Electrodynamics and Radiative Processes I

Lecture 2 – Thermal and black body radiation Line Radiative Transfer

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Lecture -1 recap
Specific Intensity or Brightness I_ν

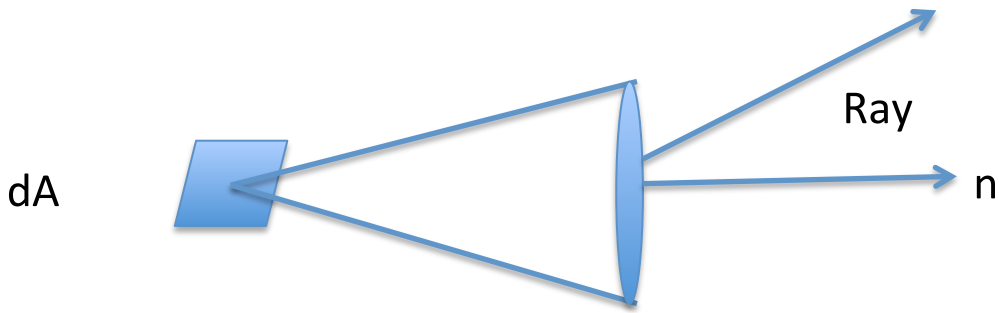


Figure: Geometry for normal incidence

Energy crossing dA in time dt in frequency range $d\nu$ and into a solid angle $d\Omega$

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

Specific Intensity or Brightness

$[I_\nu]$= unit?

Brightness does/does not decrease with distance?

Lecture-1 recap

Measured quantities :

✧ The energy in the radiation as a function of

a) Position in the sky (for extended sources)

b) Frequency

✧ The radiation's polarisation.

From these measurements we aim to determine

✧ Physical parameters of source (e.g. temperature, composition, size)

✧ The radiation mechanism

✧ The physical state of the matter

Need to understand the difference between:

Specific intensity, Specific energy density, Flux density, Luminosity.

Lecture-1 recap
Intensity, Flux density, Luminosity

Specific Intensity
$$I(\Omega,\nu) = \frac{\text{erg}}{\text{s cm}^2\,\text{Hz ster}}$$

Mean intensity
$$J(\nu) = \frac{1}{4\pi}\oint_{4\pi} I(\Omega,\nu)\,d\Omega = \frac{\text{erg}}{\text{s cm}^2\,\text{Hz ster}}$$

Flux density
$$F(\nu) = \oint_{4\pi} I(\Omega,\nu)\,d\Omega = \frac{\text{erg}}{\text{s cm}^2\,\text{Hz}}$$

Luminosity
$$L = \oint_{4\pi} I(\Omega,\nu)\,\Omega\,dA\,d\nu\,d\Omega = \frac{\text{erg}}{\text{s}}$$

Spherical black body
$$\text{L} = 4\pi\text{R}^2\sigma\text{T}^4$$

Also called Bolometric luminosity

Lecture-1 recap
Radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Absorption

$$I_\nu(s) = I_\nu(s_0)\exp\left[-\int_{s_0}^{s} \alpha_\nu(s')\,ds'\right]$$

Decrease in brightness along the ray by exponential of absorption coefficient integrated across line of sight

Emission

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')\,ds'$$

Increase in brightness
= emission coefficient integrated across line of sight

Lecture-1
Formal Solution of Radiative transfer equation

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\,ds'$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

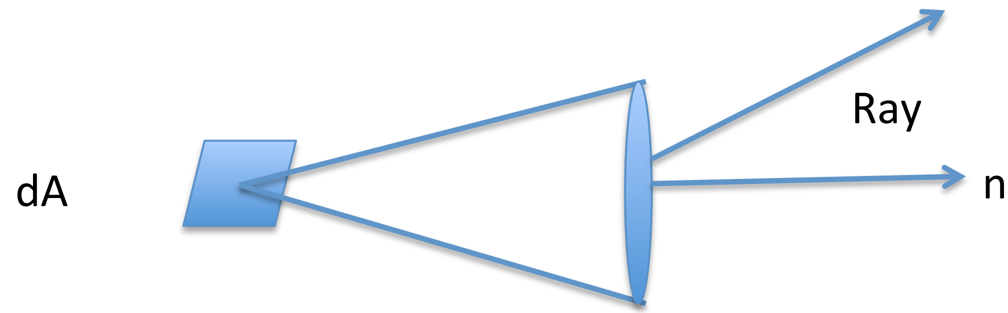
$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

1st term
Initial intensity attenuated by medium

2nd term
Medium adds to the radiation
And absorbs part of the added radiation

Lecture -1

Questions raised in the class



$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

Figure: Geometry for normal incidence

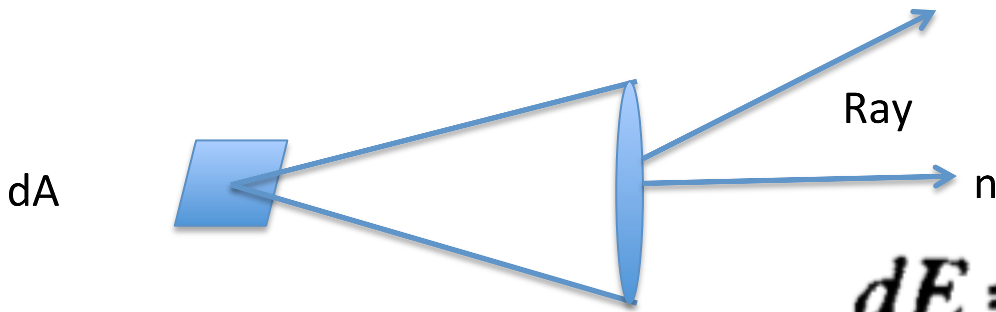
Dependence of specific intensity on distance

Dependence of specific intensity on solid angle

Frequency dependence of specific intensity

Why don't we include stimulated emission in radiative transfer equation

Lecture -1
Questions raised in the class



$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

Figure: Geometry for normal incidence

Dependence of specific intensity on distance → Distance independent

Dependence of specific intensity on solid angle → Source may/may not be isotropic

Frequency dependence of specific intensity → Depends on frequency

Why don't we include stimulated emission in radiative transfer equation

→ Revisit after Lecture 3 when we write Einstein's coefficients in terms of j_ν, α_ν

Useful radiative transfer codes

Optical/UV of the interstellar medium:

- CLOUDY http://www.nublado.org/
- Meudon PDR code http://pdr.obspm.fr/PDRcode.html
- MOCASSIN http://www.usm.uni-muenchen.de/people/ercolano/

Dust emission, absorption, scattering:

- DUSTY http://www.pa.uky.edu/~moshe/dusty/
- MC3D http://www.astrophysik.uni-kiel.de/~star/Classes/MC3D.html
- RADMC-3D http://www.ita.uni-heidelberg.de/~dullemond/software/radmc-3d/

Credit for the list : http://www.ita.uni-heidelberg.de/~dullemond/lectures/obsastro_2011/

Useful radiative transfer codes

Infrared and submillimeter lines:

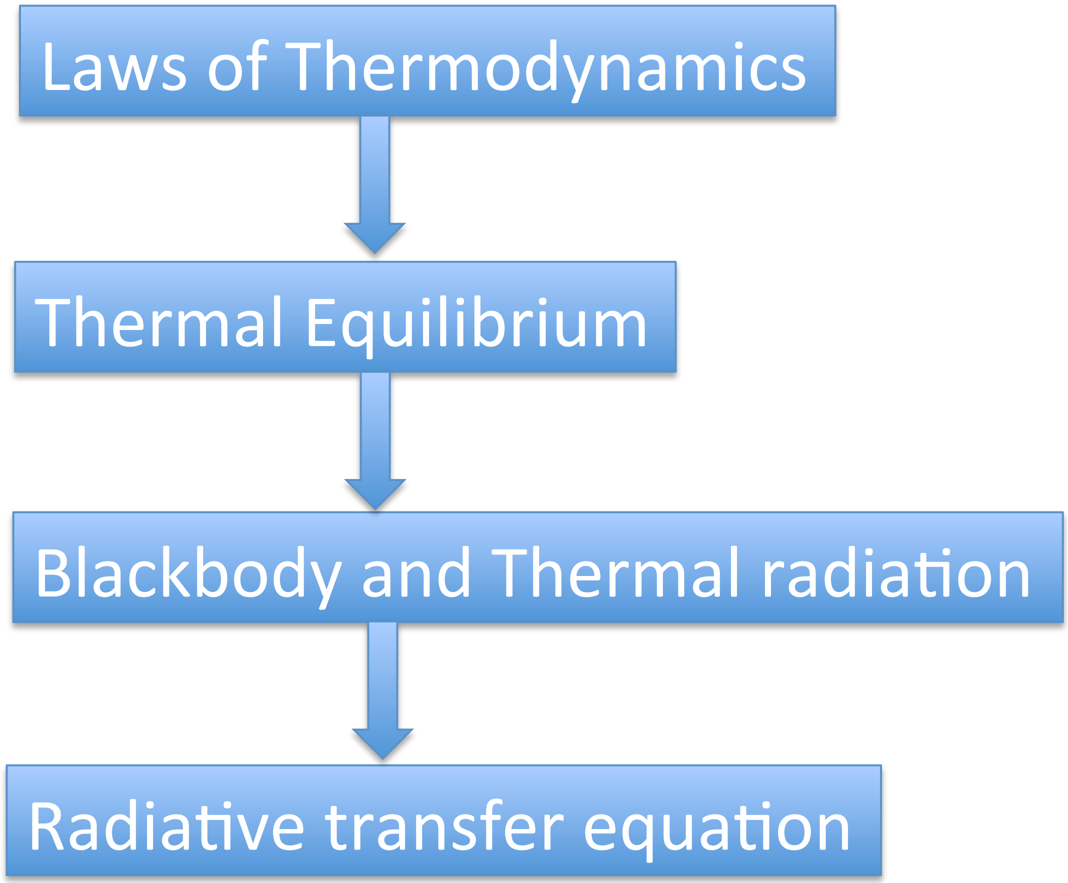
- RADEX http://www.sron.rug.nl/~vdtak/radex/radex.php
- RATRAN http://www.strw.leidenuniv.nl/~michiel/ratran/
- SIMLINE http://hera.ph1.uni-koeln.de/~ossk/Myself/simline.html

Stellar atmosphere codes:

- TLUSTY http://nova.astro.umd.edu/
- PHOENIX http://www.hs.uni-hamburg.de/EN/For/ThA/phoenix/index.html
- More codes on: http://en.wikipedia.org/wiki/Model_photosphere

Credit for the list : http://www.ita.uni-heidelberg.de/~dullemond/lectures/obsastro_2011/

Thermal radiation



Can describe radiation from Stars, Accretion disks, Nebulae, Stellar atmosphere etc

Thermal radiation

Matter in thermal equilibrium emits thermal radiation

Two physical systems are in thermal equilibrium if no heat flows between them, when they are connected by a path permeable to heat

Thermal radiation

For a plasma in thermal equilibrium,
probability distribution function of (non-relativistic) velocities is the Maxwell-Boltzmann distribution:

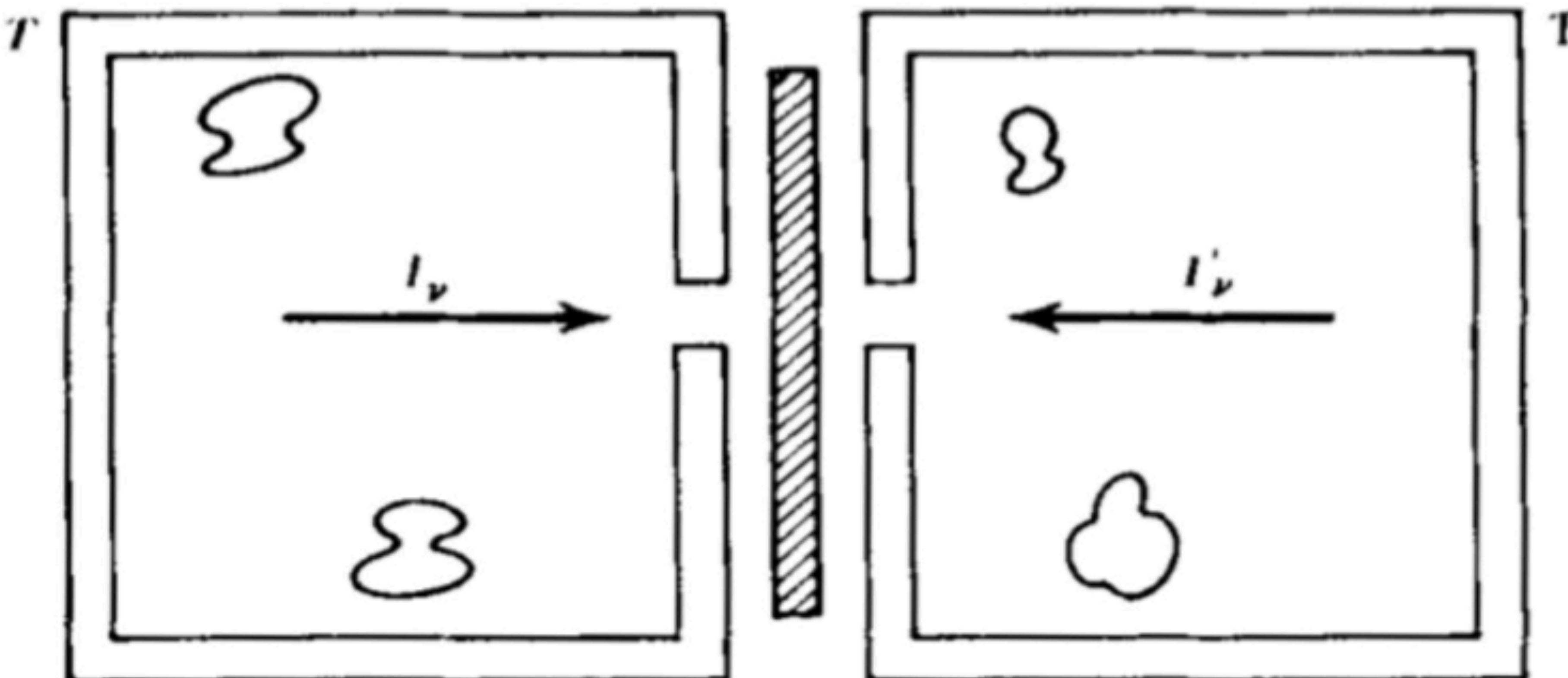
$$F(v)\,dv = 4\pi v^2 \left(\frac{m}{2\pi k T}\right)^{3/2} e^{-mv^2/2kT}\,dv$$

Valid only for non-relativistic particles. There are many astrophysical systems where particles have relativistic speeds and they emit thermal radiation.

Probability distribution function of (both relativistic and non-relativistic) velocities is the Maxwell-Boltzmann distribution:

$$F(p)\,dp = \frac{p^2 e^{-\gamma\Theta}}{\Theta m^3 c^3 K_2(1/\Theta)}\,dp \qquad p = \gamma\beta mc$$

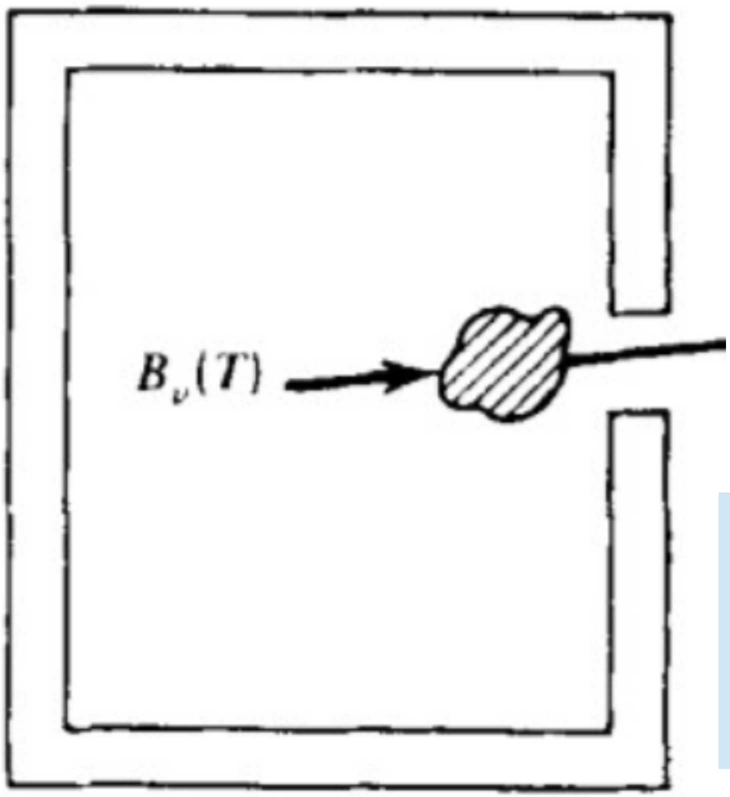
Two containers at a temperature T separated by a filter



$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Planck Function

Kirchoff's Law



$$S_\nu = B_\nu(T) \rightarrow j_\nu = \alpha_\nu B_\nu(T)$$

Thermal Radiation	$S_\nu = B_\nu(T)$
Blackbody Radiation	$I_\nu = B_\nu(T)$

Thermal radiation becomes black body radiation only for optically thick medium

Black body radiation is always thermal

Black body radiation

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Rayleigh-Jeans Law hν << kT

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

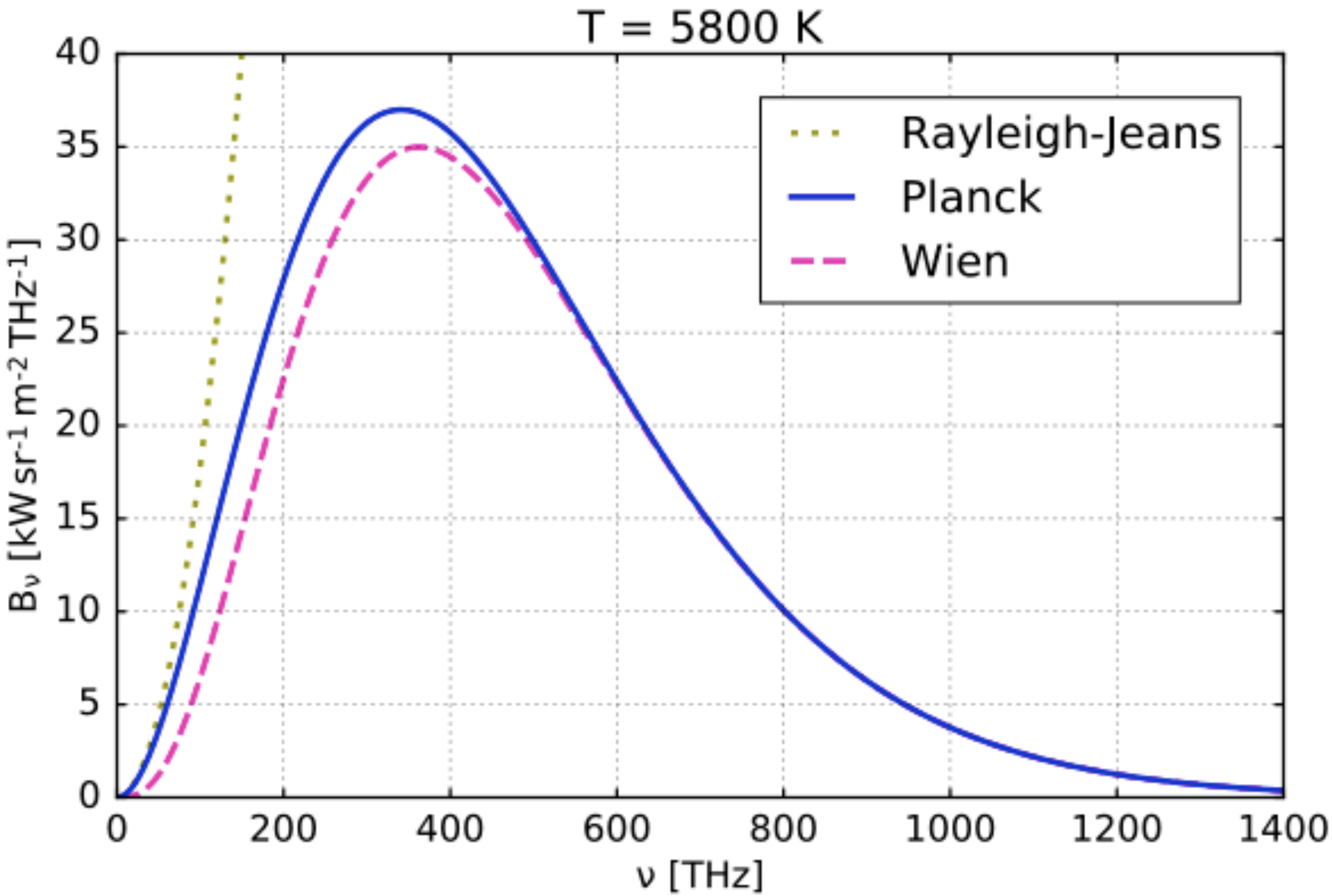
Wien Law hν >> kT

$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2} \exp\left(\frac{-h\nu}{kT}\right)$$

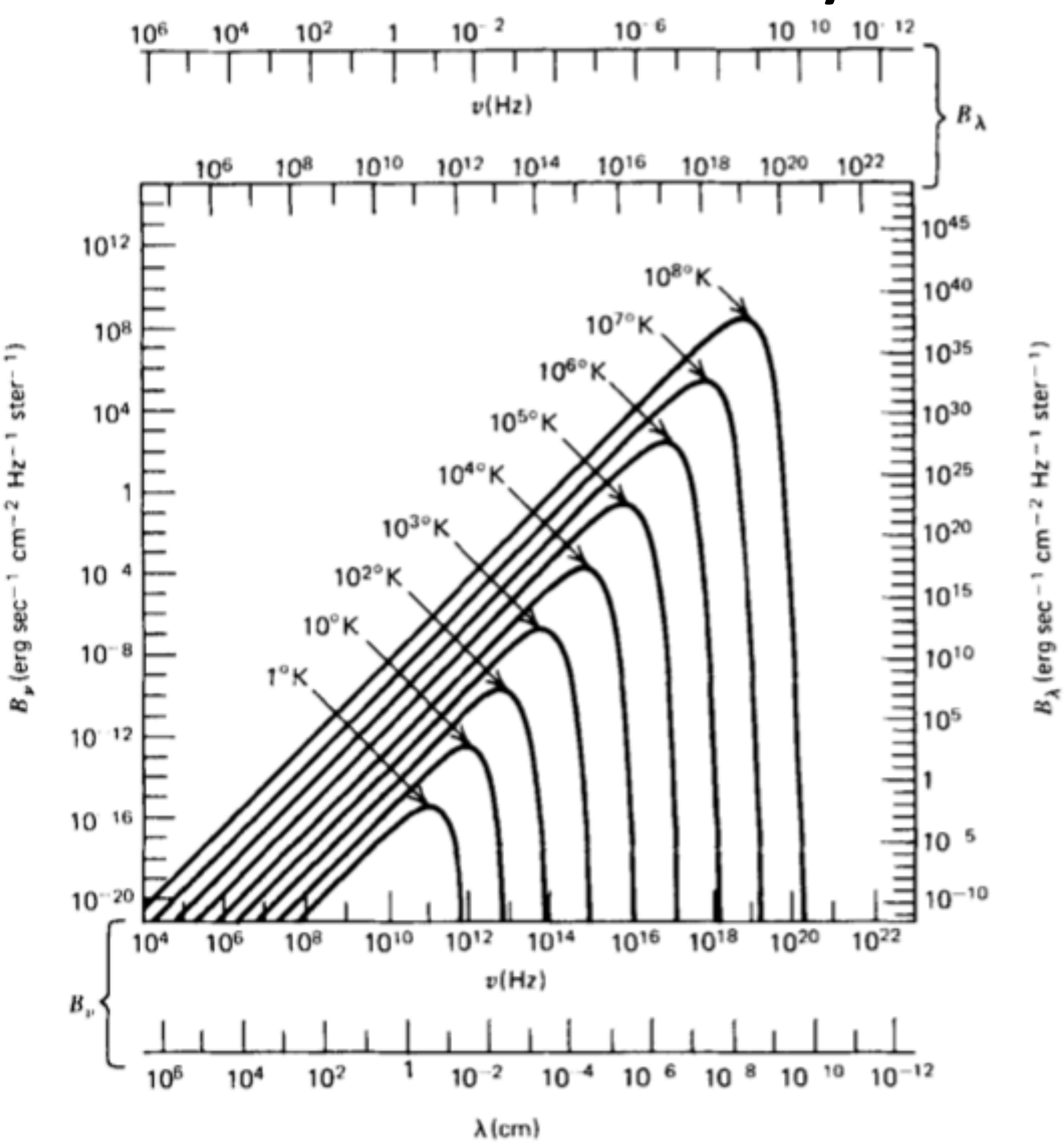
Wien's displacement Law : Max intensity at hν~ kT

$$\lambda_{\max} T = 0.290 \text{ cm deg}$$

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

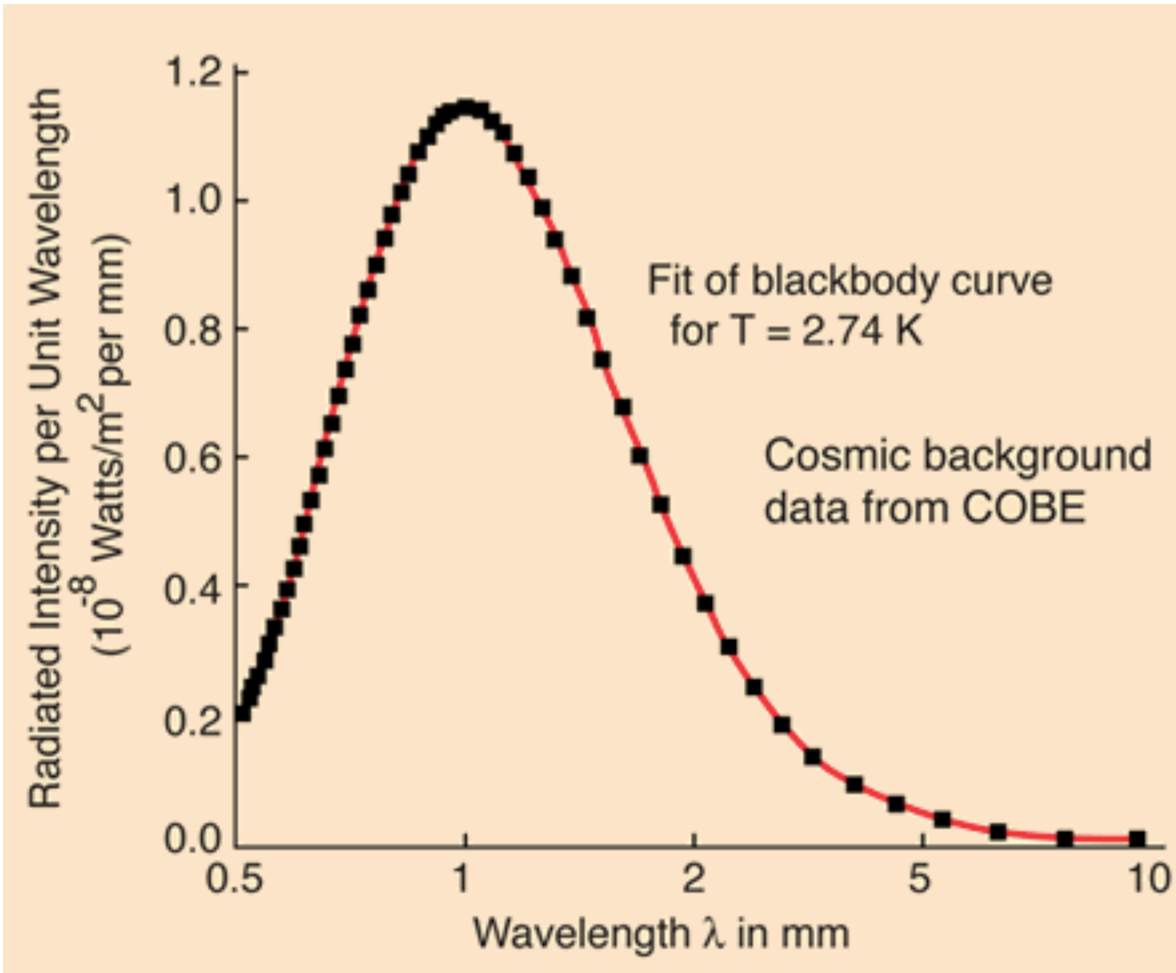


Spectrum of black body radiation

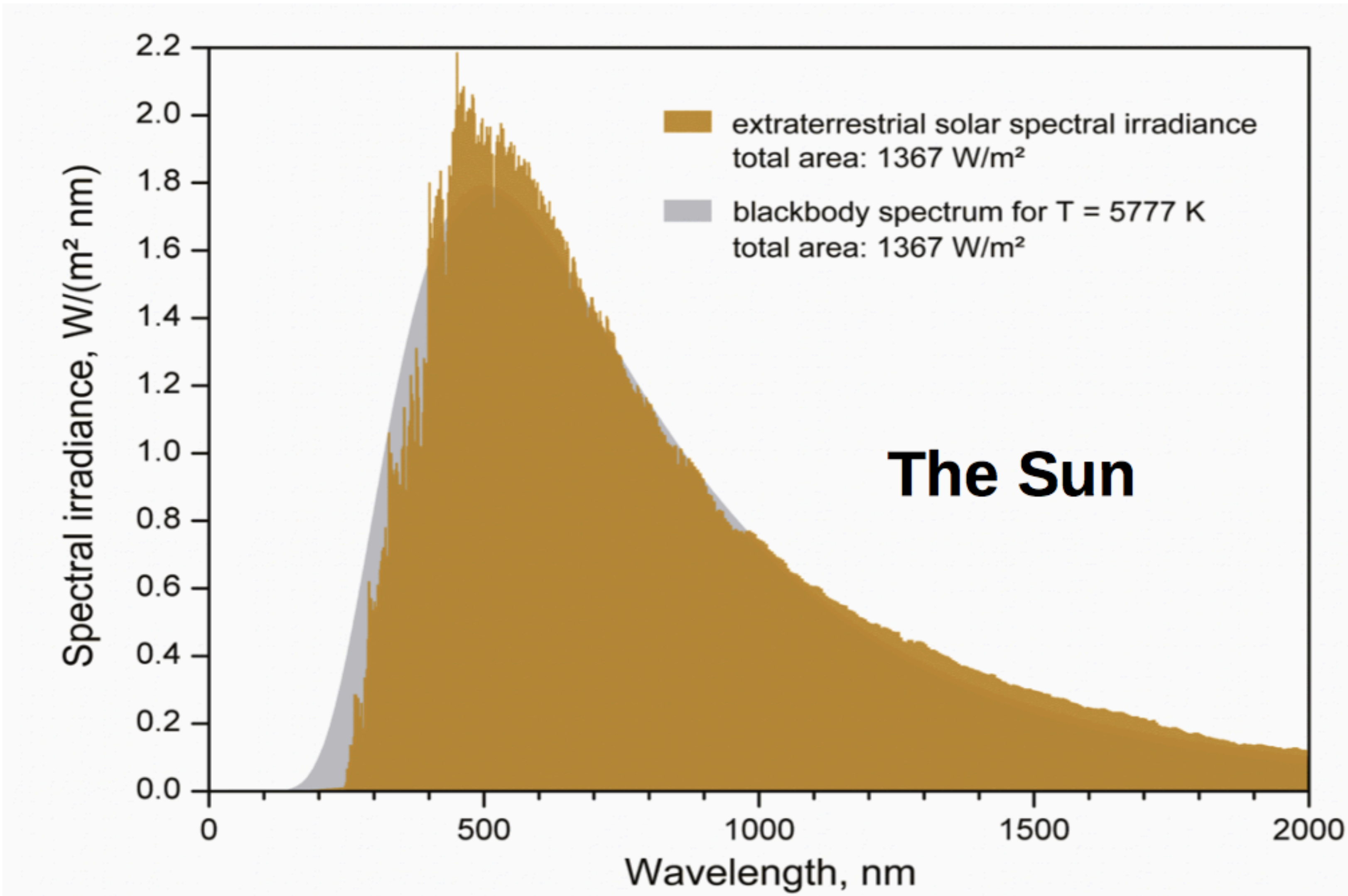


Spectrum of cosmic microwave background radiation

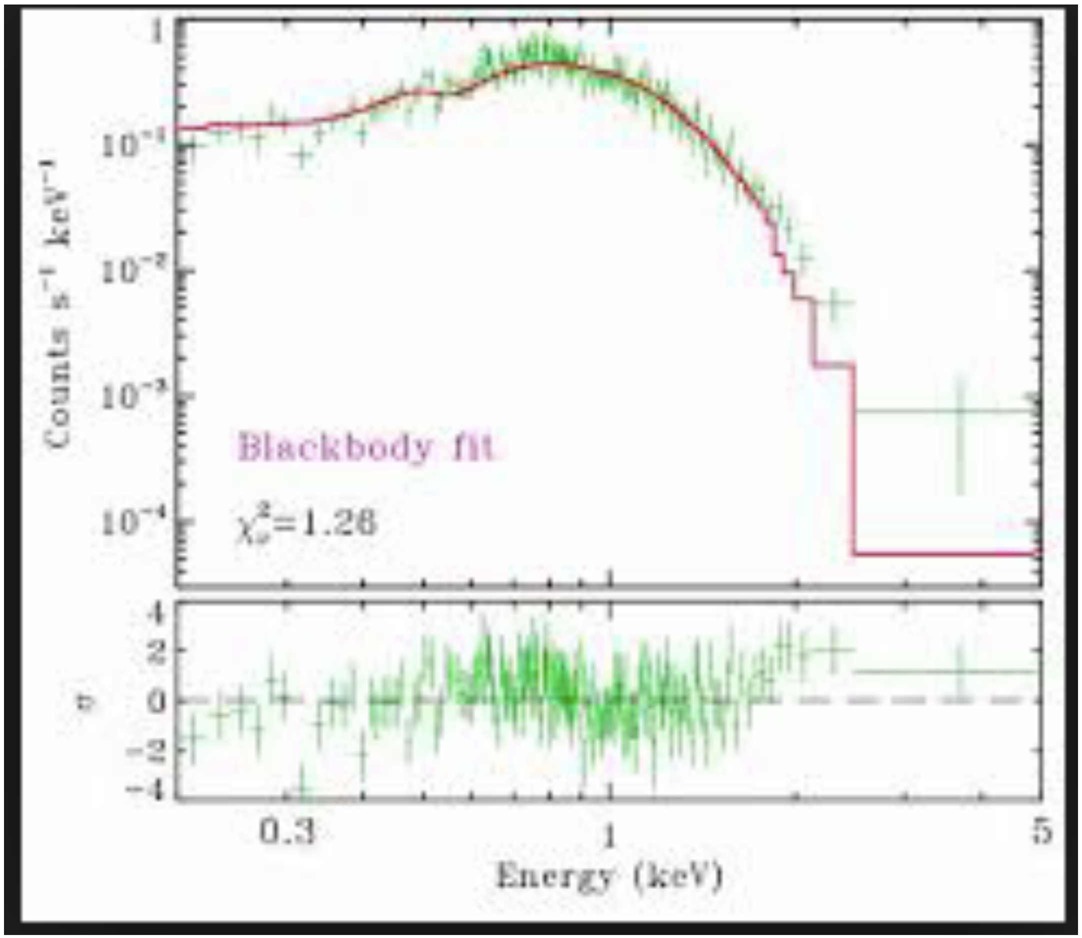
The CMB has the spectrum of a black body



Stellar spectra :black body

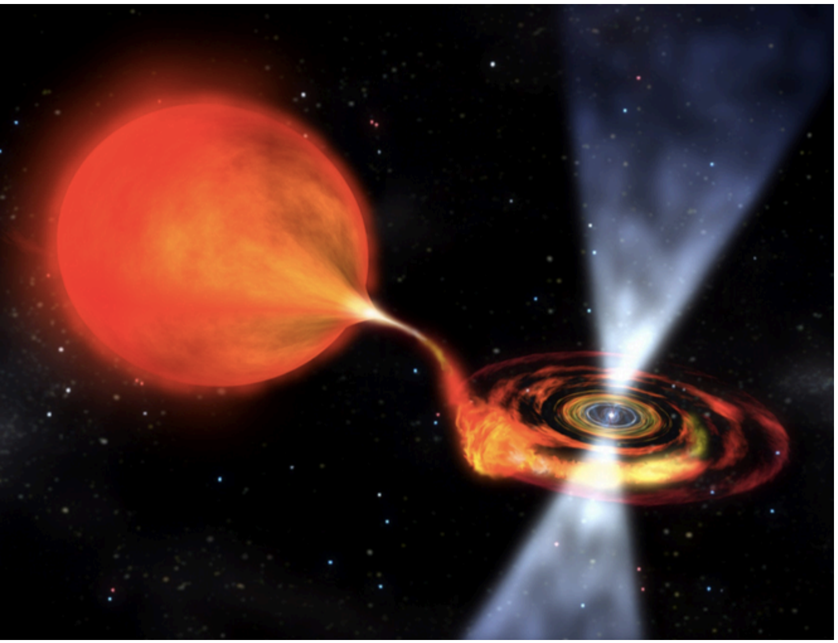


Thermal radiation from isolated neutron stars



Source: https://www.slac.stanford.edu/econf/C041213/presents/0041_TLK.PDF

Accretion disks



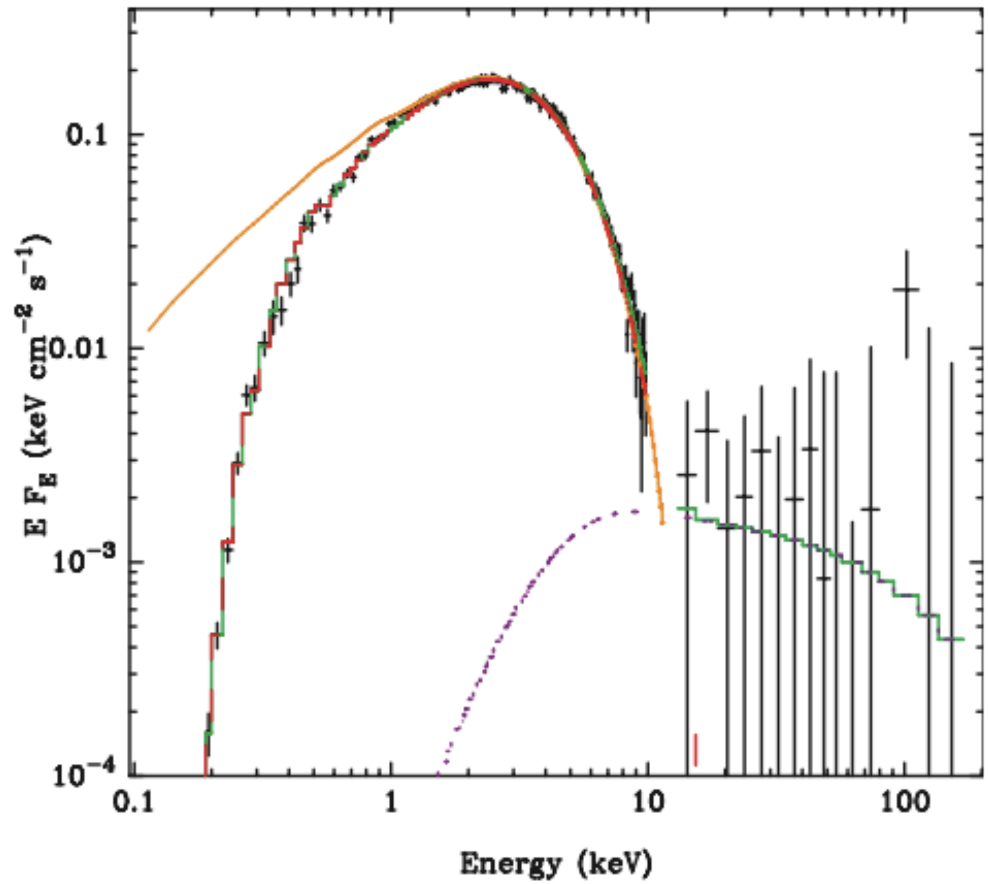
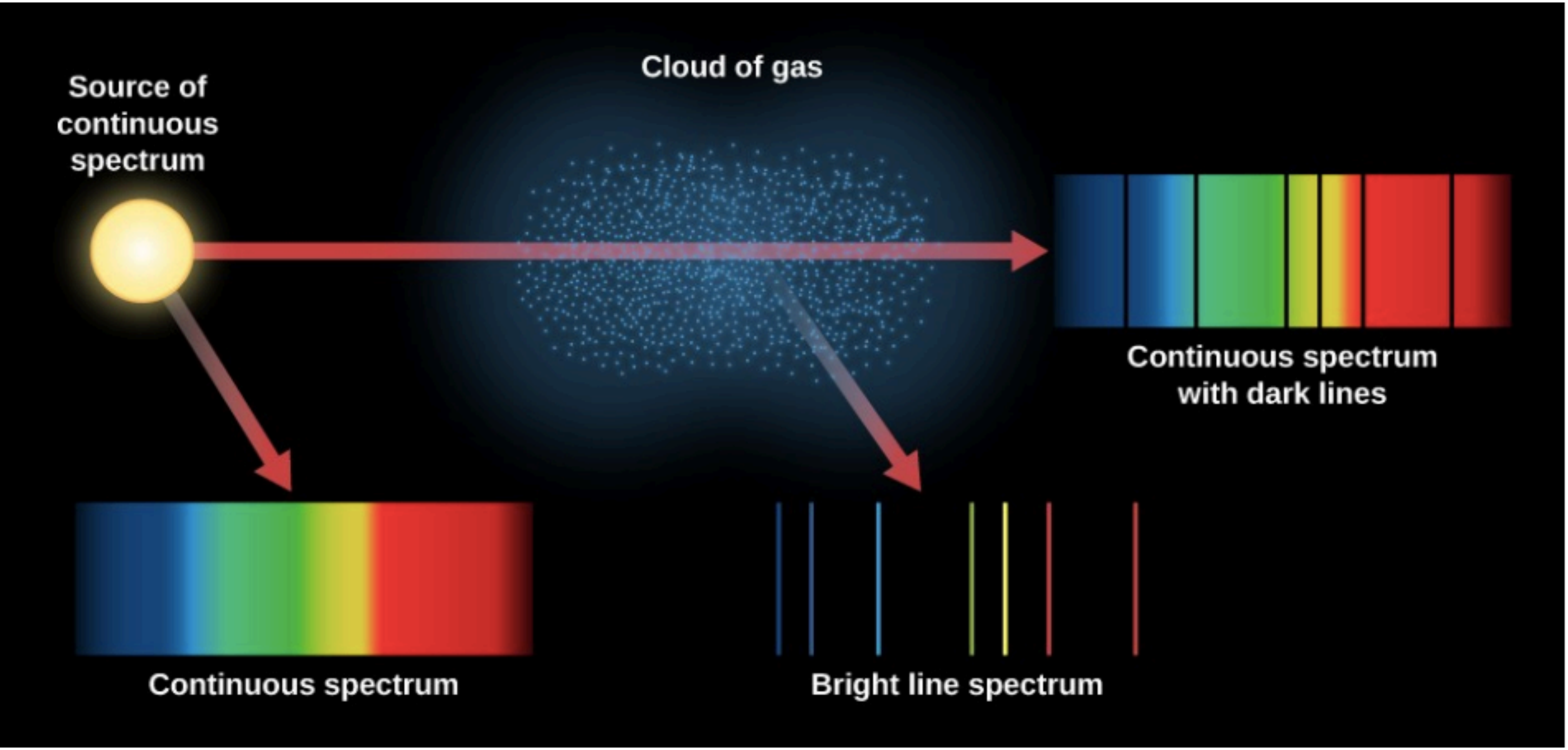


Fig credit : McClintock, Narayan & Steiner (2013) http://arxiv.org/pdf/1303.1583.pdf

Emission line and Absorption line



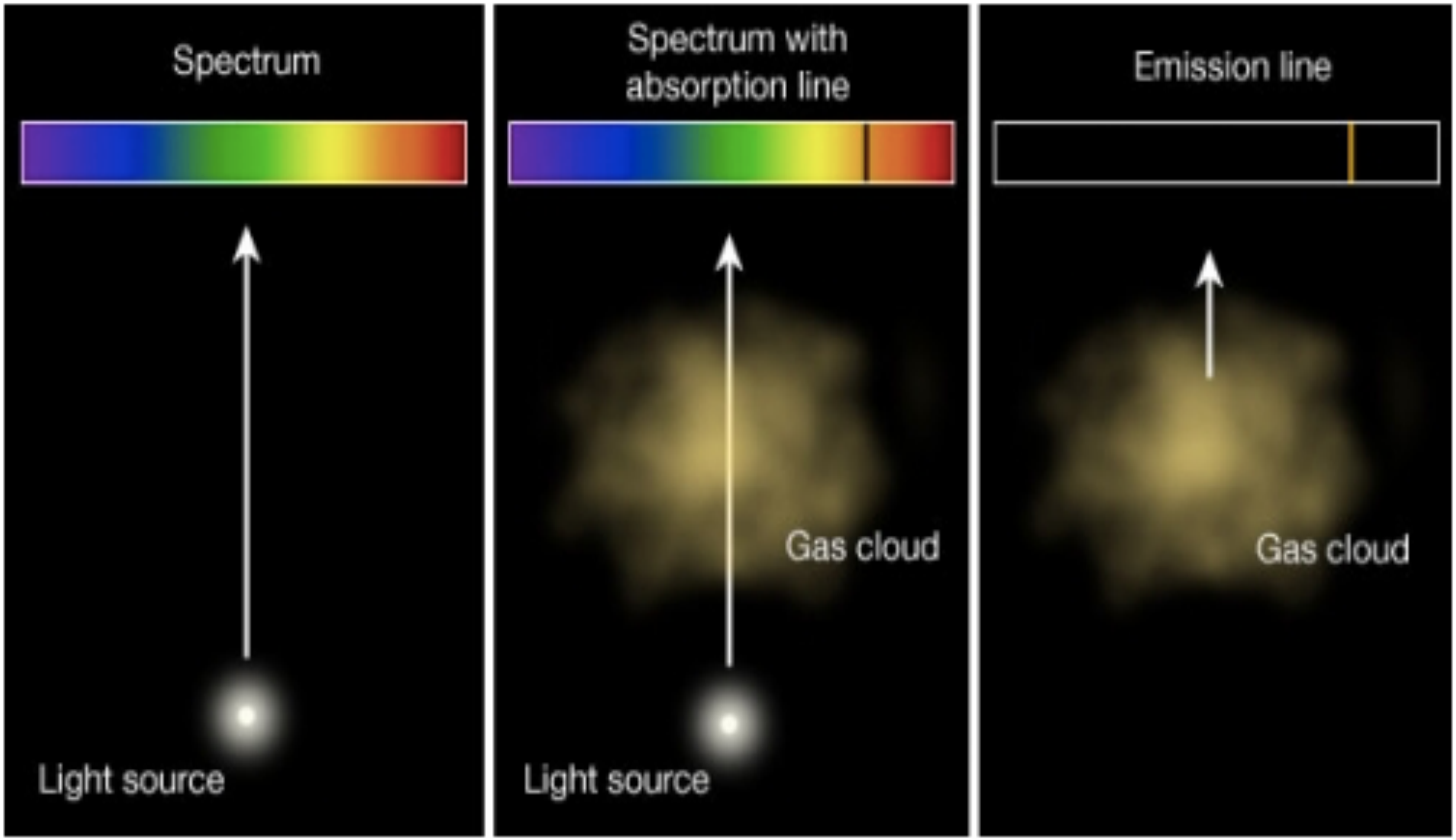
An incandescent light bulb produces a continuous spectrum.
When continuous spectrum is viewed through a thinner cloud of gas, an absorption line spectrum can be seen superimposed on the continuous spectrum.
If we look only at a cloud of excited gas atoms (with no continuous source seen behind it), we see that the excited atoms give off an emission line spectrum.

Radiation propagating through a gas is transformed by emission and absorption processes.

↓

The result is the observed spectrum including spectral lines.

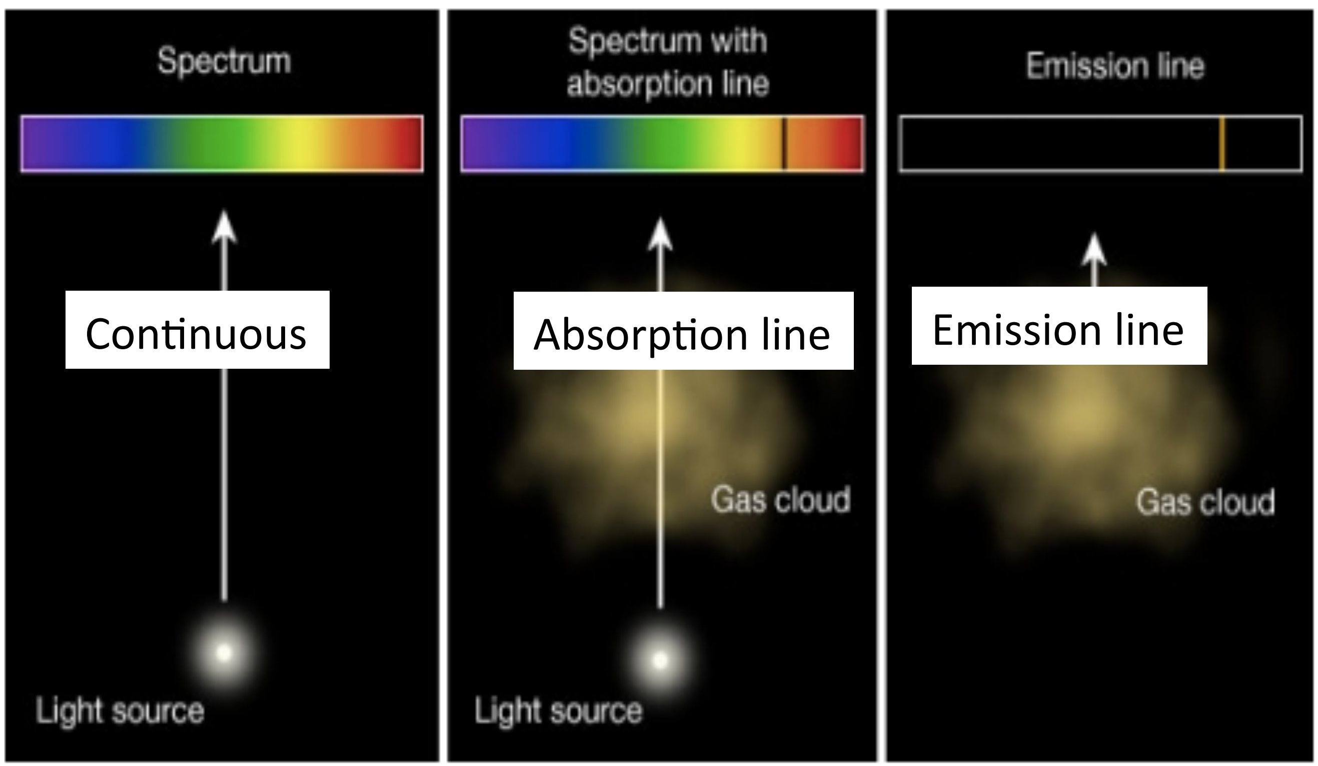
Spectrum



https://www.e-education.psu.edu/astro801/book/export/html/1549

Three main types of spectra summarized in Kirchhoff's three laws of spectroscopy:

- ✧ A luminous solid, liquid, or dense gas emits light of all wavelengths.
- ✧ A low density, cool gas in front of a hotter source of a continuous spectrum creates a DARK LINE or ABSORPTION LINE spectrum.
- ✧ A low density, hot gas seen against a cooler background emits a BRIGHT LINE or EMISSION LINE spectrum.



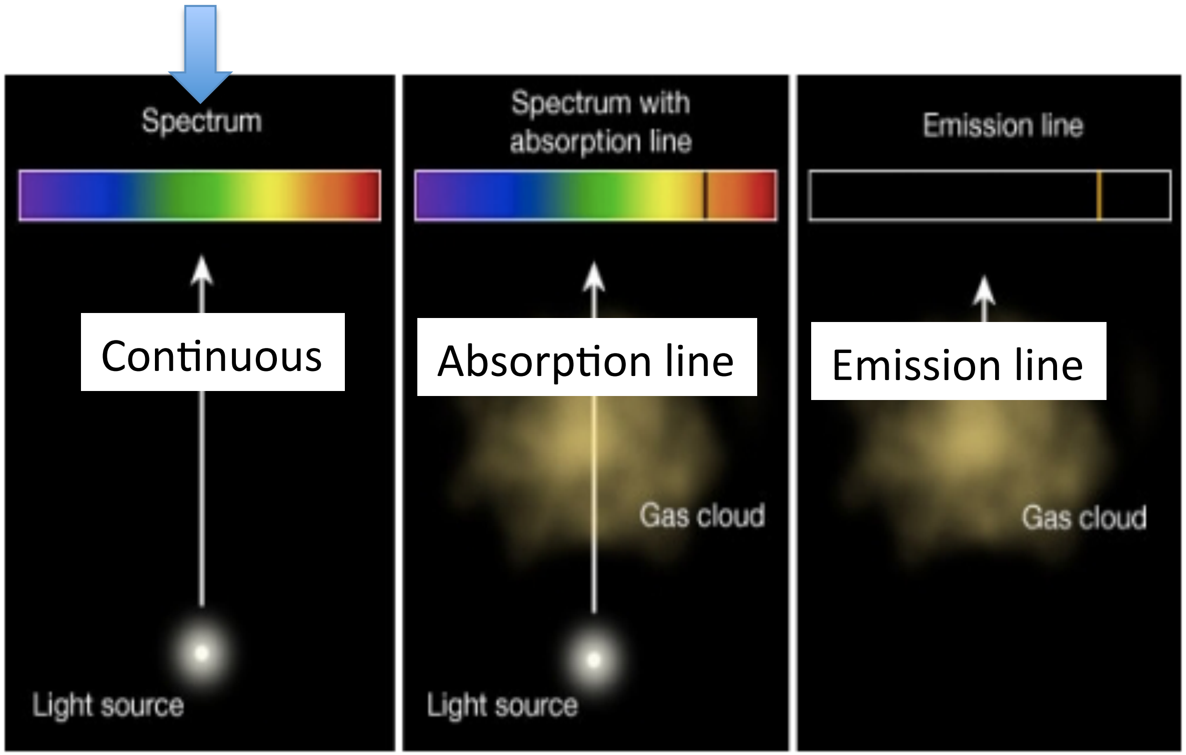
Three types of spectra in Kirchhoff's three laws of spectroscopy

1. A luminous solid, liquid, or dense gas emits light of all wavelengths

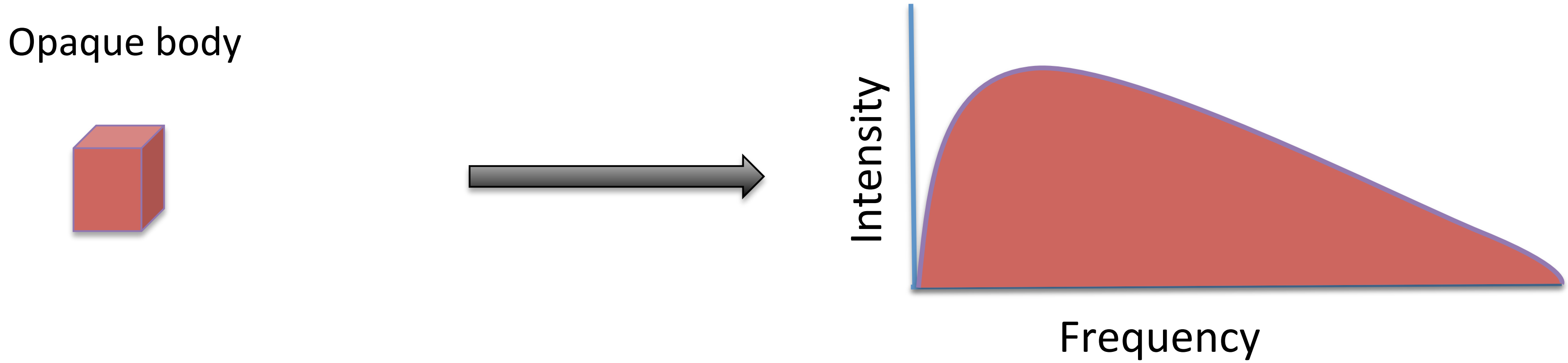
Considering initial surface brightness is zero $I_\nu(0)=0$

$$I_\nu = S_\nu = B_\nu$$

Observe a blackbody spectrum (Planck spectrum)



Spectrum



A luminous opaque body behaves like a black body
emits frequencies of all wave lengths and produces continuous spectrum

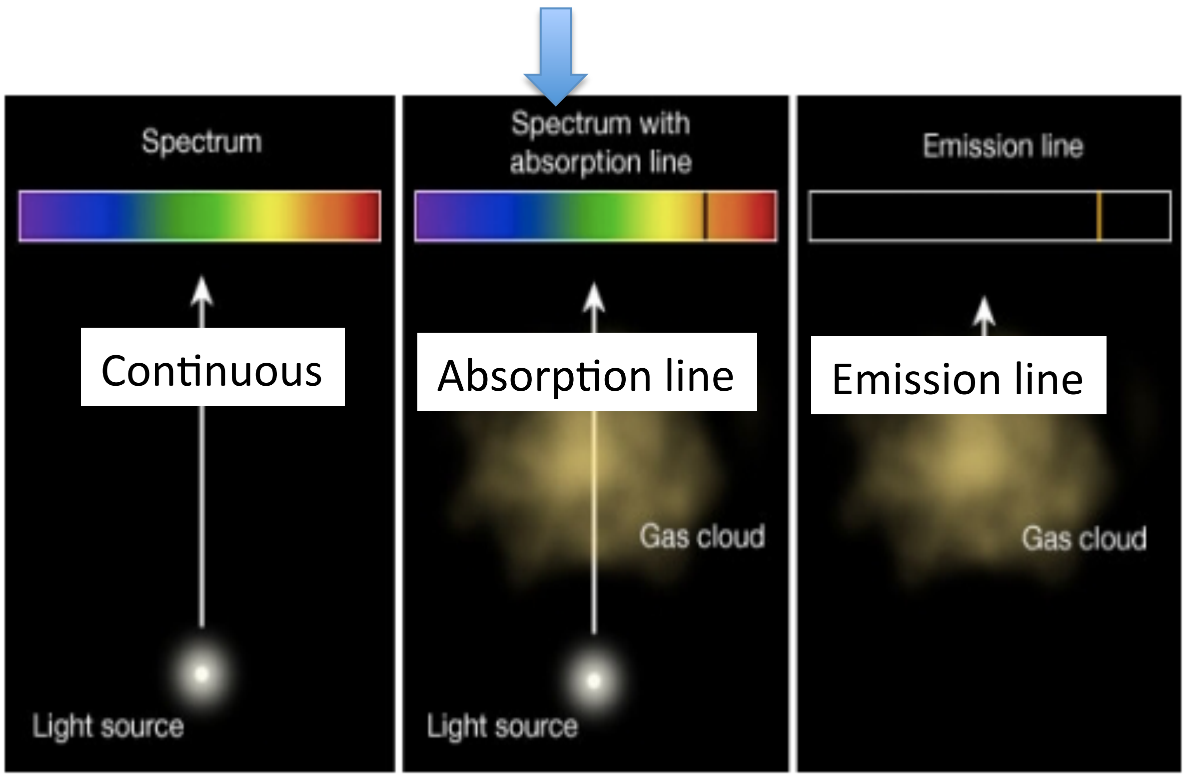
Three types of spectra in Kirchhoff's three laws of spectroscopy:

2. A low density, cool gas in front of a hotter source of a continuous spectrum creates a DARK LINE or ABSORPTION LINE spectrum.

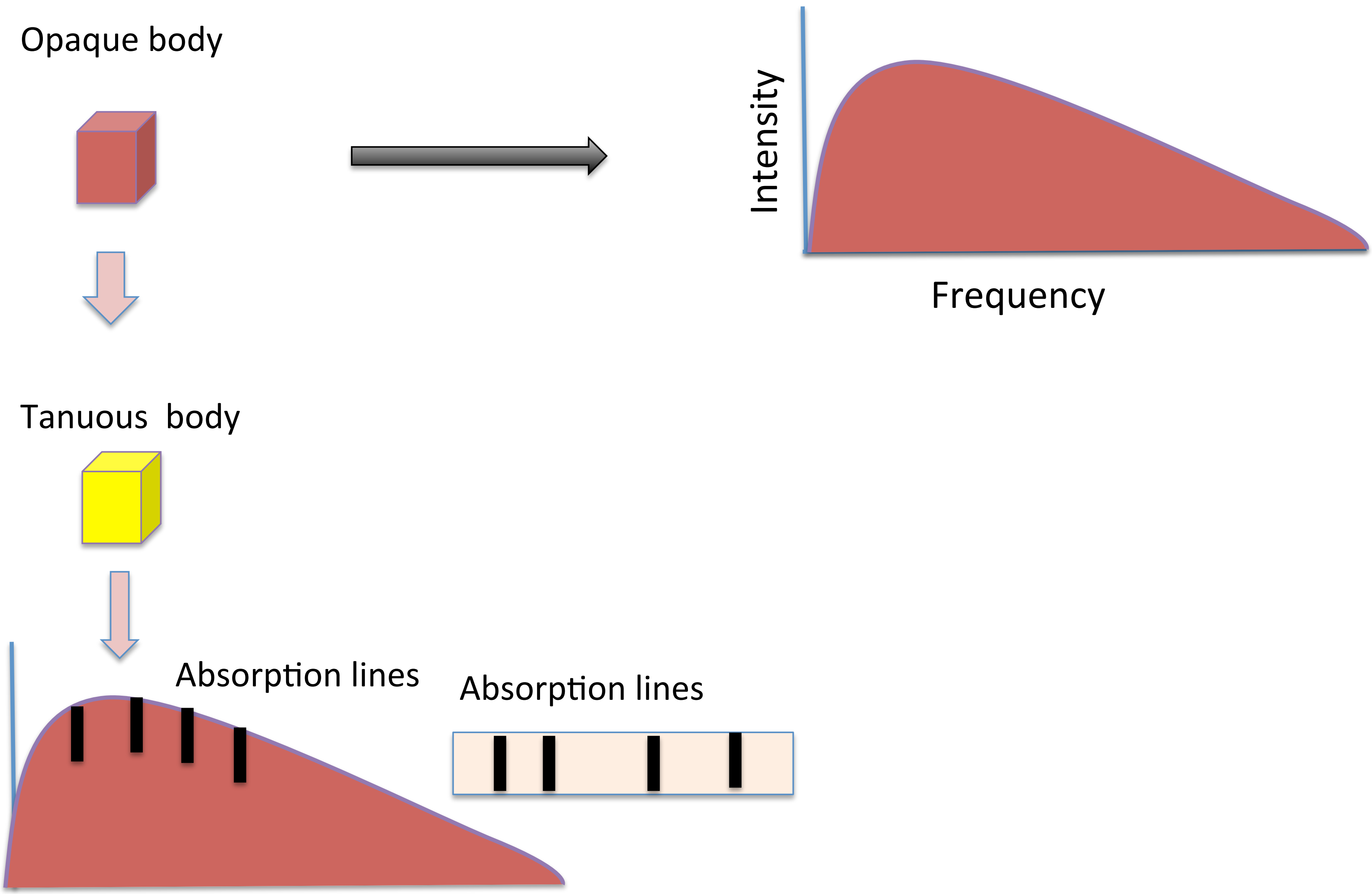
Hot gas will emit like blackbody $I_\nu(0) = B_\nu$

Cold gas will have negligible emission $S_\nu \approx 0 \longrightarrow I_\nu = B_\nu e^{-\tau_\nu}$

Planck spectrum lowered (absorption) where optical depth is higher

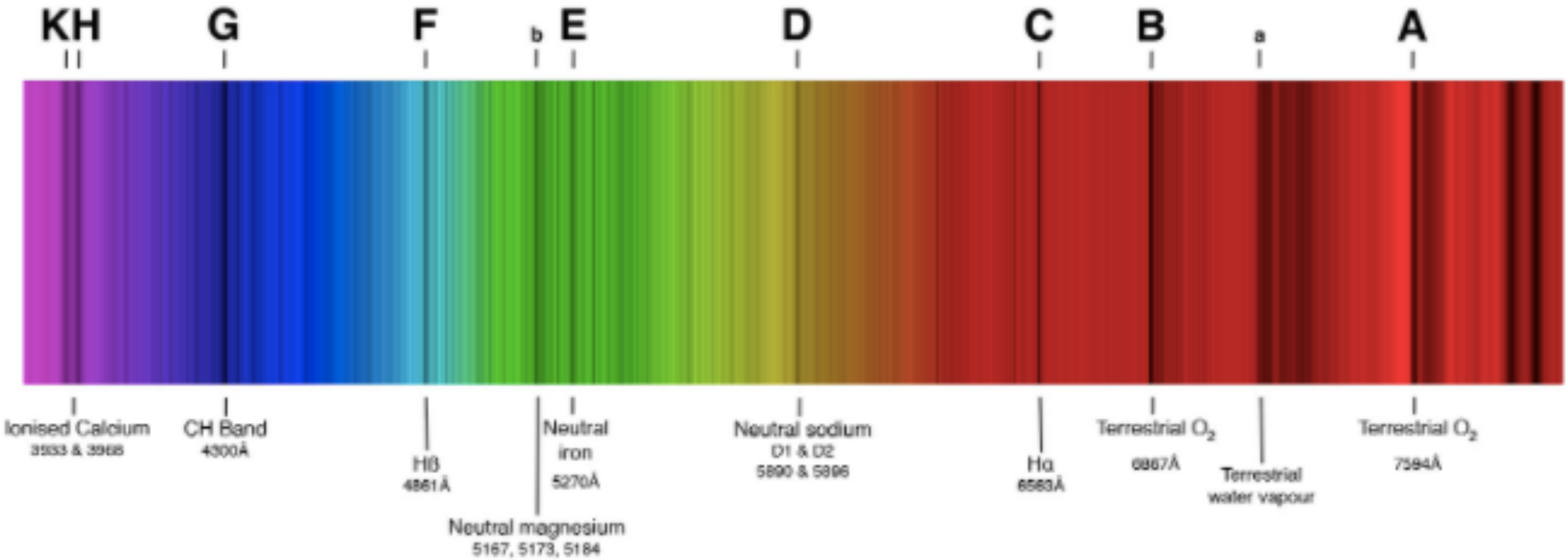


Spectrum



Opaque body
Intensity
Frequency
Tanuous body
Absorption lines
Absorption lines

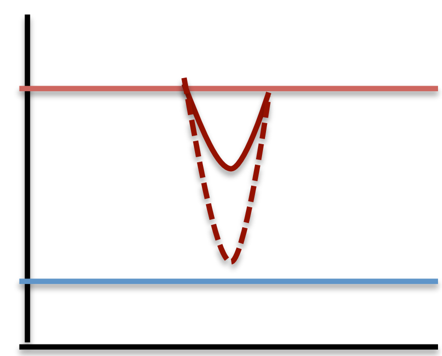
Absorption line



Fraunhofer's lines (~2500 such lines)

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

$$I_\nu = B_\nu e^{-\tau_\nu}$$



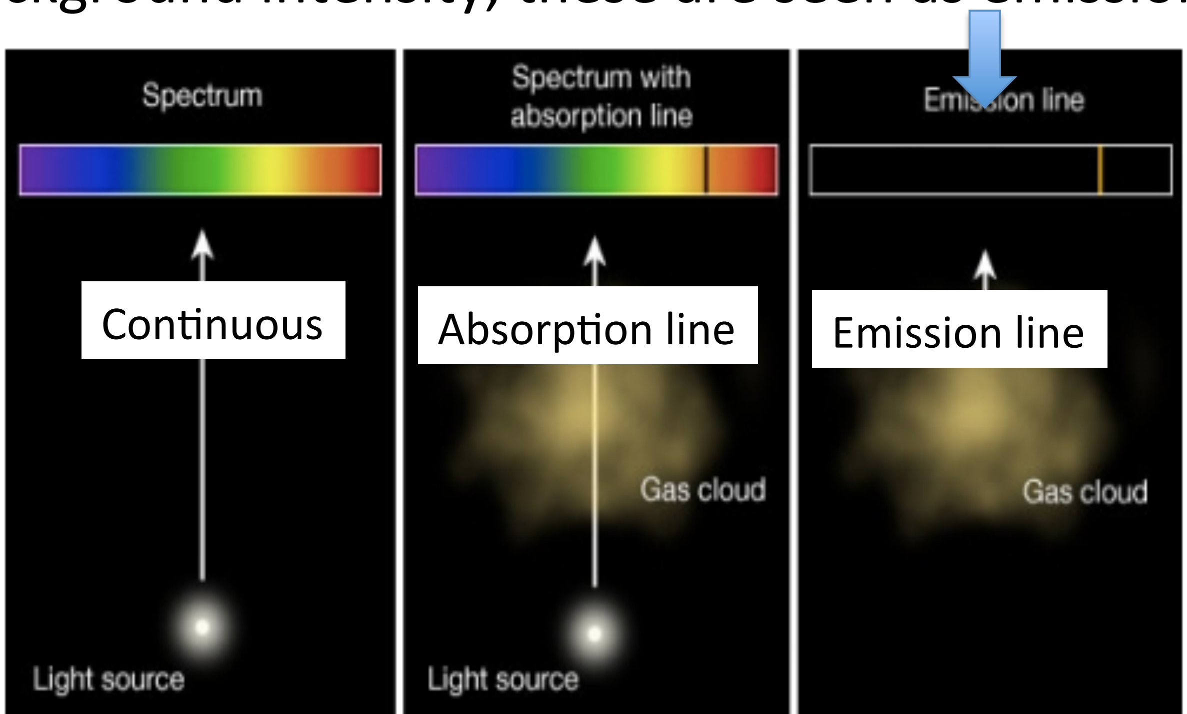
Three types of spectra in Kirchhoff's three laws of spectroscopy:

3. A low density, hot gas seen against a cooler background emits a BRIGHT LINE or EMISSION LINE spectrum.

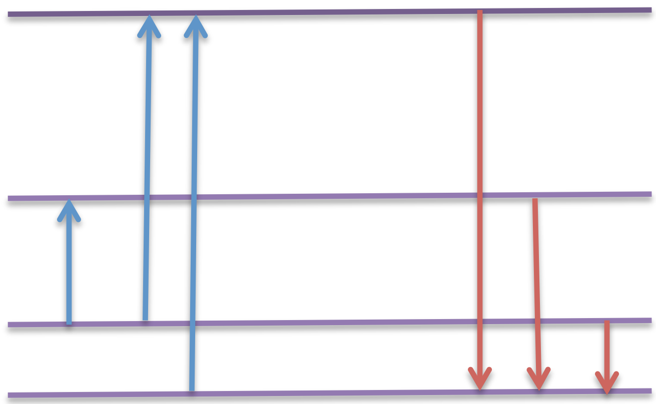
For optically thin part

$$I_\nu = S_\nu(1 - e^{-\tau_\nu}) \approx S_\nu(1 - 1 + \tau_\nu) = S_\nu \tau_\nu$$

The intensity will be high where the optical depth is high.
There is no background intensity, these are seen as emission lines.



Emission line and Absorption line



Emission and absorption coefficients depend on frequency

↓

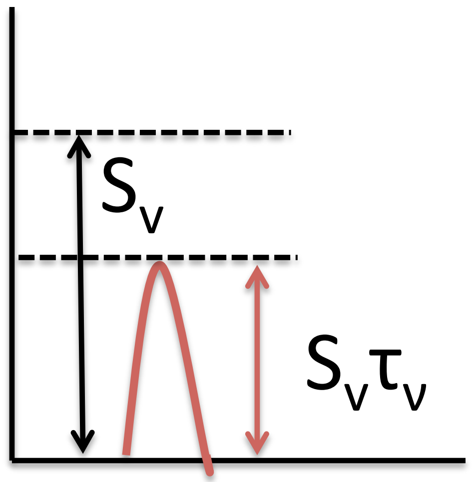
zero except at discrete frequencies

Radiative transfer equation

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

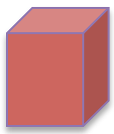
↓ Optically thin (no background radiation)

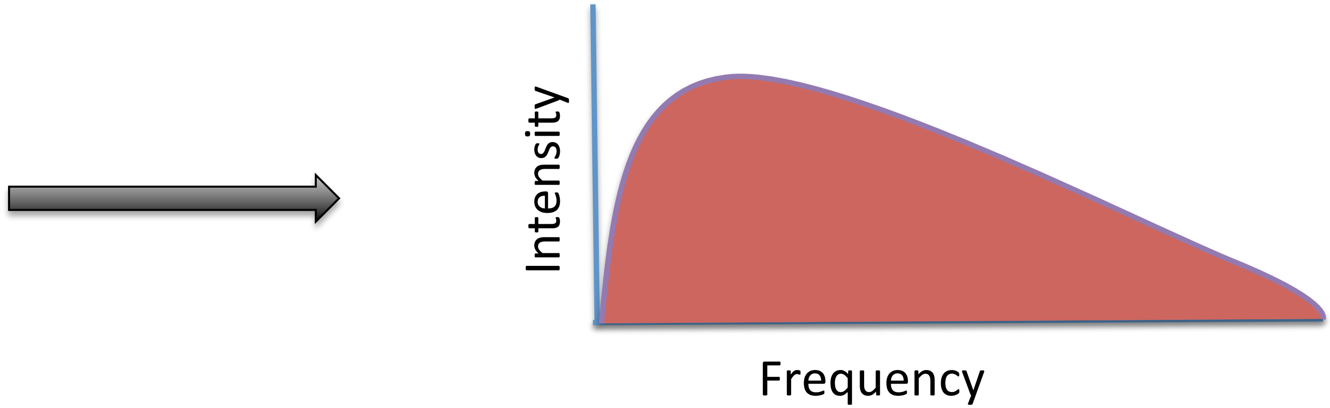
$S_\nu \tau_\nu$



Spectrum

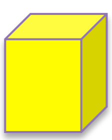
Opaque body

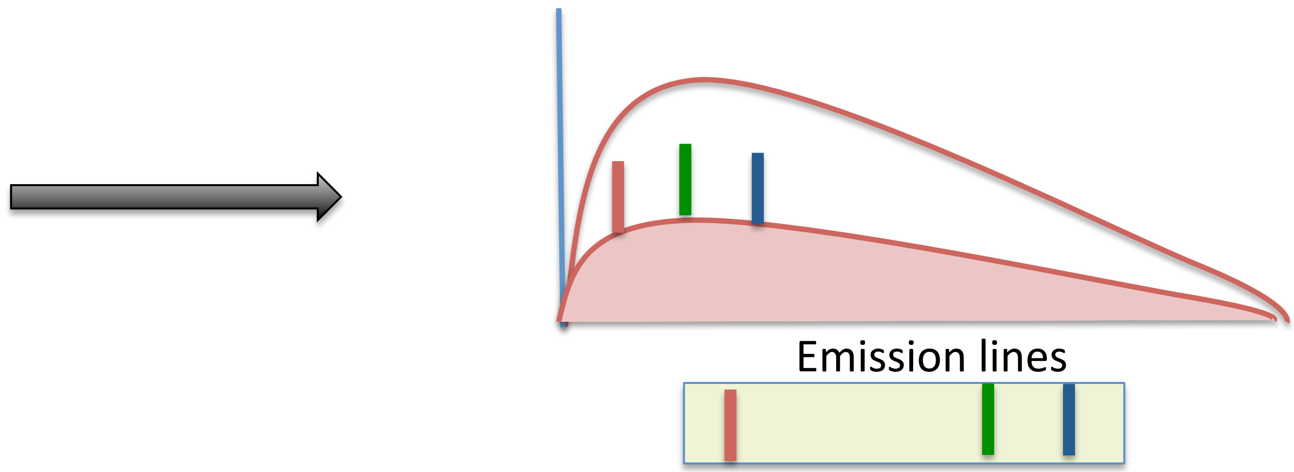




A luminous opaque body behaves like a black body
emits frequencies of all wave lengths and produces continuous spectrum

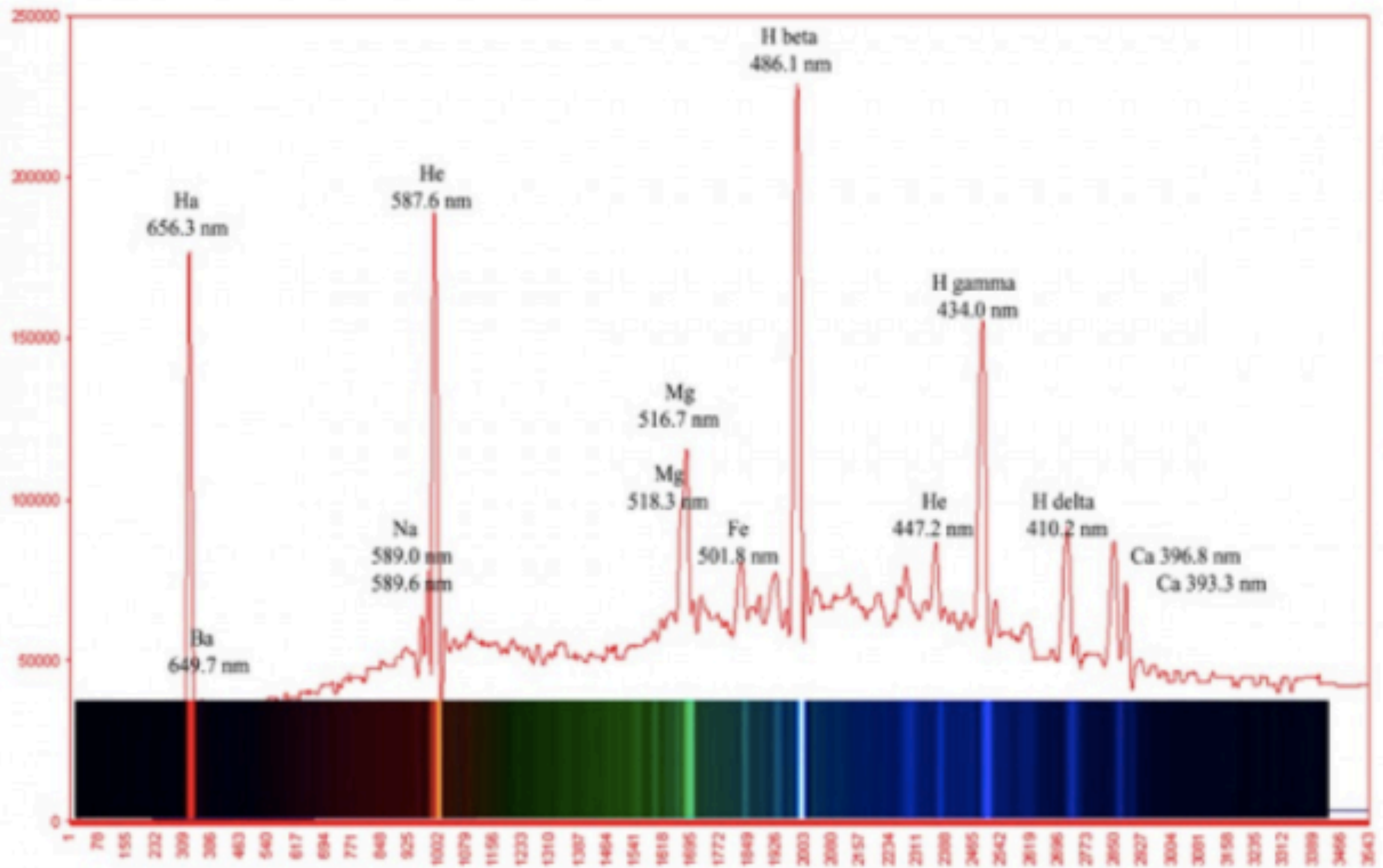
Tanuous body





Emission lines superimposed on faint continuous spectra.
Intensity of continuum or the emission lines can never exceed black body at any point

The Solar Chromosphere Spectrum (Flash Spectrum)



A new element was discovered in the flash spectrum during eclipse of 1868 in Guntur (Andhra, India). This was named “Helium” after “Helios” for the Sun in Greek.

- The energy levels of the electrons in an atom are like fingerprints—no two elements have the same set of energy levels, so the atoms of no two elements create the same pattern of absorption or emission lines.
- What this means is that if we observe absorption lines caused by a cloud of gas, we can tell what elements make up that cloud by the wavelengths or frequencies of the absorption lines.
- Tables exist that list all of the known wavelengths of the lines from a particular element as measured in the lab.

A star will create an absorption line spectrum because the continuous spectrum emitted by the dense, opaque gas that makes up most of the star passes through the cooler, transparent atmosphere of the star.

When you observe an absorption spectrum of an astronomical object, any cloud of gas between us and the object can absorb light. So, in a typical star, you see absorption lines from the atmosphere of the object, you might see absorption lines caused by intervening gas clouds between us and that star, and finally, Earth's atmosphere will also absorb some of the star's light.

Radiative Transfer

Radiative Transfer = change in I_ν as radiation propagate

✧ Radiation is ultimately produced by quantum mechanical transitions in which electrons move from one level to anothr

✧ In an ensemble of atoms/molecules occupancy of these energy levels is given By Boltzman distribution $e^{-E/KT}$ ->matter is in thermal equilibrium

✧ In diffuse matter when $\tau \ll 1$ the photons retain their signature.

✧ In an opaque body when $\tau \gg 1$ the radiation loses all its memory during the Process of multiple absorption and emission and behave like a black body.
This is why spectrum of raditation is characterised by temperature and not by any other property of matter.

✧ In most astrophysical situation matter and radiation are not in thermodynamic equilibrium and so we are not dealing with opaque matter.
(examples of opaque body : early universe and interiors of stars)

End of Lecture 2

Reference: Rybicki Lightman Chapter 1

https://www.cv.nrao.edu/course/astr534/LineRadxfer.html

https://www.astro.rug.nl/~etolstoy/astroa07/

https://apatruno.wordpress.com/about/teaching/rp-2016/

Next lecture : 14th August (Wednesday)

Topic of next Lecture:
Einstein Coefficients, Problem solving related to Radiative transfer

Preparation: Lecture1,2 + Problems from Rybicki Lightman + Revision on simulated emission, spontaneous emission and stimulated absorption