Electrodynamics and Radiative Processes I

Lecture 3 – Radiative Transfer

Bhaswati Bhattacharyya

bhaswati@ncra.tifr.res.in

IUCAA-NCRA Graduate School

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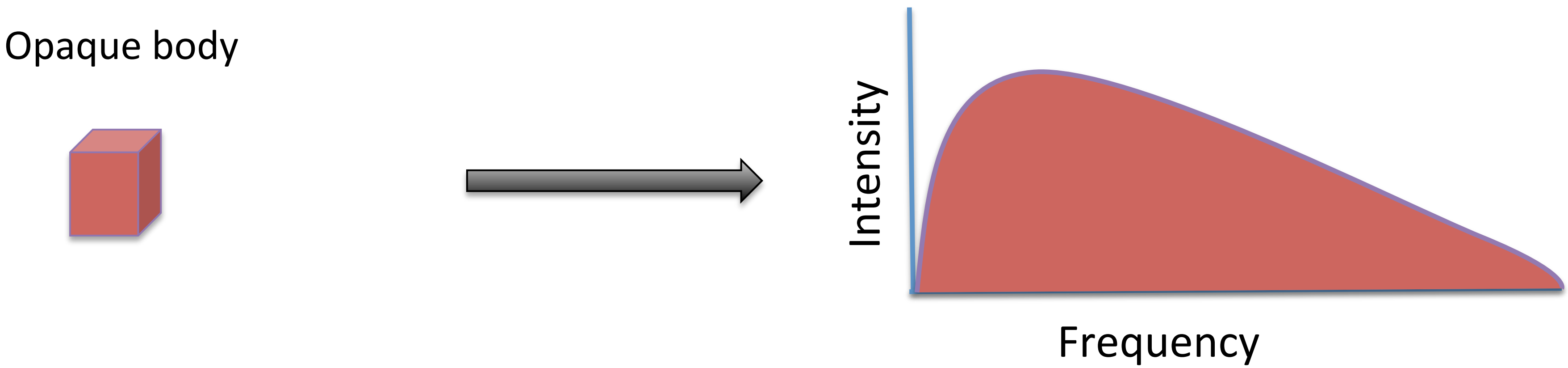
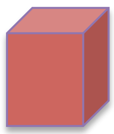
Recap Lecture-1,2

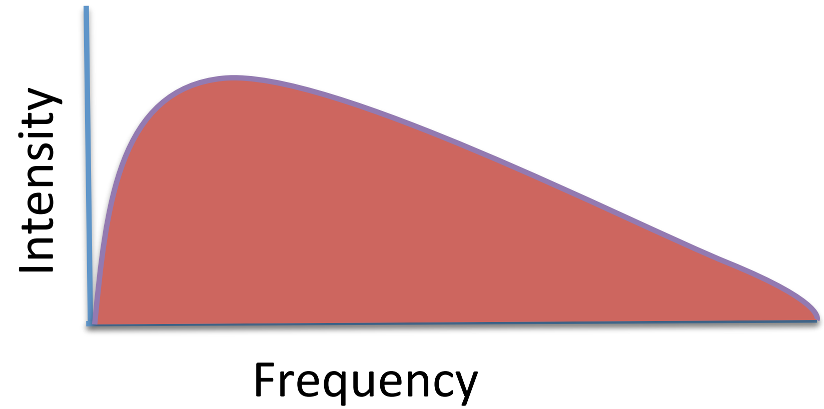


A luminous opaque body behaves like a black body
emits frequencies of all wave lengths and produces continuous spectrum

Recap Lecture-1,2

Opaque body





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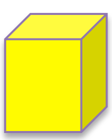
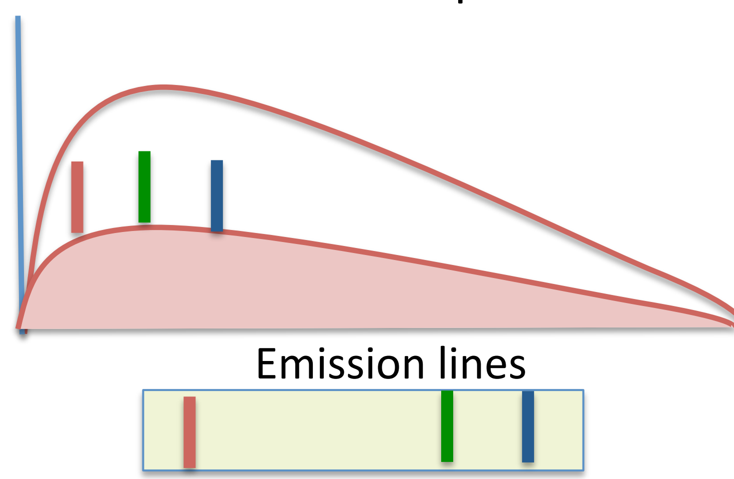
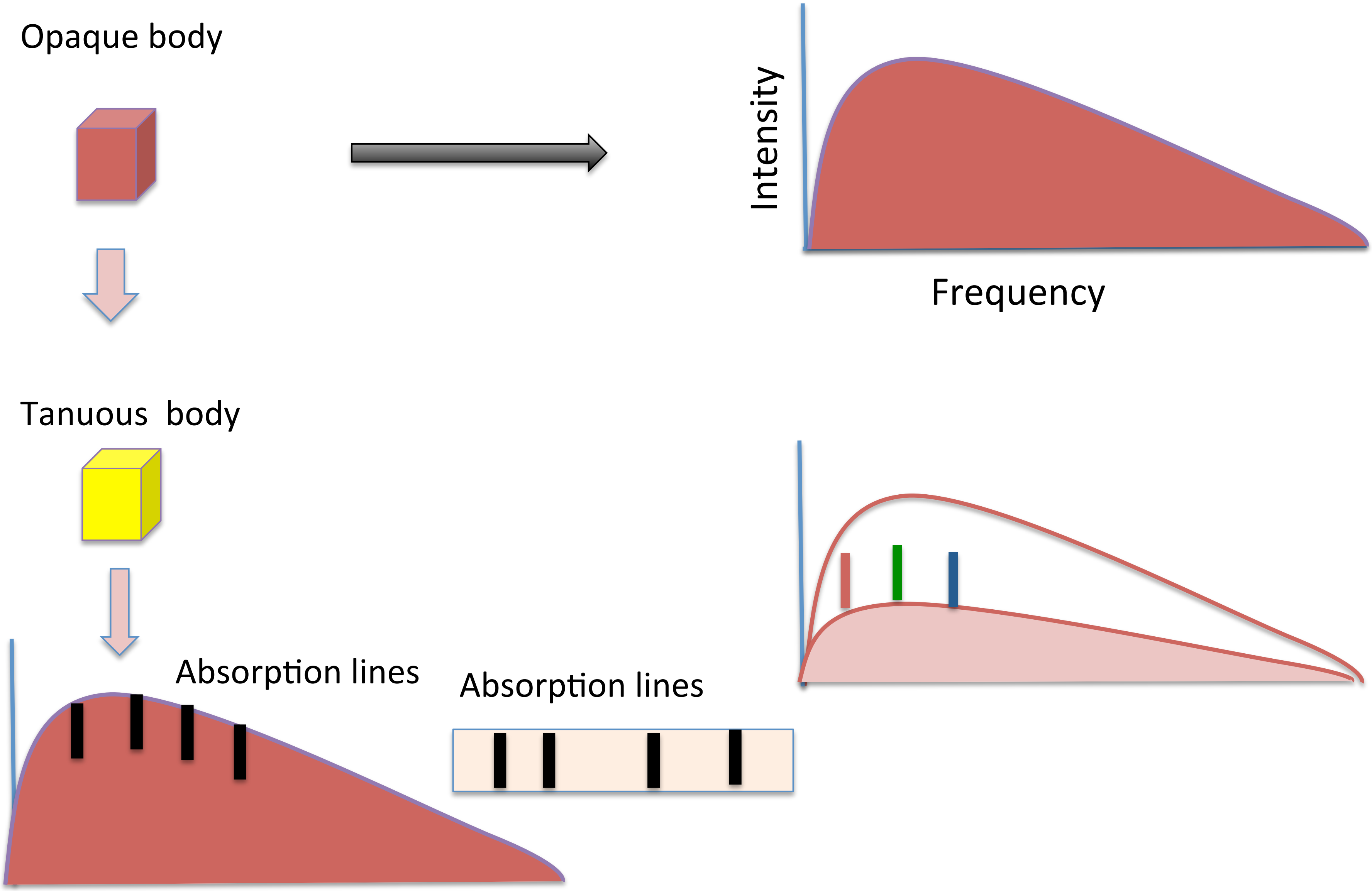
Tanuous body





Emission lines superimposed on faint continuous spectra.
Intensity of continuum or the emission lines can never exceed black body at any point

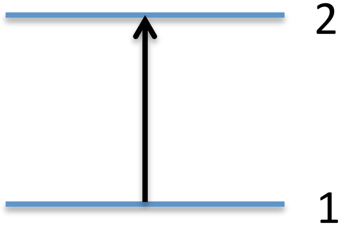
Recap Lecture-1,2



Einstein Coefficients

First derivation of Planck's function

Einstein Coefficients



Stimulated absorption

B_{12} (dependent on radiation)

This occurs in presence of photons of energy $h\nu_o$

Energy difference between two levels is not infinitely sharp

Described by a line profile function $\Phi(\nu)$

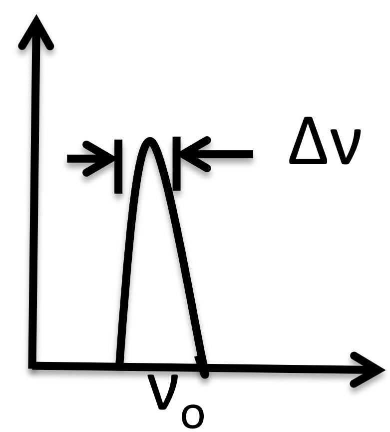
Remember (from Lecture 1, slide 16)

$$J_\nu = \frac{1}{4\pi}\int I_\nu \, d\Omega$$

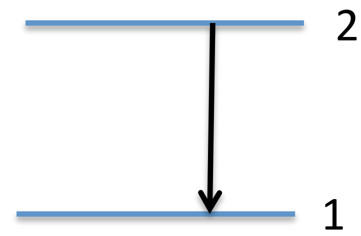
$$\int_0^\infty \phi(\nu)\, d\nu = 1$$

$$\bar{J} \equiv \int_0^\infty J_\nu \phi(\nu)\, d\nu$$

$B_{12}\bar{J}$ → Stimulated absorption rate



Einstein Coefficients



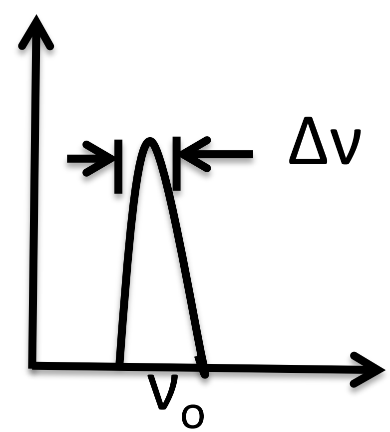
Stimulated emission

$\mathbf{B_{21}}$(dependent on radiation)

Photons of energy $h\nu_o$ is emitted.

Energy difference between two levels is not infinitely sharp

Described by a line profile function $\Phi(\nu)$



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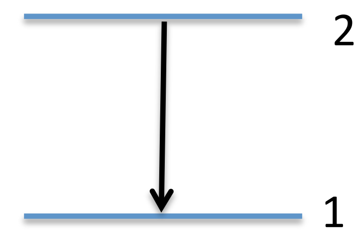
Einstein Coefficients



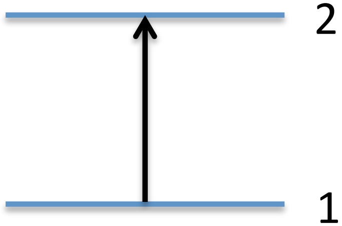
Spontaneous emission

$\mathbf{A_{21}}$(independent of radiation)

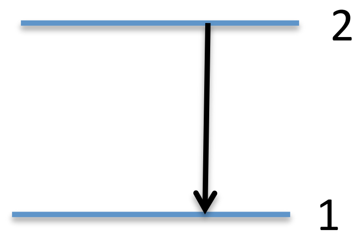
Occurs when system in level 2 goes to 1, emits a photon of energy $h\nu_o$
It occurs even in absence of radiation fields

A_{21} → Spontaneous emission rate

Einstein Coefficients



Stimulated absorption
B_{12} (dependent on radiation)



Stimulated emission
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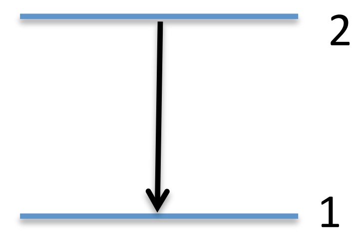
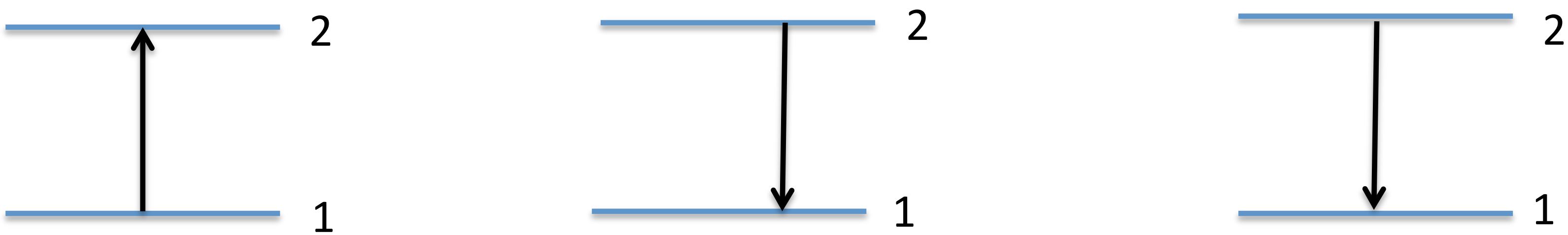


Spontaneous emission
A_{21}(independent of radiation)

Rate 1 to 2 = Rate 2 to 1 $$n_1 B_{12} \bar{J} = n_2 A_{21} + n_2 B_{21} \bar{J}.$$

Einstein Coefficients



Stimulated absorption
B_{21} (dependent on radiation)

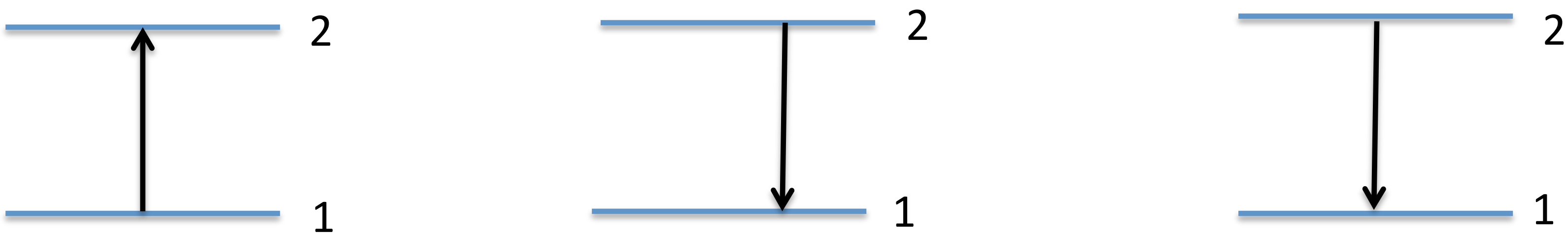
Stimulated emission
B_{12}(dependent on radiation)

Spontaneous emission
A_{21}(independent of radiation)

Rate 1 to 2 = Rate 2 to 1 $$n_1 B_{12} \bar{J} = n_2 A_{21} + n_2 B_{21} \bar{J}.$$

In thermal equilibrium $$\frac{n_1}{n_2} = \frac{g_1 \exp(-E/kT)}{g_2 \exp[-(E+h\nu_0)/kT]} = \frac{g_1}{g_2} \exp(h\nu_0/kT)$$

Einstein Coefficients



Stimulated absorption
B_{12} (dependent on radiation)

Stimulated emission
B_{21}(dependent on radiation)

Spontaneous emission
A_{21}(independent of radiation)

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Mean specific intensity

$$\bar{J} = \frac{A_{21}/B_{21}}{(g_1 B_{12}/g_2 B_{21}) \exp(h\nu_0/kT) - 1}$$ => Plank Function

Einstein Coefficients

$$\bar{J} = \frac{A_{21}/B_{21}}{(g_1 B_{12}/g_2 B_{21}) \exp(h\nu_0/kT) - 1}$$

$$\boxed{I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}}$$

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$$g_1 B_{12} = g_2 B_{21} \qquad\qquad A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

These relations must hold whether or not there is thermodynamic equilibrium.

Einstein Coefficients

$$\bar{J} = \frac{A_{21}/B_{21}}{(g_1 B_{12}/g_2 B_{21})\exp(h\nu_0/kT) - 1}$$

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$$g_1 B_{12} = g_2 B_{21}$$

$$A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

Wien Law hν >> kT

$$I_\nu^W(T) = \frac{2h\nu^3}{c^2}\exp\left(\frac{-h\nu}{kT}\right)$$

Einstein Coefficients

$$A_{21} = \frac{2h\nu^3}{c^2} B_{21}$$

Whenever there is stimulated emission, there has to be spontaneous emission.

- Einstein coefficients connect the atomic properties A_{21} , B_{21} and B_{12} and have no relation to temperature.

- If we determine any one of these coefficient then that will allow us to determine other two.

- Einstein had to include the process of simulated emission as without it he could not get Planck's Law.

Einstein Coefficients

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Whenever there is stimulated emission, there has to be spontaneous emission.

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$h\nu >> kT$ level 2 is sparsely populated compared to level 1 i.e. $n2 << n1$

Stimulated emission is unimportant compared to absorption, since these are proportional to n2 and n1.

The Quantum Theory of Radiation

A. Einstein
(Received March, 1917)

The formal similarity of the spectral distribution curve of temperature radiation to Maxwell's velocity distribution curve is too striking to have remained hidden very long. Indeed, in the important theoretical paper in which Wien derived his displacement law

$$\rho = \nu^3 f\left(\frac{\nu}{T}\right) \tag{1}$$

Emission coefficient in terms of Einstein Coefficients

Amount of energy emitted in volume dV, solid angle dΩ frequency dν and time dt

$$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu,$$ (slide 19, Lecture 1)

Each atom contributes energy $h\nu_0$ distributed over 4π solid angle

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$$dE = (h\nu_0/4\pi)\phi(\nu) n_2 A_{21} \, dV \, d\Omega \, d\nu \, dt$$

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$$j_\nu = \frac{h\nu_0}{4\pi} n_2 A_{21} \phi(\nu)$$

Absorption coefficient in terms of Einstein Coefficients

Amount of energy emitted in volume dV, solid angle dΩ frequency dv and time dt

$$dE = dV\,dt\,d\Omega\,d\nu \frac{h\nu_0}{4\pi} n_1 B_{12} \phi(\nu) I_\nu$$

Absorption coefficient in terms of Einstein Coefficients

Amount of energy emitted in volume dV, solid angle dΩ frequency dν and time dt

$$dV\,dt\,d\Omega\,d\nu\,\frac{h\nu_0}{4\pi}\,n_1 B_{12}\phi(\nu)\,I_\nu$$

↓

$$\alpha_\nu = \frac{h\nu}{4\pi}\,n_1 B_{12}\phi(\nu).$$

Absorption coefficient in terms of Einstein Coefficients

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$$dV\,dt\,d\Omega\,d\nu\,\frac{h\nu_0}{4\pi}n_1 B_{12}\phi(\nu)I_\nu$$

Absorption coefficient

$$\alpha_\nu = \frac{h\nu}{4\pi}n_1 B_{12}\phi(\nu).$$

Absorption coefficient corrected for stimulated emission

$$\alpha_\nu = \frac{h\nu}{4\pi}\phi(\nu)(n_1 B_{12} - n_2 B_{21}).$$

Radiative transfer equation in terms of Einstein Coefficients

Replacing the emission and the absorption coefficients in the radiative transfer equation

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{ds} = -\frac{h\nu}{4\pi}(n_1 B_{12} - n_2 B_{21})\phi(\nu) I_\nu + \frac{h\nu}{4\pi} n_2 A_{21}\phi(\nu)$$

Source function

$$S_\nu = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}}$$

Radiative transfer equation in terms of Einstein Coefficients

Replacing the emission and the absorption coefficients in the radiative transfer equation

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Source function
$$S_\nu = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}}$$

$$\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12}(1 - g_1 n_2 / g_2 n_1)\phi(\nu),$$

Radiative transfer equation in terms of Einstein Coefficients

Replacing the emission and the absorption coefficients in the radiative transfer equation

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$$S_\nu = \frac{2h\nu^3}{c^2}\left(\frac{g_2 n_1}{g_1 n_2} - 1\right)^{-1}$$

Special cases
1. Thermal emission

If the matter is in thermodynamic equilibrium

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} \exp\left(\frac{h\nu}{kT} \right)$$

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Correction factor due to stimulated emission

Special cases
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$$\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} \left[1 - \exp\left(\frac{-h\nu}{kT}\right)\right] \phi(\nu)$$

Correction factor due to stimulated emission

$$S_\nu = B_\nu(T)$$

Special cases
2. Non-thermal emission

For all other cases where thermal equilibrium is not achieved

$$\frac{n_1}{n_2} \neq \frac{g_1}{g_2} \exp\left(\frac{h\nu}{kT} \right)$$

Special cases
3. Inverted Populations

For a system with thermal equilibrium we have

$$\frac{n_1}{g_1} > \frac{n_2}{g_2}.$$

Such systems are called normal population

It is possible to put enough atoms in the upper state so that we have population inversion

$$\frac{n_1}{g_1} < \frac{n_2}{g_2}$$

Absorption coefficient is negative

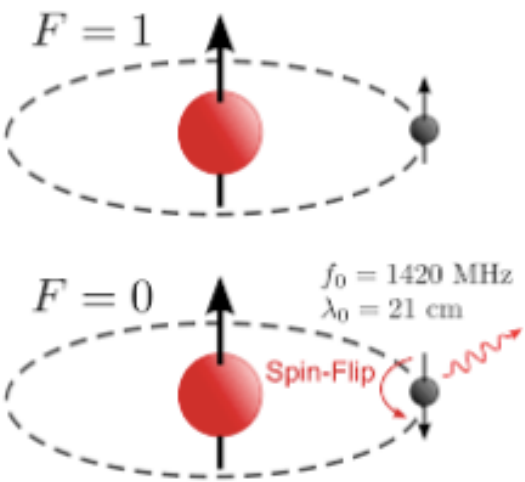
$$\alpha_\nu < 0,$$

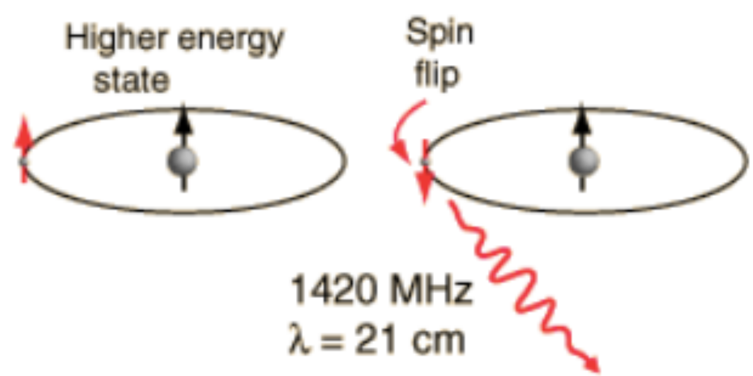
Einstein Coefficients

From a quantum electrodynamics treatment of spontaneous emission, it may be shown

$$A_{UL} \approx \frac{64\pi^4}{3hc^3}\nu_{UL}^3 |\mu_{UL}|^2$$

Radiation is due to change of dipole moment.





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↓

Radiation is due to change of dipole moment.

21cm emission line

Hydrogen is the most abundant element in the interstellar medium (ISM), but the symmetric H2 molecule has no permanent dipole moment and hence does not emit a detectable spectral line at radio frequencies.

Neutral hydrogen (HI) atoms are abundant in low-density regions of the ISM. They are detectable in the 21 cm (~1420 MHz) hyperfine line.
Two energy levels result from the magnetic interaction between quantized electron and proton spins. When the relative spins change from parallel to antiparallel, a photon is emitted.

$$A_{10} \approx 2.85 \times 10^{-15}\ \mathrm{s}^{-1}$$

$$\tau_{1/2} = A_{10}^{-1} \approx 3.5 \times 10^{14}\ \mathrm{s} \approx 11\ \text{million years}$$

However a large fraction of what we know about the universe comes from studying the universe at 21 cm

Radiative Transfer

Radiative Transfer = change in I_ν as radiation propagats

✧ Radiation is ultimately produced by quantum mechanical transitions in which electrons move from one level to another

✧ In an ensemble of atoms/molecules occupancy of these energy levels is given By Boltzman distribution $e^{-E/KT}$ ->matter is in thermal equilibrium

✧ In diffuse matter when $\tau \ll 1$ the photons retain their signature.

✧ In an opaque body when $\tau \gg 1$ the radiation loses all its memory during the Process of multiple absorption and emission and behave like a black body.
This is why spectrum of raditation is characterised by temperature and not by any other property of matter.

✧ In most astrophysical situation matter and radiation are not in thermodynamic equilibrium and so we are not dealing with opaque matter.
(examples of opaque body : early universe and interiors of stars)

Observables

From an empiricist's point of view there are 4 observables for radiation

- Energy Flux
- Direction
- Frequency
- Polarization

More on these in coming lectures

End of Lecture 3

Next lecture : 16^{th} August