Electrodynamics and Radiative Processes I

Lecture 5 – Radiation from moving charges

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Lecture -5

Questions raised in the class (update)

Why **j.E** is rate of change of mechanical energy

- ve Sign in slide 36

$$R^2(t') = \mathbf{R}^2(t')$$

$$2R(t')\dot{R}(t') = -2\mathbf{R}(t')\cdot\mathbf{u}(t'),$$

Recap Lecture 4

Maxwell's equations

Maxwell's equations in vacuum

Wave equation with **E**

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Solution of wave equation with **E** and **B**

$$\mathbf{E} = \hat{\mathbf{a}}_1 E_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

$$\mathbf{B} = \hat{\mathbf{a}}_2 B_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

Recap Lecture 4

Maxwell's Equations (Recap)
(in Gaussian units)

$$\nabla \cdot \mathbf{D} = 4\pi\rho \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

Electromagnetic Potentials

E and B are replaced by Φ(r,t) and **A(r,t)**

$$\mathbf{B} = \nabla \times \mathbf{A}.$$

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

1) One scalar plus one vector simpler than two vectors

2) Determining **A** and Φ are simpler

3) Relativistic EM theory will be simpler

Recap Lecture 4

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} + \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right) = -4\pi\rho$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla\left(\nabla\cdot\mathbf{A} + \frac{1}{c}\frac{\partial\phi}{\partial t}\right) = -\frac{4\pi}{c}\mathbf{j}$$

Scalar and vector potential are not uniquely determined by the conditions

Lorentz Gauge $\nabla\cdot\mathbf{A} + \frac{1}{c}\frac{\partial\phi}{\partial t} = 0$

Recap Lecture 4

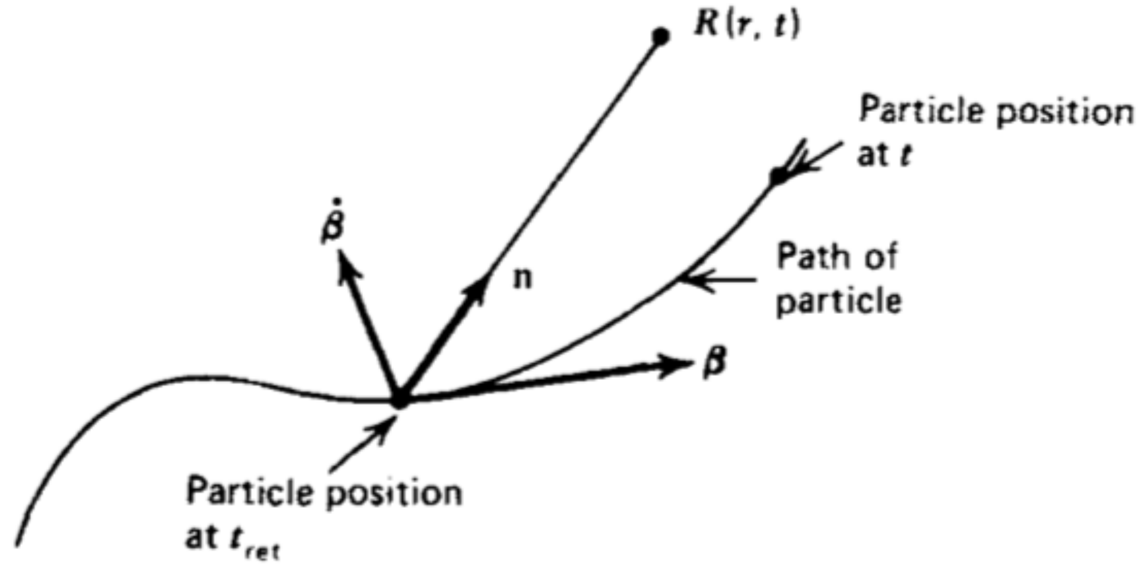
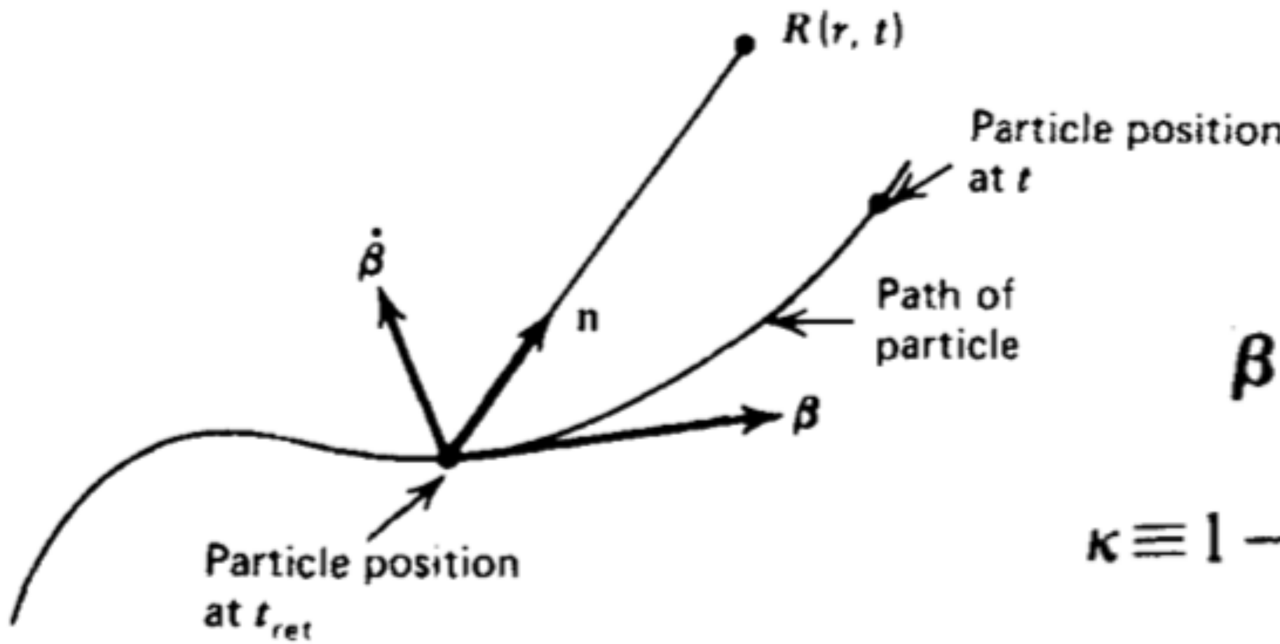


Fig : Radiation field at R from position of the radiating particle at the retarded time

$$\phi(\mathbf{r}, t) = \frac{q}{\kappa(t_{\mathrm{ret}})R(t_{\mathrm{ret}})} \qquad \kappa(t') = 1 - \frac{1}{c}\mathbf{n}(t')\cdot\mathbf{u}(t')$$

$$\boxed{\phi = \left[\frac{q}{\kappa R}\right] \qquad \mathbf{A} = \left[\frac{q\mathbf{u}}{c\kappa R}\right]}$$

Recap Lecture 4



$$\beta \equiv \frac{\mathbf{u}}{c}$$

$$\kappa \equiv 1 - \mathbf{n} \cdot \boldsymbol{\beta}.$$

Fig : Radiation field at R from position of the radiating particle at the retarded time

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right] + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R} \times \{(\mathbf{n}-\boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}\right]$$

$$\mathbf{B}(\mathbf{r},t) = [\mathbf{n} \times \mathbf{E}(\mathbf{r},t)]$$

Velocity field

Acceleration/Radiation field

Radiation field

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right] + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R} \times \{(\mathbf{n}-\boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}\right]$$

Velocity field

- $1/R^2$ dependence
- Only contributing term for particle with constant velocity
- Generalization of the Coulomb's law to moving particles approaches to coulomb's law when u<<c
- Electric filed always point towards current position of the particle

Acceleration field/Radiation field
1/R dependence
Proportional to particle's acceleration perpendicular to n

Radiation field

$$\mathbf{E}(\mathbf{r},t)=q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]+\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right]$$

Radiation field

$$\mathbf{E}_{\rm rad}(\mathbf{r},t)=\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right]$$

$$\mathbf{B}_{\rm rad}(\mathbf{r},t)=\left[\mathbf{n}\times\mathbf{E}_{\rm rad}\right]$$

E_{rad}, B_{rad}, n : mutually perpendicular

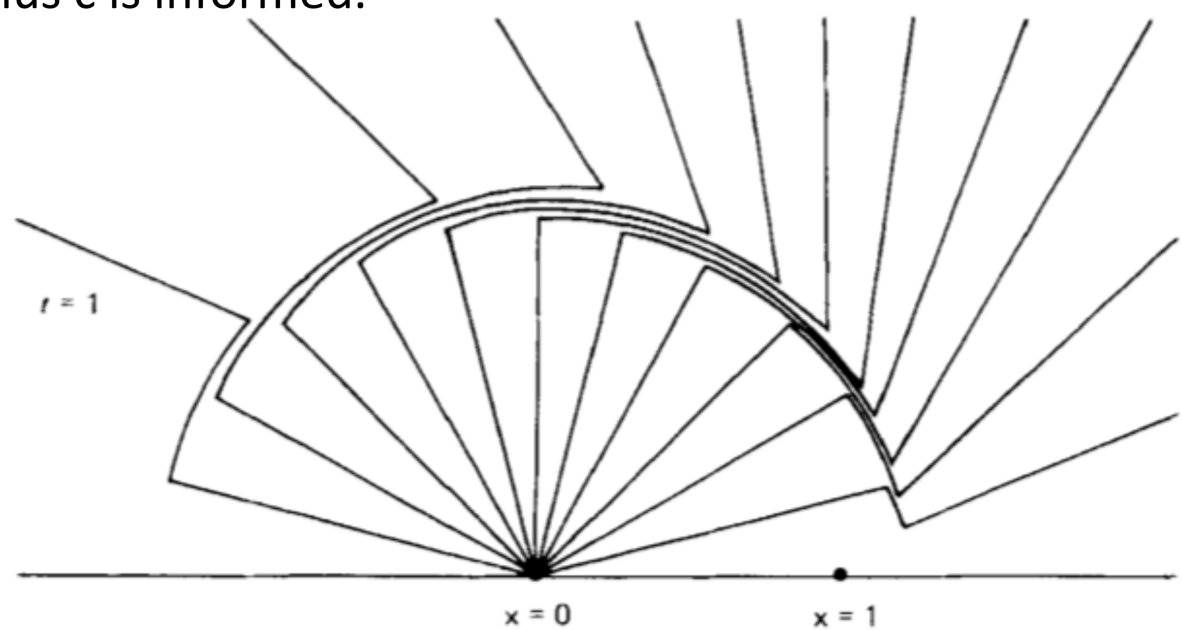
$|E_{rad}| = |B_{rad}|$

Radiation fields

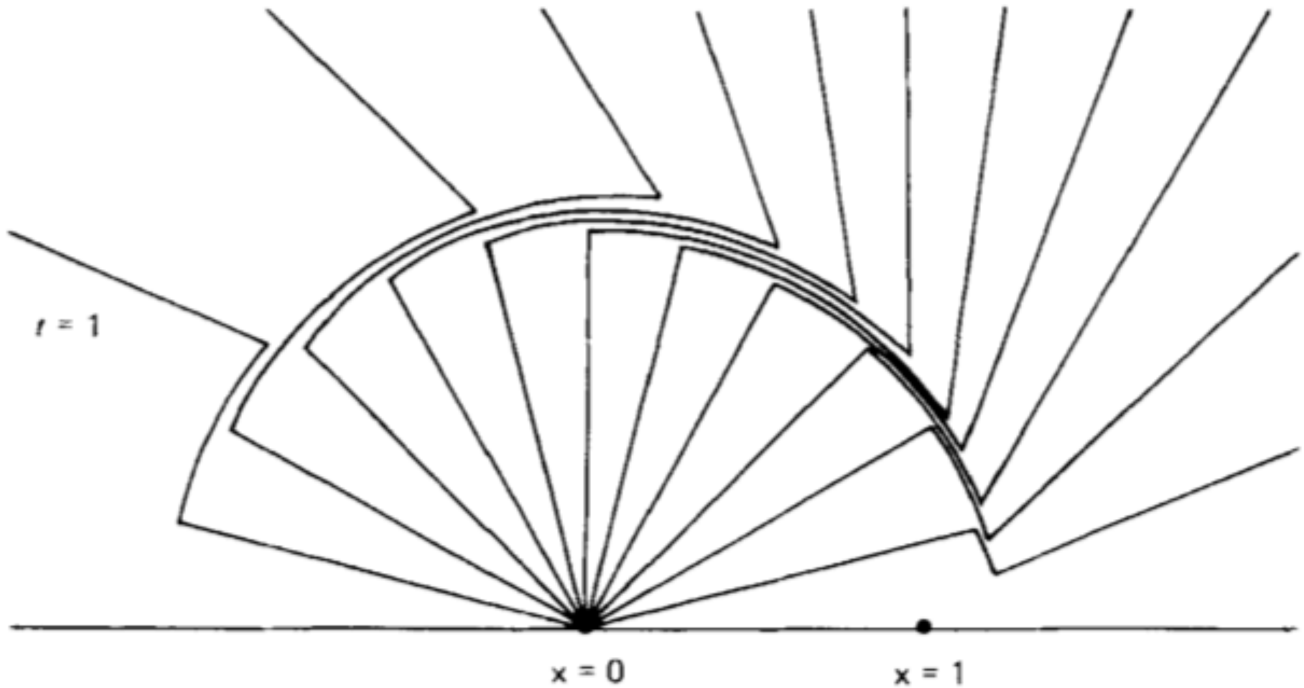
Consider a particle originally moving at constant velocity along x axis is stopped at x=0 and t=0

At t=1 the field outside of a radius c is radial and points to the position where particle would have been if there was no deceleration (since no information Is yet propagated to that distance)

But field inside the radius c is informed.



Radiation fields



These two fields can be connected with flux conservation: as shown in the figure.

Transition zone whose radial thickness is the time interval over which deceleration occurs.
This transition zone is almost transverse and much stronger.

Radiation Spectrum

Energy per unit frequency per unit solid angle corresponding to the radiation field of a single particle

$$\frac{dW}{dA\,d\omega} = c|\hat{E}(\omega)|^2 \qquad \text{(Lecture 4)}$$

$$\frac{dW}{d\omega\,d\Omega} = \frac{c}{4\pi^2}\left|\int [R\mathbf{E}(t)]e^{i\omega t}dt\right|^2$$

$$= \frac{q^2}{4\pi^2 c}\left|\int \left[\underline{\mathbf{n}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\kappa^{-3}}\right]e^{i\omega t}dt\right|^2$$

↓ Evaluated at a retarded time

Radiation Spectrum

Energy per unit frequency per unit solid angle corresponding to the radiation field of a single particle

$$\frac{dW}{d\omega\, d\Omega} = \frac{c}{4\pi^2}\left|\int [R\mathbf{E}(t)]e^{i\omega t}dt\right|^2$$

$$= \frac{q^2}{4\pi^2 c}\left|\int \left[\underline{\mathbf{n}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\kappa^{-3}}\right]e^{i\omega t}dt\right|^2$$

Evaluated at a retarded time

Changing variable from t to t'

$$t' = t - R(t')/c \qquad R(t') \approx |\mathbf{r}| - \mathbf{n}\cdot\mathbf{r}_0$$

$$dt = \kappa\, dt'$$

$$\frac{dW}{d\omega\, d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int \mathbf{n}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\kappa^{-2}\exp[i\omega(t' - \mathbf{n}\cdot\mathbf{r}_0(t')/c)]dt'\right|^2.$$

Radiation from non-relativistic systems of particles

Electric filed of moving charges

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right] + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]$$

$\mathbf{E}_{\mathbf{vel}}$ $\mathbf{E}_{\mathbf{rad}}$

Knowing the velocity and radiation fields we will be able to discuss many radiation processes involving moving charges

For the moment we will consider discussion of non relativistic particles

$$|\beta| = \frac{u}{c} \ll 1 \quad \Longrightarrow \quad \frac{E_{\mathrm{rad}}}{E_{\mathrm{vel}}} \sim \frac{R\dot{u}}{c^2}$$

Radiation from non-relativistic systems of particles

Refer to Slide 26

$$\mathbf{E}(\mathbf{r},t)=q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3R^2}\right]+\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right]$$

Considering $|\beta|=\frac{u}{c}\ll 1$

$$\frac{E_{\text{rad}}}{E_{\text{vel}}}\sim\frac{R\dot{u}}{c^2}$$

For particle with frequency of oscillation ν

$$\frac{E_{\text{rad}}}{E_{\text{vel}}}\sim\frac{Ru\nu}{c^2}=\frac{u}{c}\,\frac{R}{\lambda}.$$

R < λ → "Near zone" → Velocity field stronger than Radiation field by > c/u

R >> λ(c/u) → "Far zone" → Acceleration field dominates
Domination increase linearly with R

Larmor's Formula

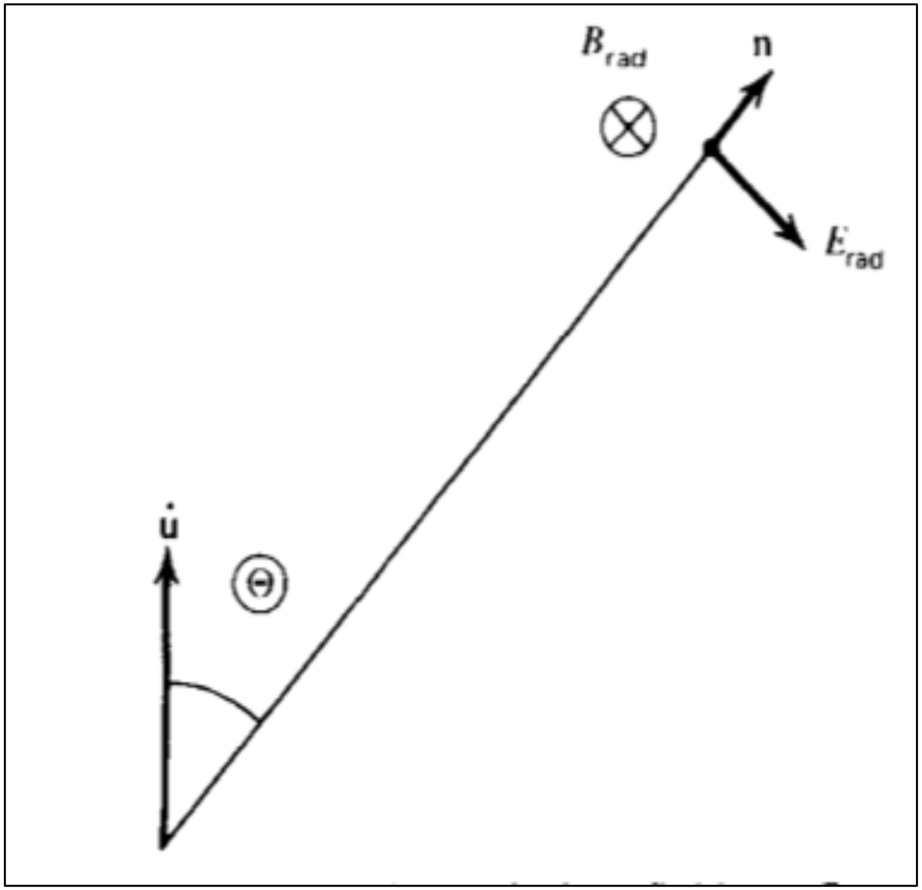
Total power radiated by a non-relativistic point charge as it accelerates

$$\mathbf{E}_{\rm rad}(\mathbf{r},t) = \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right]$$

For β<<1

$$\mathbf{E}_{\rm rad} = \left[(q/Rc^2)\mathbf{n}\times(\mathbf{n}\times\dot{\mathbf{u}})\right]$$

$$\mathbf{B}_{\rm rad} = \left[\mathbf{n}\times\mathbf{E}_{\rm rad}\right].$$



Outward flow of energy along **n**

Larmor's Formula

Total power radiated by a non-relativistic point charge as it accelerates

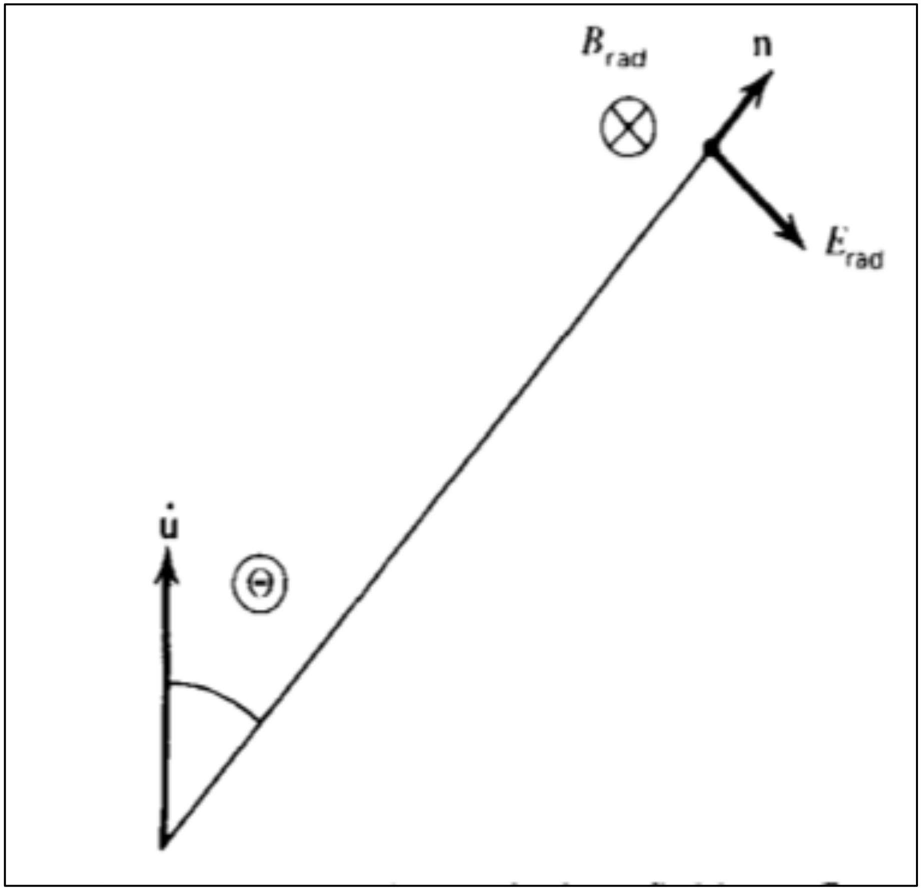
$$\mathbf{E}_{\rm rad}(\mathbf{r},t)=\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}\right]$$

For β<<1

$$\mathbf{E}_{\rm rad}=\left[(q/Rc^2)\mathbf{n}\times(\mathbf{n}\times\dot{\mathbf{u}})\right]$$

$$\mathbf{B}_{\rm rad}=\left[\mathbf{n}\times\mathbf{E}_{\rm rad}\right].$$

$$|\mathbf{E}_{\rm rad}|=|\mathbf{B}_{\rm rad}|=\frac{q\dot{u}}{Rc^2}\sin\Theta$$



Poynting Vector

$$S=\frac{c}{4\pi}E_{\rm rad}^2=\frac{c}{4\pi}\frac{q^2\dot{u}^2}{R^2c^4}\sin^2\Theta$$

Outward flow of energy along **n**

Larmor's Formula

$$S = \frac{c}{4\pi} E_{\mathrm{rad}}^2 = \frac{c}{4\pi} \frac{q^2 \dot{u}^2}{R^2 c^4} \sin^2\Theta$$

Power radiated per unit solid angle per unit time

$$\frac{dW}{dt\, d\Omega} = \frac{q^2 \dot{u}^2}{4\pi c^3} \sin^2\Theta.$$

$$P = \frac{dW}{dt} = \frac{q^2 \dot{u}^2}{4\pi c^3} \int \sin^2\Theta \, d\Omega$$

$$P = \frac{2 q^2 \dot{u}^2}{3 c^3}$$

Larmor's Formula for emission
from a single accelerated charge q

Larmor's Formula

$$P = \frac{dW}{dt} = \frac{q^2 \dot{u}^2}{4\pi c^3} \int \sin^2\Theta \, d\Omega$$

$$P = \frac{2q^2 \dot{u}^2}{3c^3}$$

- ✓ Power emitted is proportional to square of charge and square of acceleration
- ✓ Dependence on $\sin^2\Theta$: No radiation along direction of acceleration
  Max radiation perpendicular to acceleration
- ✓ Direction of E_{rad} is determined by $\mathring{\mathbf{u}}$ and **n** : If the particle accelerates along a line radiation will be 100% polarized in the plane of $\mathring{\mathbf{u}}$ and **n**

Larmor's Formula

$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

Limitations:

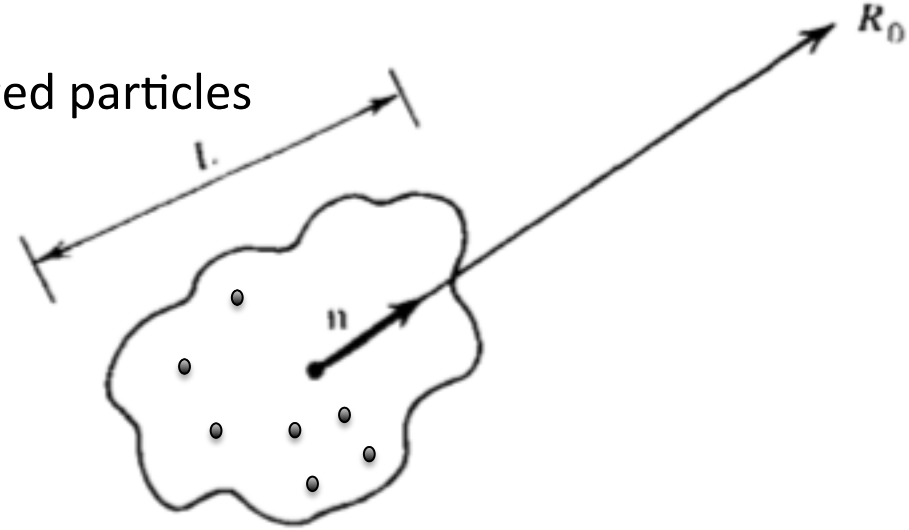
- ✓ Larmor's formula is nonrelativistic; it is valid only in frames moving at velocities v<<c with respect to the radiating particle.

- ✓ To treat particles moving at nearly the speed of light in the observer's frame, we must use Larmor's equation to calculate the radiation in the particle's rest frame and then transform the result to the observer's frame in a relativistically correct way.

- ✓ Larmor's formula does not incorporate the constraints of quantum mechanics, so it should be applied with great caution to microscopic systems such as atoms. For example, Larmor's equation incorrectly predicts that the electron in a hydrogen atom will quickly radiate away all of its kinetic energy and fall into the nucleus.

Dipole approximation

Collection of charged particles



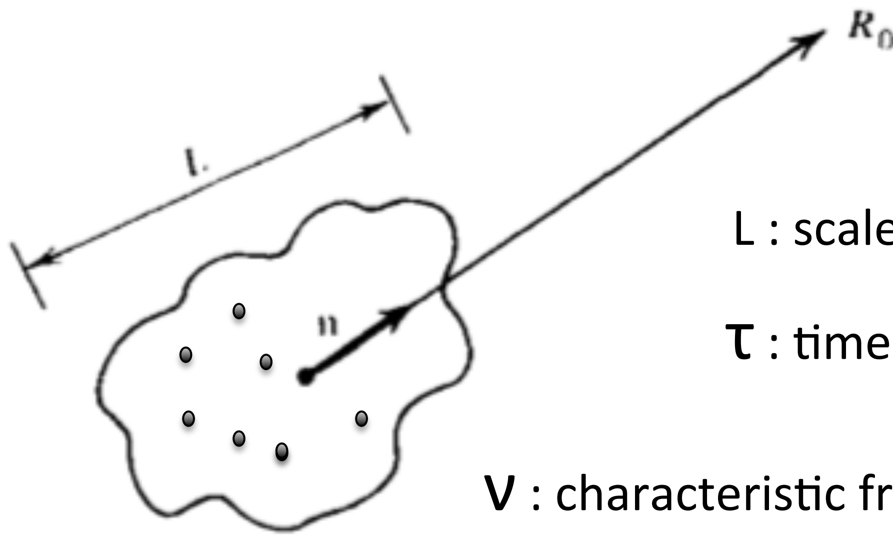
When there are many particles with position $r_{i,}$ velocities u_i, charges q_i

Radiation field at large distance ~ summation of E_{rad} for each particle

But E_{rad} for each particle is true for different retarded times

How to derive radiation field?

Dipole approximation



L : scale of the system

τ : time scale for changes

ν : characteristic frequency of E_{rad}=1/τ

Differences in retarded time across source is negligible ⟺ $\tau \gg L/c,$

$\frac{c}{\nu} \gg L,$

Differences in retarded time can be ignored if size of the system is small compared to wavelength ⟺ $\lambda \gg L,$

Dipole approximation

$$\mathbf{E}_{\rm rad} = \sum_i \frac{q_i}{c^2} \frac{\mathbf{n} \times (\mathbf{n} \times \dot{\mathbf{u}}_i)}{R_i}$$

$$\mathbf{d} = \sum_i q_i \mathbf{r}_i$$

$$\mathbf{E}_{\rm rad} = \frac{\mathbf{n} \times (\mathbf{n} \times \ddot{\mathbf{d}})}{c^2 R_0}$$

$$\frac{dP}{d\Omega} = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3} \sin^2 \Theta,$$

$$P = \frac{2\ddot{\mathbf{d}}^2}{3c^3}$$

Dipole approximation :
Larmor's formula extended for a collection of non-relativistic particles

Dipole approximation

Spectrum of radiation $\frac{dW}{d\omega}$

Assuming **d** lies in single direction

$$E(t) = \ddot{d}(t)\frac{\sin\Theta}{c^2 R_0}$$

$$\hat{E}(\omega) = -\frac{1}{c^2 R_0}\omega^2 \hat{d}(\omega)\sin\Theta$$

Electric field in frequency domain

Fourier transform of d(t)

$$d(t) = \int_{-\infty}^{\infty} e^{-i\omega t}\hat{d}(\omega)\,d\omega.$$

$$\ddot{d}(t) = -\int_{-\infty}^{\infty} \omega^2 \hat{d}(\omega) e^{-i\omega t}\,d\omega.$$

Dipole approximation

Spectrum of radiation $\frac{dW}{d\omega}$

Assuming **d** lies in single direction

Electric field in time domain → $$E(t) = \ddot{d}(t)\frac{\sin\Theta}{c^2 R_0}$$

Electric field in frequency domain → $$\hat{E}(\omega) = -\frac{1}{c^2 R_0}\omega^2 \hat{d}(\omega)\sin\Theta$$

From Lecture 4, energy per unit area $$\frac{dW}{dA} = c\int_0^\infty |\hat{E}(\omega)|^2 d\omega,$$

Energy per unit solid angle per frequency range, ($dA = R_0^2\, d\Omega$)

$$\frac{dW}{d\omega\, d\Omega} = \frac{1}{c^3}\omega^4 |\hat{d}(\omega)|^2 \sin^2\Theta.$$

Spectrum of radiation → $$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2$$

Dipole approximation
Spectrum of radiation

Spectrum of radiation → $$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2$$

↓

Rayleigh scattering formula
proportional to $1/\lambda^4$
(Reason for blue color of the sky)

Recap
Lecture 5 till now

Maxwell's equation with source terms

Introduce scalar potential $\Phi(r,t)$ and vector potential $\mathbf{A(r,t)}$

Expression of $\Phi(r,t)$ and $A(r,t)$ in terms of κ and R at retarded time

Expression of Electric field E having two components

Velocity field and Radiation field and when they are important

Total power radiated by non relativistic point charge when it accelerates

Dipole approximation

Recap

Maxwell's equation with source terms

Introduce scalar potential Φ(r,t) and vector potential **A(r,t)**

Expression of Φ(r,t) and A(r,t) in terms of κ and R at retarded time

$$\phi = \left[\frac{q}{\kappa R} \right]$$

$$\mathbf{A} = \left[\frac{q\mathbf{u}}{c\kappa R} \right]$$

Expression of Electric field E having two components

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2} \right] + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R} \times \{(\mathbf{n}-\boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\} \right]$$

Velocity field and Radiation field and when they are important

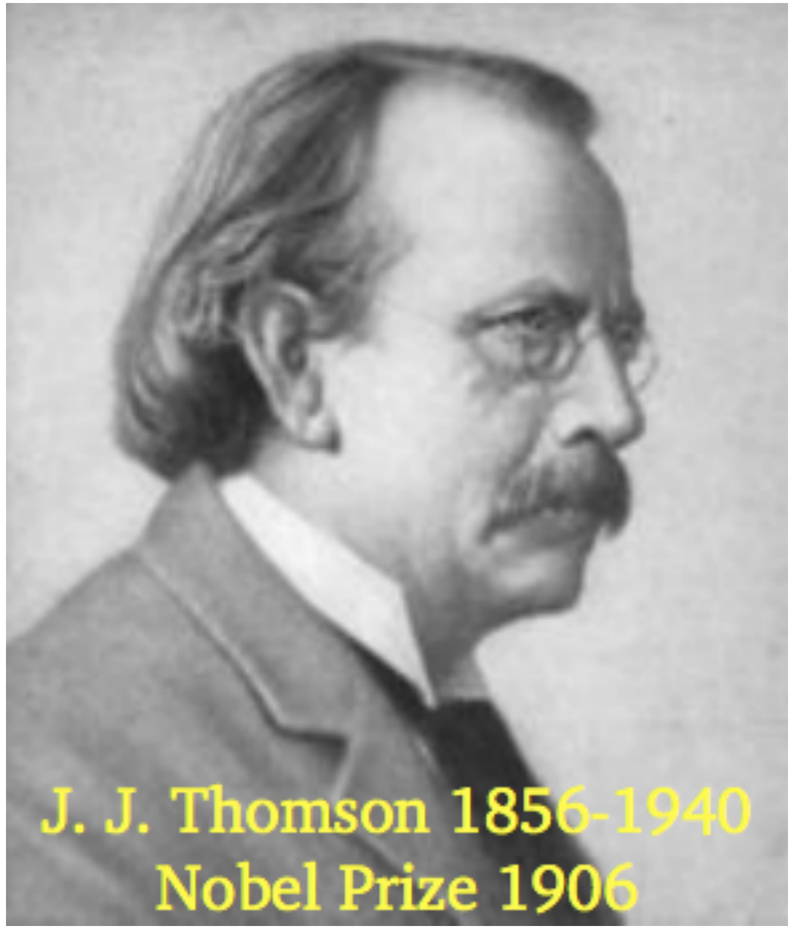
$$\frac{E_{\mathrm{rad}}}{E_{\mathrm{vel}}} \sim \frac{Ruv}{c^2} = \frac{u}{c}\frac{R}{\lambda}.$$

Total power radiated by non relativistic point charge when it accelerates

$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

Dipole approximation

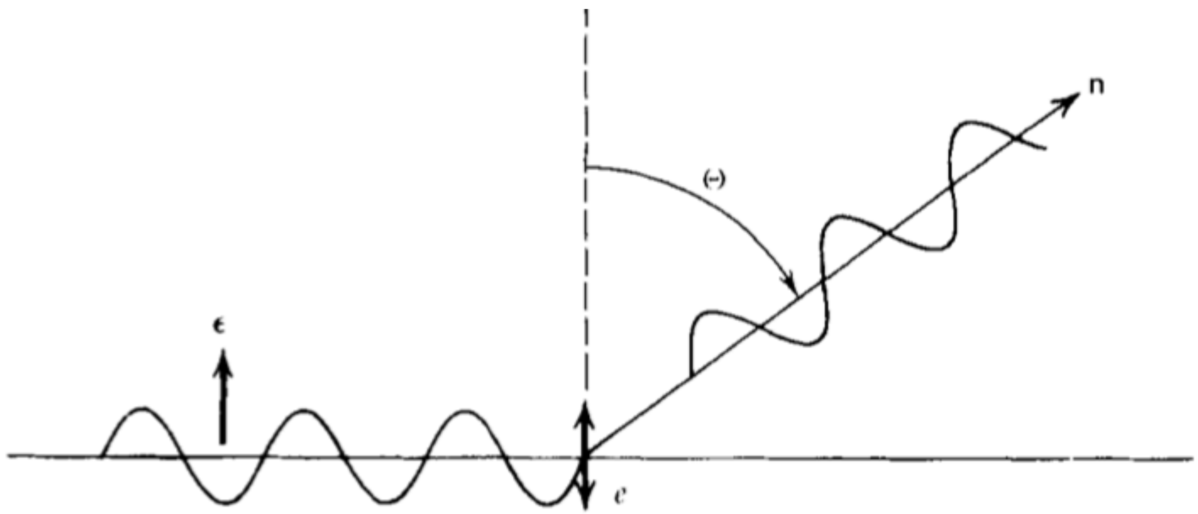
$$P = \frac{2\ddot{\mathbf{d}}^2}{3c^3}$$



"In recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases."

Thomson scattering

Let us consider application of the dipole formula in a process in which a free charge radiates in response to an incident electromagnetic wave



Process by which an electromagnetic wave is scattered by a free electron.

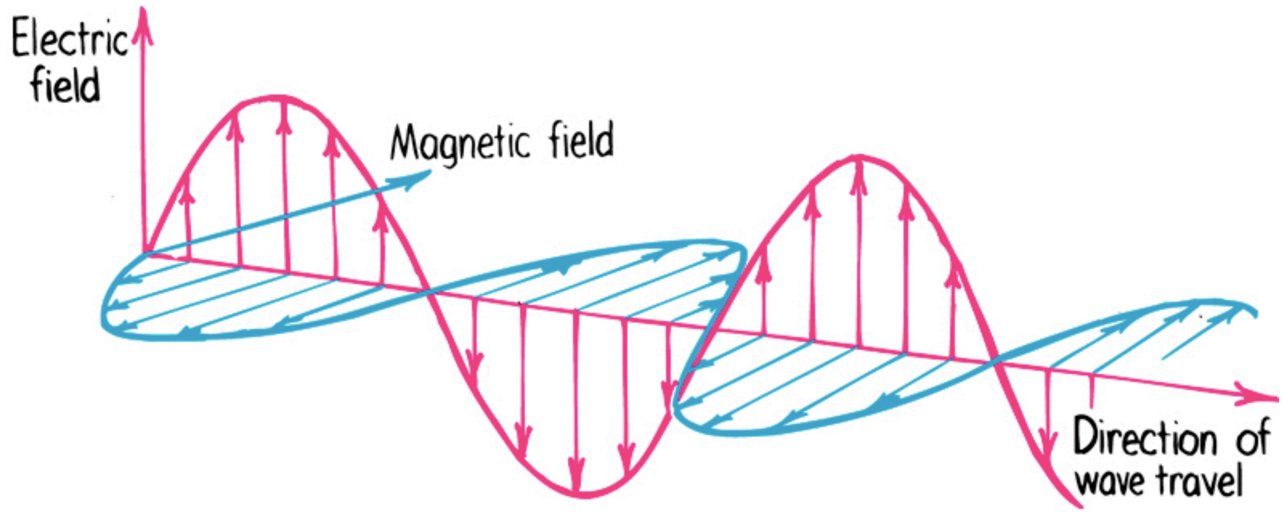
Applicable for $h\nu \ll m_e c^2$

Thomson scattering

Process by which an electromagnetic wave is scattered by a free electron.

Applicable for $h\nu << m_e c^2$

Consider a linearly polarized electromagnetic wave incident on a free electron



Force on the electron

$$\mathbf{F} = q\left(\mathbf{E} + \underbrace{\frac{\mathbf{v}}{c} \times \mathbf{B}}\right)$$

Negligible as v<<c

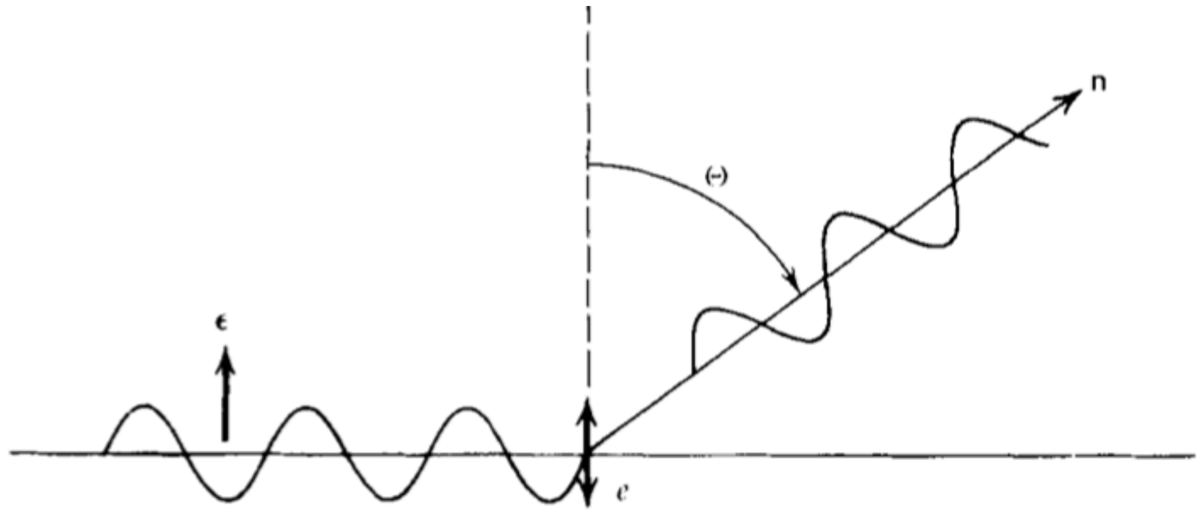
Thomson scattering

Force of a linearly polarized wave acting on a electron

$$\mathbf{F} = e\boldsymbol{\epsilon} E_0 \sin \omega_0 t$$

$$\downarrow$$

$$m\ddot{\mathbf{r}} = e\boldsymbol{\epsilon} E_0 \sin \omega_0 t$$

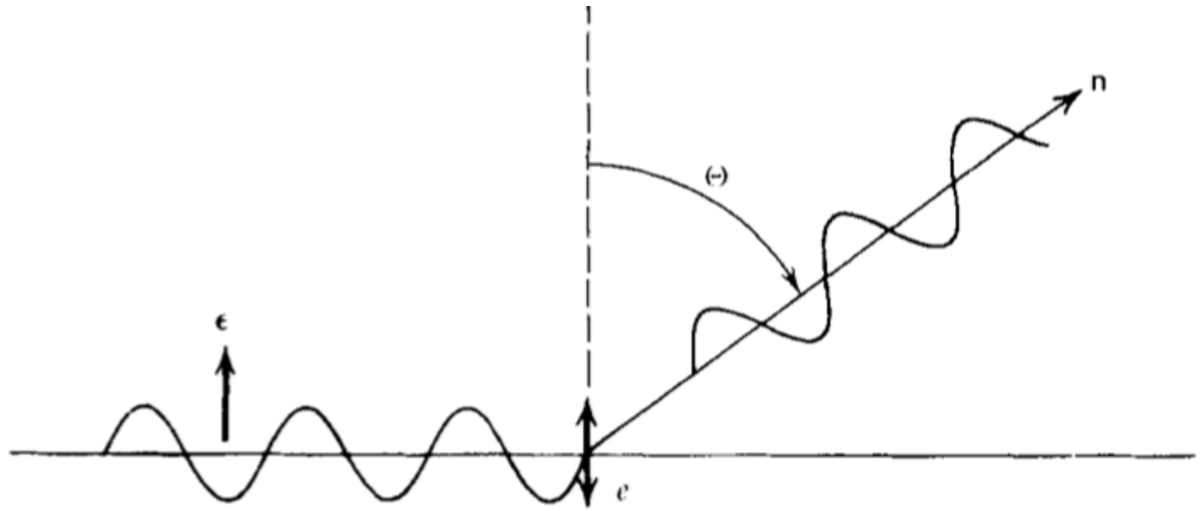


Thomson scattering

Force of a linearly polarized wave acting on a electron

$$\mathbf{F} = e\boldsymbol{\epsilon} E_0 \sin \omega_0 t.$$

$$m\ddot{\mathbf{r}} = e\boldsymbol{\epsilon} E_0 \sin \omega_0 t.$$



Dipole moment is defined by $\mathbf{d} = e\mathbf{r},$

$$\ddot{\mathbf{d}} = \frac{e^2 E_0}{m} \boldsymbol{\epsilon} \sin \omega_0 t,$$

$$\mathbf{d} = -\left(\frac{e^2 E_0}{m\omega_0^2} \right) \boldsymbol{\epsilon} \sin \omega_0 t,$$

Oscillating dipole of amplitude $\mathbf{d}_0 = \frac{e^2 E_0}{m\omega_0^2} \boldsymbol{\epsilon}.$

Thomson scattering

Dipole approximation

Radiation from a non relativistic system of particles (λ>>L)

Second derivative of dipole moment

$$\mathbf{d} = \sum_i q_i \mathbf{r}_i.$$

Power radiated per unit solid angle

$$\frac{dP}{d\Omega} = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3}\sin^2\Theta,$$

Total Power radiated

$$P = \frac{2\ddot{\mathbf{d}}^2}{3c^3}$$

Thomson scattering

Electron subject to electromagnetic wave ($h\nu \ll mc^2$)

Second derivative of dipole moment

$$\ddot{\mathbf{d}} = \frac{e^2 E_0}{m}\boldsymbol{\epsilon}\sin\omega_0 t,$$

Power radiated per unit solid angle

$$\frac{dP}{d\Omega} = \frac{e^4 E_0^2}{8\pi m^2 c^3}\sin^2\Theta$$

(time average of $\sin^2 \omega_o t$ gives a factor ½)

Total Power radiated

$$P = \frac{e^4 E_0^2}{3m^2 c^3}$$

Thomson scattering

Remember time averaged pointing flux is defined as $\langle S \rangle = \frac{c}{8\pi} E_0^2$

Define differential cross section dσ for scattering in to dΩ

$$\frac{dP}{d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega} = \frac{cE_0^2}{8\pi} \frac{d\sigma}{d\Omega}$$

$$\frac{dP}{d\Omega} = \frac{e^4 E_0^2}{8\pi m^2 c^3} \sin^2\Theta$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{polarized}} = \frac{e^4}{m^2 c^4} \sin^2\Theta = r_0^2 \sin^2\Theta$$

$$r_0 \equiv \frac{e^2}{mc^2}$$

→ **Classical electron radius**

Thomson scattering

Classical electron radius

$$r_0 \equiv \frac{e^2}{mc^2}$$

Measure of the size of the point charge
(assuming the rest energy is purely electromagnetic)
~2.8×10^{-13} cm

Total cross-section is obtained after integrating over solid angle,

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi r_0^2 \int_{-1}^{1} (1-\mu^2) d\mu = \frac{8\pi}{3} r_0^2 = \sigma_T \sim 0.66 \times 10^{-24}\ \text{cm}^2$$

↓

Thomson Scattering cross section

- ✧ Frequency independent, so scattering is equally effective at all frequencies.
- ✧ Valid for lower frequencies. Not valid for high frequencies $h\nu > mc^2$

Thomson scattering

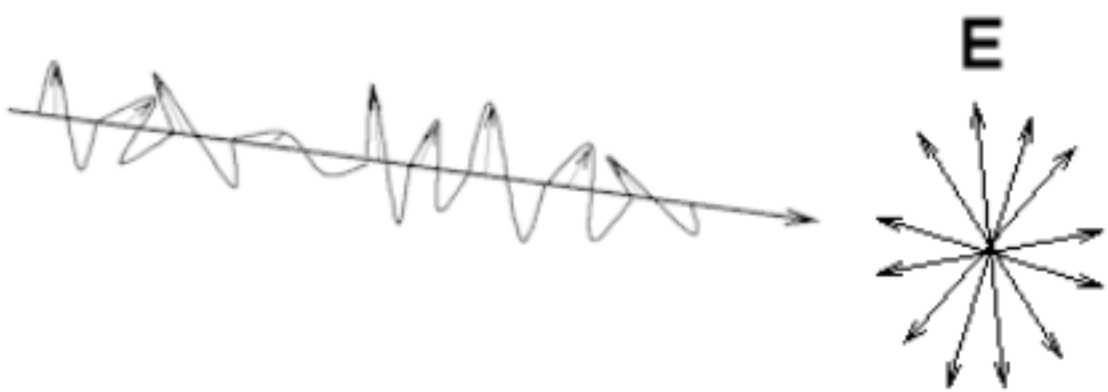
Calculated Thomson scattering cross-section for an electron and Polarized EM wave

↓

Incoming wave linearly polarized along **ε**

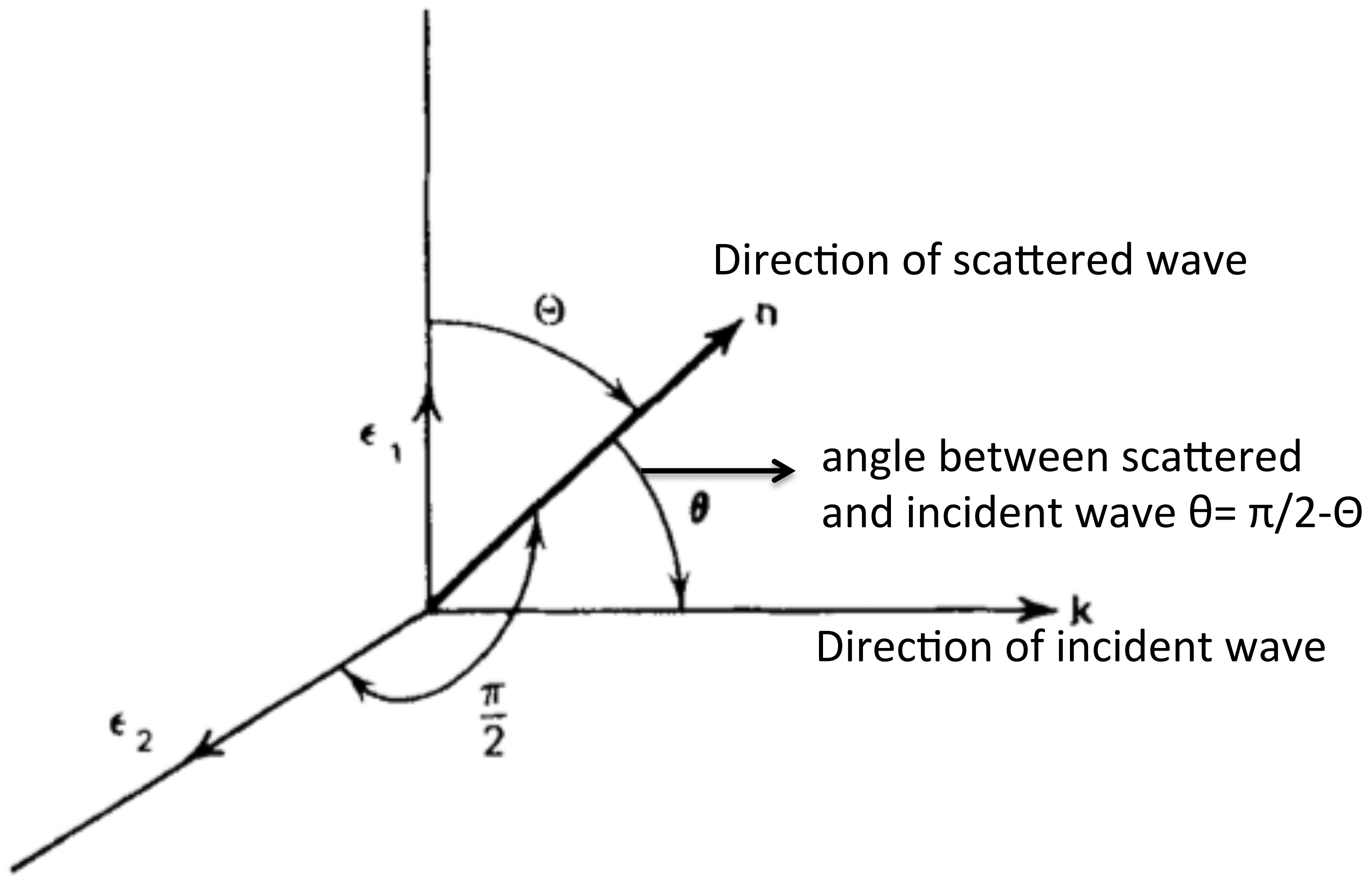
Outgoing EM wave is also linearly polarized in the plane defined by **ε** and **n**

Unpolarized EM (better randomly polarized) wave can be regarded as superposition of two linearly polarized beams with perpendicular axes



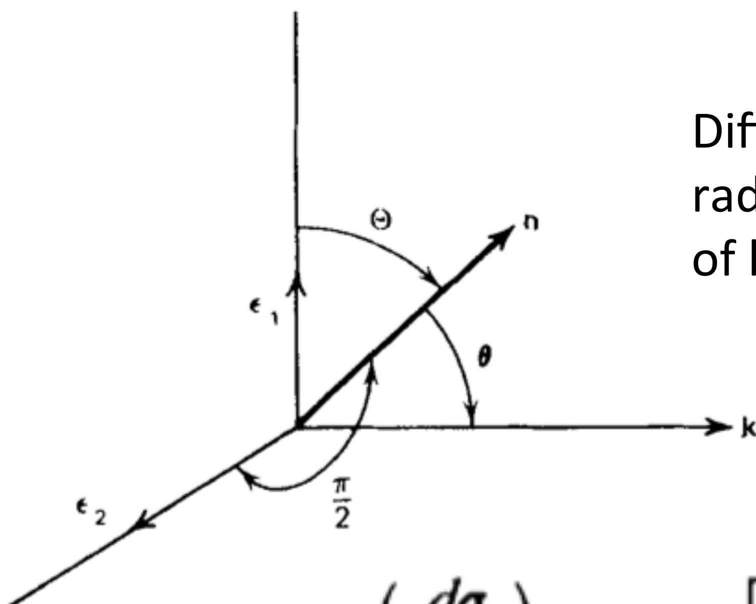
Thomson scattering

Unpolarized EM wave can be regarded as superposition of two linearly polarized beams with perpendicular axes ε_1 and ε_2



Thomson scattering

Unpolarized EM wave can be regarded as superposition of two linearly polarized beams with perpendicular axes



Differential cross section for unpolarized radiation is the average of the cross sections of linear-polarized radiation through Θ and π/2

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left[\left(\frac{d\sigma(\Theta)}{d\Omega}\right)_{\text{pol}} + \left(\frac{d\sigma(\pi/2)}{d\Omega}\right)_{\text{pol}}\right]$$
$$= \frac{1}{2} r_0^2 (1 + \sin^2\Theta)$$
$$= \frac{1}{2} r_0^2 (1 + \cos^2\theta)$$

Thomson scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left[\left(\frac{d\sigma(\Theta)}{d\Omega}\right)_{\text{pol}} + \left(\frac{d\sigma(\pi/2)}{d\Omega}\right)_{\text{pol}}\right]$$

↓

intensities in two perpendicular directions in the plane normal to **n** arising from two perpendicular components of the incident wave

✓ Forward-backward symmetry : The scattering cross section is symmetric under the reflection θ → -θ

✓ Total cross section: The total scattering cross-section of unpolarized incident radiation is same as that for polarized incident radiation.
Since electron at rest has no direction intrinsically defined.

$$\sigma_{\text{unpol}} = \sigma_{\text{pol}} = (8\pi/3)r_0^2$$

Thomson scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left[\left(\frac{d\sigma(\Theta)}{d\Omega}\right)_{\text{pol}} + \left(\frac{d\sigma(\pi/2)}{d\Omega}\right)_{\text{pol}}\right]$$

intensities in two perpendicular directions in the plane normal to **n** arising from two perpendicular components of the incident wave

Polarization intensities in the plane and perpendicular are $\cos^2\theta$:1

For partially polarized light degree of polarization of the scattered wave

$$\Pi = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \longrightarrow \Pi = \frac{1 - \cos^2\theta}{1 + \cos^2\theta}$$

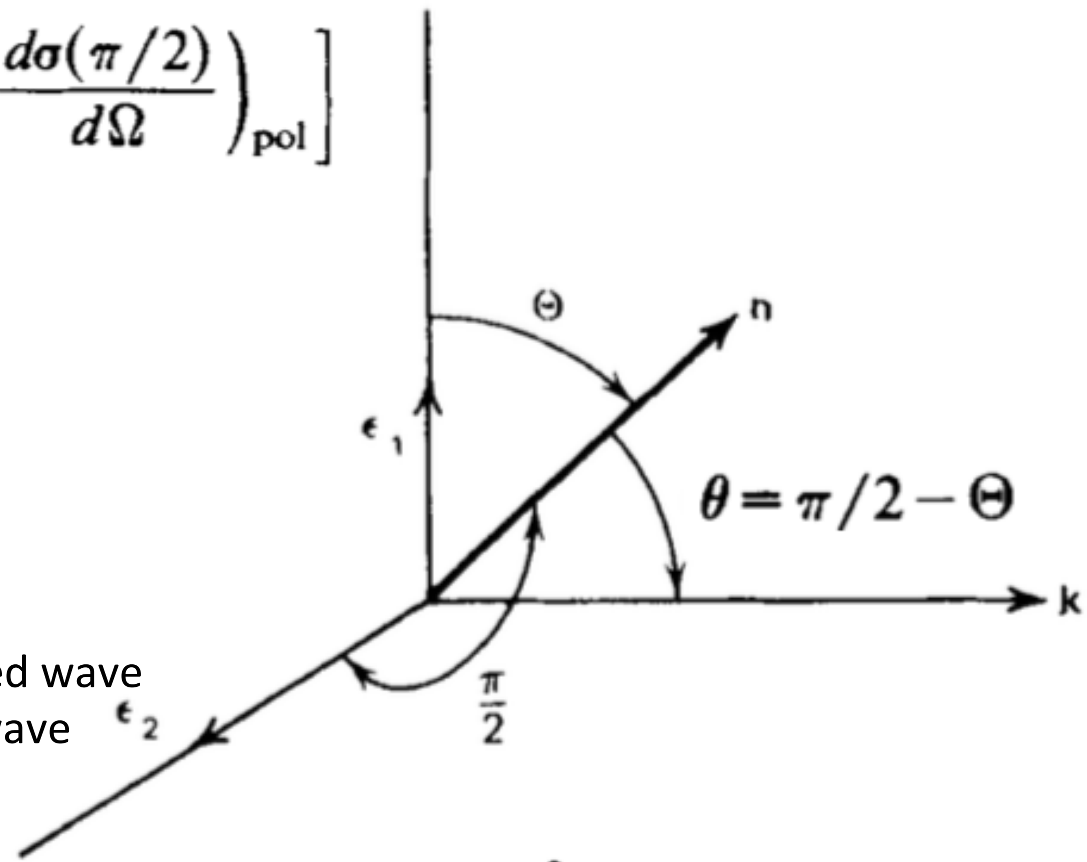
Thomson scattering

k direction of incoming e.m. wave

n direction of scattered wave

Total scattering cross-section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left[\left(\frac{d\sigma(\Theta)}{d\Omega}\right)_{\text{pol}} + \left(\frac{d\sigma(\pi/2)}{d\Omega}\right)_{\text{pol}}\right]$$
$$= \frac{1}{2} r_0^2 (1 + \sin^2\Theta)$$
$$= \frac{1}{2} r_0^2 (1 + \cos^2\theta),$$



Reflection θ → -θ

Scattering cross-section is same

✓ Scattering cross section for unpolarized wave

= Scattering cross-section for polarized wave

$$\sigma_{\text{unpol}} = \sigma_{\text{pol}} = (8\pi/3) r_0^2.$$

✓ Degree of polarization of scattered wave

$$\Pi = \frac{1 - \cos^2\theta}{1 + \cos^2\theta}$$

Thomson scattering

$$\Pi = \frac{1-\cos^2\theta}{1+\cos^2\theta}$$

Since $\Pi > 0$ electron scattering of a completely unpolarized incident wave produces scattered wave with some degree of polarization intensities. The degree depend on θ

Example

Absorption coefficient

$$\alpha = n\sigma_T$$

Number density of particles (n)

Thomson scattering cross section (σ_T) ~6.62x 10^{-25} cm^2

So Thompson scattering is significant only when number density is high

The cosmic microwave background is linearly polarized as a result of Thomson scattering (as measured by Degree angular scale interferometer(DASI) and more recent experiments).

The solar K-corona is the result of the Thomson scattering of solar radiation from solar coronal electrons.

Example

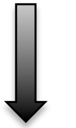
Optical depth

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\, ds'.$$

$$\tau = n \sigma_T R$$

Now considering a nebula having n =10,000 and
At a distance of R=10^{19} cm

Then we can get estimate of $\tau = 10{,}000 \times 10^{19} \times 6.25 \mathrm{X} 10^{-25} = 0.07$



Optically thin

Example

The cross-section for Thomson scattering is tiny and therefore Thomson scattering is most important when the density of free electrons is high, as in the early Universe or in the dense interiors of stars.

Radiation reaction

Force acting on a particle by virtue of the radiation it produces

→ Radiation reaction force

Let T be the time interval over which kinetic energy of the particle is changed substantially by the emission of radiation

$$T \sim \frac{mv^2}{P_{rad}} \sim \frac{3mc^3}{2e^2}\left(\frac{v}{a}\right)^2$$

$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

$1/\tau$

$$\tau \equiv \frac{2e^2}{3mc^3} \sim 10^{-23} s$$

Radiation reaction

$$\tau \equiv \frac{2e^2}{3mc^3} \sim 10^{-23} s$$

$\tau \sim r_o/c$ → time for radiation to cross a distance comparable to classical electron radius

As long as we are considering processes that occur on a time scale much longer than τ, we can treat radiation reaction as a perturbation.

Radiation reaction

Energy radiated compensated by work done against radiation reaction force F_{rad}.

$$-\mathbf{F}_{rad}\cdot\mathbf{u} = \frac{2e^2\dot{u}^2}{3c^3}$$

$$-\int_{t_1}^{t_2}\left(\mathbf{F}_{rad} - \frac{2e^2\ddot{\mathbf{u}}}{3c^3}\right)\cdot\mathbf{u}\,dt = 0.$$

$$\mathbf{F}_{rad} = \frac{2e^2\ddot{\mathbf{u}}}{3c^3} = m\tau\ddot{\mathbf{u}},$$

⇓

Abraham–Lorentz force

Radiation reaction

$$\mathbf{F}_{\rm rad} = \frac{2e^2\ddot{\mathbf{u}}}{3c^3} = m\tau\ddot{\mathbf{u}},$$

↓

Recoil force acting on the charge

Proportional to the acceleration

Valid for non relativistic cases.
Dirac proposed relativistic version

End of Lecture 5

Reference: Rybicki Lightman Chapter 3

Link to lecture on radiation from accelerated charges by Prof. G Srinibasan
: https://www.youtube.com/watch?v=GIYMHkkFGhc&list=PL04QVxpjcnjidFFZjZ3m0aJ6pnKfBN5sA

Next lecture : 26th August

Topic of next Lecture:
Relativity in Electrodynamics
(Chapter 4 of Rybicki & Lightman)
Preparation: special relativity