Electrodynamics and Radiative Processes I

Lecture 12 – Compton/Inverse-Compton scattering

Bhaswati Bhattacharyya
bhaswati@ncra.tifr.res.in

August-September 2019

IUCAA-NCRA Graduate School

Reference :
1) Rybicki and Lightman
2) Ghisellini: http://www.brera.inaf.it/utenti/gabriele/total.pdf

Date : 16th September 2019

Compton Scattering

Photon scattering by electrons - Overview

	Low energy photons $\hbar\omega \ll m_e c^2$	**High energy photons** $\hbar\omega \geq m_e c^2$
$v \ll c$	Thomson scattering Classical treatment frequency unchanged	Compton scattering Quantum treatment incorporating photon momentum frequency decreases
$v \sim c$	$\gamma\hbar\omega \ll m_e c^2$ Inverse Compton Photons gain energy from relativistic electrons Approximate with classical treatment in electron rest frame Frequency increases	$\gamma\hbar\omega \geq m_e c^2$ Inverse Compton Quantum treatment in electron rest frame Photons gain energy from relativistic electrons

Thomson Scattering
(low-energy photons)

1. It occurs when the photon's energy is << electron rest mass

2. The electrons move non-relativistically: v<<c

The incoming and outgoing photon has the same energy and the electron does not change energy in the scattering process (elastic or coherent scattering).

Thomson Scattering

For low photon energies the scattering of radiation from free charges reduces the classical Thomson scattering (Lecture 3,4).

$$\epsilon = \epsilon_1,$$

$\longrightarrow$ Equal energy for incident and scattered photons
Elastic scattering

$$\frac{d\sigma_T}{d\Omega} = \frac{1}{2} r_0^2 (1 + \cos^2\theta),$$

$$\sigma_T = \frac{8\pi}{3} r_0^2.$$

$= \sigma_T \sim 0.66 \times 10^{-24}\ \mathrm{cm}^2$

- ✧ Frequency independent, so scattering is equally effective at all frequencies.
- ✧ Valid for lower frequencies. Not valid for high frequencies $h\nu > mc^2$

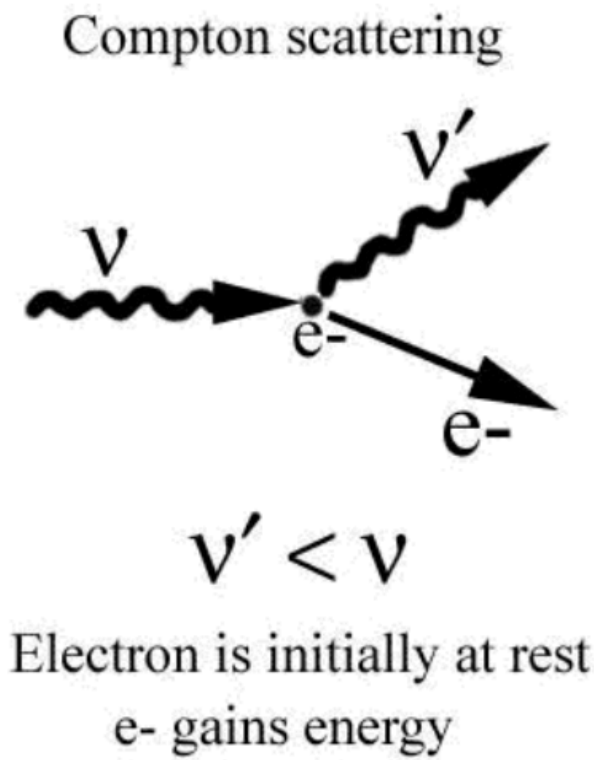
Compton Scattering

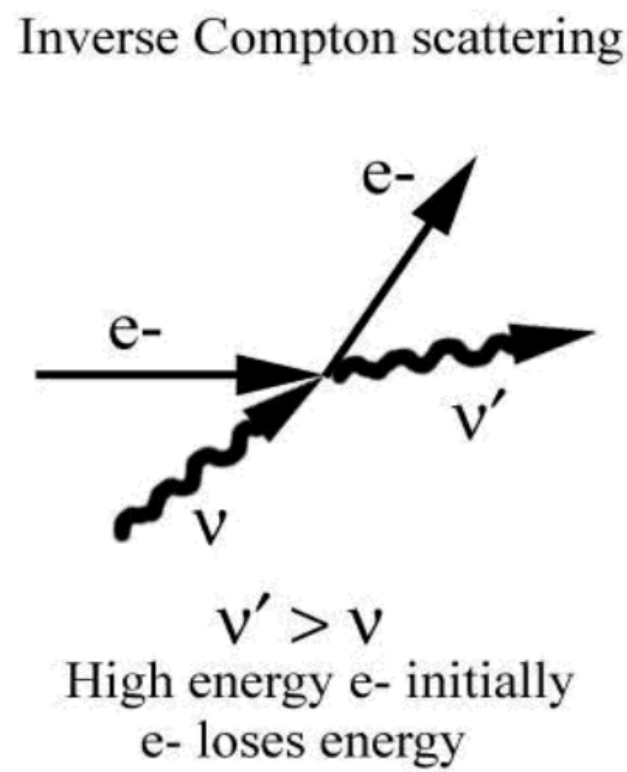
"In physics, Compton scattering or the Compton effect, is the decrease in energy (increase in wavelength) of an X-ray or gamma ray photon, when it interacts with matter.
Inverse Compton scattering also exists, where the photon gains energy (decreasing in wavelength) upon interaction with matter. The amount the wavelength increases by is called the Compton shift."

Compton effect was observed by Arthur Holly Compton in 1923, for which he earned the 1927 Nobel Prize in Physics.

The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon.
Thomson scattering, the classical theory of charged particles scattered by an electromagnetic wave, cannot explain any shift in wavelength. Light must behave as if it consists of particles in order to explain the Compton scattering.

Compton Scattering





Direct : Photon loses energy
electron gains energy

Inverse : Photon gains energy
electron loses energy

Compton Scattering (notation)

-- The prime symbol ' means that the quantity is calculated in the rest frame K' (i.e. the electron's rest frame in this case).

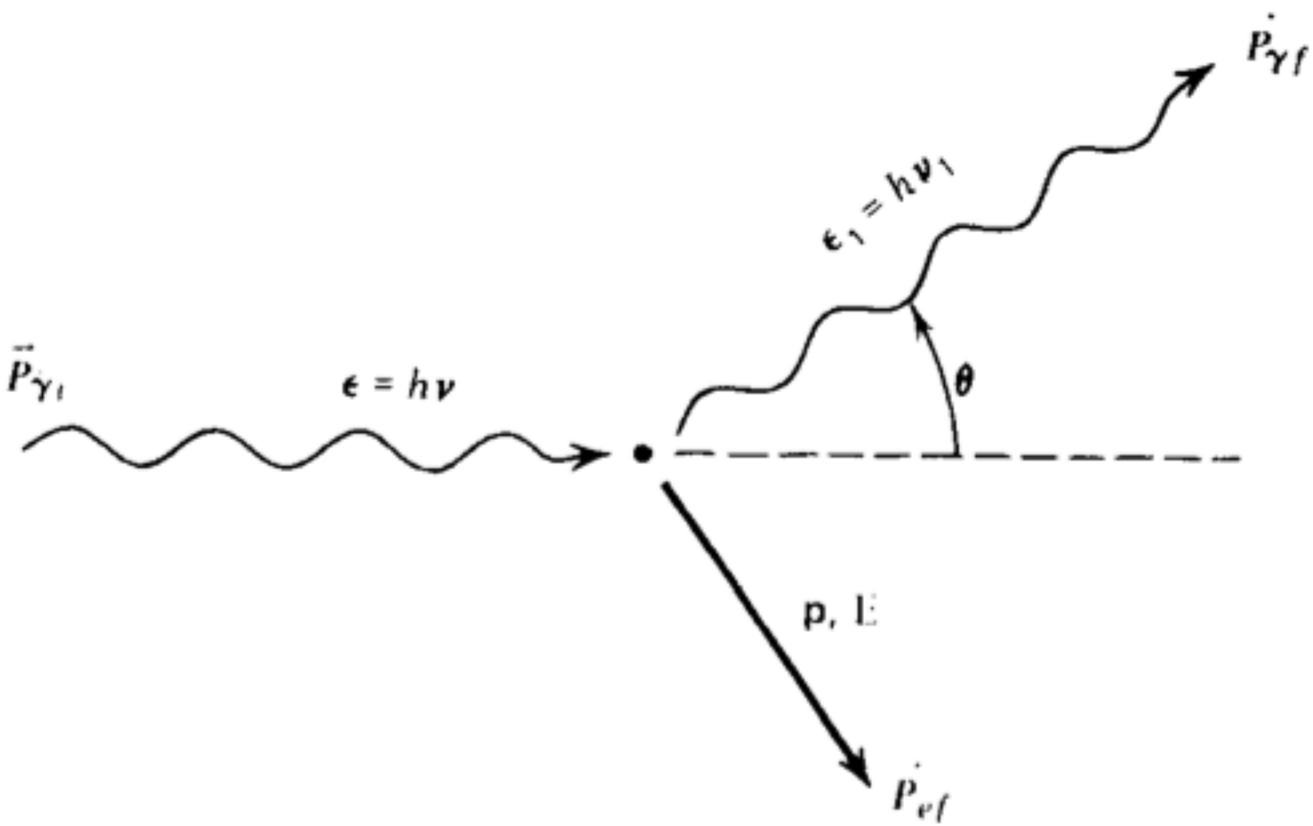
– No prime symbol means the quantity is calculated in the lab frame K (i.e. observer frame).

– The under-script 1 means that the quantity is calculated after the scattering has already occurred.

– No under-script means that the quantity is calculated before the scattering.

ε → energy before the scattering in K
ε_1 → energy after the scattering in K
ε' → energy before the scattering in K'
ε'_1 → energy after the scattering in K'

Compton Scattering



Compton Scattering

Let's start by looking at the momentum and energy of the photon and electrons.

In Thomson scattering the photon has no momentum (classical electrodynamics).

However, from quantum mechanics we do know that a photon has a momentum.

This means that scattering process cannot be purely elastic since the electron will recoil due to the momentum of the photon.

The photon has initial energy ε and final energy ε_1 .
The photon has initial momentum ε/c and final momentum ε_1/c.
The electron has initial energy mc^2 and final energy E/c.
The electron has initial momentum 0 and final momentum p.

Compton Scattering

Using the conservation of energy and momentum it can be shown that the final and initial photon energies are related,

$$\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1-\cos\theta)}.$$

In terms of wavelength this can be written as,

$$\lambda_1 - \lambda = \lambda_c(1-\cos\theta)$$

Compton wavelength is defined as,

$$\lambda_c \equiv \frac{h}{mc} \qquad = 0.02426 \text{ Å for electrons}$$

Compton Scattering

$$\lambda_1 - \lambda = \lambda_c(1 - \cos\theta)$$

The photon always looses energy, unless θ = 0, and the scattering is closely elastic ($h\nu \ll m c^2$)

When the photons involved in the collision have large energies, the scattering become less efficient and quantum electrodynamics effects reduce the cross section.

The Thomson cross section becomes the Klein-Nishina cross section.

Compton Scattering

When the wavelength of the incoming photon is smaller than the Compton wavelength then the Compton scattering is important.

The net effect is to decrease the energy of the photon. When the wavelength is larger than the Compton wavelength then elastic scattering (i.e., Thomson scattering) is a good approximation and the photon does not change wavelength (or energy).

Direct Compton

$$\lambda_1 - \lambda = \lambda_c (1 - \cos\theta)$$

Thompson Scattering

$$\lambda_1 - \lambda = 0$$

Compton Scattering

Quantum effects appear in two ways

(1) Kinematics of the scattering process :
Since the photon has a momentum $h\nu/c$ and energy $h\nu$, scattering will not be elastic because of recoil of charge

Need to consider energy momentum relations.

(2) Alteration of scattering cross section

Compton Scattering

Differential scattering cross-section of unpolarized radiation is given by Klein-Nishna formula

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\frac{\epsilon_1^2}{\epsilon^2}\left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta\right)$$

Note for $\varepsilon_1 = \varepsilon$, the scattering cross-section reduces to classical expression

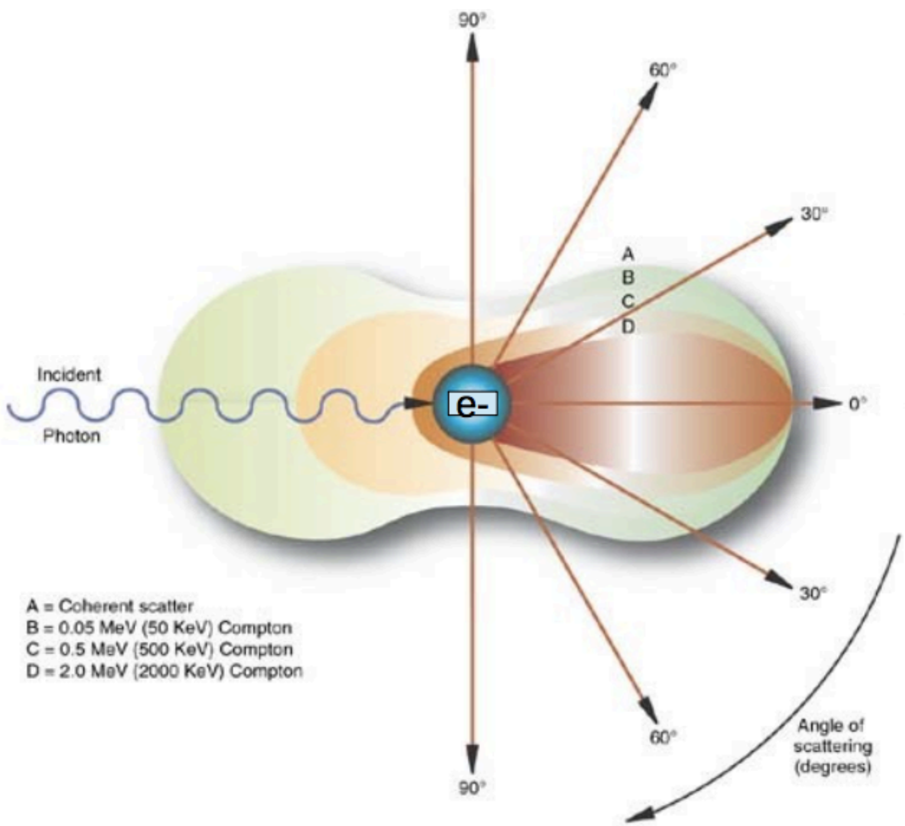
In the non relativistic regime we have (considering $x = h\nu/mc^2$)

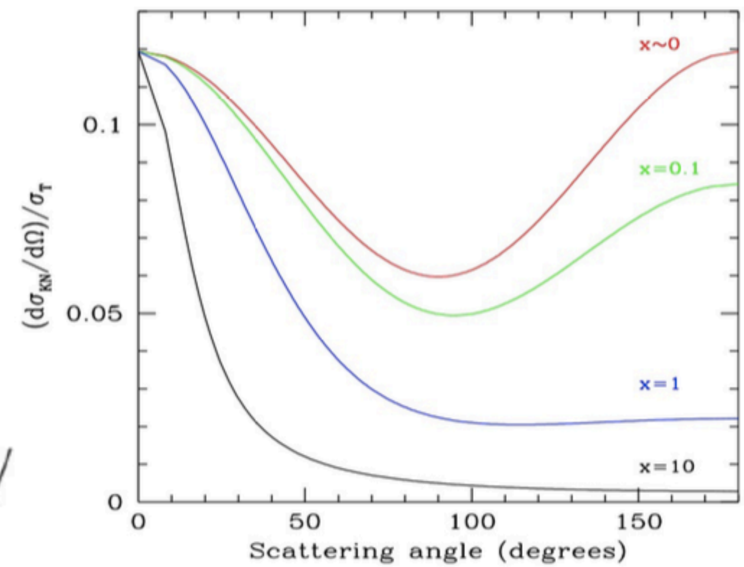
$$\sigma \approx \sigma_T\left(1 - 2x + \frac{26x^2}{5} + \cdots\right) \qquad x \ll 1$$

In extreme relativistic regime we have

$$\sigma = \frac{3}{8}\sigma_T x^{-1}\left(\ln 2x + \frac{1}{2}\right) \qquad x \gg 1$$

Compton Scattering





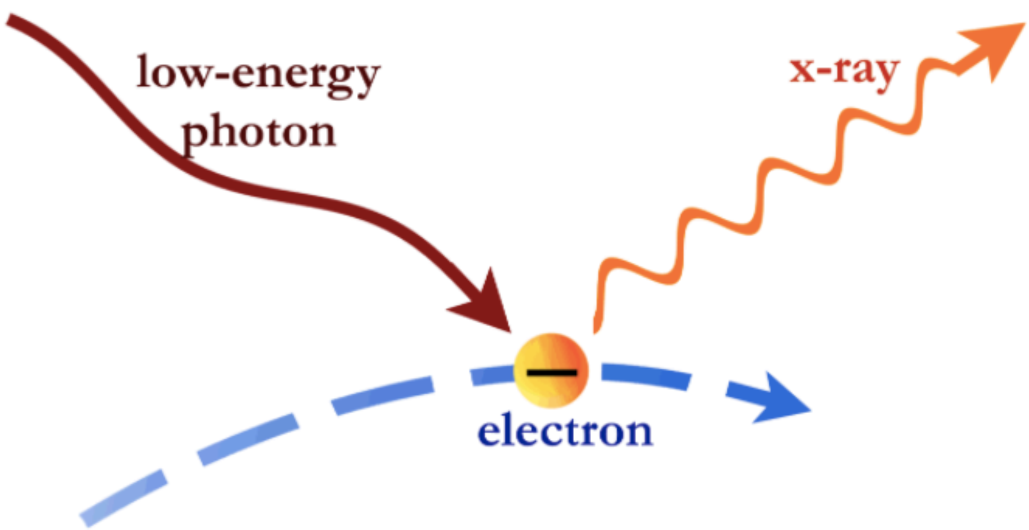
The green “peanut shape” pattern is the Thomson scattering (x~0, coherent scattering)
As the energy is increased the peanut shape disappears and the scattering becomes elongated in the “forward” direction.

Inverse Compton Scattering

The direct Compton (or simply Compton) scattering is not a very common process in astrophysics, but its inverse process (inverse Compton) is more common.

Whenever moving electron has sufficient kinetic energy compared to the photon, net energy can be transferred from the electron to the photon : inverse Compton

How does the initial photon energy change after a collision with the relativistic electron?



Inverse Compton Scattering (notation)

-- The prime symbol ' means that the quantity is calculated in the rest frame K' (i.e. the electron's rest frame in this case).

– No prime symbol means the quantity is calculated in the lab frame K (i.e. observer frame).

– The under-script 1 means that the quantity is calculated after the scattering has already occurred.

– No under-script means that the quantity is calculated before the scattering.

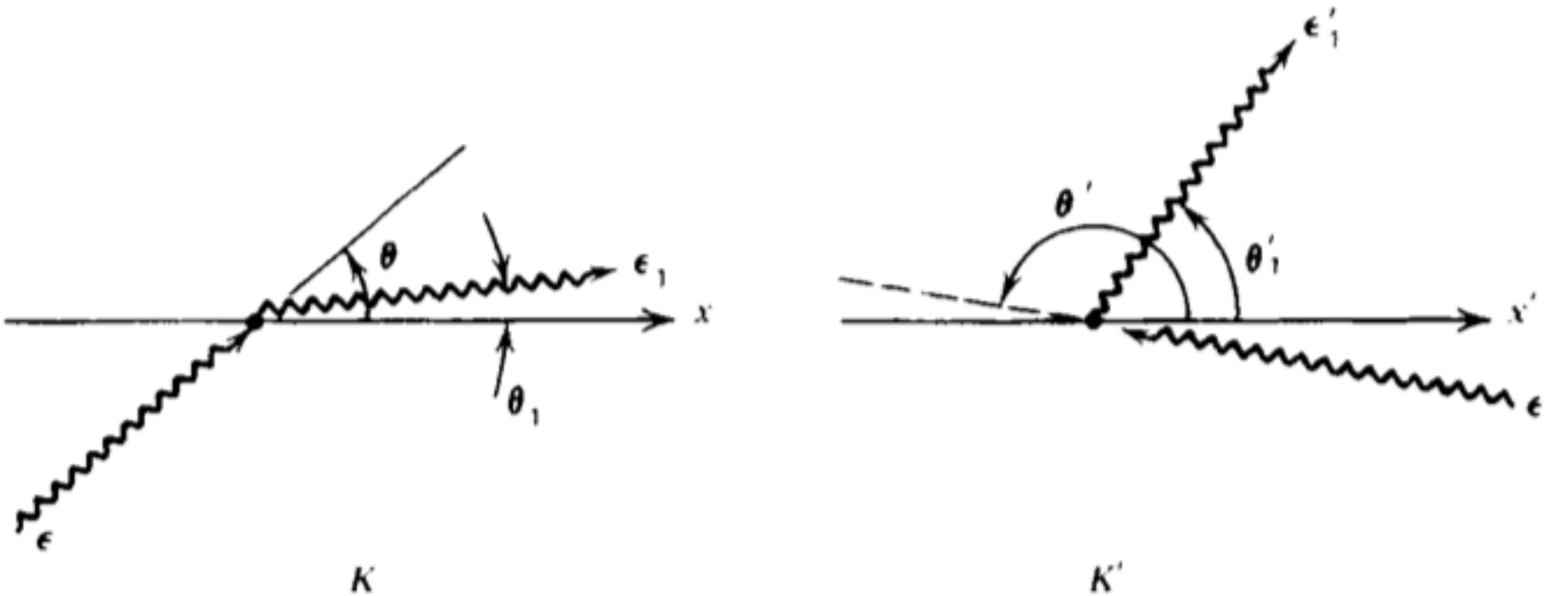
ε → energy before the scattering in K

ε_1 → energy after the scattering in K

ε' → energy before the scattering in K'

ε'_1 → energy after the scattering in K'

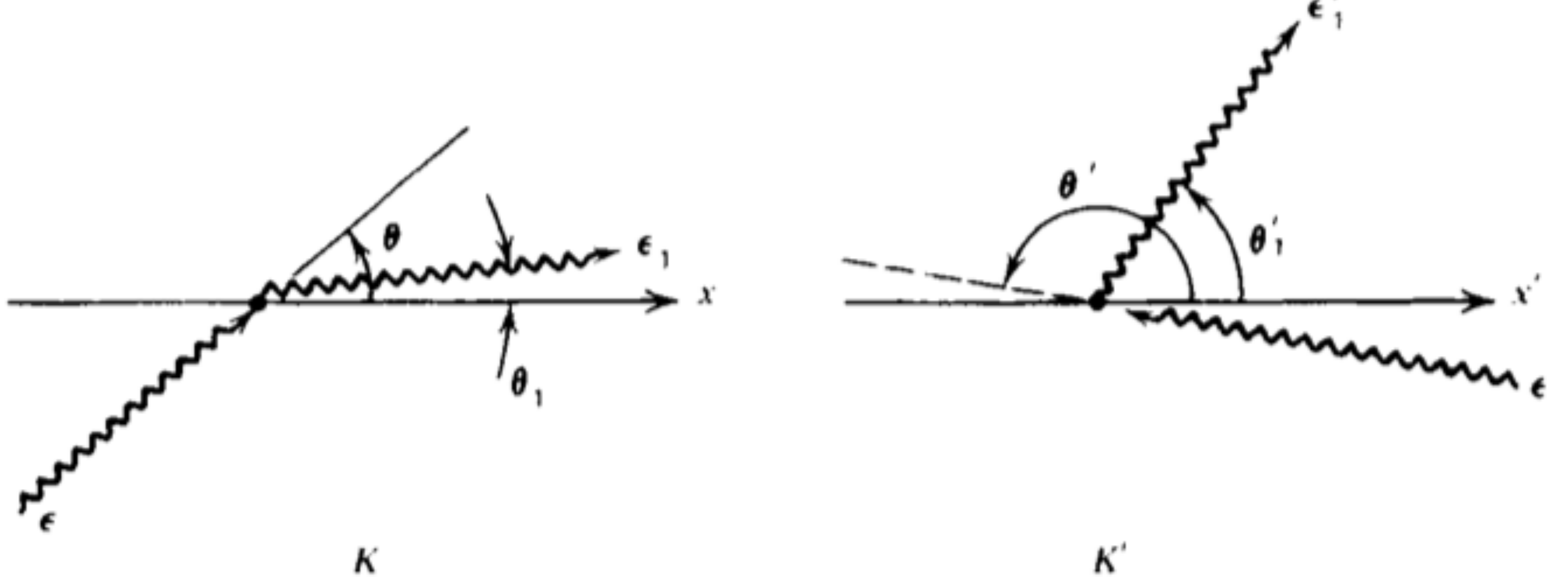
Inverse Compton Scattering



Scattering geometries in the observer's frame K and the electron rest frame K'

Step -1 : Photon and electron in lab frame to electron's rest frame
Step-2 : Photon and electron interact in electron rest frame
Step-3 : Back to the lab frame

Inverse Compton Scattering



Scattering geometries in the observer's frame K and the electron rest frame K'

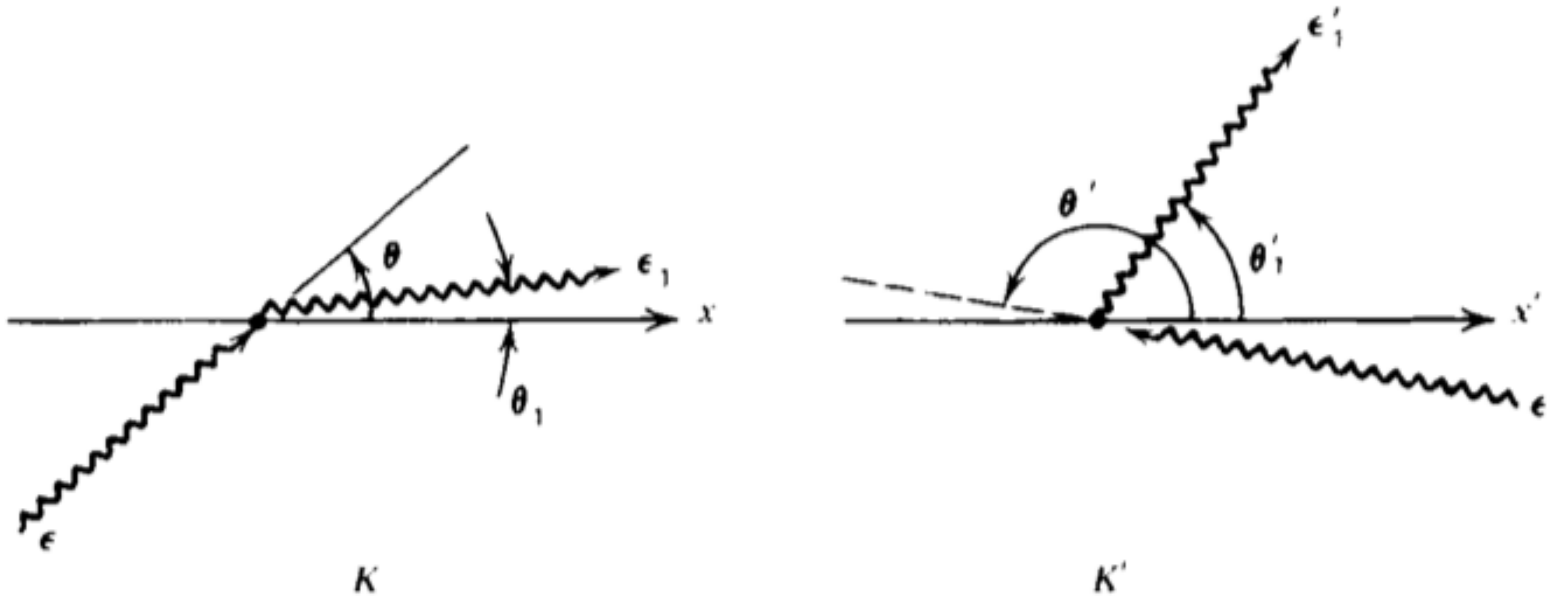
Step -1 : Photon and electron in lab frame and go to electron's rest frame

$\varepsilon' = \varepsilon\gamma(1-\beta\cos\theta)$, ε is energy of photon in K

ε' is energy of photon in K'

Energy of photon is increased in electron's rest frame due to relativistic Doppler boost

Inverse Compton Scattering



Scattering geometries in the observer's frame K and the electron rest frame K'

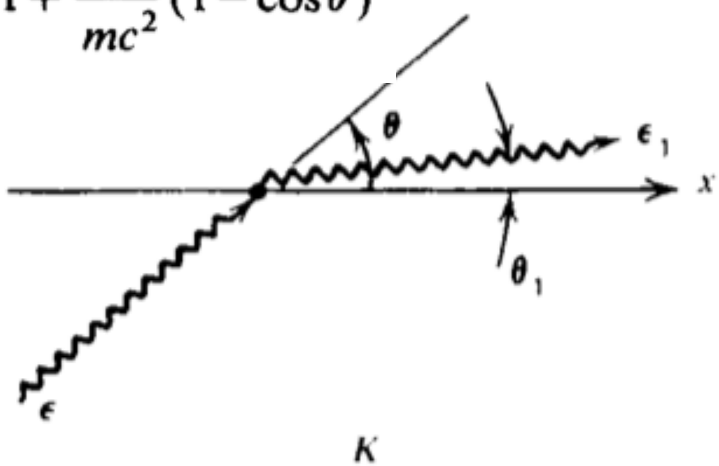
Step -2 : Photon and electron interact in electron's rest frame

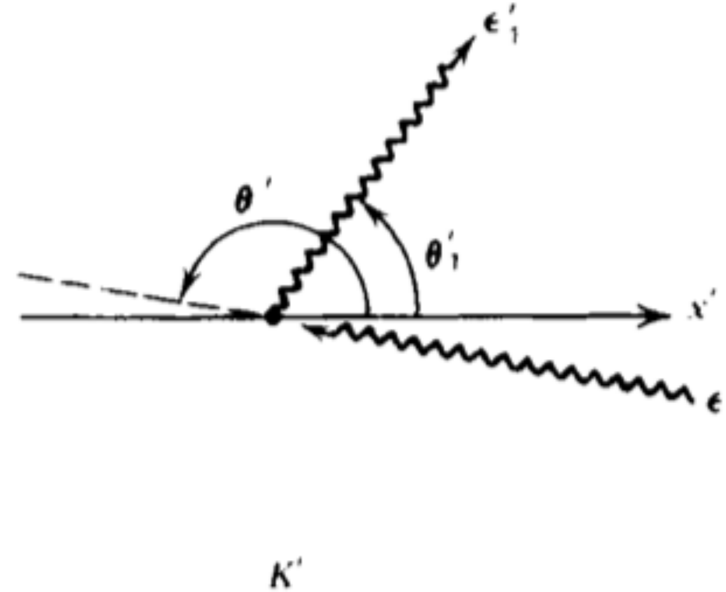
$$\epsilon_1' = \frac{\epsilon'}{1 + \dfrac{\epsilon'(1-\cos\Theta)}{mc^2}} \approx \epsilon'\left[1 - \frac{\epsilon'}{mc^2}(1-\cos\Theta)\right]$$

We are now in electron's rest frame so we can use normal Compton formula

Inverse Compton Scattering

$$\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1 - \cos\theta)}.$$





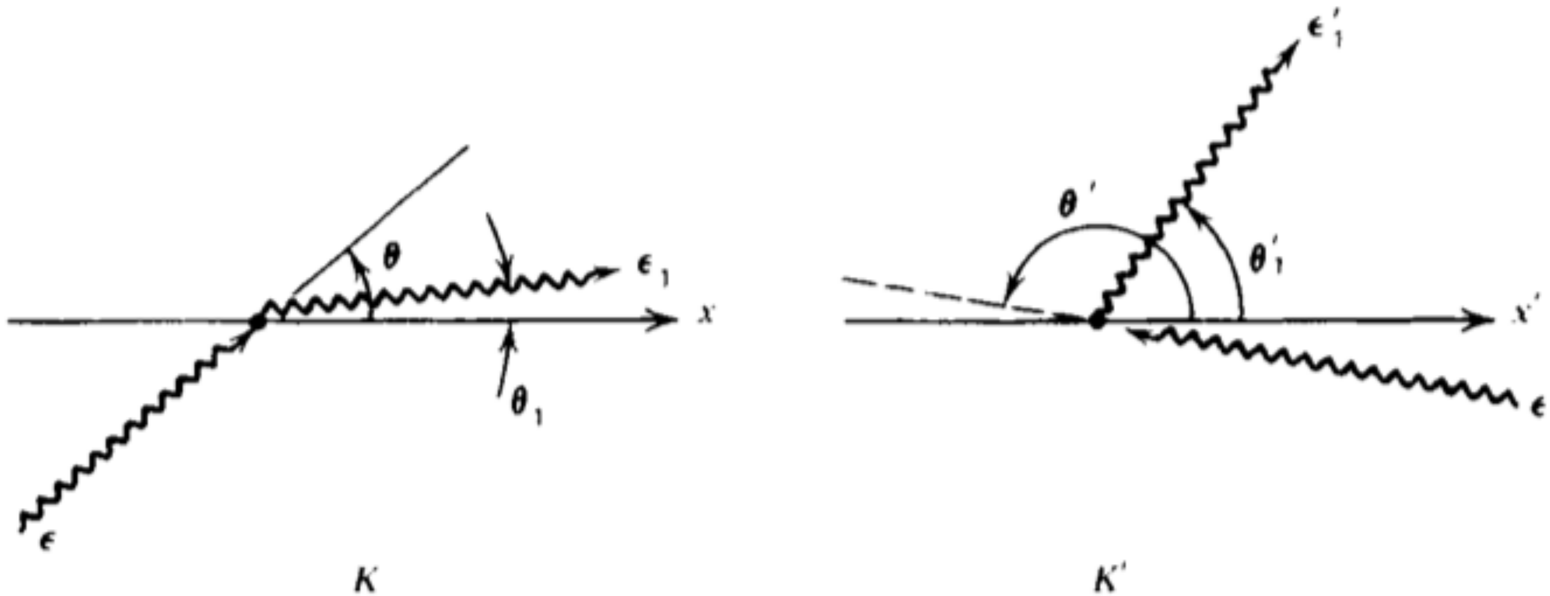
Scattering geometries in the observer's frame K and the electron rest frame K'

Step -2 : Photon and electron interact in electron's rest frame

$$\epsilon_1' = \frac{\epsilon'}{1 + \frac{\epsilon'(1-\cos\Theta)}{mc^2}} \approx \epsilon'\left[1 - \frac{\epsilon'}{mc^2}(1-\cos\Theta)\right]$$

The photon energy has decreased in this process since some energy was given away to the electron

Inverse Compton Scattering



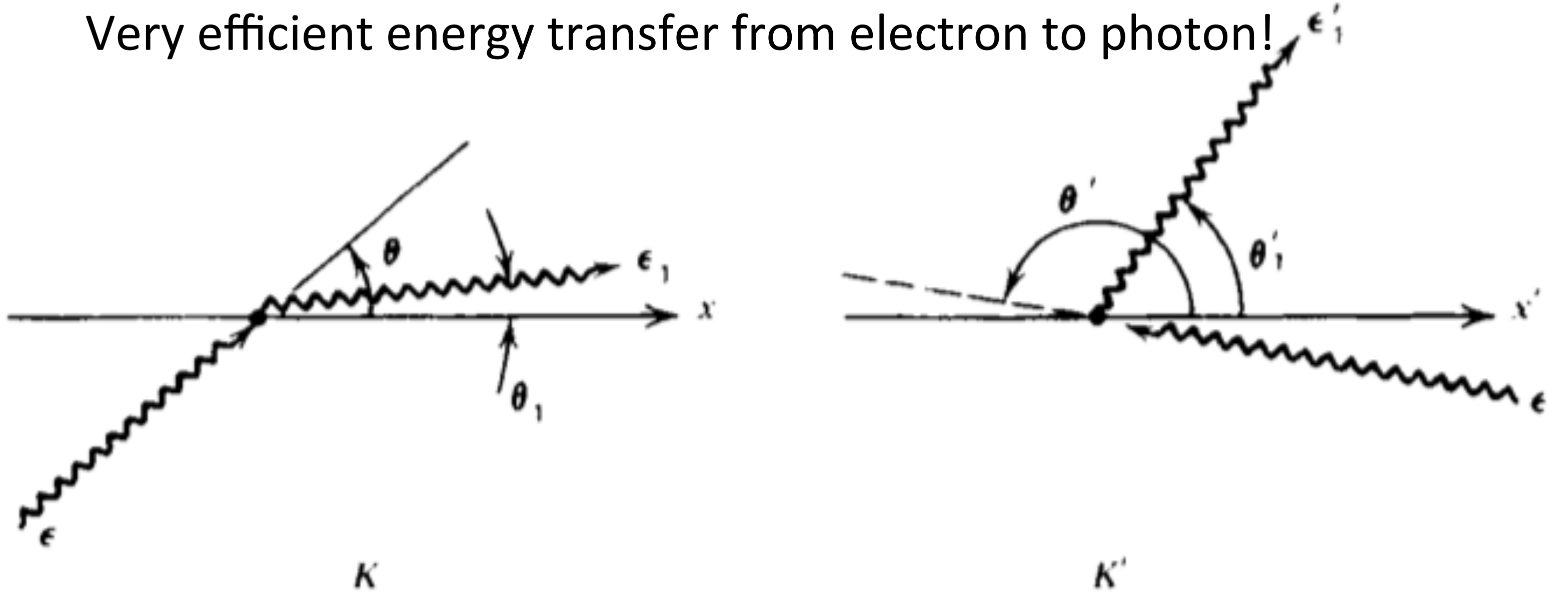
Scattering geometries in the observer's frame K and the electron rest frame K'

Step-3 : Back to the lab frame $\epsilon_1 = \epsilon_1{'}\,\gamma\left(1+\beta\cos\theta_1{'}\right)$

We have a second Doppler boost because now we go back to the lab frame. The photon energy has increased again (second relativistic Doppler boost)

Inverse Compton Scattering

Very efficient energy transfer from electron to photon!

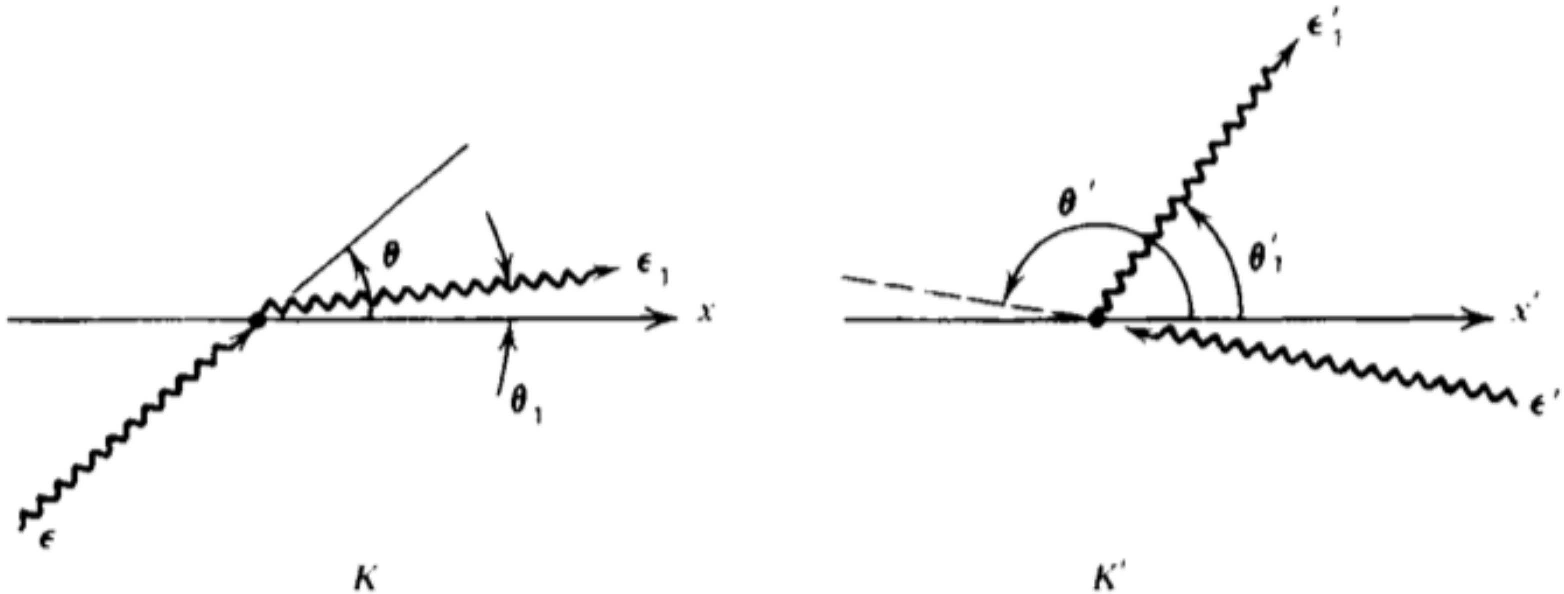


Scattering geometries in the observer's frame K and the electron rest frame K'

Summary:
Step 1. K → K' (1st Rel. Doppler boost)
Step 2. Compton Scattering (photon loses energy to the electron)
Step 3. K' → (2nd Rel. Doppler boost)

The photon gains an energy in step 1 and 3 by a factor gamma (so in total it gains a factor γ^2 because of the double Relativistic Doppler boost).

Inverse Compton Scattering



Scattering geometries in the observer's frame K and the electron rest frame K'

Step-3 : Back to the lab frame $\epsilon_1 = \epsilon_1{}' \gamma \left(1 + \beta \cos \theta_1{}'\right)$

Not all the quantities are calculated in lab frame K

By transforming angle s from K' to K

$$\epsilon_1 = \epsilon \frac{1 - \beta \cos \theta}{1 - \beta \cos \theta_1}$$

Inverse Compton Scattering Minimum and Maximum energy

$$\epsilon_1 = \epsilon \frac{1-\beta\cos\theta}{1-\beta\cos\theta_1}$$

ε_1 maximum when $\theta=\pi$ and $\theta_1=0$

$$\epsilon_1 = \epsilon \frac{1+\beta}{1-\beta}$$

Photon is scattered along velocity vector (head-on)

ε_1 minimum when $\theta=0$ and $\theta_1=\pi$

$$\epsilon_1 = \epsilon \frac{1-\beta}{1+\beta}$$

Photon is scattered from behind velocity vector (tail-on)

Inverse Compton Scattering Minimum and Maximum energy

$$\epsilon_1 = \epsilon \frac{1 - \beta \cos\theta}{1 - \beta \cos\theta_1}$$

$\varepsilon 1$ maximum when $\theta=\pi$ and $\theta_1=0$ $$\epsilon_1 = \epsilon \frac{1+\beta}{1-\beta}$$ Photon is scattered along velocity vector (head-on)

$$\epsilon_1 = \epsilon \gamma^2 (1+\beta)^2 \approx 4 \gamma^2 \epsilon$$

$$\nu' = 4/3\, \gamma^2 \nu$$

In case it is possible to measure both the initial and final photon energy/ frequency, then γ^2 can be deduced

Inverse Compton Scattering

The net power emitted by the electron in Compton scattering is,

$$P_{\text{compt}} = \frac{dE_{\text{rad}}}{dt} = \tfrac{4}{3}\sigma_T c\gamma^2\beta^2 U_{\text{ph}}.$$

Photon energy

Net power emitted in Synchrotron emission,

$$P_{\text{synch}} = \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_B.$$

Very similar

$$\frac{P_{\text{synch}}}{P_{\text{compt}}} = \frac{U_B}{U_{\text{ph}}}$$

Inverse Compton Scattering

$$\boxed{\frac{P_{\text{synch}}}{P_{\text{compt}}} = \frac{U_B}{U_{\text{ph}}}}$$

Radiation losses due to synchrotron emission and to Compton effect are in the Same ratio as the magnetic field density versus photon energy density

This is true for arbitrary value of electron velocities

Inverse Compton Scattering

$$P_{\text{compt}} = \frac{dE_{\text{rad}}}{dt} = \tfrac{4}{3}\sigma_T c\gamma^2\beta^2 U_{\text{ph}}.$$

$$P_{\text{synch}} = \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_B.$$

Why are these two powers so similar for very different physical mechanism operating?

The energy loss rate depends upon the electric field which accelerates the electron in its rest frame and it does not matter what the origin of that field is.

Single particle spectrum

We want to know spectrum of the scattered radiation emerging after a collision with a single electron

Energy of the electron after scattering $\epsilon_1 = \epsilon \frac{1 - \beta\cos\theta}{1 - \beta\cos\theta_1}$

Maximum Energy of the electron after scattering $\epsilon_1 = \epsilon \frac{1 + \beta}{1 - \beta}$

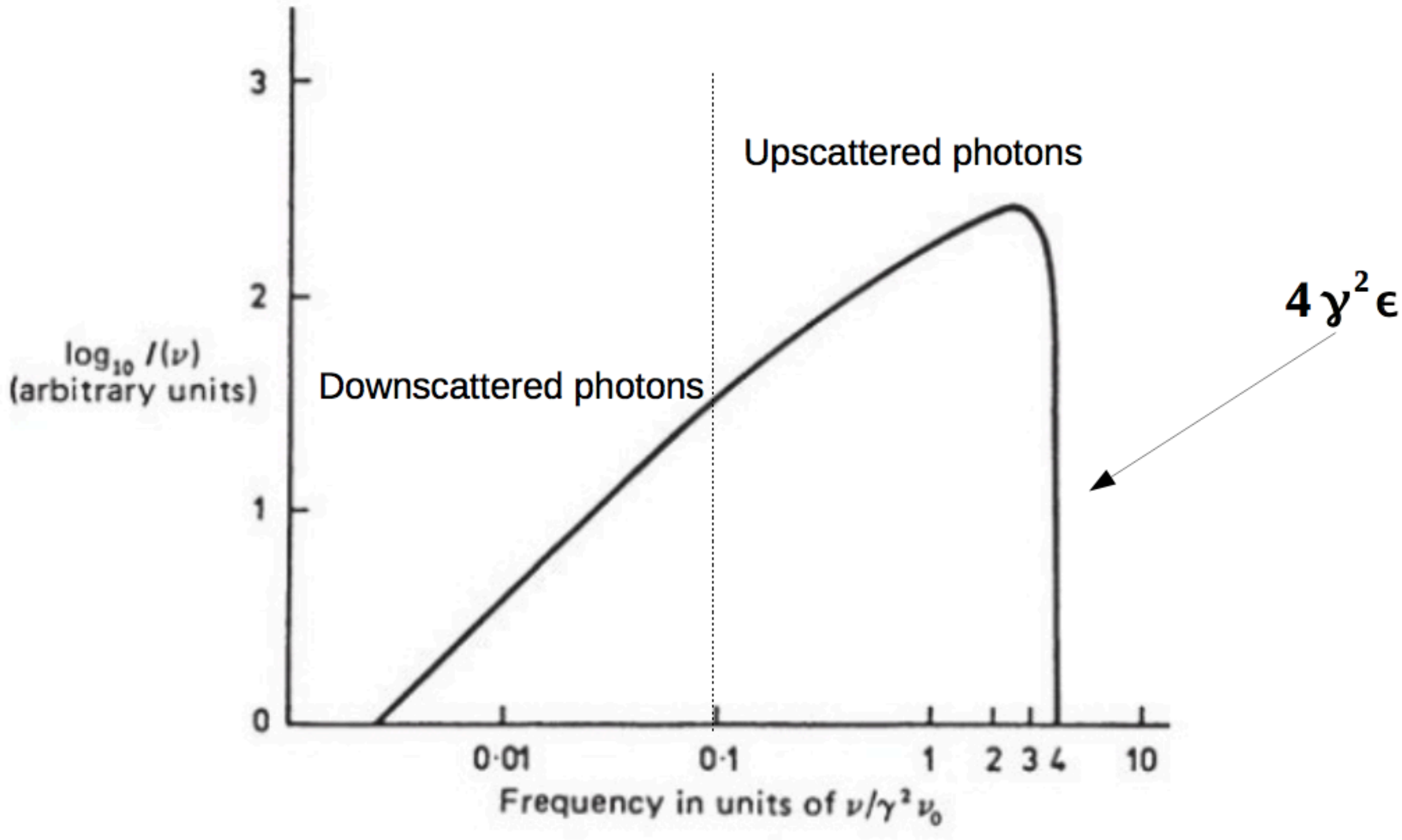
$$\epsilon_1 = \epsilon\,\gamma^2 (1+\beta)^2 \approx 4\gamma^2 \epsilon$$

So Inverse Compton spectra for single electron scattering should fall off at $4\gamma^2\,\varepsilon$

Average photon energy for single electron scattering ~ $(4/3)\,\gamma^2\,\varepsilon$

Multiple Scattering

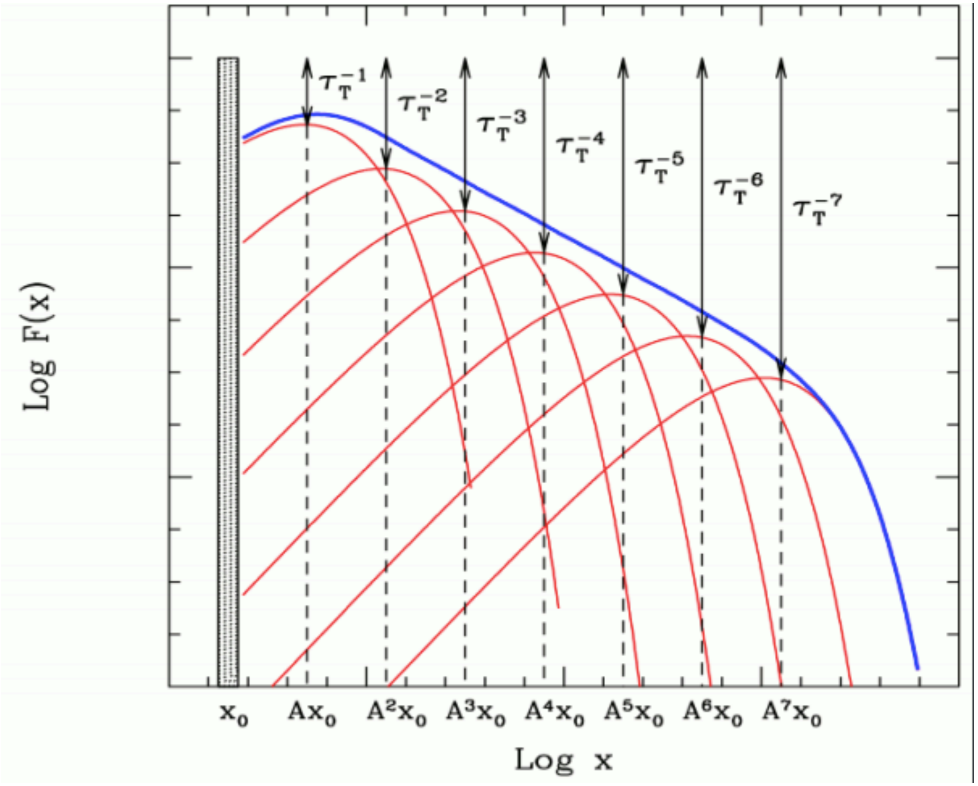
Single particle spectrum



Multi-scattering spectrum

Now, if the medium is of small optical depth the probability of a photon undergoing k scatterings before escaping the Comptonizing cloud is approximately:
Prob.$\approx\tau^k$

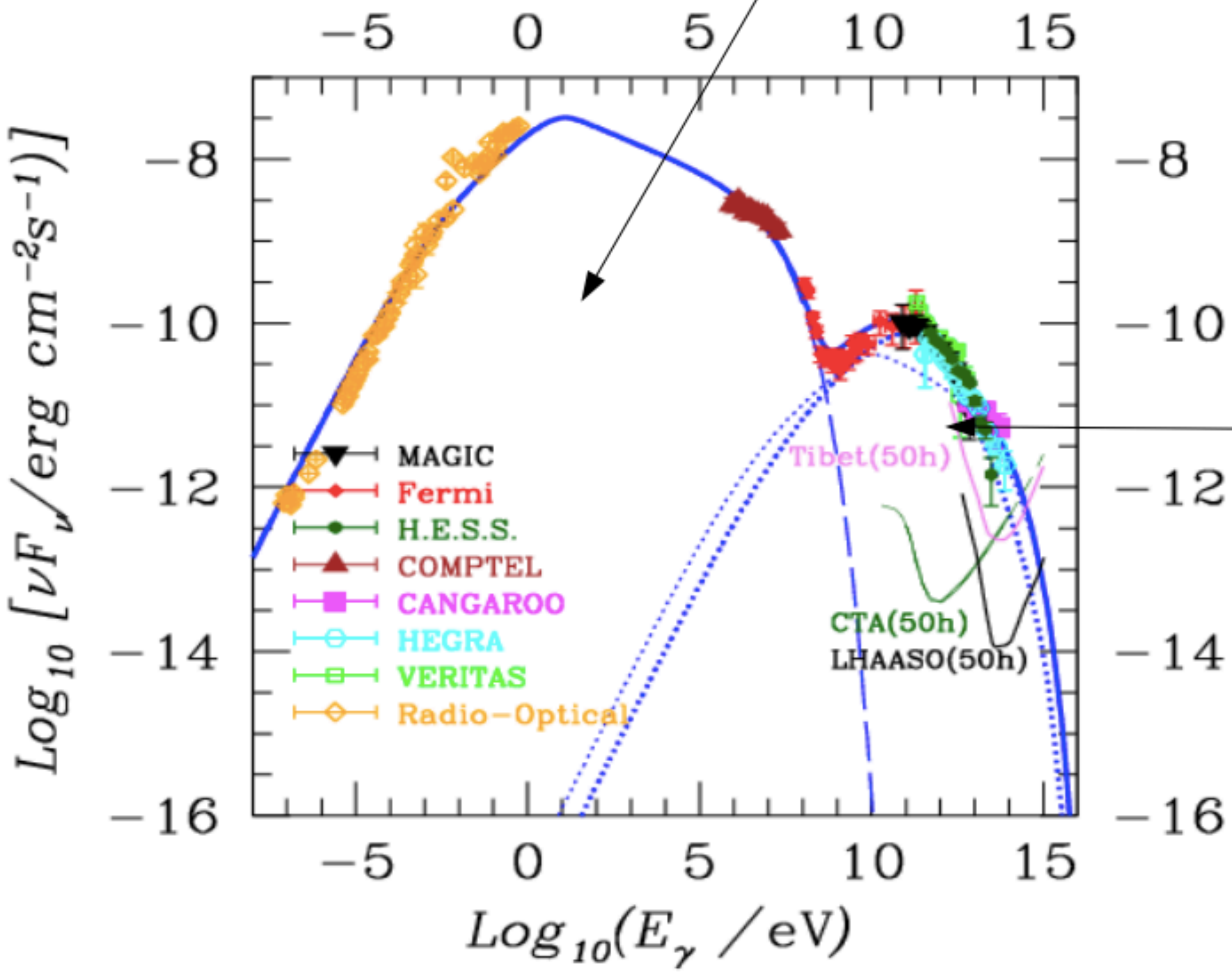
The intensity of the emerging Compton radiation will therefore decrease by a factor tau as a function of energy (or frequency) of the radiation.



($x=h\nu/mc^2$)

Refer to: https://apatruno.files.wordpress.com/2016/09/lecture101.pdf

Synchrotron spectrum of the Crab Nebula.



Synchrotron
Self Compton

Synchrotron + inverse compton

The same relativistic electrons radiate via synchrotron and inverse Compton; their contributions add up.

The increased cooling rate implies that the electron's radiative lifetime is consequently reduced.

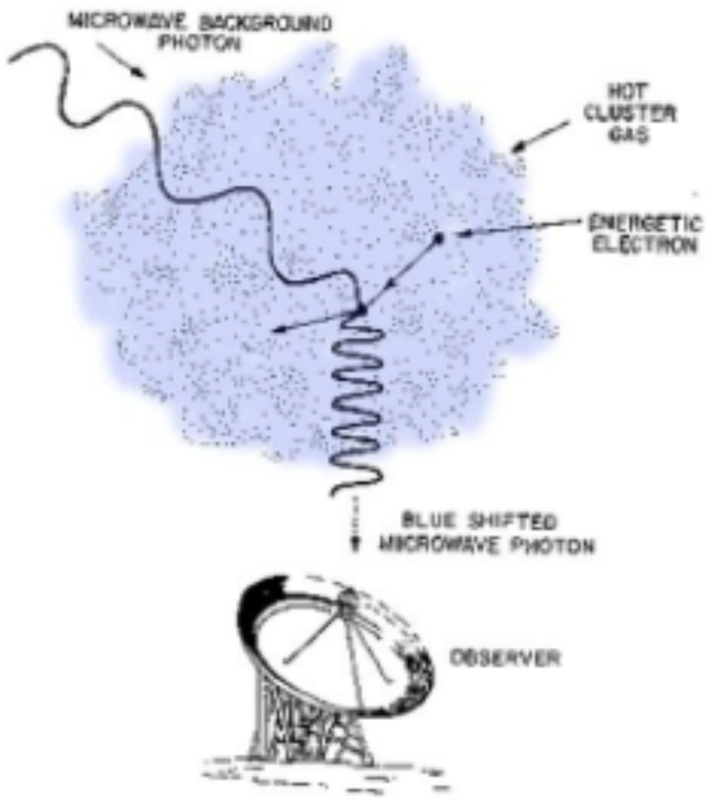
Astrophysical application
Sunyaev-Zeldovich effect

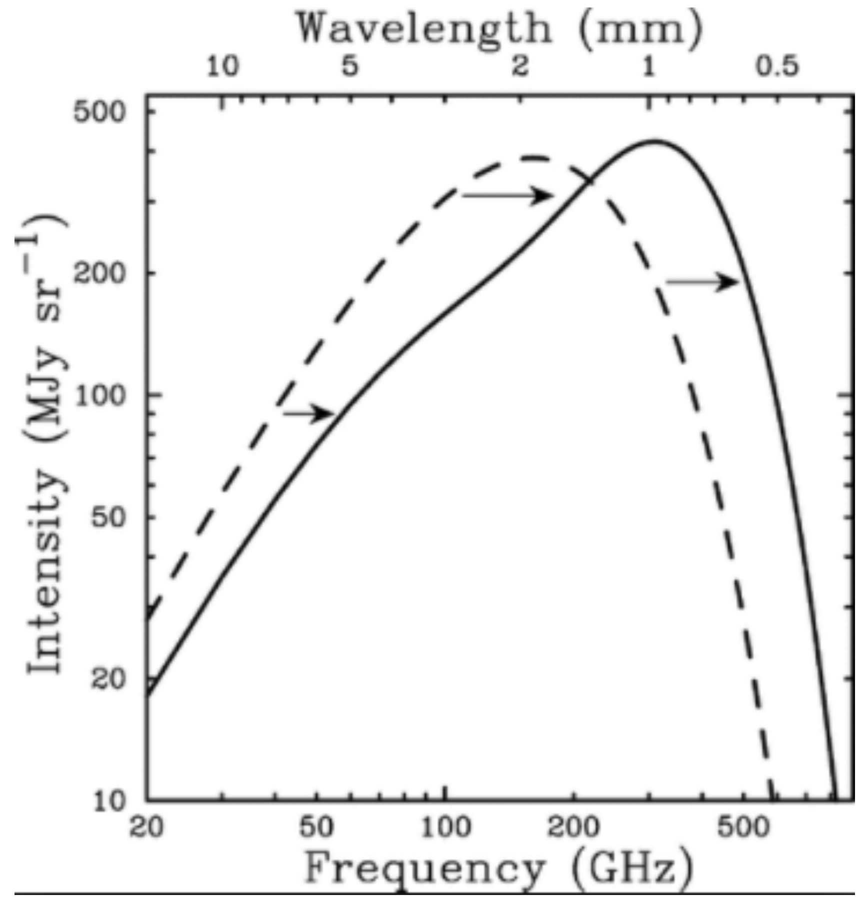
Sunyaev–Zel'dovich effect (named after Rashid Sunyaev and Yakov B. Zel'dovich and often abbreviated as the SZ effect) is the distortion of the cosmic microwave background radiation (CMB) through inverse Compton scattering by high energy electrons in galaxy clusters, in which the low energy CMB photons receive an average energy boost during collision with the high energy cluster electrons.

SZ effect arises from inverse Compton scattering of cosmic microwave background (CMB) photons off energetic free electrons in the hot, ionized gas within galaxy clusters. This creates a small fluctuation in the CMB temperature along the line of sight towards a cluster.

Observed distortions of the cosmic microwave background spectrum are used to detect the density perturbations of the universe. Using the Sunyaev–Zel'dovich effect, dense clusters of galaxies have been observed.

Astrophysical application Sunyaev-Zeldovich effect

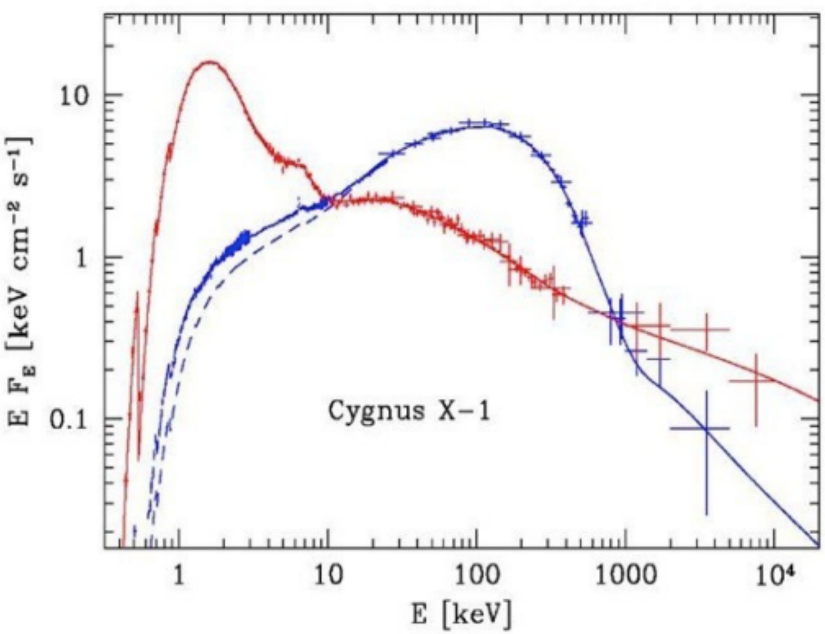




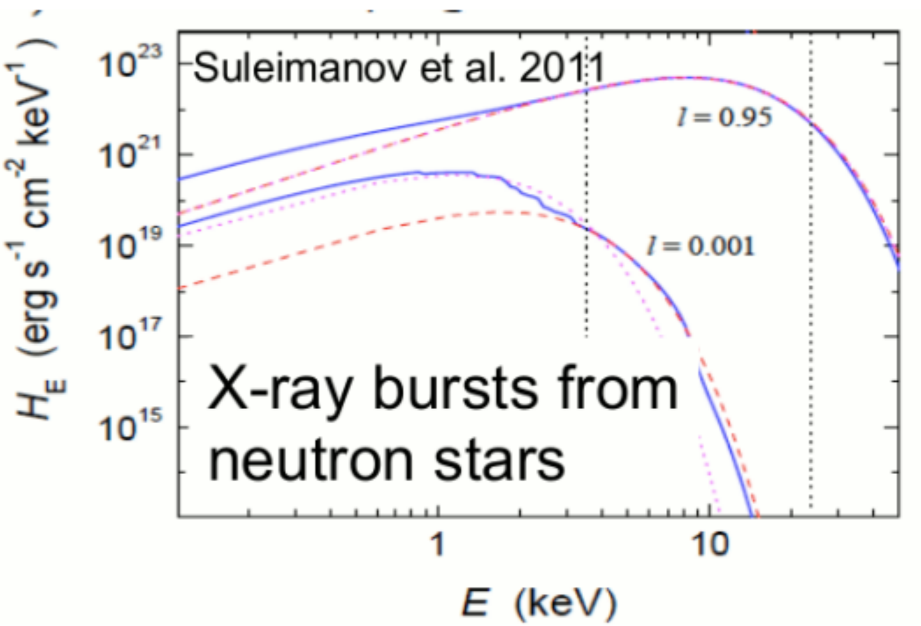
Refer: https://astro.uni-bonn.de/~bertoldi/projects/sz/ringberg/img1.html

Other astrophysical applications

Accreting Black Holes & Neutron Stars



Thermonuclear bursts on Neutron stars



End of Lecture 12