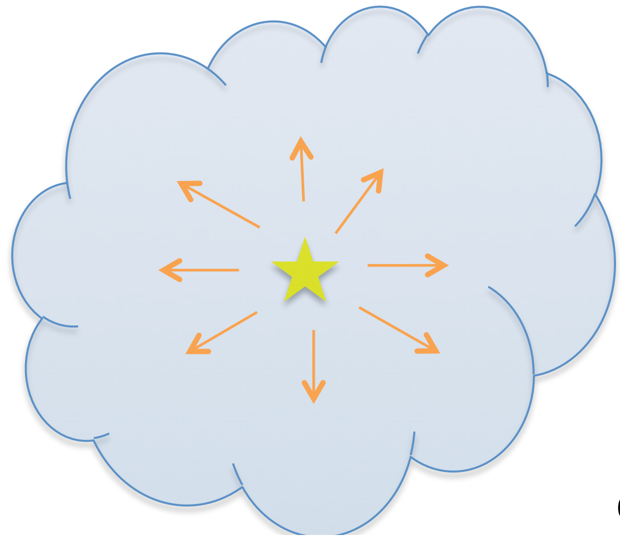
Image courtesy of U. Briel, MPE Garching, Germany and ESA

Bremsstrahlung (free-free) emission
Example

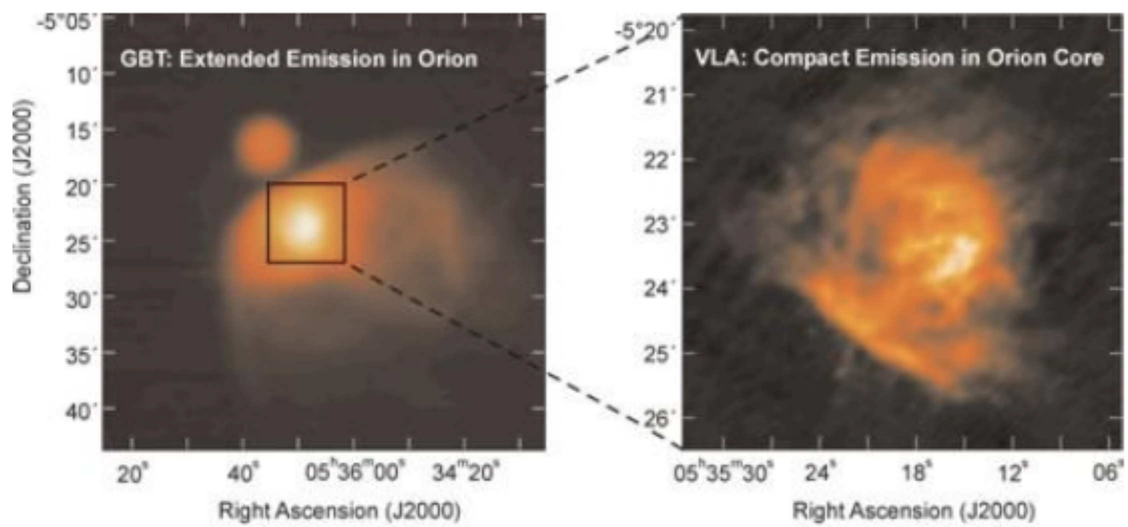
HII region around OB stars

uv photons from the OB star ionises the surrounding gas

Temperature T~ 10 [4]



Credit:NRAO/AUI



Galactic HII region:
M42 (Orion nebula)
distance ~ 0.5 kpc
angular size ~ 1.1°

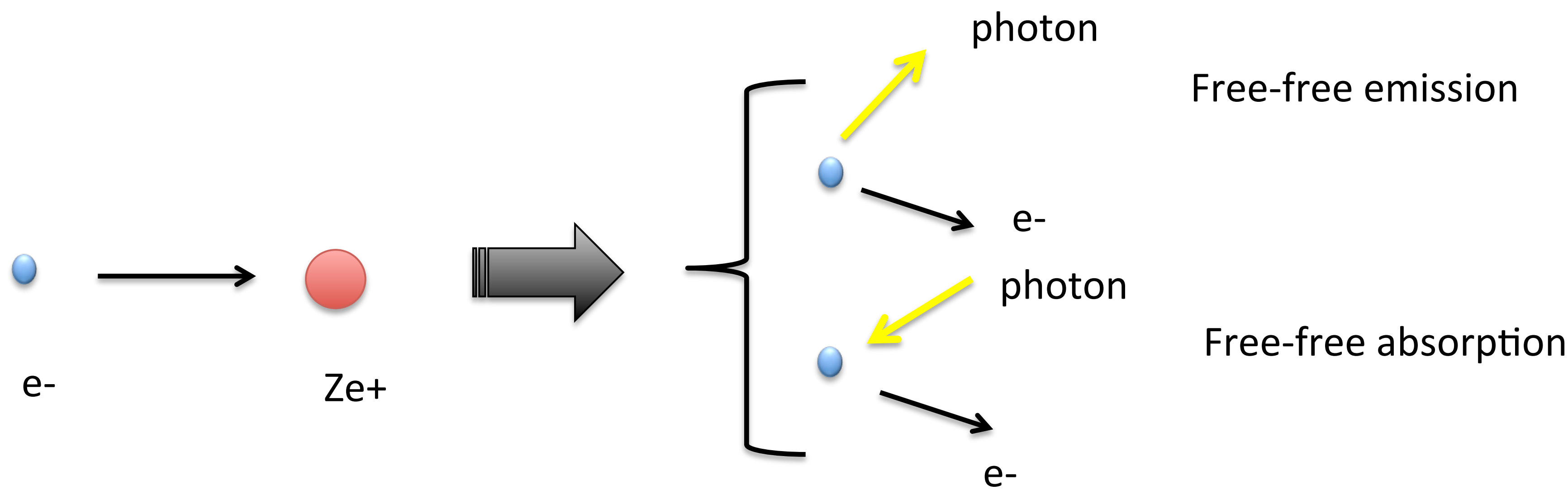
- Bremsstrahlung can be thermal or non-thermal.
- Bremsstrahlung becomes blackbody when optical depth >>1

Bremsstrahlung emissivity in hot plasmas

	T (K)	Obs. of ε_{ν}^{ff}
Solar flare	10^7 (~ 1keV)	radio→ flat X-ray → exponential
H II region	10^5	radio→ flat
Orion	10^4	radio →flat
Sco X-1	10^8	optical-flat X-ray → flat/exp.
Coma Cluster ICM	10^8	X-ray → flat/exp.

Credit : George F. Smoot

Bremsstrahlung (free-free) absorption



Free-free absorption :

Absorption of radiation by an electron moving in the field of an ion due to the bremsstrahlung process

Bremsstrahlung (free-free) absorption

Absorption of radiation by an electron moving in the field of an ion due to to the bremsstrahlung process

Let us consider thermal free-free absorption

Kirchhoff's law

$$j_\nu^{ff} = \alpha_\nu^{ff} B_\nu(T).$$

j_ν^{ff} → emission coefficient

α_ν^{ff} → Free-free absorption coefficient

However we have

$$\varepsilon_\nu^{ff} \Longrightarrow \frac{dW}{dt\, dV\, d\nu} = 4\pi j_\nu^{ff}.$$

Bremsstrahlung (free-free) absorption

$$j_\nu^{ff} = \alpha_\nu^{ff} B_\nu(T).$$

$$\frac{dW}{dt\, dV\, d\nu} = 4\pi j_\nu^{ff}.$$

$$\frac{dW}{dV\, dt\, d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT} \bar{g}_{ff}$$

$$I_\nu = B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Free-free absorption coefficient

$$\alpha_\nu^{ff} = \frac{4e^6}{3mhc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3}\left(1 - e^{-h\nu/kT}\right)\bar{g}_{ff}.$$

Bremsstrahlung (free-free) absorption

Free-free absorption coefficient

$$\alpha_\nu^{ff} = \frac{4e^6}{3mhc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3}(1-e^{-h\nu/kT})\bar{g}_{ff}.$$

Evaluating the constants in C.G.S. units

$$\alpha_\nu^{ff} = 3.7\times 10^8 T^{-1/2} Z^2 n_e n_i \nu^{-3}(1-e^{-h\nu/kT})\bar{g}_{ff}.$$

↓

Unit cm^{-1}

Bremsstrahlung (free-free) absorption

Free-free absorption coefficient in C.G.S. units

$$\alpha_\nu^{ff} = 3.7\times 10^8 T^{-1/2} Z^2 n_e n_i \nu^{-3}(1-e^{-h\nu/kT})\bar{g}_{ff}.$$

↓

Unit cm^{-1}

Can find the optical depth now

Case-1 $h\nu \gg kT$ (e.g. X-rays)

The exponential is negligible and α_ν is proportional to $T^{-1/2}\ \nu^{-3}$ is very small unless n_e is very large

In X-rays, thermal bremsstrahlung emission can be treated as *optically thin* (except in stellar interiors)

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$, We are in Rayleigh-Jeans regime: Radio frequencies

$$\alpha_\nu^{ff} = \frac{4e^6}{3mkc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff},$$

Evaluating the constants in C.G.S. units

$$\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}.$$

Absorption can be important, even for relatively low n_e in the radio regime.

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$, Rayleigh-Jeans regime: Radio frequencies

$$\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}.$$

Calculate optical depth $\tau_\nu = \int \alpha_\nu ds$

If optically thin, spectrum is as calculated before : flat until turnover.

If optically thick, spectrum is effectively blackbody.

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$

We are in Rayleigh-Jeans regime: Radio frequencies

$$\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}.$$

Optical depth

$$\tau \propto \int \frac{n^2\, \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}} dl$$

Recap

$$I_\nu = (1 - e^{-\tau_\nu}) B_\nu(T_e)$$

At low ν, $\tau_\nu >> 1$

$$I_\nu \propto B_\nu(T_e) \propto \nu^2$$

Black body like spectrum

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$, We are in Rayleigh-Jeans regime: Radio frequencies

Optical depth
$$\tau \propto \int \frac{n^2 \, \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}} dl$$

In this regime $\bar{g}_{ff}(\nu) \propto \nu^{-0.1} T^{0.15}$

Recap $I_\nu = (1 - e^{-\tau_\nu}) B_\nu(T_e)$

- ✓ At low ν, τ_ν >>1 $I_\nu \propto B_\nu(T_e) \propto \nu^2$ Black body like spectrum
- ✓ At low ν, τ_ν <<1 $I_\nu \propto \tau B_\nu(T_e) \propto \nu^{-0.1}$ Flat spectrum

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT,$ We are in Rayleigh-Jeans regime: Radio frequencies

Optical depth

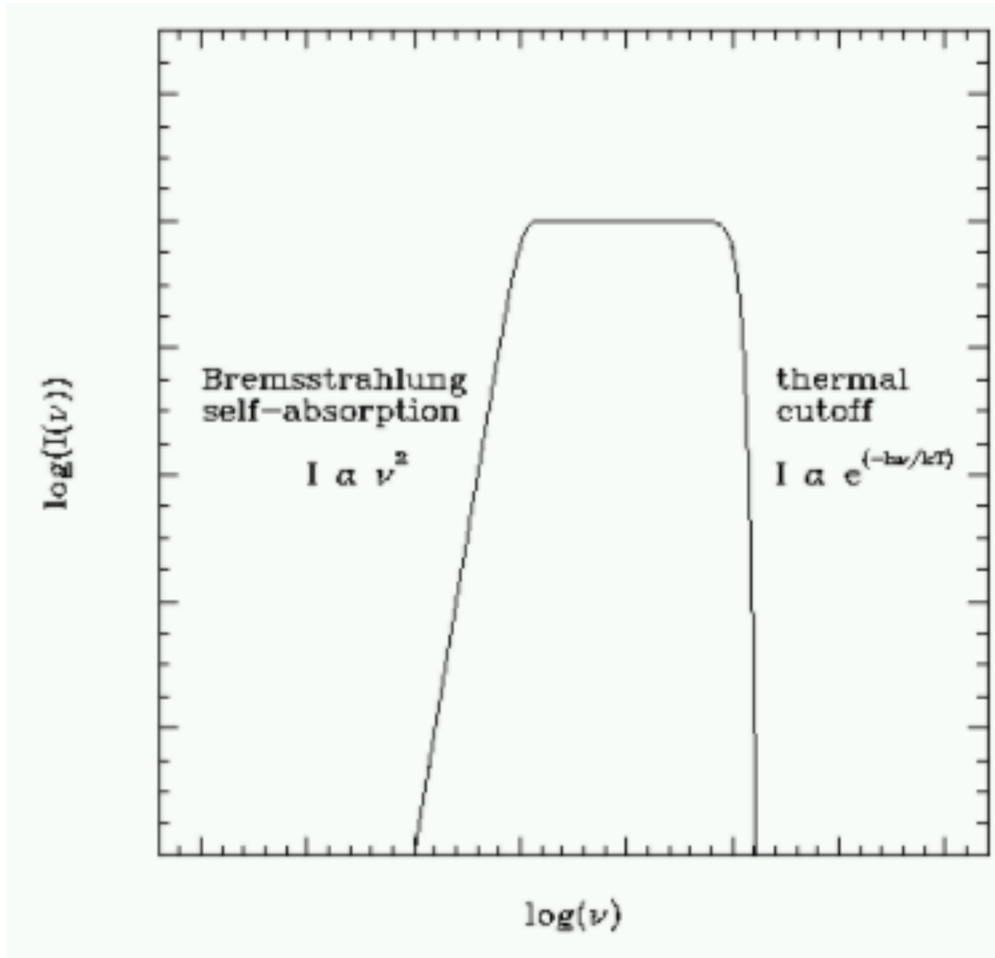
$$\tau \propto \int \frac{n^2 \, \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}} dl$$

$$I_\nu = (1 - e^{-\tau_\nu}) B_\nu(T_e)$$

- ✓ At low ν, τ_ν >>1 $I_\nu \propto B_\nu(T_e) \propto \nu^2$ Black body like spectrum
- ✓ At low ν, τ_ν <<1 Flat spectrum

Turnover when τ_ν ~1, ν~ 1 GHz for Orion

Bremsstrahlung Spectra



Effect of optical thickness on the Bremsstrahlung spectrum.
At low frequencies due to self-absorption spectrum follow the Raleigh-Jeans law.
(spectrum is typically seen in dense ionised gas such as found in star formation regions)

Relativistic Bremsstrahlung Emission

For a plasma with electron and ion densities n_e and n_i frequency integrated power

$$\frac{dW}{dV\,dt} = 1.4\times 10^{-27} T^{1/2} Z^2 n_e n_i \bar{g}_B (1 + 4.4\times 10^{-10} T)$$

Relativistic correction
Significant for very high temperatures

Recap : we calculated emissivity for non-relativistic case

$$\epsilon^{ff} \equiv \frac{dW}{dt\,dV} = 1.4\times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B$$

Relativistic Bremsstrahlung Emission

The typical velocities of the particles are now relativistic and energy (velocity) distribution is described by a power law

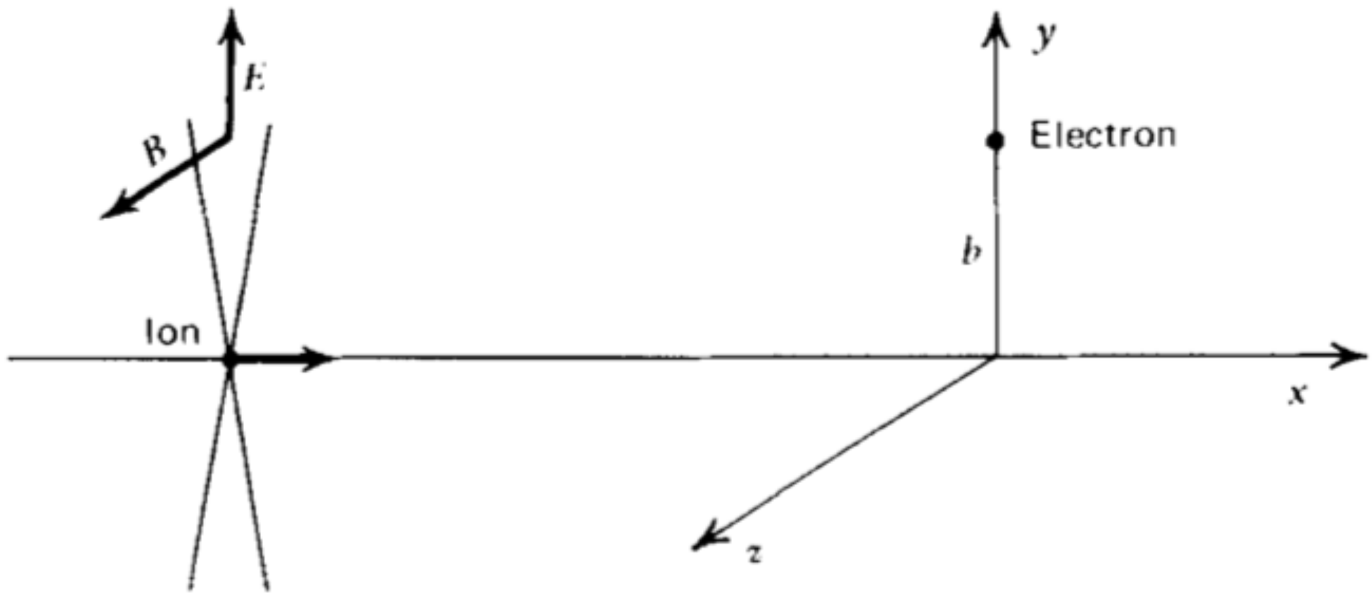
Analysis will be performed with the method of virtual quanta

Consider collision between an electron and a heavy ion of charge Ze

Normally ion moves slower than electrons.
Let us consider a frame where electron is initially at rest and in that case the ion appears to move rapidly towards electron.

Relativistic Bremsstrahlung Emission

Ion moves along x-axis with velocity v. Electron is initially rest on y-axis at a distance b from the origin.



Electrostatic field of the ion is transformed in to a transverse pulse with |E|=|B| .
To the electron this is a pulse of electromagnetic radiation.

Relativistic Bremsstrahlung Emission

This radiation then Compton scatters off electron to produce the detected radiation

Transforming back to the rest frame of the ion (lab frame) we obtain the bremsstrahlung emission of the ion.

Thus relativistic bramsstrahlung can be regarded as the Compton scattering of the virtual quanta of the ion's electrostatic field as seen in the electron's frame.

In the primed frame (the rest frame of electrons) the spectrum of the pulse of virtual quanta has the following form (as derived in Lecture −6, slide 48), with v=c in ultra relativistic limit

$$\frac{dW'}{dA'\,d\omega'}(\mathrm{erg\ cm^{-2}\ Hz^{-1}}) = \frac{(Ze)^2}{\pi^2 b'^2 c}\left(\frac{b'\omega'}{\gamma c}\right)^2 K_1^2\left(\frac{b'\omega'}{\gamma c}\right)$$

Relativistic Bremsstrahlung Emission

In the primed frame, i.e. in the rest frame of the electrons the spectrum of the pulse of virtual quanta has the following form (as derived in Lecture −6),
with v=c in ultra relativistic limit

$$\frac{dW'}{dA'\,d\omega'}(\mathrm{erg\ cm^{-2}\ Hz^{-1}}) = \frac{(Ze)^2}{\pi^2 b'^2 c}\left(\frac{b'\omega'}{\gamma c}\right)^2 K_1^2\left(\frac{b'\omega'}{\gamma c}\right)$$

In primed frame (rest frame of the electron)

$\hbar\omega' \lesssim mc^2$ Virtual quanta are scattered by electron according to Thomson scattering

$\hbar\omega' > mc^2$ Virtual quanta are scattered by electron according to Compton scattering

Relativistic Bremsstrahlung Emission

Now since energy and frequency transform identically in Lorentz transformations,

$$dW = \gamma \, dW'$$

$$dW / d\omega = dW' / d\omega'$$

Thus the emission in lab frame is

$$\frac{dW}{d\omega} = \frac{8Z^2e^6}{3\pi b^2c^5m^2}\left(\frac{b\omega}{\gamma^2c}\right)^2 K_1\left(\frac{b\omega}{\gamma^2c}\right)$$

Replacing
b=b' and ω=γω'

Energy per unit frequency emitted by collision of an ion
and a relativistic electron at a impact parameter b

Thermal Bremsstrahlung
Emission from multiple single speed electron

Total spectrum for a medium with ion density n_i electron density n_e and fixed electron speed v

$$\frac{dW}{d\omega\, dV\, dt} = n_e n_i 2\pi v \int_{b_{min}}^{\infty} \frac{dW(b)}{d\omega} b\, db$$

Flux of electrons (electrons per unit area per unit time) incident on one ion is n_e v

The element of area is $2\pi b db$ about a single ion.

b_{min} is minimum value of impact parameter

Relativistic Bremsstrahlung Emission

For a plasma with electron and ion densities n_e and n_i we repeat the same treatment which we did for thermal bremsstrahlung

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

→ Thermal Bremsstrahlung

v is replaced by c with electron and ion densities n_e and n_i we repeat the same treatment which we did for thermal bremsstrahlung

$$\frac{dW}{dt\, dV\, d\omega} \sim \frac{16Z^2e^6n_e n_i}{3c^4m^2} \ln\left(\frac{0.68\gamma^2 c}{\omega b_{\min}}\right)$$

→ Non-thermal Bremsstrahlung
In low-frequency limit $h\nu \ll \gamma mc^2$

Relativistic Bremsstrahlung Emission

For a plasma with electron and ion densities n_e and n_i frequency integrated power

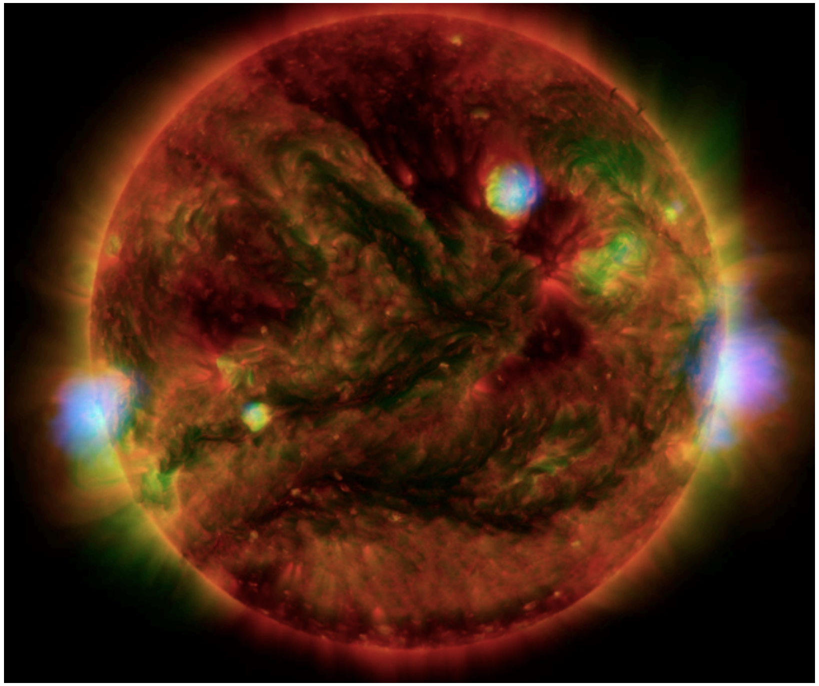
$$\frac{dW}{dV\,dt} = 1.4 \times 10^{-27} T^{1/2} Z^2 n_e n_i \bar{g}_B (1 + 4.4 \times 10^{-10} T)$$

Relativistic correction
Significant for very high temperatures

Relativistic Bremsstrahlung Emission Example

Solar wind:
ne ~ 4 – 7 cm^{-3}
v_{swind}~ 300 – 900 km s^{-1}
T ~ 150, 000 K

Solar Flares: brief and intense emission from sun's surface relativistic particles with power–law energy distribution



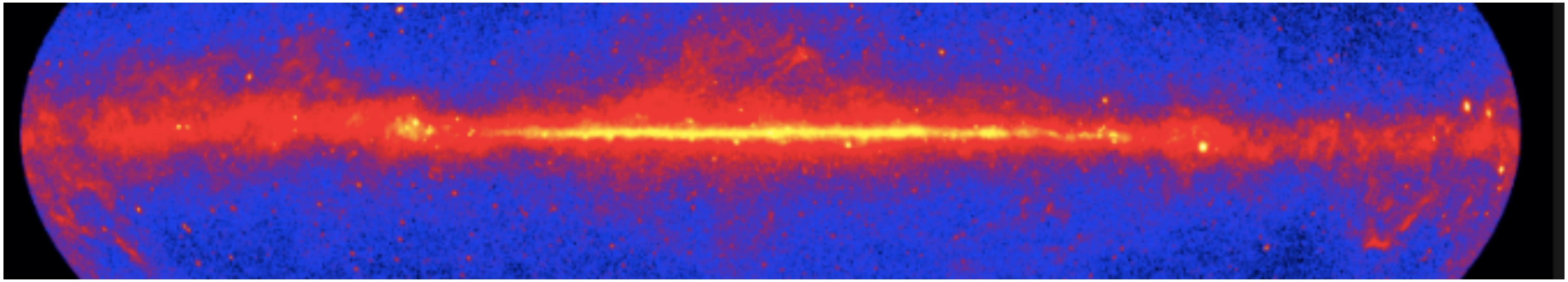
Flaring, active regions sun are highlighted in this image combining observations from several telescopes. High-energy X-rays from NASA's Nuclear Spectroscopic Telescope Array (NuSTAR) are shown in blue; low-energy X-rays from Japan's Hinode spacecraft are green; and extreme ultraviolet light from NASA's Solar Dynamics Observatory (SDO) is yellow and red.

Image credit: NASA/JPL-Caltech/GSFC/JAXA

Relativistic Bremsstrahlung Emission Example

Gamma-rays from the Galaxy

Gamma-ray emission is detected from our Galaxy is thought to arise from relativistic Bremsstrahlung from high energy electrons.



Milky way in Gamma-rays (Fermi LAT image)

The radiative energy is carried by photons with $h\nu \sim E_e$

Energies in the range ~100 MeV, suggesting many relativistic electrons with γ~100

Polarisation of Bremsstrahlung Emission

Free-free emission is intrinsically unpolarised.

Coulomb interaction are random in orientation, so are in general unpolarised.

- ✓ Individual interaction produces polarized radiation (fixed direction of E and B fields of the e-m wave wrt the acceleration)

- ✓ Overall the emission is unpolarized (random orientation of the plane of interaction)

Bremsstrahlung Emission
Summary

- ✓ Photon is emitted from accelerating electron in the Coulomb field of ion
- ✓ The energy of the photon can never exceed the kinetic energy of electron
- ✓ Generally the ''classical'' description of the phenomenon is adequate, the quantum mechanics requires an appropriate Gaunt Factor
- ✓ Overall the emission is unpolarized (random orientation of the plane of interaction)
- ✓ We have ''thermal'' or ''relativistic'' bremsstrahlung, depending on velocity/energy distribution of electrons.

Simple Examples

Hydrogen Plasma

A common case is that of an optically thin hydrogen plasma, so n_e=n_i & Z=1

$$I_\nu \propto \int n_e^2 \, T^{-1/2} \, dl$$

$$\boxed{\int n_e^2 \, dl}$$ → Emission Measure
Unit $cm^{-6}pc$

Simple Examples

Cooling Time T_{cool} = $\dfrac{\text{Energy content of a gas}}{\text{Rate at which energy is being radiated}}$

For fully ionised pure hydrogen gas $\varepsilon_{ff} = 1.7 \times 10^{-27} T^{1/2} n_e^2$

$$T_{cool} = 7900 \, \frac{T^{1/2}}{n_e} \, years$$

For HII region $n_e = 10^2$-10^3 cm^{-3}, T= 10^3-10^4 K, T_{cool} ~100-1000 years

For Galaxy clusters $n_e = 10^{-3}$ cm^{-3}, T= 10^8 K, T_{cool} ~10^{10} years

Lecture -7
Questions raised in the class

1. Definition of dipole moment for unequal charges?
2. Why free-free emission from like particles (e-e, p-p) is zero?
3. Why electron radiate in electron-ion bremsstrahlung?
4. Change of time of interaction with b?
5. Polarisation of Bresmsstahlung radiation?

End of Lecture 8

http://demonstrations.wolfram.com/RadiationPulseFromAnAcceleratedPointCharge/

Next Lecture : 5th September

Topic of next Lecture:
Cyclotron to Synchrotron Radiation
(Chapter 6 of Rybicki & Lightman)