Electrodynamics and Radiative Processes I

Lecture 9 – Bremsstrahlung

Bhaswati Bhattacharyya

bhaswati@ncra.tifr.res.in

IUCAA-NCRA Graduate School

August-September 2018

Date : 6th September 2018

Bremsstrahlung
Layout

(1) Emission from single speed electron
 pick rest frame of ion
 calculate dipole radiation
 correct for quantum effects (Gaunt factor)

(2) Emission from collection of electron
 Thermal bremsstrahlung
 Free-Free Absorption
 Non-thermal bremsstrahlung

(3) Relativistic bremsstrahlung (Virtual Quanta)

Thermal Bremsstrahlung Recap

Free electrons are accelerated (decelerated) in the coulomb field of ionised nuclei and radiate energy)

Consider a charged particle at a specific impact parameter(b) and velocity(v).

When a charged particle accelerates it emits radiation.

Acceleration is a function of b, v and Z.

Intensity spectrum via Fourier Transform.

Integrate (exact details tricky – gives rise to the Gaunt Factor $\bar{g}_{ff}$, which is a function of ν,T,Z).

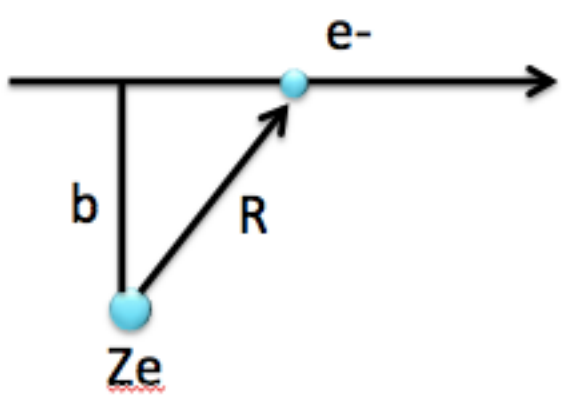
Include term for collision rate (depends on number densities n_e and n_i). Integrate over v .

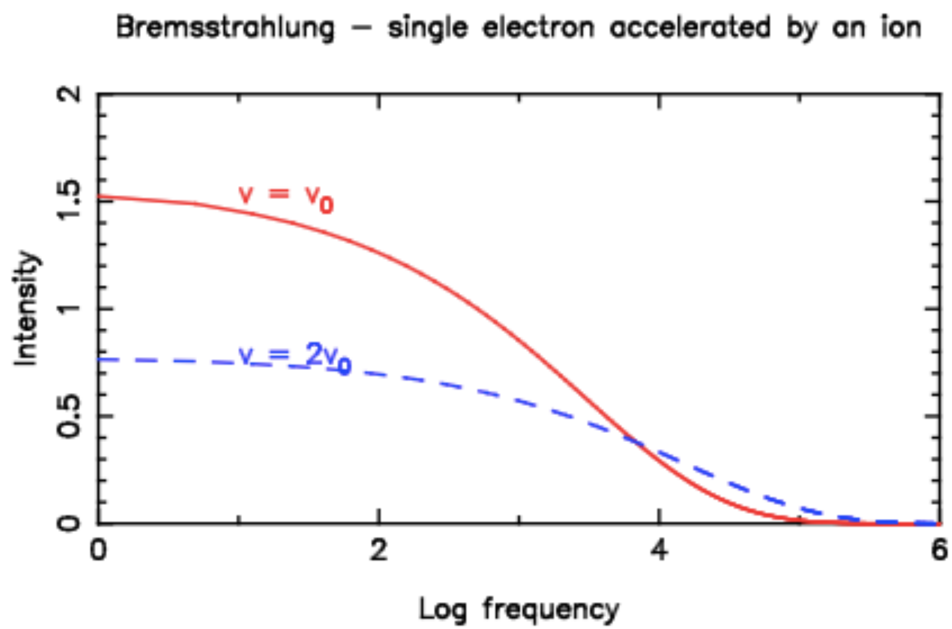
Assume plasma in thermal equilibrium →Maxwellian distribution of v .

Bremsstrahlung (Recap)
Emission from a single speed electron

Emission from a single speed electron

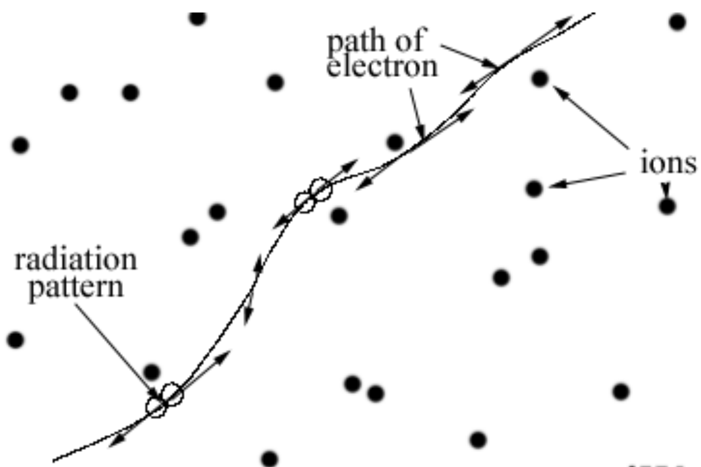
$$\frac{dW(b)}{d\omega} = \begin{cases} \dfrac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2}, & b \ll v/\omega \\ 0, & b \gg v/\omega. \end{cases}$$





Bremsstrahlung(Recap)
Emission from multiple single speed electron

Bunch of electrons, all with the same speed, v, which interact with a bunch of ions.



$$\frac{dW}{d\omega\, dV\, dt} = n_e n_i 2\pi v \int_{b_{\min}}^{\infty} \frac{dW(b)}{d\omega} b\, db$$

$$b_{\max} \cong \frac{v}{\omega} \qquad b_{\min}^{(1)} = \frac{4Ze^2}{\pi m v^2} \qquad b_{\min}^{(2)} = \frac{h}{mv}$$

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16 e^6}{3c^3 m^2 v} n_e n_i Z^2 \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{16 e^6}{3c^3 m^2 v} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

$$\frac{dW}{d\omega\, dV\, dt} = \frac{16\pi e^6}{3\sqrt{3}\, c^3 m^2 v} n_e n_i Z^2 g_{ff}(v, \omega)$$

Thermal Bremsstrahlung Emission

$$\boxed{\frac{dW}{dV\,dt\,d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT} \bar{g}_{ff}}$$

particle energy

high energy cutoff

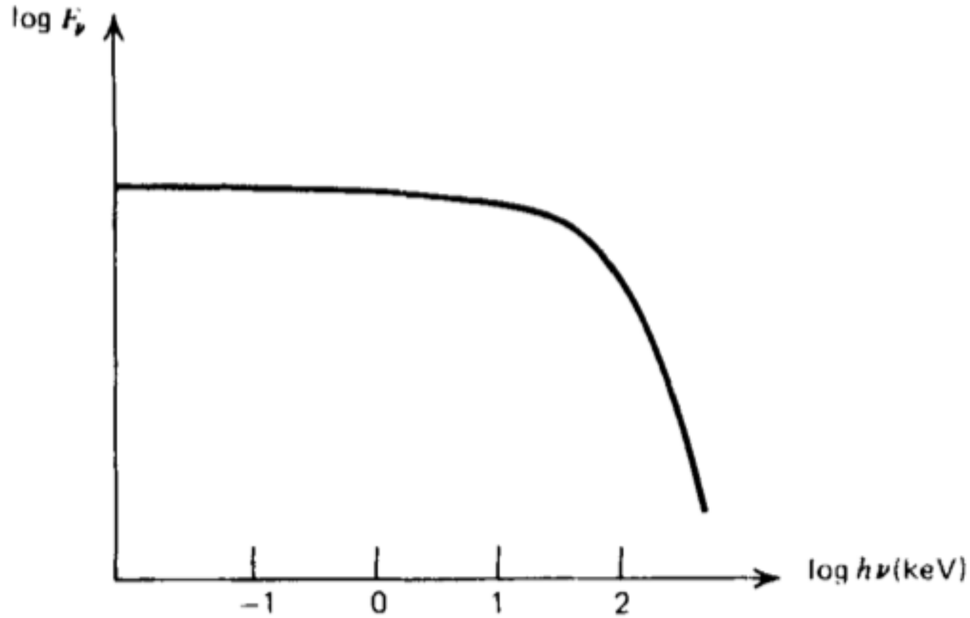
Lower limit of electron velocity from the condition $h\nu \leqslant \frac{1}{2} m v^2$

$$v_{\min} \equiv (2h\nu/m)^{1/2}$$

$$\boxed{\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8\times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}} = 4\pi j_\nu^{ff}.$$

$$\boxed{\varepsilon^{ff} \equiv \frac{dW}{dt\,dV} = 1.4\times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B}$$

Thermal Bremsstrahlung spectra



Thermal Bremsstrahlung Emission
Photon frequency vs temperature and cut off frequency

Consider an electron moving at a speed of v =1000 km s^{-1}.

Thermal Bremsstrahlung Emission

Photon frequency vs temperature and cut off frequency

Consider an electron moving at a speed of $v = 1000\ \mathrm{km\ s^{-1}}$.

In case it would radiate all its kinetic energy in a single interaction
$\Delta E = h\nu = 1/2\ mv^2$

So frequency(maximum) of the emitted radiation is $\Delta E/h \sim 7\times10^{14}$ Hz

Now assuming that the electron is part of population of particles for which v is the typical temperature $T = \Delta E/k \sim 3\times10^{4}$ K

Thermal Bremsstrahlung Emission

Photon frequency vs temperature and cut off frequency

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

Cut off frequency depends only and is set when the exponential is equal to 1

$h\nu = kT$

$$\nu_c = \frac{k}{h} T \quad \sim 2x10^{10}\ \mathrm{Hz}$$

So one can determine cut-off frequency for warm plasma (e.g. HII region, T~10^4 K), and hot plasma (in clusters of galaxies, T~10^8 K), and cut off for hot plasma will be at a higher frequency.

Thermal Bremsstrahlung Emission
Photon frequency vs temperature and cut off frequency

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8\times10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

Cut off frequency depends only and is set when the exponential is equal to 1

$h\nu = kT$ $\qquad$ $\nu_c = \frac{k}{h} T \quad \sim 2\text{x}10^{10}\text{ Hz}$

So one can determine cut-off frequency for warm plasma (e.g. HII region, T~10^4 K), and hot plasma (in clusters of galaxies, T~10^8 K), and cut off for hot plasma will be at a higher frequency.

Observationally one measures the the cutoff frequency and calculate the plasma temperature and hence velocity

Total emissivity does not depend on frequency (except the cut – off) and spectrum is flat

Thermal Bremsstrahlung Emission (Cooling time)

$3/2(n_e+n_p)\ kT$

$$\text{Cooling Time } T_{cool} = \frac{\text{Energy content of a gas}}{\text{Rate at which energy is being radiated}}$$

$$\boxed{\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8\times10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}}$$

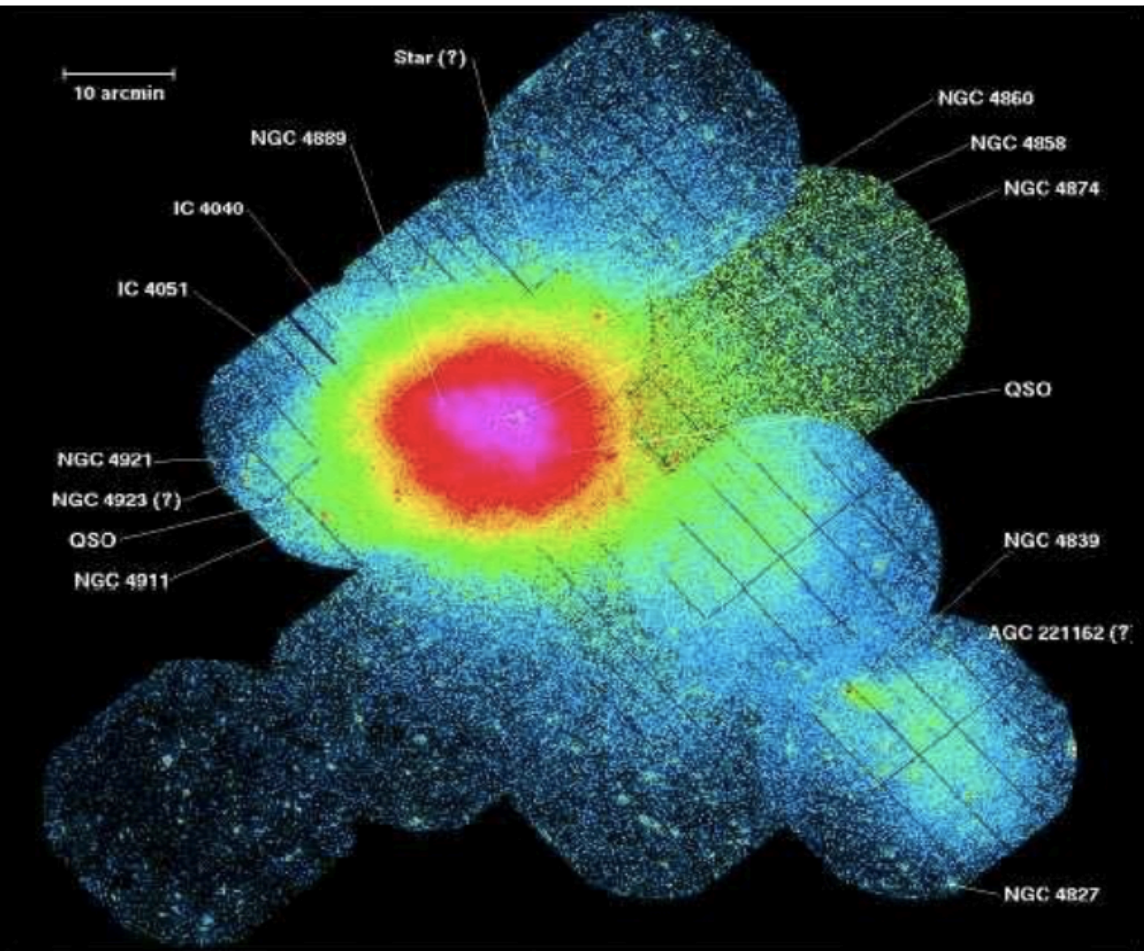
Bremsstrahlung is the main cooling process at temperatures above T~ 10^7 K

All galaxy clusters have bremsstrahlung emission

Thermal Bremsstrahlung Example
Cluster of galaxies

A galaxy cluster, or cluster of galaxies, is a structure consisting hundreds to thousands of galaxies bound together by gravity.

Coma cluster (ROSAT image)



hot gas →thermal bremsstrahlung

Radiative cooling time> 10 Gyr

Radio + X- ray emitters

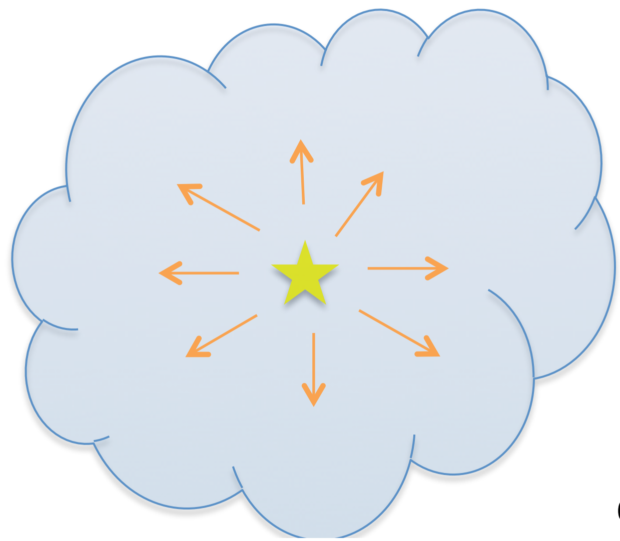
Image courtesy of U. Briel, MPE Garching, Germany and ESA

Bremsstrahlung (free-free) emission
Example

HII region around OB stars

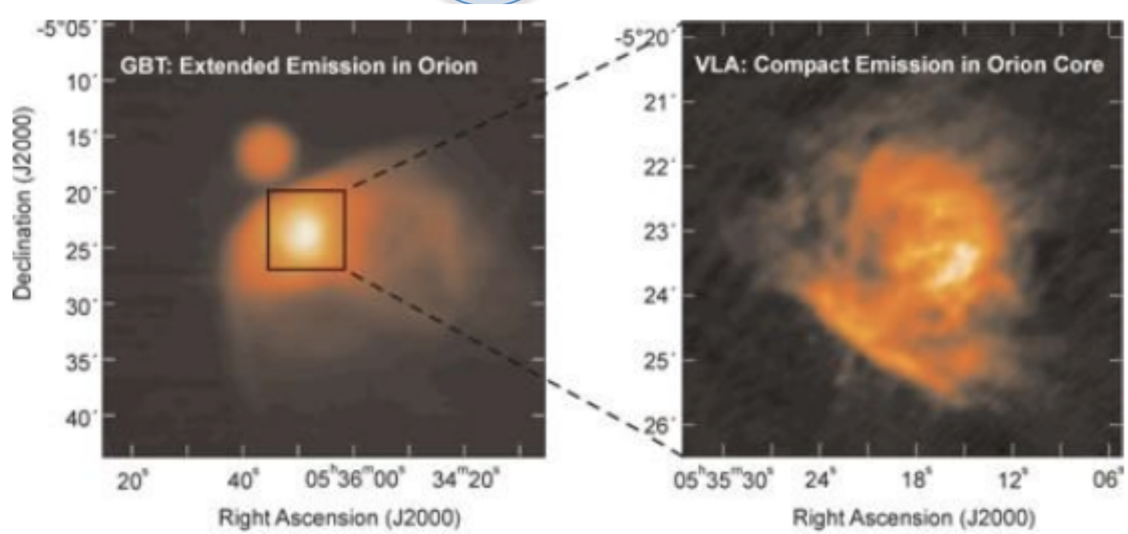
uv photons from the OB star ionises the surrounding gas

Temperature T~ 10 [4]



Credit:NRAO/AUI

Galactic HII region:
M42 (Orion nebula)
distance ~ 0.5 kpc
angular size ~ 1.1°

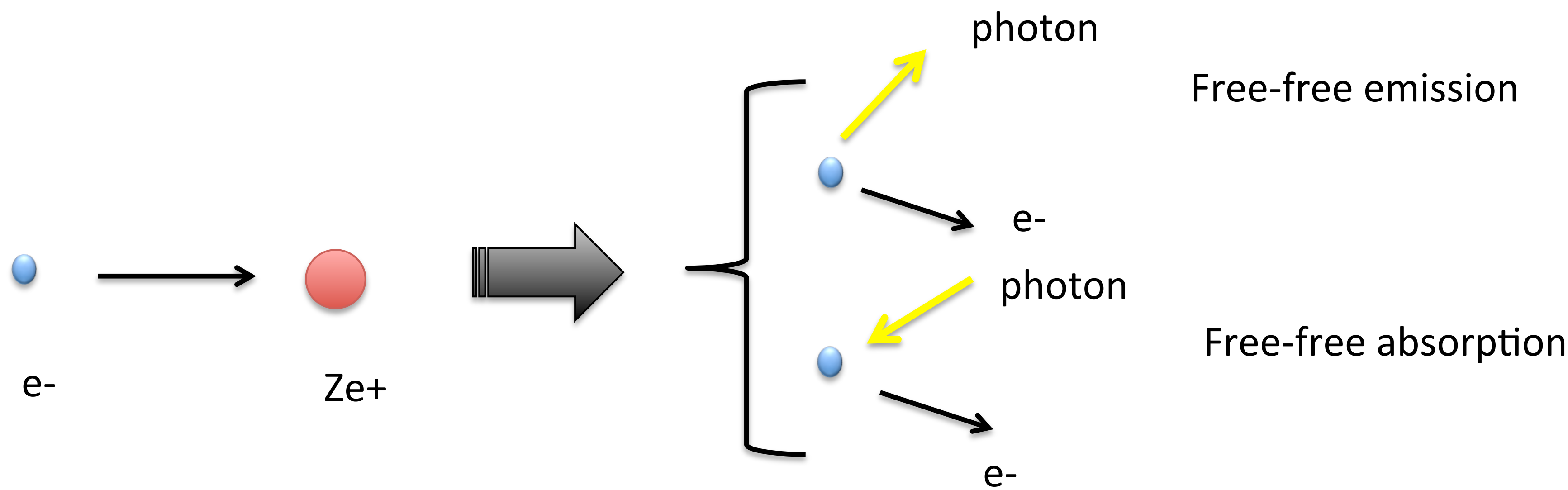


Bremsstrahlung emissivity in hot plasmas

	T (K)	Obs. of ε_ν^{ff}
Solar flare	10^7 (~ 1keV)	radio→ flat X-ray → exponential
H II region	10^5	radio→ flat
Orion	10^4	radio →flat
Sco X-1	10^8	optical-flat X-ray → flat/exp.
Coma Cluster ICM	10^8	X-ray → flat/exp.

Credit : George F. Smoot

Bremsstrahlung (free-free) absorption



Free-free absorption :

Absorption of radiation by an electron moving in the field of an ion due to to the bremsstrahlung process

Bremsstrahlung (free-free) absorption

Absorption of radiation by an electron moving in the field of an ion due to to the bremsstrahlung process

Let us consider thermal free-free absorption

Kirchhoff's law

$$\boxed{j_\nu^{ff} = \alpha_\nu^{ff} B_\nu(T).}$$

emission coefficient

Free-free absorption coefficient

However we have

$$\frac{dW}{dt\,dV\,d\nu} = 4\pi j_\nu^{ff}.$$

Bremsstrahlung (free-free) absorption

$$j_\nu^{ff} = \alpha_\nu^{ff} B_\nu(T).$$

$$\frac{dW}{dt\,dV\,d\nu} = 4\pi j_\nu^{ff}.$$

$$\frac{dW}{dV\,dt\,d\nu} = \frac{2^5 \pi e^6}{3mc^3}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i e^{-h\nu/kT} \bar{g}_{ff}$$

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Free-free absorption coefficient

$$\alpha_\nu^{ff} = \frac{4e^6}{3mhc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3}\left(1 - e^{-h\nu/kT}\right)\bar{g}_{ff}.$$

Bremsstrahlung (free-free) absorption

Free-free absorption coefficient

$$\alpha_\nu^{ff} = \frac{4e^6}{3mhc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_e n_i \nu^{-3}\left(1 - e^{-h\nu/kT}\right)\bar{g}_{ff}.$$

Evaluating the constants in C.G.S. units

$$\alpha_\nu^{ff} = 3.7 \times 10^8 T^{-1/2} Z^2 n_e n_i \nu^{-3}\left(1 - e^{-h\nu/kT}\right)\bar{g}_{ff}.$$

Unit cm^{-1}

Bremsstrahlung (free-free) absorption

Free-free absorption coefficient in C.G.S. units

$$\alpha_\nu^{ff} = 3.7 \times 10^8 T^{-1/2} Z^2 n_e n_i \nu^{-3} (1 - e^{-h\nu/kT}) \bar{g}_{ff}.$$

Unit cm^{-1}

Can find the optical depth now

Case-1 $h\nu \gg kT$ (e.g. X-rays)

The exponential is negligible and α_ν is proportional to $T^{-1/2}\ \nu^{-3}$ is very small unless n_e is very large

In X-rays, thermal bremsstrahlung emission can be treated as *optically thin* (except in stellar interiors)

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$, We are in Rayleigh-Jeans regime: Radio frequencies

$$\alpha_\nu^{ff} = \frac{4e^6}{3mkc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff},$$

Evaluating the constants in C.G.S. units

$$\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}.$$

Absorption can be important, even for relatively low n_e in the radio regime.

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$, Rayleigh-Jeans regime: Radio frequencies

$$\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}.$$

Calculate optical depth $\tau_\nu = \int \alpha_\nu ds$

If optically thin, spectrum is as calculated before : flat until turnover.

If optically thick, spectrum is effectively blackbody.

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$

We are in Rayleigh-Jeans regime: Radio frequencies

$$\alpha_\nu^{ff} = 0.018\, T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}.$$

Optical depth

$$\tau \propto \int \frac{n^2\, \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}} dl$$

Recap

$$I_\nu = (1 - e^{-\tau_\nu}) B_\nu(T_e)$$

At low ν, $\tau_\nu >> 1$ $\quad I_\nu \propto B_\nu(T_e) \propto \nu^2$

Black body like spectrum

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$, We are in Rayleigh-Jeans regime: Radio frequencies

Optical depth
$$\tau \propto \int \frac{n^2\, \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}} dl$$

In this regime
$$\bar{g}_{ff}(\nu) \propto \nu^{-0.1} T^{0.15}$$

Recap
$$I_\nu = (1 - e^{-\tau_\nu}) B_\nu(T_e)$$

- ✓ At low ν, τ_ν >>1 $I_\nu \propto B_\nu(T_e) \propto \nu^2$ Black body like spectrum
- ✓ At low ν, τ_ν <<1 $I_\nu \propto \tau B_\nu(T_e) \propto \nu^{-0.1}$ Flat spectrum

Bremsstrahlung (free-free) absorption

Case-2 $h\nu \ll kT$, We are in Rayleigh-Jeans regime: Radio frequencies

Optical depth

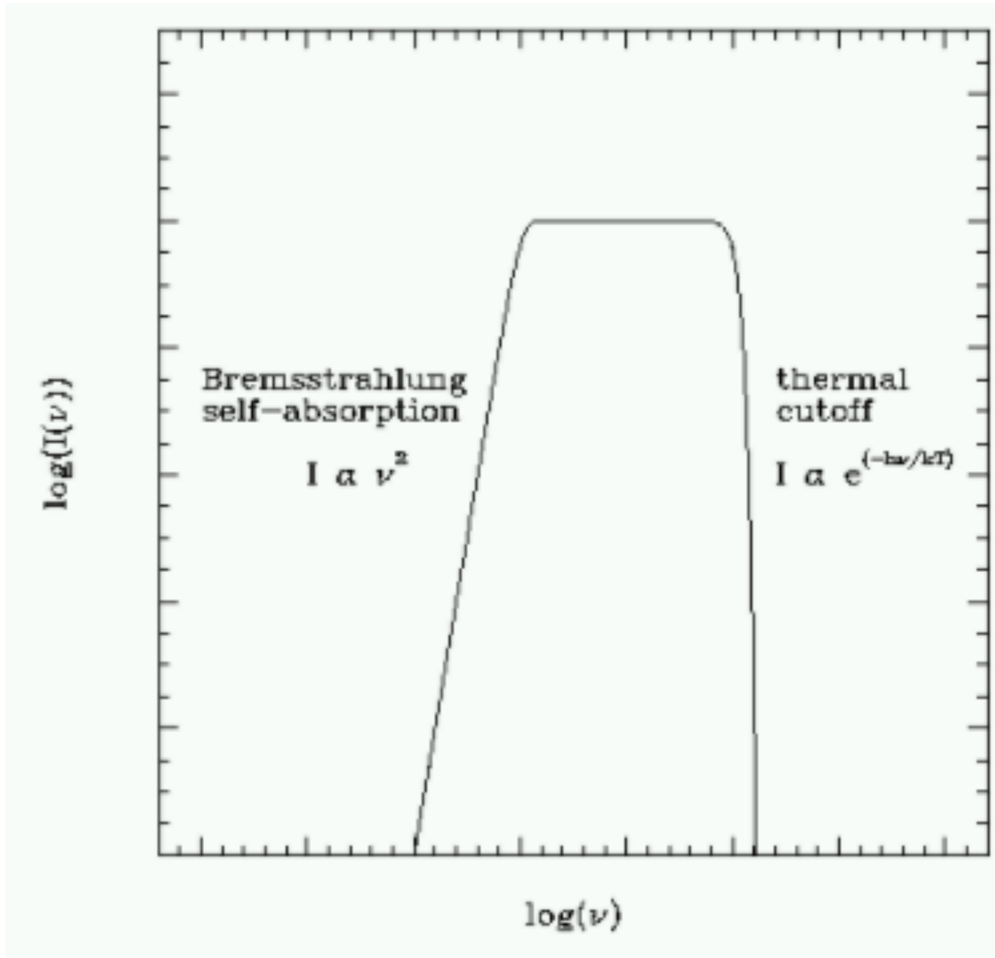
$$\tau \propto \int \frac{n^2 \, \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}} dl$$

$$I_\nu = (1 - e^{-\tau_\nu}) B_\nu(T_e)$$

- ✓ At low ν, τ_ν >>1 $I_\nu \propto B_\nu(T_e) \propto \nu^2$ Black body like spectrum
- ✓ At low ν, τ_ν <<1 Flat spectrum

Turnover when τ_ν ~1, ν~ 1 GHz for Orion

Bremsstrahlung Spectra



Effect of optical thickness on the Bremsstrahlung spectrum.
At low frequencies due to self-absorption spectrum follow the Raleigh-Jeans law.
(spectrum is typically seen in dense ionised gas such as found in star formation regions)

Relativistic Bremsstrahlung Emission

T is not a relevant concept any more, v must be used

The typical velocities of the particles are now relativistic and energy (velocity) distribution is described by a power law

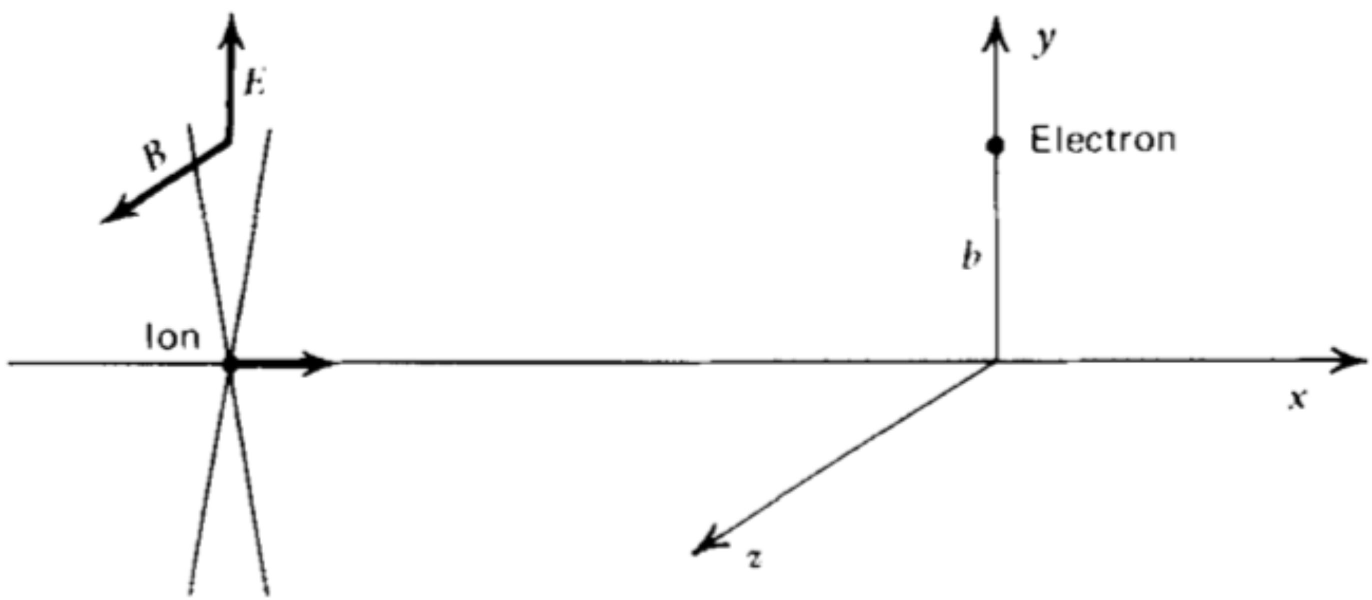
Analysis will be performed with the method of virtual quanta

Consider collision between an electron and a heavy ion of charge Ze

Normally ion moves slower than electrons.
Let us consider a frame where electron is initially at rest and in that case the ion appears to move rapidly towards electron.

Relativistic Bremsstrahlung Emission

Ion moves along x-axis with velocity v. Electron is initially rest on y-axis at a distance b from the origin.



Electrostatic field of the ion is transformed in to a transverse pulse with |E|=|B| . To the electron this is a pulse of electromagnetic radiation.

Relativistic Bremsstrahlung Emission

This radiation then Compton scatters off electron to produce the detected radiation

Transforming back to the rest frame of the ion (lab frame) we obtain the bremsstrahlung emission of the ion.

Thus relativistic bramsstrahlung can be regarded as the Compton scattering of the virtual quanta of the ion's electrostatic field as seen in the electron's frame.

In the primed frame (the rest frame of electrons) the spectrum of the pulse of virtual quanta has the following form (as derived in Lecture −7), with v=c in ultra relativistic limit

$$\frac{dW'}{dA'\,d\omega'}(\mathrm{erg\ cm^{-2}\ Hz^{-1}}) = \frac{(Ze)^2}{\pi^2 b'^2 c}\left(\frac{b'\omega'}{\gamma c}\right)^2 K_1^2\left(\frac{b'\omega'}{\gamma c}\right)$$

Relativistic Bremsstrahlung Emission

In the primed frame, i.e. in the rest frame of the electrons the spectrum of the pulse of virtual quanta has the following form (as derived in Lecture −7), with v=c in ultra relativistic limit

$$\frac{dW'}{dA'd\omega'}(\mathrm{erg\ cm^{-2}\ Hz^{-1}}) = \frac{(Ze)^2}{\pi^2 b'^2 c}\left(\frac{b'\omega'}{\gamma c}\right)^2 K_1^2\left(\frac{b'\omega'}{\gamma c}\right)$$

In primed frame (rest frame of the electron)

$\hbar\omega' \lesssim mc^2$ Virtual quanta are scattered by electron according to Thomson scattering

$\hbar\omega' > mc^2$ Virtual quanta are scattered by electron according to Compton scattering

Relativistic Bremsstrahlung Emission

Now since energy and frequency transform identically in Lorentz transformations,

$$dW/d\omega = dW'/d\omega'$$

Thus the emission in lab frame is

$$\frac{dW}{d\omega} = \frac{8Z^2e^6}{3\pi b^2c^5m^2}\left(\frac{b\omega}{\gamma^2c}\right)^2 K_1\left(\frac{b\omega}{\gamma^2c}\right)$$

Replacing
b=b’ and ω=γω’

Energy per unit frequency emitted by collision of an ion
and a relativistic electron at a impact parameter b

Thermal Bremsstrahlung (slide 16 of lecture 8) Emission from multiple single speed electron

Total spectrum for a medium with ion density n_i electron density n_e and fixed electron speed v

$$\frac{dW}{d\omega\, dV\, dt} = n_e n_i 2\pi v \int_{b_{\min}}^{\infty} \frac{dW(b)}{d\omega}\, b\, db$$

Flux of electrons (electrons per unit area per unit time) incident on one ion is n_e v

The element of area is $2\pi b db$ about a single ion.

b_{min} is minimum value of impact parameter

Relativistic Bremsstrahlung Emission

For a plasma with electron and ion densities n_e and n_i we repeat the same treatment which we did for thermal bremsstrahlung

$$\boxed{\frac{dW}{d\omega\, dV\, dt} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)} \rightarrow$$

Thermal Bremsstrahlung

v is replaced by c with electron and ion densities n_e and n_i we repeat the same treatment which we did for thermal bremsstrahlung

$$\boxed{\frac{dW}{dt\, dV\, d\omega} \sim \frac{16Z^2e^6n_en_i}{3c^4m^2} \ln\left(\frac{0.68\gamma^2c}{\omega b_{\min}}\right)} \longrightarrow$$

Non-thermal Bremsstrahlung
In low-frequency limit $h\nu \ll \gamma mc^2$

Relativistic Bremsstrahlung Emission

For a plasma with electron and ion densities n_e and n_i frequency integrated power

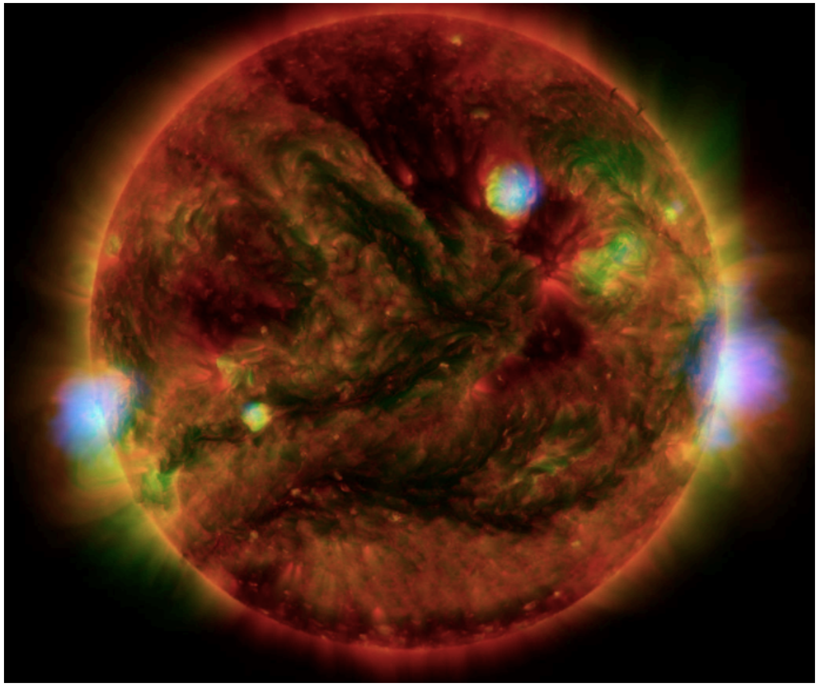
$$\frac{dW}{dV\,dt} = 1.4\times 10^{-27} T^{1/2} Z^2 n_e n_i \bar{g}_B (1 + 4.4\times 10^{-10} T)$$

Relativistic correction
Significant for very high temperatures

Relativistic Bremsstrahlung Emission Example

Solar wind:
ne ~ 4 – 7 cm^{-3}
v_{swind}~ 300 – 900 km s^{-1}
T ~ 150, 000 K
(not consistent with v!)

Solar Flares: brief and intense emission from sun's surface
relativistic particles with power–law energy distribution



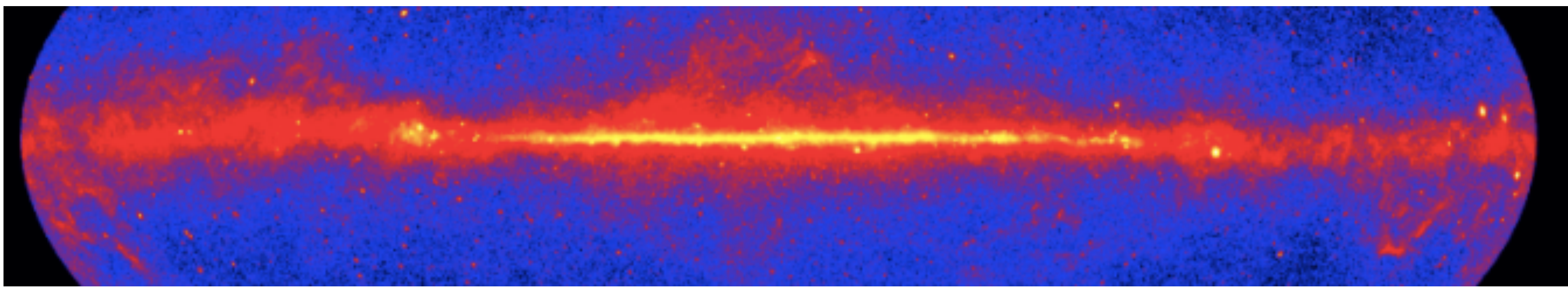
Flaring, active regions sun are highlighted in this image combining observations from several telescopes. High-energy X-rays from NASA's Nuclear Spectroscopic Telescope Array (NuSTAR) are shown in blue; low-energy X-rays from Japan's Hinode spacecraft are green; and extreme ultraviolet light from NASA's Solar Dynamics Observatory (SDO) is yellow and red.

Image credit: NASA/JPL-Caltech/GSFC/JAXA

Relativistic Bremsstrahlung Emission Example

Gamma-rays from the Galaxy

Gamma-ray emission is detected from our Galaxy is thought to arise from relativistic Bremsstrahlung from high energy electrons.



Milky way in Gamma-rays (Fermi LAT image)

The radiative energy is carried by photons with $h\nu \sim E_e$

Energies in the range ~100 MeV, suggesting many relativistic electrons with $\gamma \sim 100$

Bremsstrahlung Emission
Summary

- ✓ Photon is emitted from accelerating electron in the Coulomb field of ion
- ✓ The energy of the photon can never exceed the kinetic energy of electron
- ✓ Generally the ''classical'' description of the phenomenon is adequate, the quantum mechanics requires an appropriate Gaunt Factor
- ✓ Individual interaction produces polarized radiation (fixed direction of E and B fields of the e-m wave wrt the acceleration)
- ✓ Overall the emission is unpolarized (random orientation of the plane of interaction)
- ✓ We have ''thermal'' or ''relativistic'' bremsstrahlung, depending on velocity/energy distribution of electrons.

End of Lecture 9

Next Lecture : 11^{th} September