Electrodynamics and Radiative Processes I

Lecture 1 – Radiation & Radiative Transfer

Bhaswati Bhattacharyya

bhaswati@ncra.tifr.res.in

IUCAA-NCRA Graduate School

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Radiation

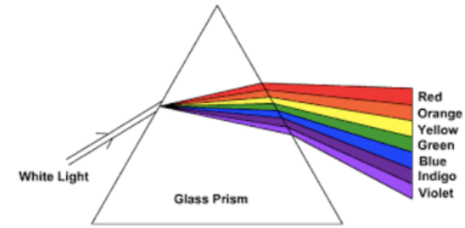
Astrophysics → study of radiation received from astronomical body

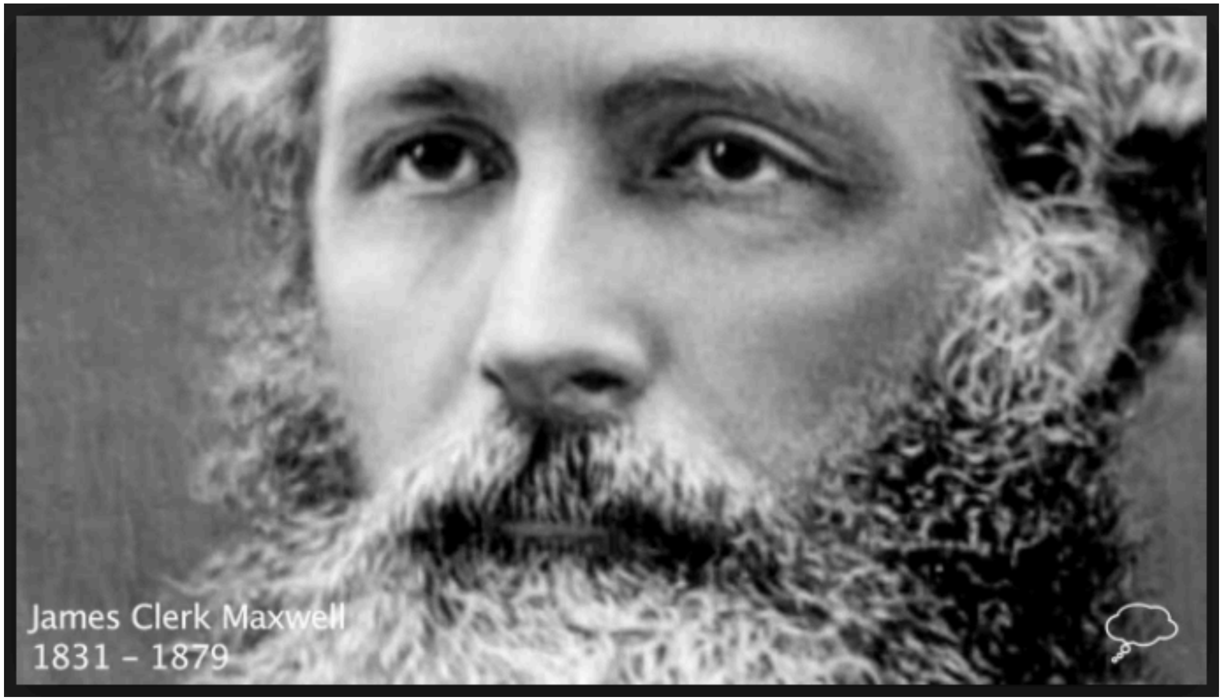
Production of radiation & **Interaction of radiation with matter**

Radiation travelling through matter gets modified:

a) Absorbed
b) Scattered
c) Cause stimulated emission
d) Get modified by spontaneous emission AND MORE

Electromagnetic radiation





Existence of electromagnetic wave predicted by **Maxwell** confirmed by **Hertz**

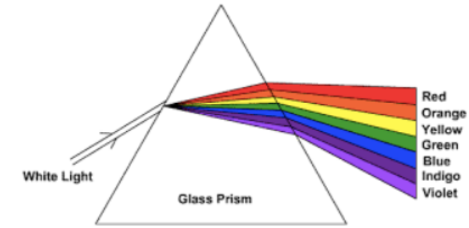
↓

Experiments showed oscillating electric charge radiate electromagnetic wave.

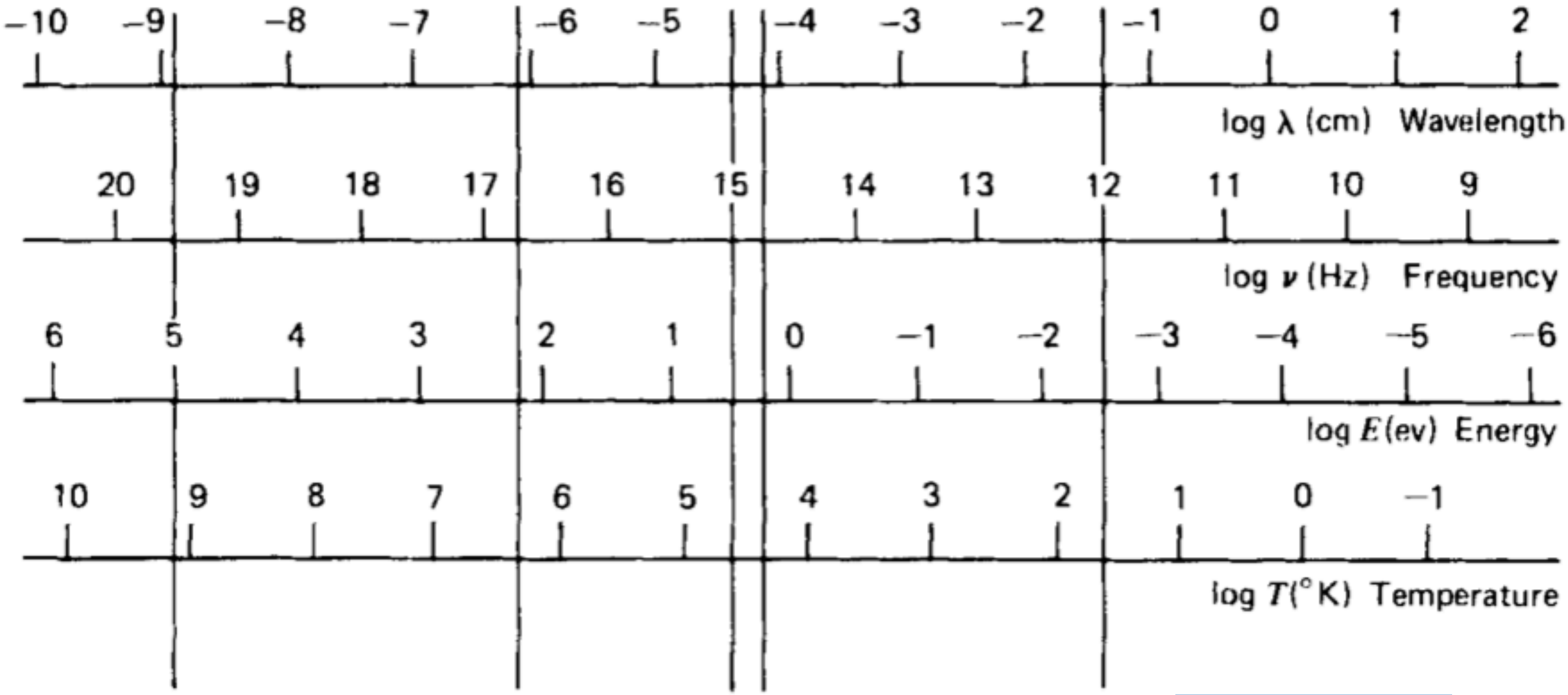
Energy of EM waves is proportional to energy of oscillation of electric charge.

Electromagnetic spectrum

$\lambda\nu = c$ $\qquad E = h\nu$ $\qquad T = E/k$



Electromagnetic radiation can be decomposed into spectrum.



γ-rays	x-rays	UV	IR	Radio
$T > 10^9$ K	10^9 K<T<10^6K	10^6 K<T<10^5K	10^4 K<T<100K	10 K<T<1 K

Electromagnetic Spectrum

Refraction, diffraction & interference indicate EM-radiation behaves as waves:

$$\lambda = c/\nu$$

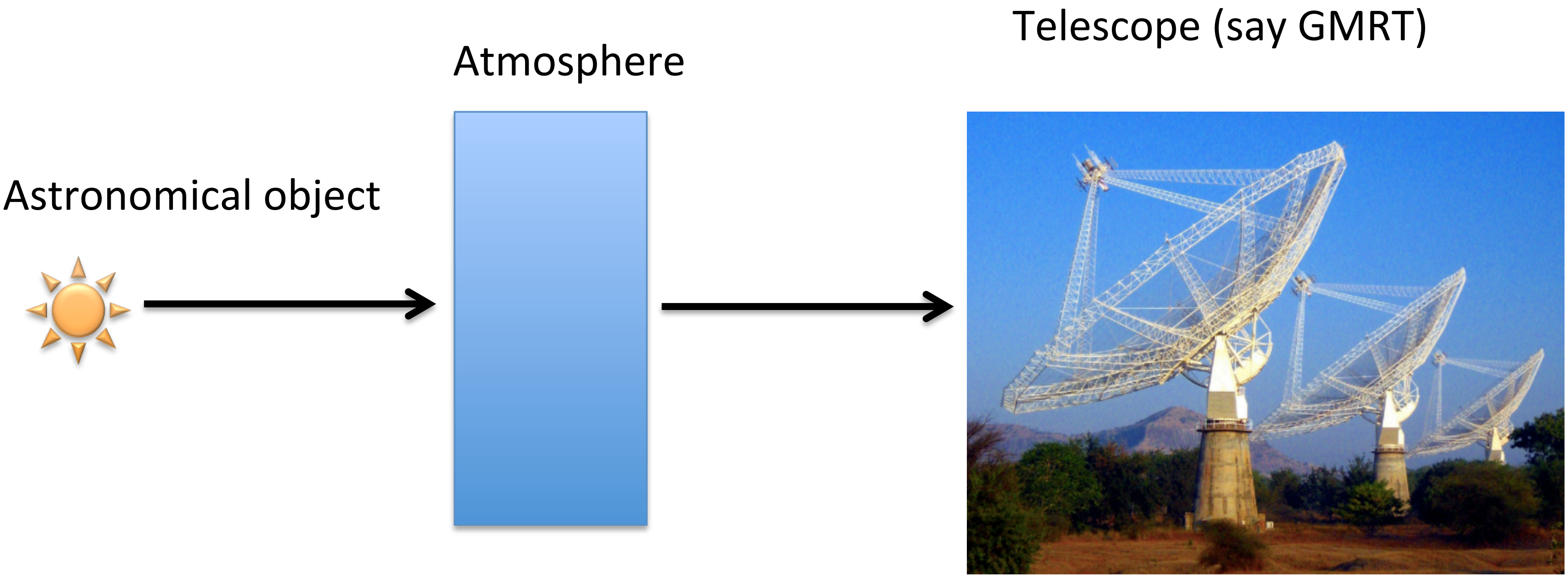
Photo-electric effect shows energy is given to or taken from radiation field in discrete quanta, or photons, with energy:

$$E = h\nu$$

For thermal energy emitted by matter in thermo-dynamic equilibrium, the characteristic photon energy is related to the temperature of the emitting material:

$$T = E/k$$

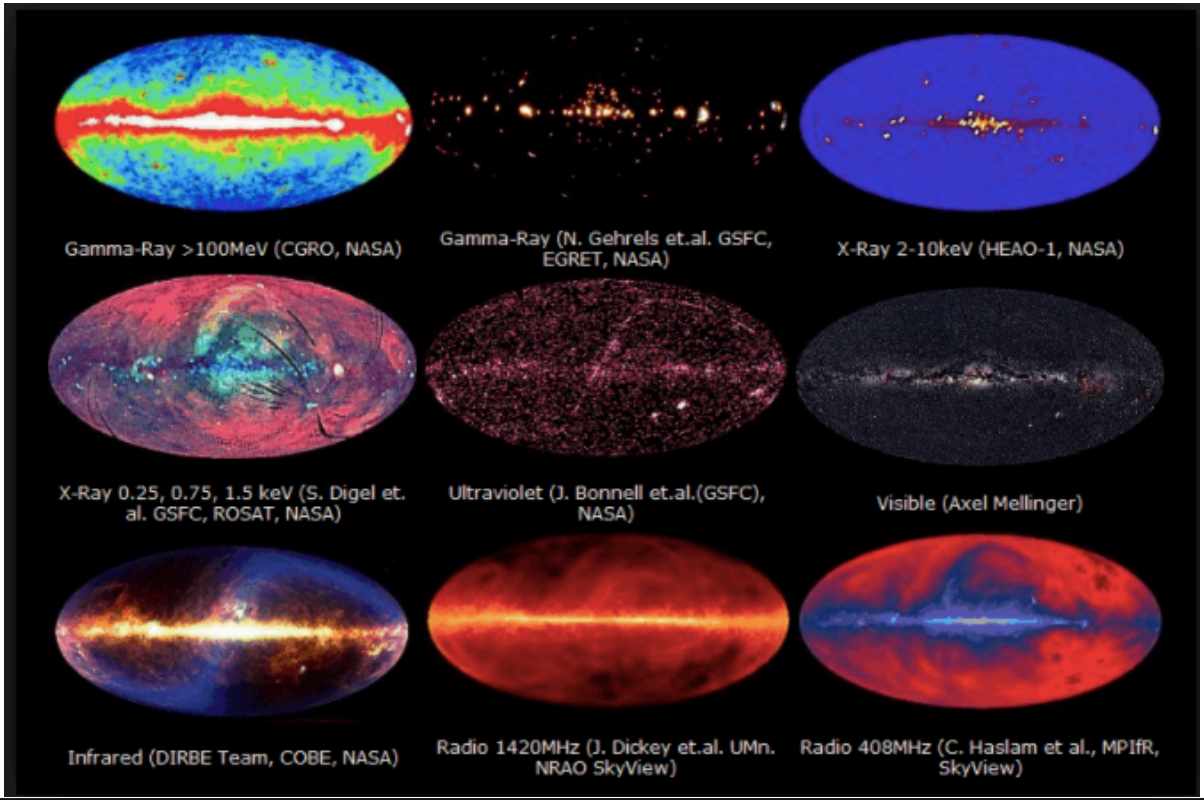
Radiation



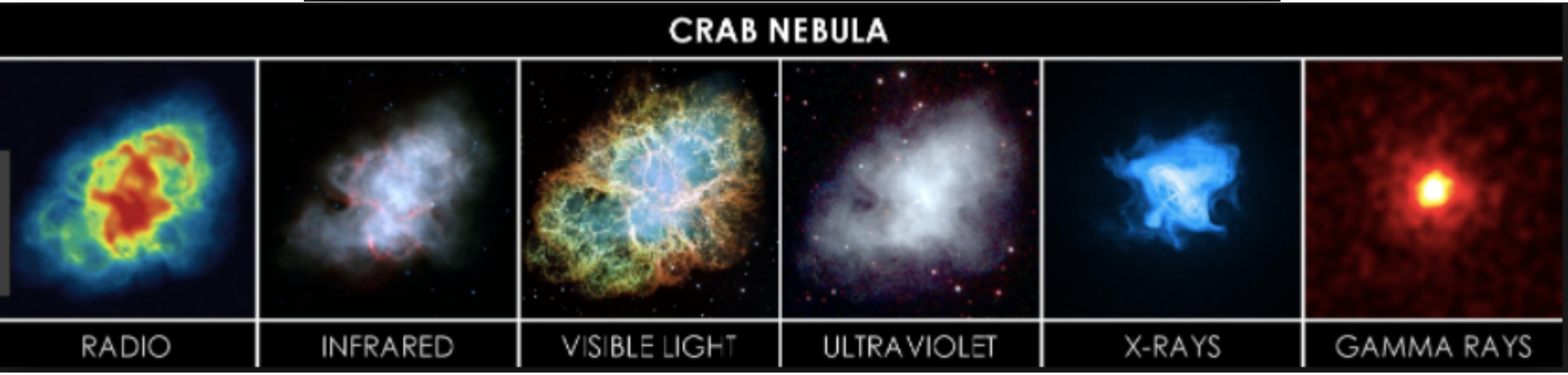
We are most sensitive to electromagnetic radiation.

Radiation can be considered to travel in straight lines for practical purposes.

Radiation

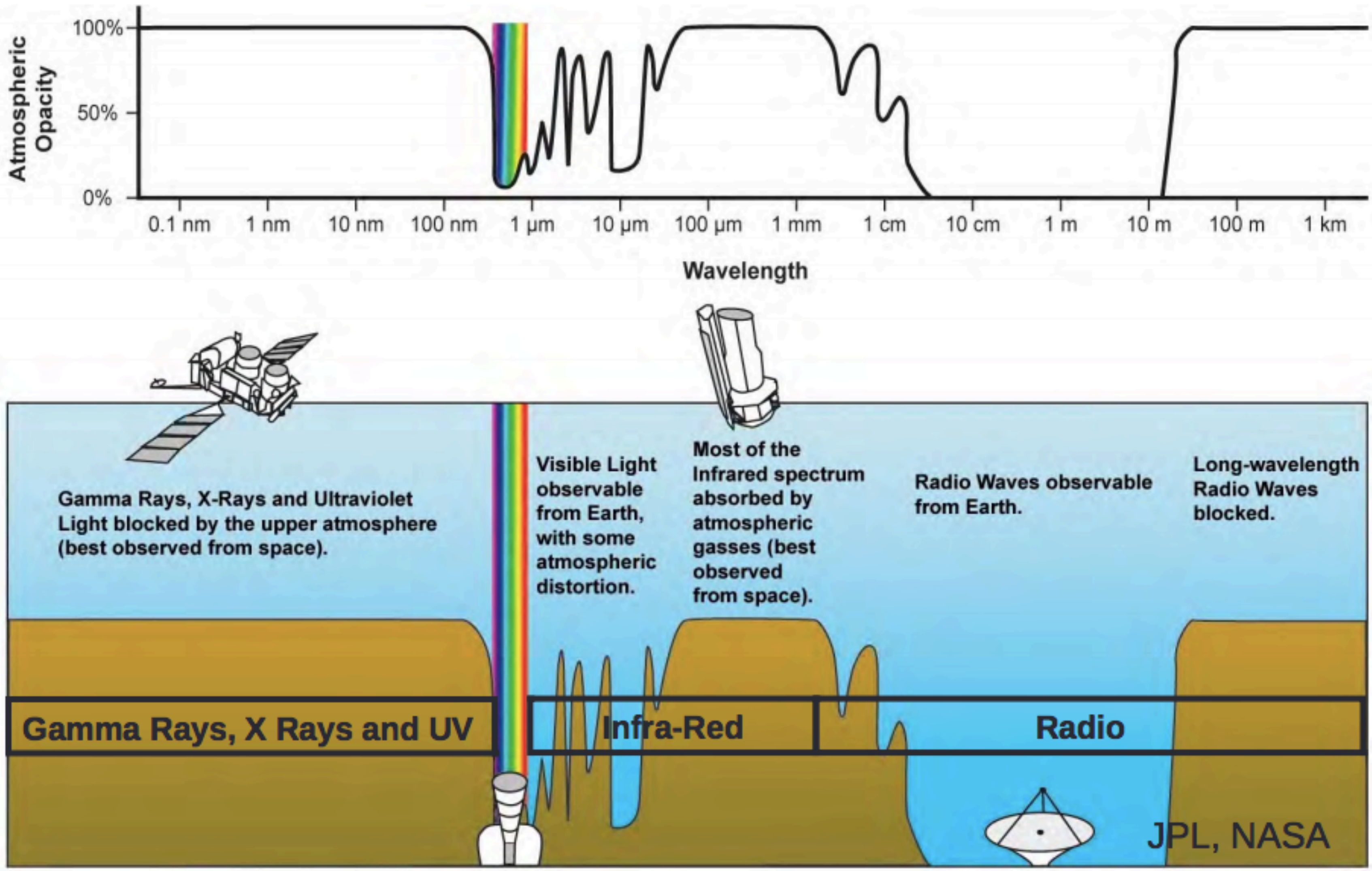


Gamma-Ray >100MeV (CGRO, NASA)
Gamma-Ray (N. Gehrels et.al. GSFC, EGRET, NASA)
X-Ray 2-10keV (HEAO-1, NASA)
X-Ray 0.25, 0.75, 1.5 keV (S. Digel et. al. GSFC, ROSAT, NASA)
Ultraviolet (J. Bonnell et.al.(GSFC), NASA)
Visible (Axel Mellinger)
Infrared (DIRBE Team, COBE, NASA)
Radio 1420MHz (J. Dickey et.al. UMn. NRAO SkyView)
Radio 408MHz (C. Haslam et al., MPIfR, SkyView)



CRAB NEBULA
RADIO
INFRARED
VISIBLE LIGHT
ULTRAVIOLET
X-RAYS
GAMMA RAYS

Radiation received at Earth



Radiative transfer

Energy transfer in the form of electromagnetic radiation

Propagation of emission through a medium is effected by absorption, emission and scattering processes

Radiative transfer equation describe this mathematically

Specific Intensity or Brightness I_ν
Transport of energy via radiation

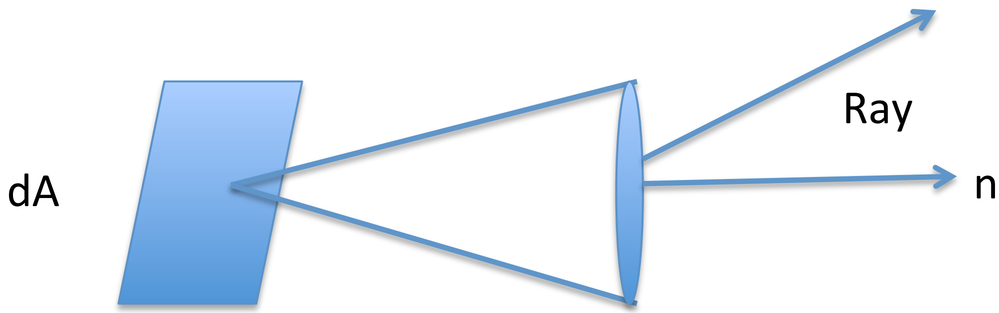


Figure: Geometry for normal incidence

Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

Specific Intensity or Brightness

$[I_\nu]$= erg cm^{-2} s^{-1} Hz $^{-1}$ ster^{-1}

Brightness does not decrease with distance

Specific Intensity or Brightness I_ν

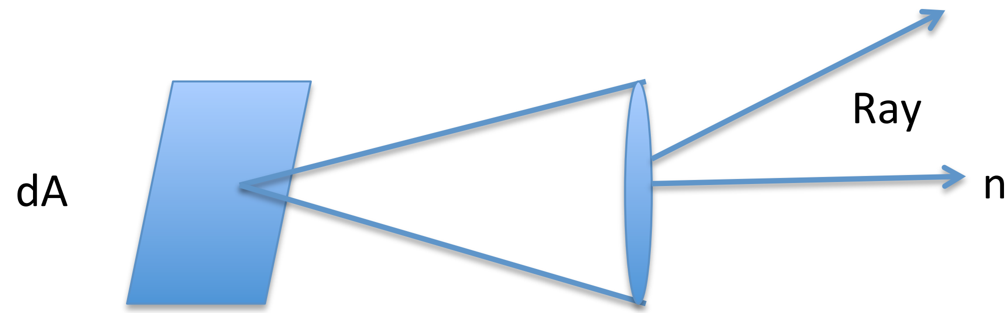


Figure: Geometry for normal incidence

Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ

$$dE = F\, dA\, dt$$

Energy flux

$[F] = \text{erg cm}^{-2}\,\text{s}^{-1}$

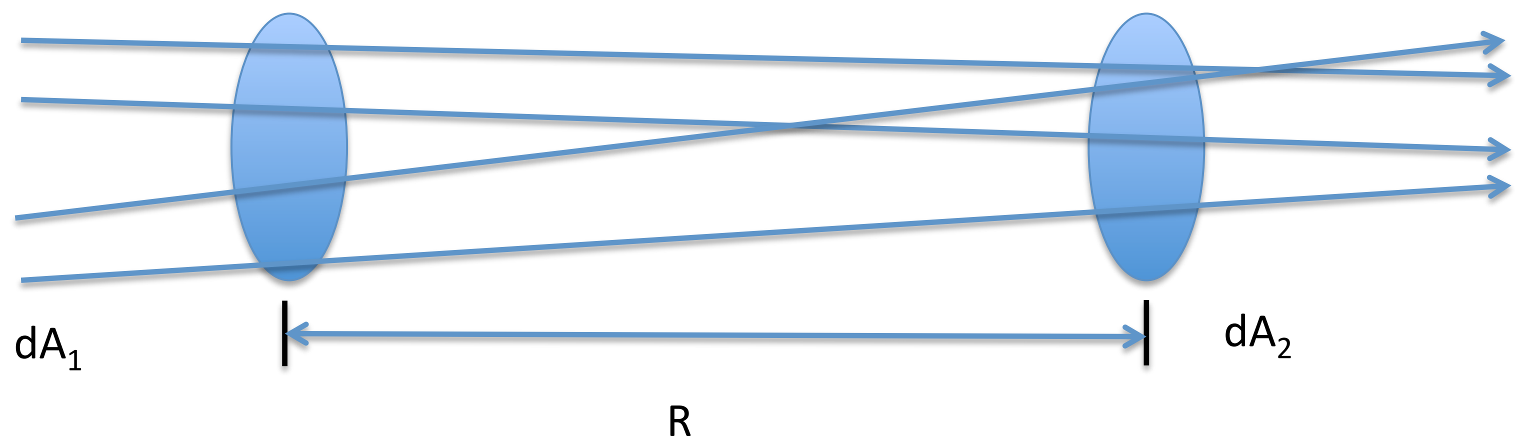
$$dE = I_\nu\, dA\, dt\, d\Omega\, d\nu$$

Specific Intensity or Brightness

$[I_\nu] = \text{erg cm}^{-2}\,\text{s}^{-1}\,\text{Hz}^{-1}\,\text{ster}^{-1}$

Brightness does not decrease with distance

Specific Intensity constant across a ray



$$dE_1 = I_{\nu_1}\, dA_1\, dt\, d\Omega_1\, d\nu_1 = dE_2 = I_{\nu_2}\, dA_2\, dt\, d\Omega_2\, d\nu_2.$$

$$d\Omega_1 = dA_2/R^2,\ d\Omega_2 = dA_1/R^2$$

$$I_{\nu_1} = I_{\nu_2}.$$

Specific Intensity for oblique incidence

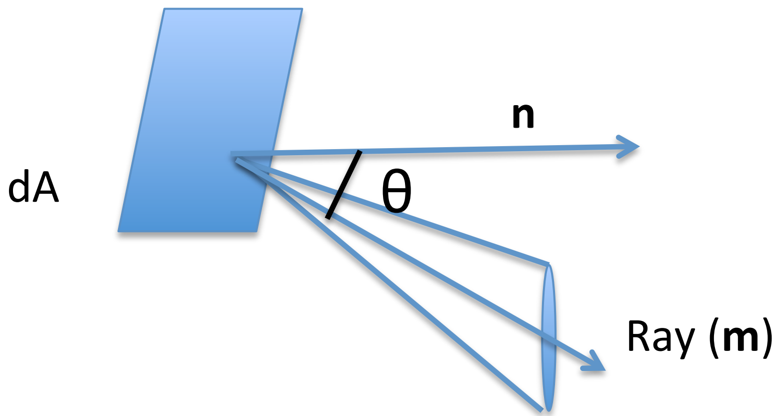


Figure: Geometry for oblique incidence

Flux through area dA (orientation n) and incident ray (orientation m) will be reduced by a factor of n.m=cosθ

$$dF_\nu(\text{erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}) = I_\nu \cos\theta \, d\Omega$$

Net flux

$$F_\nu = \int I_\nu \cos\theta \, d\Omega$$

Net flux is zero for isotropic radiation

Total flux

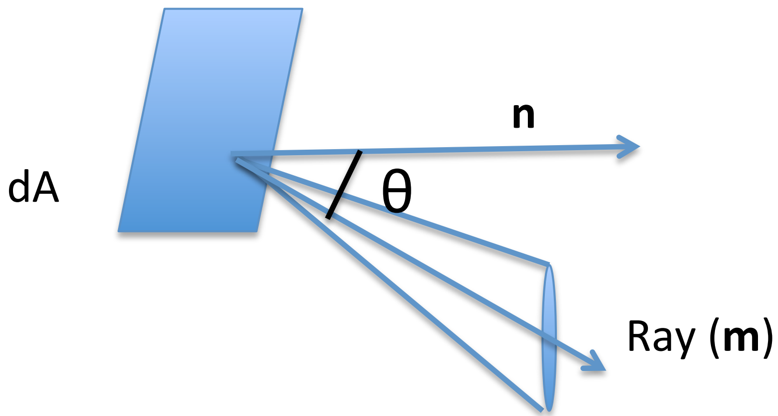


Figure: Geometry for oblique incidence

Integrate over frequency to get the total flux

$$F(\mathrm{erg\ s^{-1}\ cm^{-2}}) = \int F_\nu d\nu$$

Integrate over frequency to get the total Intensity

$$I(\mathrm{erg\ s^{-1}\ cm^{-2}\ ster^{-1}}) = \int I_\nu d\nu$$

Specific Intensity and total intensity

- ✓ Intrinsic property of the source
- ✓ Independent of the distance from the source

Total Intensity - Specific Intensity integrated over frequency

$$I(\text{erg s}^{-1}\text{ cm}^{-2}\text{ ster}^{-1}) = \int I_\nu d\nu$$

Flux density -
Total spectral power received (from a source) by a detector of unit area

Flux density - Dependent on distance to the source

Specific energy density
Radiative energy density

Specific energy density u_ν is defined as energy per unit volume per unit frequency range.

Let us first consider $u_\nu(\Omega)$; $dE = u_\nu(\Omega) dV\, d\Omega\, d\nu$; $dv = c\, dt\, dA$

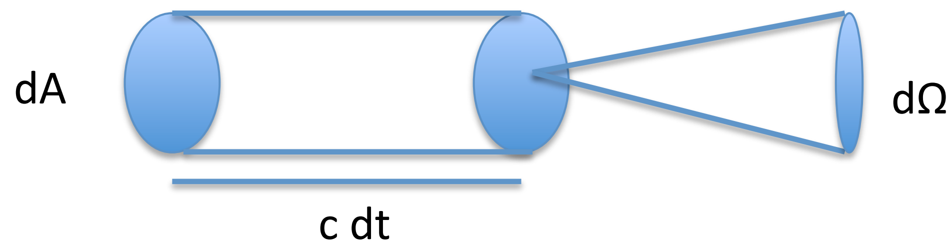


Figure: Electromagnetic energy in a cylinder

Energy crossing dA in time dt in frequency range $d\nu$ and into a solid angle $d\Omega$

$$dE = u_\nu(\Omega)\, dA\, c\, dt\, d\Omega\, d\nu.$$

Specific energy density
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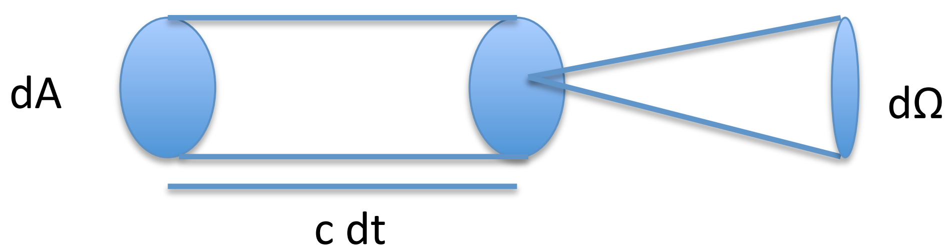


Figure: Electromagnetic energy in a cylinder

Energy crossing dA in time dt in frequency range $d\nu$ and into a solid angle $d\Omega$

$$dE = u_\nu(\Omega)\, dA\, c\, dt\, d\Omega\, d\nu \quad \Longleftrightarrow \quad dE = I_\nu\, dA\, dt\, d\Omega\, d\nu$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

Specific energy density
Radiative energy density

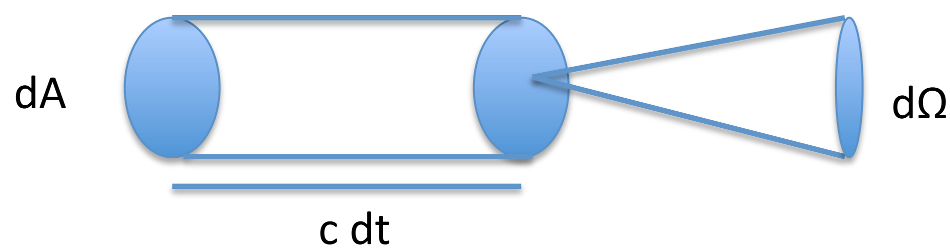


Figure: Electromagnetic energy in a cylinder

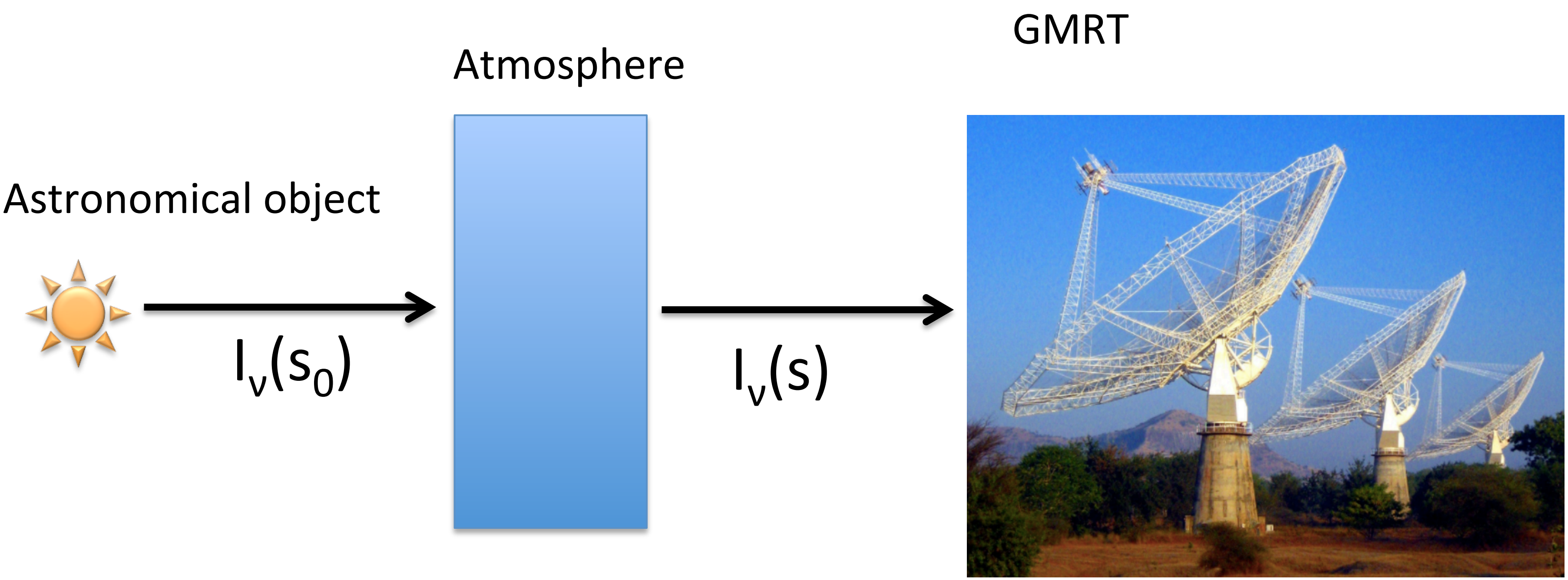
$$u_\nu(\Omega) = \frac{I_\nu}{c} \longrightarrow u_\nu = \int u_\nu(\Omega)\, d\Omega = \frac{1}{c}\int I_\nu\, d\Omega,$$

Mean Intensity

$$J_\nu = \frac{1}{4\pi}\int I_\nu\, d\Omega$$

$$u_\nu = \frac{4\pi}{c} J_\nu$$

Radiative transfer

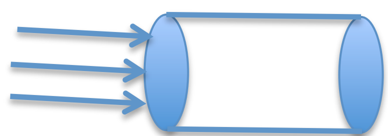


Radiative transfer is phenomenon of energy transfer from electromagnetic radiation.

Radiative Transfer
Emission Coefficient j_ν

If a ray passes through matter energy may be added or subtracted from it and specific intensity will not be constant : Emission and Absorption

Considering only Emission



$$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu,$$

Spontaneous emission coefficient :
Energy emitted per unit time per unit solid angle per unit volume

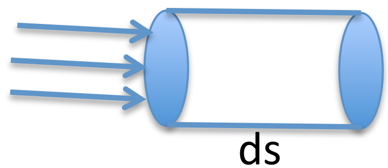
$[j_\nu]$= erg cm^{-3} s^{-1} Hz^{-1} Sr^{-1}

Specific Intensity added to the beam by spontaneous emission

$$dI_\nu = j_\nu \, ds$$

Radiative Transfer
Emission Coefficient j_ν

Considering only Emission



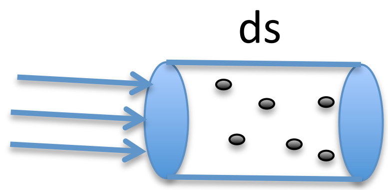
$$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu \quad \longleftrightarrow \quad dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

$$dI_\nu = j_\nu \, ds$$

Specific Intensity added to the beam by spontaneous emission

Absorption Coefficient α_ν

Loss of brightness in a beam as it travels a distance ds is



$$dI_\nu = -\alpha_\nu I_\nu \, ds \qquad (4)$$

Absorption coefficient

$[\alpha_\nu] = cm^{-1}$

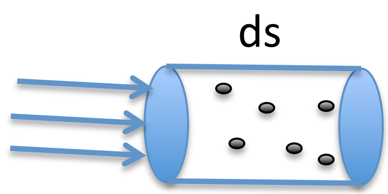
Consider n particles per unit volume each with cross-section σ_ν $\quad \alpha_\nu = n\sigma_\nu$

$$\alpha_\nu = \rho\kappa_\nu$$

Mass density

Mass absorption coefficient ($cm^2 \, g^{-1}$)

Radiative Transfer Equation



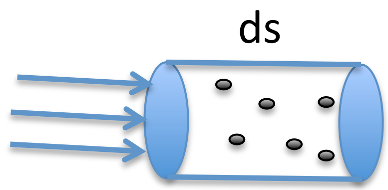
$$dI_\nu = j_\nu \, ds$$

$$dI_\nu = -\alpha_\nu I_\nu \, ds$$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Fundamental equation of Radiative transfer

Radiative Transfer Equation



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Emission only

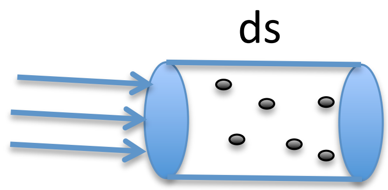
$$\frac{dI_\nu}{ds} = j_\nu$$

Solution

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')\,ds'$$

Increase in brightness
= emission coefficient integrated across line of sight

Radiative Transfer Equation



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Absorption only

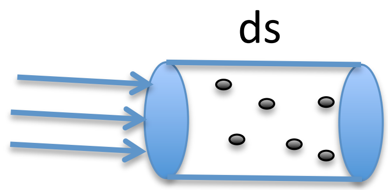
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

Solution

$$I_\nu(s) = I_\nu(s_0)\exp\left[-\int_{s_0}^{s}\alpha_\nu(s')\,ds'\right]$$

Decrease in brightness along the ray by exponential of absorption
Coefficient integrated across line of sight

Radiative Transfer Equation



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Emission only

$$\frac{dI_\nu}{ds} = j_\nu$$

Solution

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')\,ds'$$

Absorption only

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

Solution

$$I_\nu(s) = I_\nu(s_0)\exp\left[-\int_{s_0}^{s} \alpha_\nu(s')\,ds'\right]$$

Brightness and Flux density

Brightness $I_\nu(A, t, \nu, \Omega)$

- Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ
- Also called Specific intensity, spectral radiance
- Property of the astronomical source and conserved along a ray in empty space
- Unit $[I_\nu]$= erg cm^{-2} s^{-1} Hz $^{-1}$ $ster^{-1}$

Flux density $F_\nu(A, \nu, t)$

- Energy crossing dA in frequency range dν
- Spectral power received from a source by a detector of unit area
- Not intrinsic property of the source but dependent on the distance to the source
- Unit $[F_\nu]$= erg cm^{-2} s^{-1}Hz $^{-1}$= W m^{-2} Hz^{-1}, 1 Jy =10^{-26} W m $^{-2}$ Hz^{-1}

When do we use spectral Brightness and when flux density to describe a source?

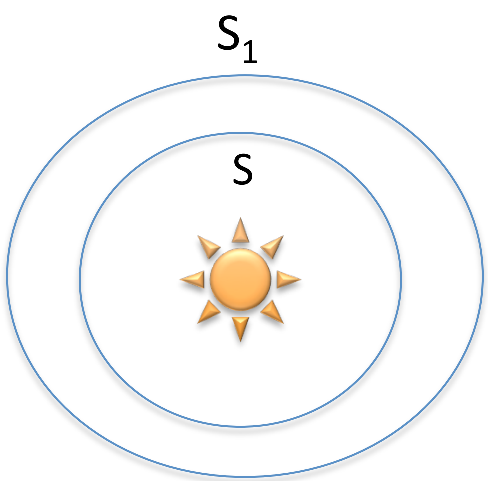
Flux density for compact sources:
If a source is unresolved, i.e much smaller in angular size than the point-source response of the eye or telescope observing it, its flux density can be measured.

Spectral Brightness for larger sources:
If a source is much larger than the point-source response, its spectral brightness at any position on the source can be measured directly, but its flux density must be calculated by integrating the observed spectral brightness over the source solid angle. Consequently, flux densities are normally used to describe only relatively compact sources.

Flux from isotropic source

Inverse square law

A source of radiation is isotropic if it emits energy equally in all directions.



Put imaginary spherical surfaces at S_1 and S at radii r and r_1

Conservation of energy :
Total energy passing through S_1, S will be same

$$F(r_1)\cdot 4\pi r_1^2 = F(r)\cdot 4\pi r^2,$$

$$F(r) = \frac{F(r_1) r_1^2}{r^2}$$

$$F = \frac{\text{constant}}{r^2}$$

Optical depth τ_ν

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\, ds'$$

Dimension of τ_ν ?

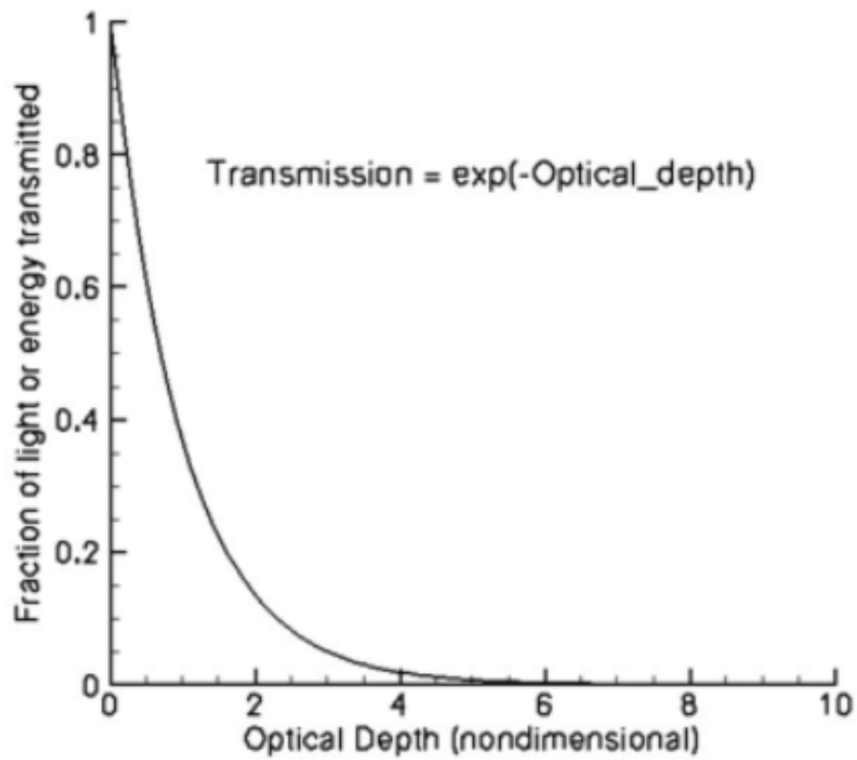
Optically **THICK** medium $\tau_\nu > 1$ → Medium is opaque
Radiation gets absorbed

Optically **THIN** medium $\tau_\nu < 1$ → Medium is transparent
Radiation does not gets absorbed

Optical depth τ_ν

Optically **THIN** medium $\tau_\nu < 1$

Optically **THICK** medium $\tau_\nu > 1$



Radiative transfer equation

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Dividing both side by α_ν

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Source function S_ν

$$S_\nu \equiv \frac{j_\nu}{\alpha_\nu}$$

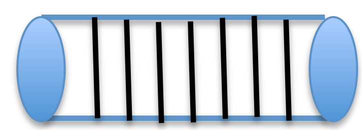
Specific intensity radiated by body

Solution of Radiative transfer equation

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Formal solution of radiative transfer equation

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \qquad \text{(5)}$$

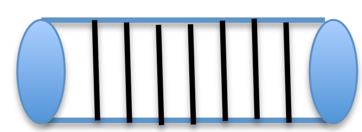


Solution of Radiative transfer equation

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Formal solution of radiative transfer equation

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \qquad (5)$$



1st term
Initial intensity attenuated by medium

2nd term
Medium adds to the radiation
And absorbs part of the added radiation

Solution of Radiative transfer equation
(a) τ_ν >>1 limit

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

Optically depth τ_ν >>1 i.e. Medium is Opaque

$$I_\nu(\tau_\nu) = \cancel{I_\nu(0)e^{-\tau_\nu}} + S_\nu(1 \cancel{- e^{-\tau_\nu}})$$

No Initial intensity attenuated by medium

Black-body radiation

Solution of Radiative transfer equation
(a) τ_ν >>1 limit

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

Optically depth τ_ν >>1 i.e. Medium is Opaque

$$I_\nu(\tau_\nu) = \cancel{I_\nu(0)e^{-\tau_\nu}} + S_\nu(1 \cancel{- e^{-\tau_\nu}})$$

No Initial intensity attenuated by medium

Specific intensity tries to approach Black body-radiation

Black-body radiation

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Black body radiation
(I will assume you are familiar)

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Rayleigh-Jeans Law $h\nu \ll kT$

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

Wien Law $h\nu \gg kT$

$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2} \exp\left(\frac{-h\nu}{kT}\right)$$

Black body radiation
(I will assume you are familiar)

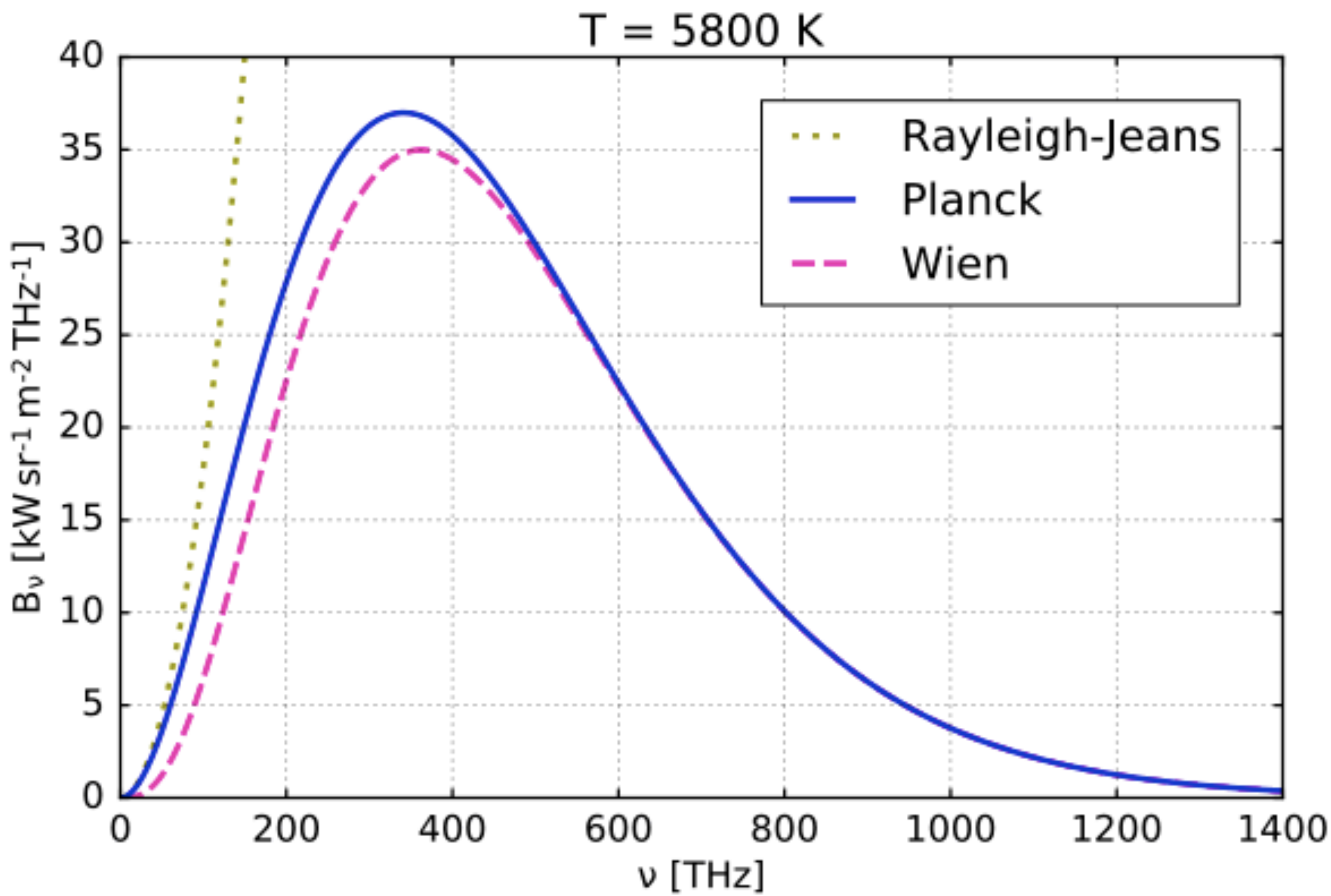
$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Rayleigh-Jeans Law hν << kT

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

Wien Law hν >> kT

$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2} \exp\left(\frac{-h\nu}{kT} \right)$$



T = 5800 K
Rayleigh-Jeans
Planck
Wien
Bν [kW sr-1 m-2 THz-1]
ν [THz]
0
5
10
15
20
25
30
35
40
0
200
400
600
800
1000
1200
1400

Solution of Radiative transfer equation
(b) τ_ν <<1 limit

$$I_\nu(\tau_\nu) = \cancel{I_\nu(0)e^{-\tau_\nu}} + S_\nu(1 - e^{-\tau_\nu})$$

Considering no background

$S_\nu \tau_\nu$

Small fraction of black body radiation

Brightness temperature T_b

Temperature of a black-body having the same brightness at that frequency

$$I_\nu = B_\nu(T_b).$$

Brightness Temperature

Low-frequency regime

$$I_\nu = \frac{2\nu^2}{c^2} k T_b \quad \longrightarrow \quad T_b = \frac{c^2}{2\nu^2 k} I_\nu$$

Brightness temperature T_b

Temperature of a black-body having the same brightness at that frequency

$$I_\nu = B_\nu(T_b).$$

Brightness Temperature

Low-frequency regime

$$I_\nu = \frac{2\nu^2}{c^2} k T_b \quad \longrightarrow \quad T_b = \frac{c^2}{2\nu^2 k} I_\nu$$

Radiative Transfer equation in terms of Brightness temperature

Solution

$$\frac{dT_b}{d\tau_\nu} = -T_b + T, \quad \longrightarrow \quad T_b = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

Radiative transfer

$$F_\nu = \int I_\nu \cos\theta \, d\Omega$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

$$dI_\nu = j_\nu \, ds \quad \longrightarrow \quad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s') \, ds'$$

$$dI_\nu = -\alpha_\nu I_\nu \, ds \quad \longrightarrow \quad I_\nu(s) = I_\nu(s_0) \exp\left[-\int_{s_0}^{s} \alpha_\nu(s') \, ds' \right]$$

$$\alpha_\nu = n\sigma_\nu$$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \quad \xrightarrow{\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s') \, ds'} \quad \frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

End of Lecture 1

Reference: Rybicki Lightman Chapter 1.1,1.2,1.3,1.4

Next lecture : 8th August

Topic of next Lecture:
Thermal and black body radiation
Line Radiative Transfer
Preparation : Lecture 1