Electrodynamics and Radiative Processes I

Lecture 2 – Radiation & Radiative Transfer

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Lecture -1 recap

Specific Intensity or Brightness I_ν

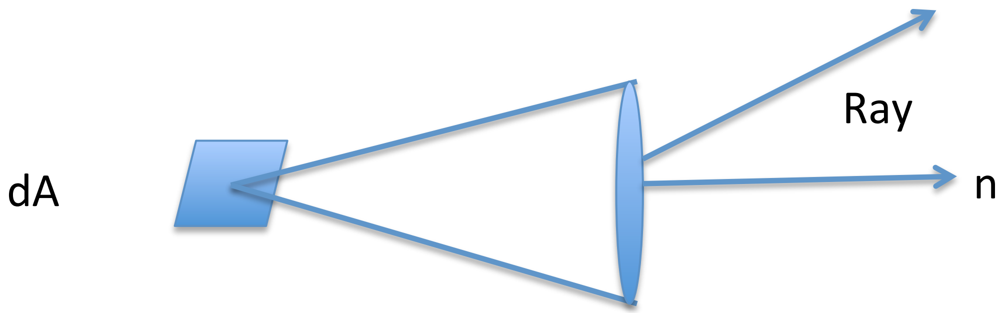

Figure: Geometry for normal incidence

Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

Specific Intensity or Brightness

$[I_\nu]$= unit?

Brightness does/does not decrease with distance?

Lecture -1 recap
Specific Intensity or Brightness I_ν

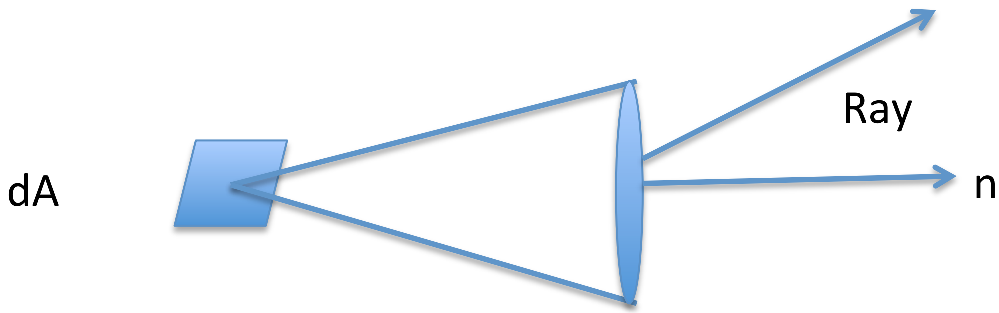

Figure: Geometry for normal incidence

Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ

$$dE = I_\nu \, dA \, dt \, d\Omega \, d\nu$$

Specific Intensity or Brightness

$[I_\nu]$= erg cm^{-2} s^{-1} Hz $^{-1}$ ster^{-1}

Brightness does not decrease with distance

Lecture-1 recap
Radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Absorption only

$$I_\nu(s) = I_\nu(s_0)\exp\left[-\int_{s_0}^{s}\alpha_\nu(s')\,ds'\right]$$

Emission only

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')\,ds'$$

Lecture -1 recap

Brightness and Flux density

Brightness $\mathbf{I_\nu(A, t, \nu, \Omega\,)}$

- Energy crossing dA in time dt in frequency range dv and into a solid angle dΩ
- Also called Specific intensity, spectral radiance
- Property of the astronomical source and conserved along a ray in empty space
- Unit $[I_\nu]$= erg $cm^{-2}\,s^{-1}\,Hz^{-1}\,ster^{-1}$

“The "brightness" of the Sun appears to be about the same over most of the Sun's surface, which looks like a nearly uniform disk even though it is a sphere. This means, for example, that a photograph of the Sun would expose the film equally across the Sun's disk. It also turns out that the exposure would not change if photographs were made at different distances from the Sun, from points near Mars, the Earth, and Venus, for example.”

Reference : https://www.cv.nrao.edu/course/astr534/Brightness.html

Lecture -1 recap

Brightness and Flux density

Brightness $\mathbf{I_\nu(A, t, \nu, \Omega)}$

- Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ
- Also called Specific intensity, spectral radiance
- Property of the astronomical source and conserved along a ray in empty space
- Unit $[I_\nu]$= erg cm^{-2} s^{-1} Hz $^{-1}$ $ster^{-1}$

Flux density $\mathbf{F_\nu(A, \nu, t)}$

- Energy crossing dA in frequency range dν
- Spectral power received from a source by a detector of unit area
- Not intrinsic property of the source but dependent on the distance to the source
- Unit $[F_\nu]$= erg cm^{-2} s^{-1}Hz $^{-1}$= W m^{-2} Hz^{-1}, 1 Jy =10^{-26} W m^{-2} Hz^{-1}

Lecture -1 recap

Brightness and Flux density

Brightness $\mathbf{I_\nu(A, t, \nu, \Omega\,)}$

- Energy crossing dA in time dt in frequency range dν and into a solid angle dΩ
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- Property of the astronomical source and conserved along a ray in empty space
- Unit ?

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- Energy crossing dA in frequency range dν
- Spectral power received from a source by a detector of unit area
- Not intrinsic property of the source but dependent on the distance to the source
- Unit ?

"Electrical disturbances apparently of extraterrestrial origin” -- Jansky

Lecture -1 recap

Brightness and Flux density

Flux density $F_\nu(A, \nu, t)$

$$F_\nu = \int I_\nu \cos\theta d\Omega, \quad F = \int d\nu F_\nu$$

- Energy crossing dA in frequency range dν
- Spectral power received from a source by a detector of unit area
- Not intrinsic property of the source but dependent on the distance to the source
- Unit $[F_\nu]$= erg cm^{-2} s^{-1}Hz^{-1}= W m^{-2} Hz^{-1}, 1 Jy =10^{-26} W m^{-2} Hz^{-1}

"Only the angular size of the Sun changes with the distance between the Sun and the observer. The number of photons falling on the film per unit area per unit time per unit solid angle does not depend on the distance between the source and the observer. The total number of photons falling on the film per unit area per unit time (or the total energy absorbed per unit area per unit time) does decrease with increasing distance. Thus we distinguish between the brightness of the Sun, which does not depend on distance, and the apparent flux, which does."

When do we use spectral Brightness and when flux density to describe a source?

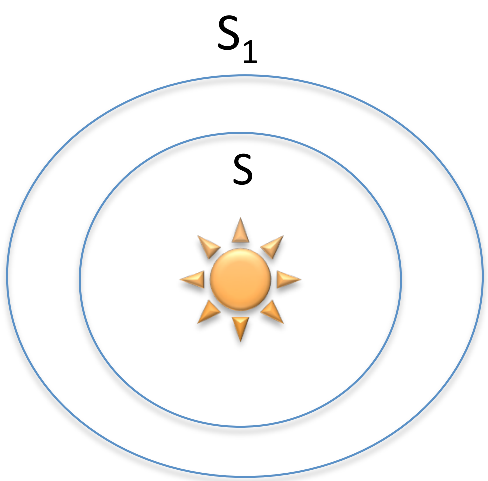
- If a source is unresolved, i.e much smaller in angular size than the point-source response of the eye or telescope observing it, its flux density can be measured.

- If a source is much larger than the point-source response, its spectral brightness at any position on the source can be measured directly, but its flux density must be calculated by integrating the observed spectral brightnesses over the source solid angle. Consequently, flux densities are normally used to describe only relatively compact sources.

Flux from isotropic source

Inverse square law

A source of radiation is isotropic if it emits energy equally in all directions.



Put imaginary spherical surfaces at S_1 and S at radii r and r_1

Conservation of energy :
Total energy passing through S_1, S will be same

$$F(r_1) \cdot 4\pi r_1^2 = F(r) \cdot 4\pi r^2,$$

$$F(r) = \frac{F(r_1) r_1^2}{r^2}$$

$$F = \frac{\text{constant}}{r^2}$$

Optical depth τ_ν

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\, ds'.$$

Dimension of τ_ν ?

Optically **THICK** medium $\tau_\nu > 1$ → Medium is opaque
Radiation gets absorbed

Optically **THIN** medium $\tau_\nu < 1$ → Medium is transparent
Radiation does not gets absorbed

Optical depth τ_ν

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\, ds'.$$

τ_ν dimension less number

Optical depth is integrated absorption coefficient over a column length that goes from observer to the source

- Optically thin medium: typical photon of frequency ν can traverse the medium without being absorbed

- Optically thick medium: an average photon of frequency ν can not traverse the medium with out being absorbed

Radiative transfer equation

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Dividing both side by α_ν

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Source function S_ν

$$S_\nu \equiv \frac{j_\nu}{\alpha_\nu}$$

↓

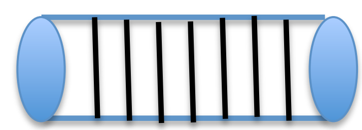
Specific intensity radiated by body

Solution of Radiative transfer equation

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Formal solution of radiative transfer equation

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \qquad \text{——— (5)}$$

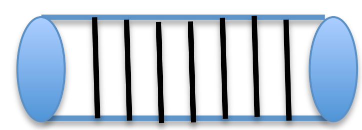


Solution of Radiative transfer equation

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Formal solution of radiative transfer equation

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \qquad (5)$$



1st term

Initial intensity attenuated by medium

2nd term

Medium adds to the radiation

And absorbs part of the added radiation

Solution of Radiative transfer equation
(a) τ_ν >>1 limit

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

Optically depth τ_ν >>1 i.e. Medium is Opaque

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

No Initial intensity attenuated by medium

Black-body radiation

Solution of Radiative transfer equation
(a) τ_ν >>1 limit

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

Optically depth τ_ν >>1 i.e. Medium is Opaque

$$I_\nu(\tau_\nu) = \cancel{I_\nu(0)e^{-\tau_\nu}} + S_\nu(1 \cancel{- e^{-\tau_\nu}})$$

No Initial intensity attenuated by medium

Specific intensity tries to approach Black body-radiation

Black-body radiation

$$\boxed{I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}}$$

Black body radiation
(I will assume you are familiar)

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Rayleigh-Jeans Law $h\nu \ll kT$

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

Black body radiation
(I will assume you are familiar)

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Rayleigh-Jeans Law $h\nu \ll kT$

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

Wien Law $h\nu \gg kT$

$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2} \exp\left(\frac{-h\nu}{kT}\right)$$

Black body radiation
(I will assume you are familiar)

$$I_\nu = B_\nu(T) = \frac{2\,h\,\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Rayleigh-Jeans Law hν << kT

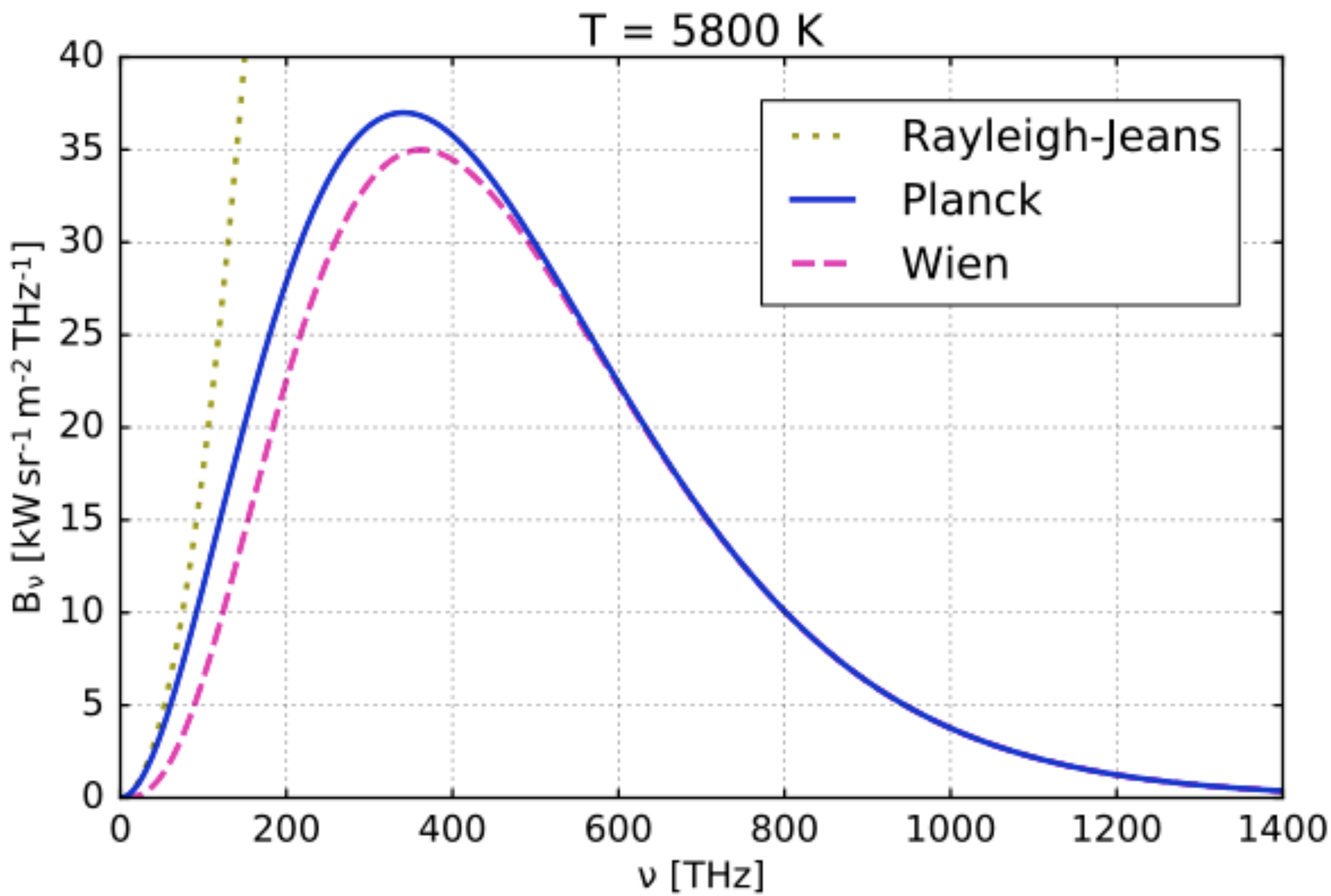
$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

Wien Law hν >> kT

$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2} \exp\left(\frac{-h\nu}{kT}\right)$$

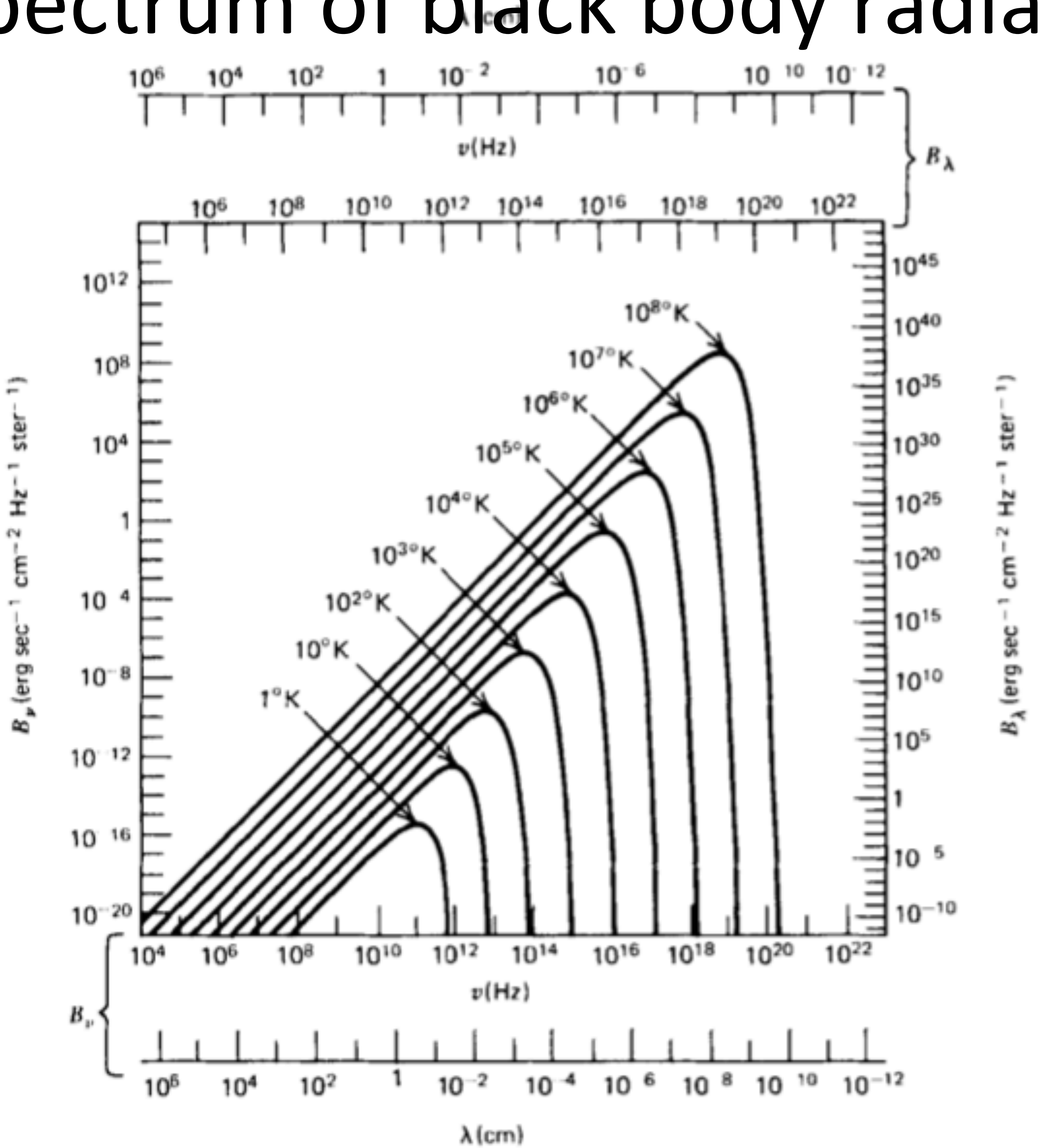
Wien's displacement Law : Max intensity at hν~ kT

$$\lambda_{\max} T = 0.290 \text{ cm deg.}$$

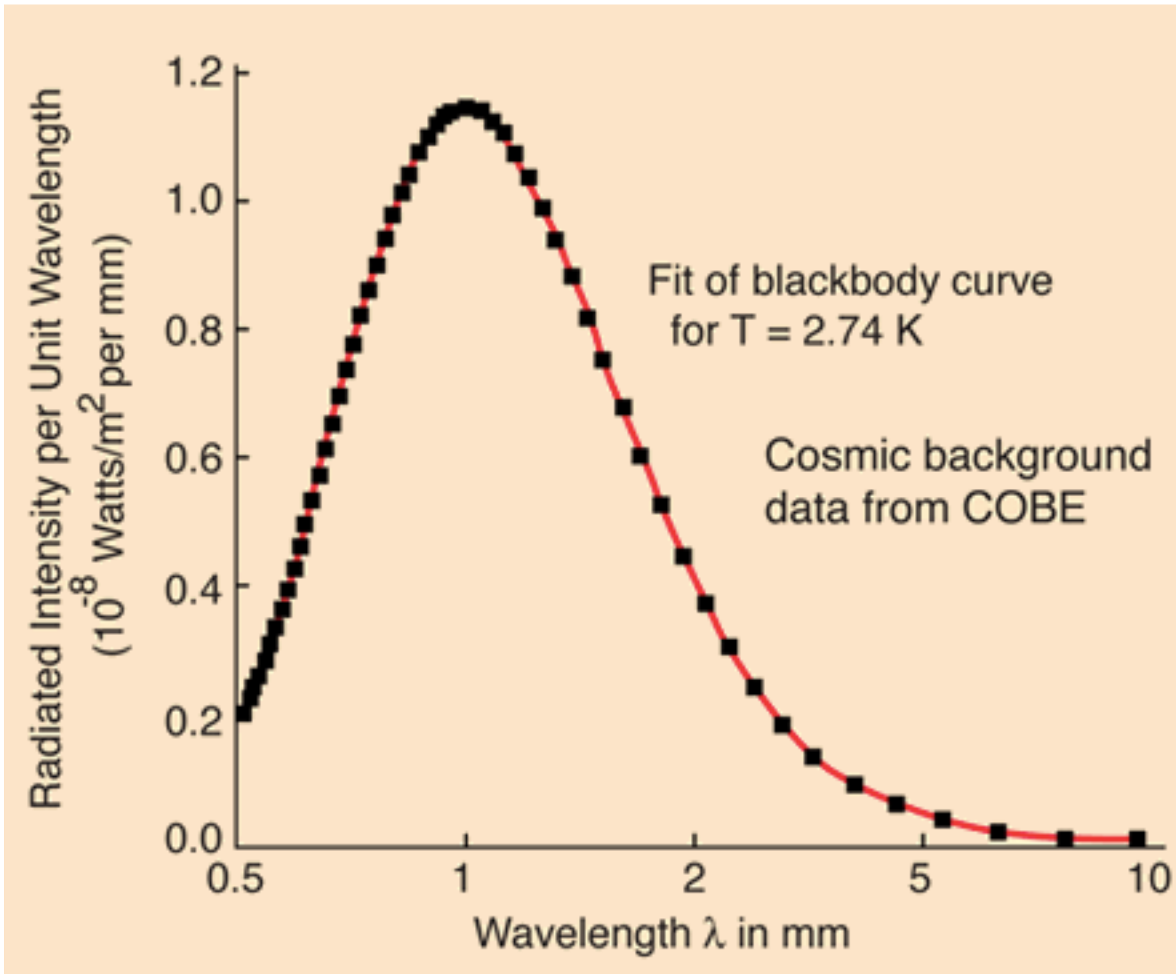


T = 5800 K
Rayleigh-Jeans
Planck
Wien
Bν [kW sr-1 m-2 THz-1]
ν [THz]
0
5
10
15
20
25
30
35
40
0
200
400
600
800
1000
1200
1400

Spectrum of black body radiation



Spectrum of cosmic microwave background radiation



Solution of Radiative transfer equation
(b) $\tau_\nu << 1$ limit

$$I_\nu(\tau_\nu) = \cancel{I_\nu(0)e^{-\tau_\nu}} + S_\nu(1 - e^{-\tau_\nu})$$

Considering no background

$S_\nu \tau_\nu$

Small fraction of black body radiation

Brightness temperature T_b

Temperature of a black-body having the same brightness at that frequency

$$I_\nu = B_\nu(T_b).$$

↓

Brightness Temperature

Low-frequency regime

$$I_\nu = \frac{2\nu^2}{c^2} k T_b \quad \longrightarrow \quad T_b = \frac{c^2}{2\nu^2 k} I_\nu$$

Brightness temperature T_b

Temperature of a black-body having the same brightness at that frequency

$$I_\nu = B_\nu(T_b).$$

Brightness Temperature

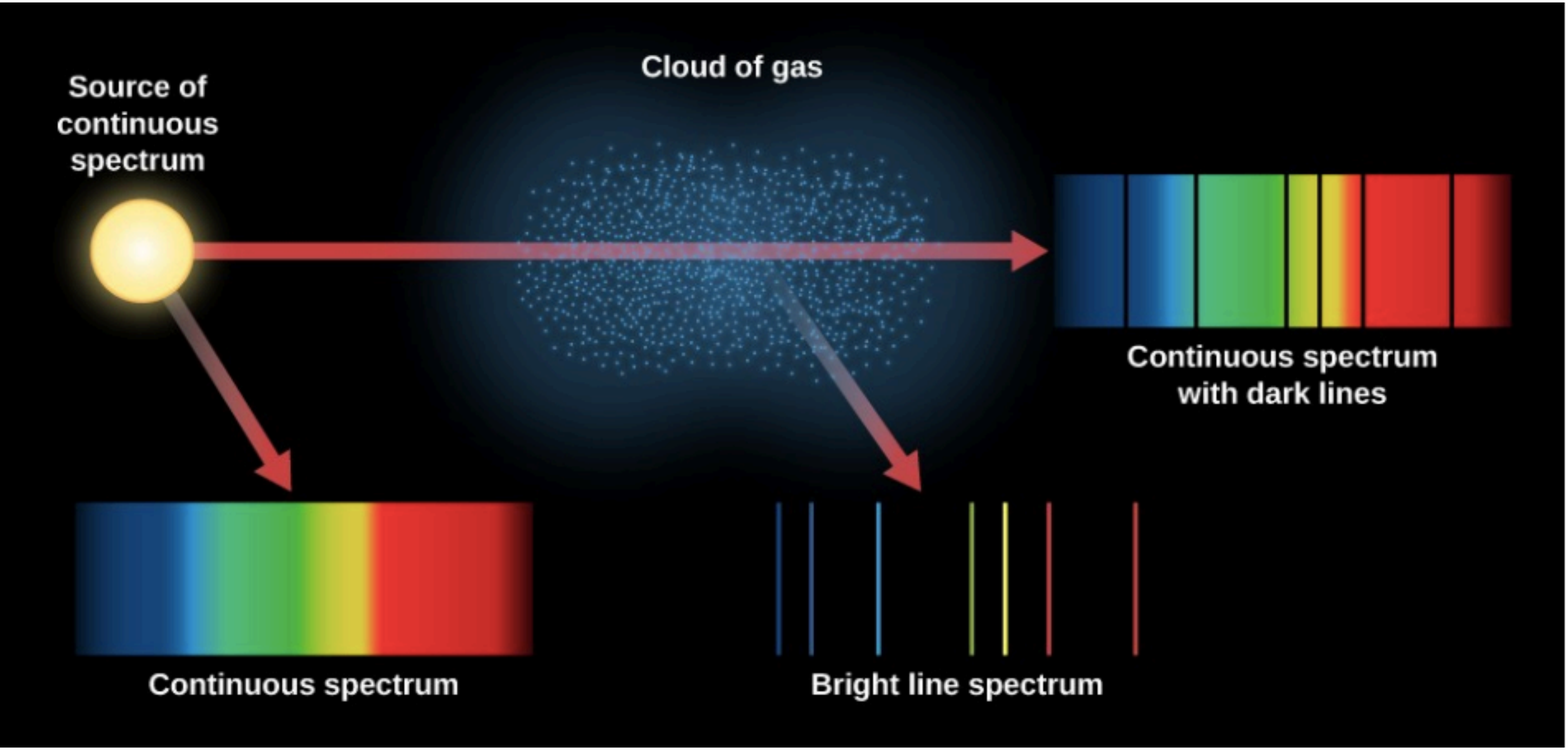
Low-frequency regime

$$I_\nu = \frac{2\nu^2}{c^2} k T_b \longrightarrow T_b = \frac{c^2}{2\nu^2 k} I_\nu$$

Radiative Transfer equation in terms of Brightness temperature

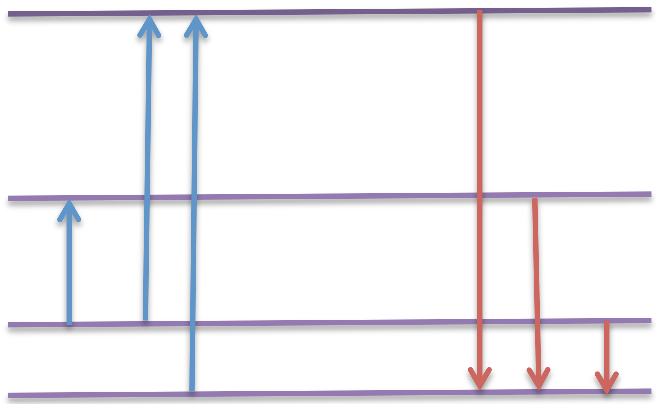
$$\frac{dT_b}{d\tau_\nu} = -T_b + T, \quad \xrightarrow{\text{Solution}} \quad T_b = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

Emission line and Absorption line



An incandescent light bulb produces a continuous spectrum.
When continuous spectrum is viewed through a thinner cloud of gas, an absorption line spectrum can be seen superimposed on the continuous spectrum.
If we look only at a cloud of excited gas atoms (with no continuous source seen behind it), we see that the excited atoms give off an emission line spectrum.

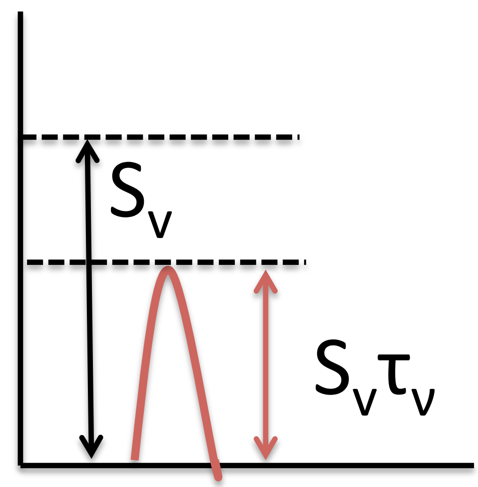
Emission line and Absorption line



Emission and absorption coefficients depend on frequency

↓

zero except at discrete frequencies



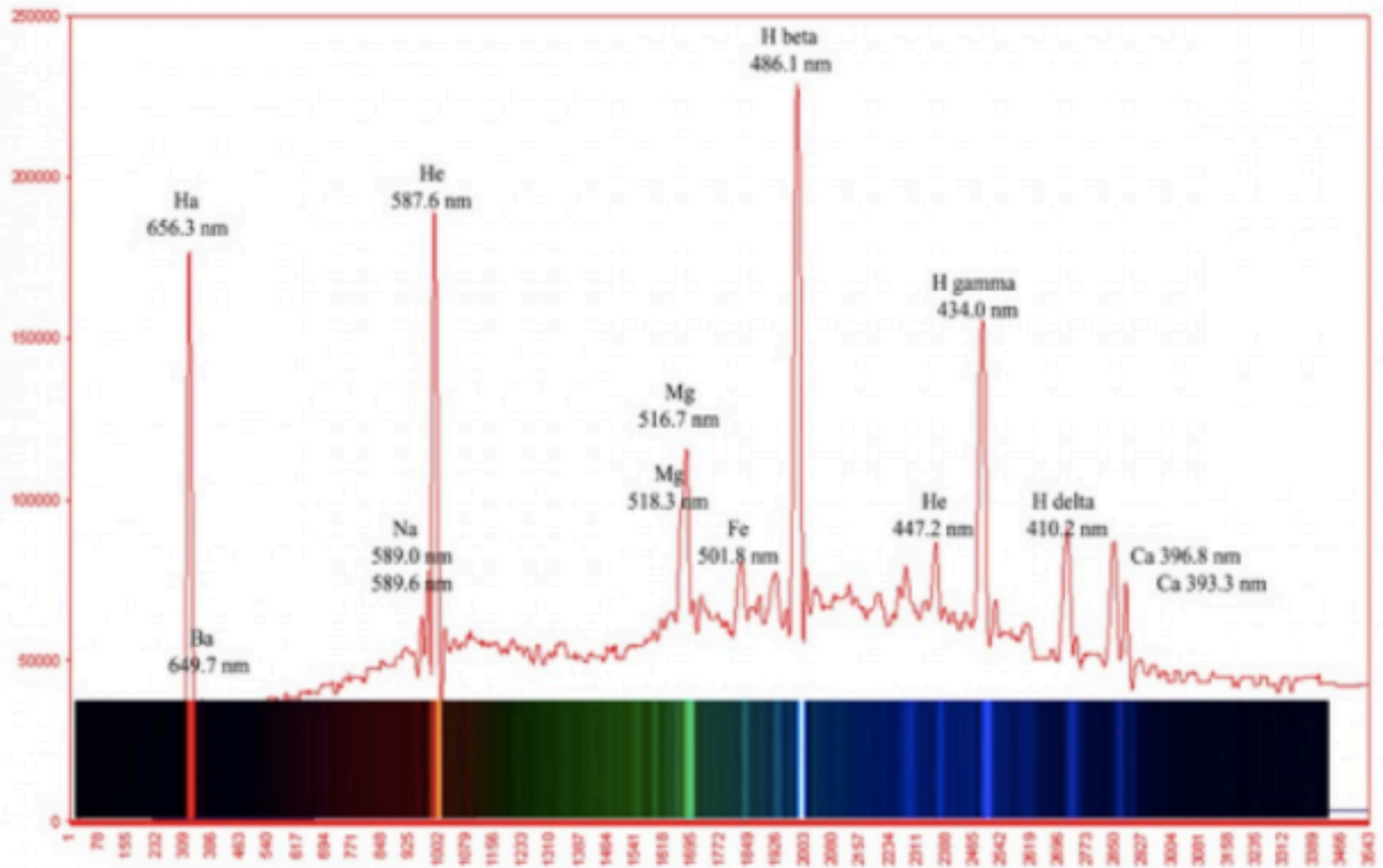
Radiative transfer equation

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

↓ Optically thin (no background radiation)

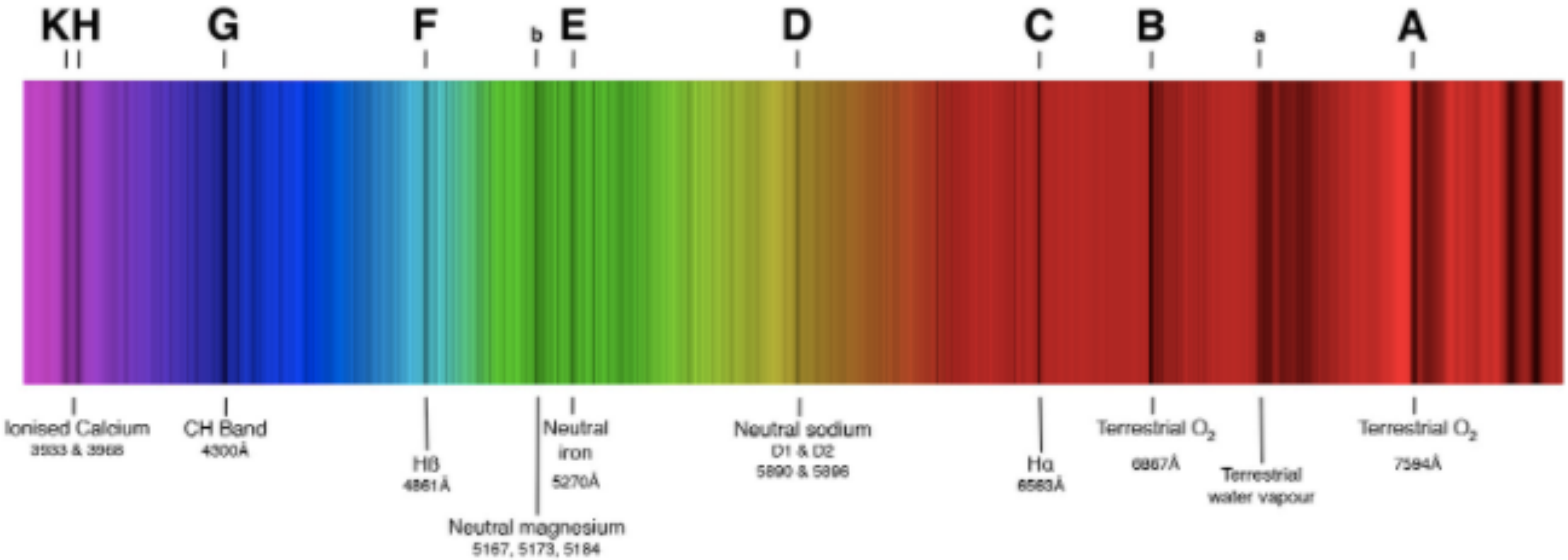
$S_\nu \tau_\nu$

The Solar Chromosphere Spectrum (Flash Spectrum)



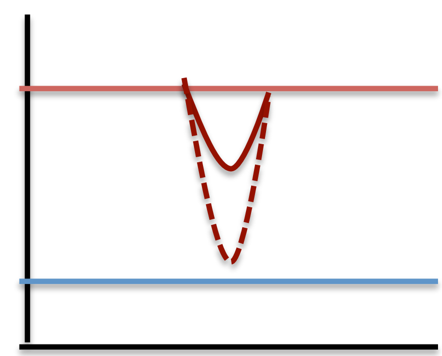
A new element was discovered in the flash spectrum during eclipse of 1868 in Guntur (Andhra, India). This was named "Helium" after "Helios" for the Sun in Greek.

Absorption line



Fraunhofer's lines (~2500 such lines)

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$



Expanding this equation for small optical depths

$$I_\nu = I_\nu(0) - \tau_\nu \, [I_\nu(0) - S_\nu]$$

End of Lecture 2

Next lecture : 9th August